



Actuarial Modelling For Diabetes Mellitus Insurance

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ABSTRACT

Diabetes mellitus is a hereditary disease with an increasing number of cases globally and has become one of the leading causes of death from critical illness in Indonesia. Several researchers have highlighted that genetic and social factors play a significant role in the development of diabetes mellitus. Consequently, financial planning and health insurance are essential. In this paper, a diabetes mellitus insurance model will be determined along with its numerical simulation, so that it can be a reference for the community and insurance companies. In this study, a reconstruction of the previously existing diabetes mellitus disease dynamics model will be carried out by adding a premium payment and compensation payment model. Furthermore, based on the new model, several actuarial calculations will be carried out using the equivalence principle such as the calculation of premiums and premium reserves from diabetes mellitus insurance. There are several numerical methods used to assist in the calculations in this study, such as the fourth-order Runge Kutta method used to determine the overall value of the population in the future period, and the trapezoidal integration method is used to help the calculation of premiums and premium reserves.

Keywords: Premium; Reserve; Diabetes Mellitus; Insurance

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INTRODUCTION

Financial planning is an important element to ensure the welfare and stability of the family in the future [1]. In financial planning, it is essential to consider various unexpected risks that may disrupt financial stability, such as critical illness, accidents, or death. Insurance serves as an important tool in managing these risks. With insurance protection, families are better prepared to face various potential hardships [2]. Life insurance is a form of financial protection designed to provide financial support to families in the event of unfortunate circumstances, such as the death of the primary breadwinner [3]. Life insurance provides a sum of money that is paid to the insurance company, allowing the family to continue their lives without being worry by financial problem.

One of the health risks that should be considered in family financial planning is critical illness, such as diabetes mellitus [4]. As stated by the World Health Organization (WHO) [5], diabetes is a condition marked by issues related to the insulin hormone,

which is naturally produced by the pancreas to assist the body in utilizing glucose and fat, as well as storing some of these substances. Based on the most recent statistics from the International Diabetes Federation (IDF) as outlined in the 10th edition of the *Atlas Diabetes 2021* [6], diabetes is a steadily increasing health issue. Currently, over 530 million individuals globally are living with diabetes, which accounts for 9.8% of the adult population. It is projected that more than 783 million individuals will be affected by diabetes by the year 2045. Diabetes mellitus is a hereditary disease that affects many family members across generations [7]. In addition to its impact on physical health, diabetes also imposes significant financial consequences due to the long-term and ongoing treatment costs. The presence of this disease within a family increases the financial risks they face, making it crucial for families to have insurance that includes critical illness coverage, such as for diabetes mellitus. In insurance practice, insurance companies must manage funds effectively. They need to carefully handle the management of funds and the structure of insurance contracts, particularly regarding premium amounts and the benefits provided. Therefore, determining the calculation model for premiums and reserves before establishing insurance contracts with clients presents a unique challenge for insurance companies.

The development of actuarial dynamic models for epidemics was initially conducted to study insurance for infectious diseases, using the SARS epidemic as a case study [8]. They applied the SIR epidemic model, adjusted with actuarial calculations to determine appropriate premium rates based on benefit types and analyze the required premium reserves. Over time, these actuarial models have evolved and been applied to other epidemics and pandemics, such as the dengue fever epidemic in Sri Lanka [9], and the COVID-19 pandemic (see [10]-[12]). Previous studies have only applied actuarial models to epidemic and pandemic compartment models. Therefore, this research discusses an actuarial model for a new type of disease: hereditary diseases, particularly diabetes mellitus. The actuarial model in this study aims to determine premiums and reserves. The compartment model for diabetes mellitus used in this research follows the PEDC model, which considers diabetes based on social and genetic factors [13]. Class P represents individuals in the pre-diabetic stage due to genetic factors, Class E represents pre-diabetic individuals due to social factors or unhealthy habits such as consuming unhealthy food, Class D represents individuals with diabetes but without complications, and Class C represents individuals with diabetes accompanied by complications, such as the development of foot ulcers. The compartmental model of diabetes mellitus from kouidere et al. [13] is illustrated in Figure 1.

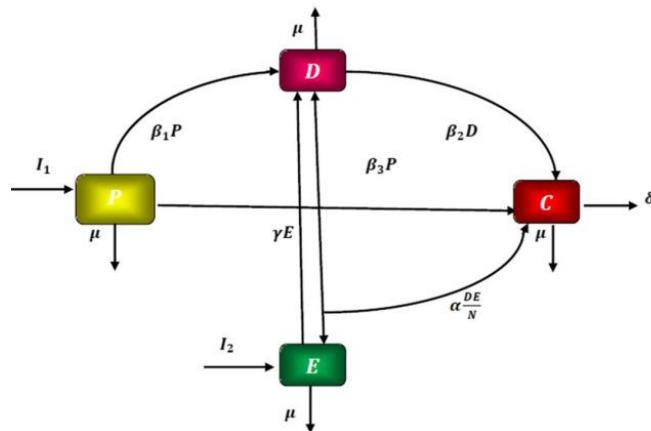


Figure 1. Dynamical model of PEDC for diabetes mellitus [13]

Based on dynamic model in Figure 1, the parameters influencing the model are defined as follows. μ represents the natural death rate of individuals. I_1 indicates the number of pre-diabetes cases with genetic factors in a population. I_2 indicates the number of pre-diabetes cases due to social factors in a population. β_1 represents the transition rate from the pre-diabetic class to diabetes without complications. β_2 represents the transition rate from diabetes without complications to diabetes with complications. β_3 represents the transition rate from pre-diabetes with genetic factors to diabetes with complications. γ represents the transition rate from pre-diabetes due to social factors to diabetes without complications. α represents the influence of social factors on diabetes, and δ represents the death rate for individuals with diabetes and complications.

In dynamic analysis, numerical approaches are often employed to find approximate solution values. The most basic numerical method for solving ordinary differential equations is the Euler method, which is very easy to implement in the numerical solution of a dynamic system (see [14]). In addition to the Euler method, there is a more effective method for solving the numerical solution of a system, namely the Fourth-Order Runge-Kutta method or RK4. RK4 has a major advantage in providing much higher accuracy compared to the Euler method with the same number of steps, without significantly increasing computational complexity (see [15]). Apart from the RK4 method which is used for numerical solutions for dynamic analysis, there is also another numerical method used to solve integration problems in actuarial calculations, namely the trapezoidal method. The trapezoidal method is a simple and effective numerical integration technique, especially in estimating the area under the curve by dividing the curve into trapezoidal segments (see [16]).

This paper aims to obtain a premium payment and compensation model for diabetes mellitus insurance, so the first step of this paper will modify the dynamic model in Figure 1 into an actuarial dynamic model for diabetes mellitus insurance. On the other hand, this paper will show that the actuarial model applied to the dynamic model [8-12] can also be applied to hereditary disease models such as diabetes mellitus insurance. After obtaining the actuarial dynamic model, several actuarial calculation concepts will be used in this study to determine the actuarial present value of premium payments, benefit payments, and premium reserves (see [17] and [18]).

METHODS

The research method in this study is divided into several steps, namely the reconstruction of the actuarial dynamic model, the determination of the premium and compensation calculation model, and the determination of the premium reserve calculation model. At the model reconstruction stage, the model in Figure 1 will be reconstructed by adding actuarial concepts such as premium payments and benefits. Premium payments in this model must be made by the population in classes P , E , and D . The population in class C will receive hospital benefits, and those who died of diabetes mellitus will receive death benefits from the insurance company. A new autonomous system will be formed from the new model by adding the concept of probability so that the premium level and premium reserves from the model can be calculated. At the actuarial calculation step, several numerical methods are needed to assist in the calculation, the RK4 method is used to obtain an approximation of the value of each class in the coming period, and the trapezoidal integration method is to help find the final

result of the calculation of premiums and premium reserves for diabetes mellitus insurance.

RESULTS AND DISCUSSION

Actuarial Model for Diabetes Mellitus

In this section, the compartment model by Koudere et al. [13] will be reconstructed by adding diabetes premium payments made by policyholders and benefit payments made by insurance companies. The previously existing model is reconstructed so that the model can be used to calculate premiums and premium reserves for diabetes mellitus insurance. The actuarial dynamics model of diabetes mellitus insurance can be seen in Figure 2.

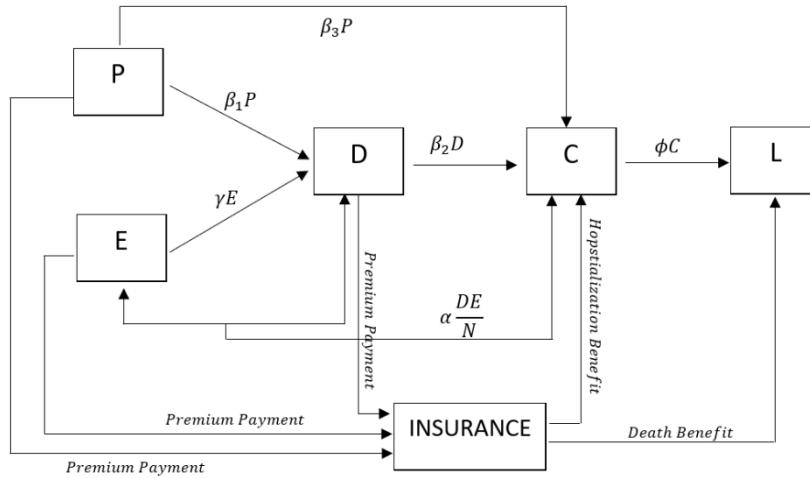


Figure 2. Actuarial model of PEDC for diabetes mellitus Insurance

In this section, it is assumed that the total population is constant, thus the birth rate and natural death rate are eliminated. From Figure 2 Class L represents individuals who have died due to diabetes mellitus, and ϕ is the death rate cause to diabetes mellitus. The autonomous $PEDCL$ system with the given assumptions can be observed in the following differential equations.

$$\frac{dP(t)}{dt} = -(\beta_1 + \beta_3)P(t) \quad (1)$$

$$\frac{dE(t)}{dt} = -\gamma E(t) \quad (2)$$

$$\frac{dD(t)}{dt} = \beta_1 P(t) + \gamma E(t) - \alpha \frac{D(t)E(t)}{N} - \beta_2 D(t) \quad (3)$$

$$\frac{dC(t)}{dt} = \beta_3 P(t) + \beta_2 D(t) + \alpha \frac{D(t)E(t)}{N} - \phi C(t) \quad (4)$$

$$\frac{dL(t)}{dt} = \phi C(t) \quad (5)$$

With given initial value $P(0) = P_0, E(0) = E_0, D(0) = D_0, C(0) = C_0, L(0) = L_0$, and total population $(N) = P_0 + E_0 + D_0 + C_0 + L_0$.

In actuarial calculations, an individual's survival probability is important to

consider. Therefore, the survival probability for each class can be defined as the ratio of each class to the total population. The survival probabilities for classes P, E, D , and C can be represented by the following functions.

$$p(t) = \frac{P(t)}{N} \quad (6)$$

$$e(t) = \frac{E(t)}{N} \quad (7)$$

$$d(t) = \frac{D(t)}{N} \quad (8)$$

$$c(t) = \frac{C(t)}{N} \quad (9)$$

$$l(t) = \frac{L(t)}{N} \quad (10)$$

By substituting Equations (6)-(10) into Equations (1)-(5) we obtain,

$$p'(t) = -(\beta_1 + \beta_3)p(t) \quad (11)$$

$$e'(t) = -\gamma e(t) \quad (12)$$

$$d'(t) = (\beta_1 p(t) + \gamma e(t) - \alpha d(t)e(t) - \beta_2 d(t)) \quad (13)$$

$$c'(t) = (\beta_3 p(t) + \beta_2 d(t) + \alpha d(t)e(t) - \phi c(t)) \quad (14)$$

$$l'(t) = \phi c(t) \quad (15)$$

with initial value $p(0) = p_0, e(0) = e_0, d(0) = d_0, c(0) = c_0, l(0) = l_0 = 0$, and $p_0 + e_0 + d_0 + c_0 = 1$.

Premium and Benefit Payments for Diabetes Mellitus Insurance

This section assumes that all citizens are enrolled in the diabetes mellitus insurance program. Premium payment obligations are fulfilled by individuals without diabetes complications, encompassing members of classes P, E , and D . The insurance benefits in this study are assumed to include hospital benefits for class C and death benefits for class L . By using the principles of international actuarial notation (see [8]), the actuarial present value (APV) for each class for the period of t -years is denoted as follows $\bar{a}_{\infty|}^p, \bar{a}_{\infty|}^e, \bar{a}_{\infty|}^d, \bar{a}_{\infty|}^c$. Thus, the APV equation for each class is obtained as follows:

$$\bar{a}_{\infty|}^p = \int_0^{\infty} e^{-\delta t} \cdot p(t) dt \quad (16)$$

$$\bar{a}_{\infty|}^e = \int_0^{\infty} e^{-\delta t} \cdot e(t) dt \quad (17)$$

$$\bar{a}_{\infty|}^d = \int_0^{\infty} e^{-\delta t} \cdot d(t) dt \quad (18)$$

$$\bar{a}_{\infty|}^c = \int_0^{\infty} e^{-\delta t} \cdot c(t) dt \quad (19)$$

Based on Figure 2, it can be seen that the benefits provided by the insurance company include hospital benefits and death benefits. First, a model will be established

to calculate the APV of the hospital benefits provided to Class C . If the hospital benefits are given annually reimbursed to policyholders, then the form of payment from the insurance company to the policyholders is in the form of an annuity, making the APV of the hospital benefits equivalent to the discrete annuity of Class C . Furthermore, the death benefits are provided to Class L , thus this calculation is influenced by the number of individuals transitioning from Class C to Class L . The model for the APV of death benefits can be seen as follows:

$$\bar{A}_{\infty|}^L = \phi \bar{a}_{\infty|}^c \quad (20)$$

Premium rate and Premium Reserves for Diabetes Mellitus Insurance

Premium rate and premium reserve calculations is very important to maintaining the financial health of insurance companies and protecting the interests of policyholders. Premium rates are useful as a reference in determining the amount of payment that must be paid by policyholders to obtain insurance protection. Meanwhile, premium reserves function as funds set aside to meet future claim payment obligations, helping to reduce liquidity risk and ensuring that the company is able to meet claims that fall due. Thus, let y represent the annual hospital benefit amount, and z represent the death benefit amount provided to the policyholder. Using the principle of equivalence, and the following equation is obtained, allowing us to conclude the equivalence of premium and benefit as follows.

$$P \left(\bar{a}_{\infty|}^p + \bar{a}_{\infty|}^e + \bar{a}_{\infty|}^d \right) = y a_{\infty|}^c + z \bar{A}_{\infty|}^L,$$

so that the premium level equation is obtained as follows,

$$P = \frac{y a_{\infty|}^c + z \phi \bar{a}_{\infty|}^c}{\bar{a}_{\infty|}^p + \bar{a}_{\infty|}^e + \bar{a}_{\infty|}^d}. \quad (21)$$

Based on Equation (21), there is no definitive amount for the diabetes mellitus premium rate. This is because the method of benefit payment is categorized into two types: hospital benefits as annual payments and death benefits paid upon the policyholder's death. There are several methods to determine premium reserves, one of which is the retrospective method [17]. Based on the retrospective method, the premium reserve $V(t)$ for diabetes mellitus insurance can be modeled as follows:

$$V'(t) = \left(\frac{y a_{\infty|}^c + z \phi \bar{a}_{\infty|}^c}{\bar{a}_{\infty|}^p + \bar{a}_{\infty|}^e + \bar{a}_{\infty|}^d} \right) (p(t) + e(t) + d(t)) - (c(t) + \phi c(t)) + \delta V(t).$$

Let us define $g(t) = \left(\frac{y a_{\infty|}^c + z \phi \bar{a}_{\infty|}^c}{\bar{a}_{\infty|}^p + \bar{a}_{\infty|}^e + \bar{a}_{\infty|}^d} \right) (p(t) + e(t) + d(t)) - (c(t) + \phi c(t))$, then we get

$$V'(t) - \delta V(t) = g(t)$$

By multiplying both sides by $e^{-\delta t}$, then

$$\begin{aligned} V'(t) \cdot e^{-\delta t} - \delta V(t) \cdot e^{-\delta t} &= g(t) \cdot e^{-\delta t} \\ \frac{d(V(t) \cdot e^{-\delta t})}{dt} &= g(t) \cdot e^{-\delta t} \end{aligned}$$

or equivalently,

$$V(t) = \left(\int_0^t e^{-\delta x} \left(\left(\frac{y \alpha_{\infty}^c + z \phi \bar{\alpha}_{\infty}^c}{\bar{\alpha}_{\infty}^p + \bar{\alpha}_{\infty}^e + \bar{\alpha}_{\infty}^d} \right) (p(t) + e(t) + d(t)) - (c(t) + \phi c(t)) \right) dx \right) e^{\delta t} + V(0) \cdot e^{\delta t} \quad (22)$$

From equation (22), the premium reserves amount at time t is based on the total premium payments, total benefit payments, and the initial value of the premium reserves.

Numerical Simulation

In this section, we will analyze the numerical simulation of the actuarial model of diabetes mellitus insurance. The age for people entering diabetes mellitus insurance in this simulation between 20 and 79 years [6]. The data used in this paper are the parameters data of the diabetes mellitus disease model from [13] and the initial value data of each class from International Diabetes Federation (IDF) for Indonesian country [6]. The expected benefits in this paper are based on one of the diabetes mellitus insurance programs in Indonesia [19], the amount for death benefits is 250 million Rupiah and hospital benefits of 2.5 million Rupiah. The initial value data and parameters used in this paper can be seen in table 1.

Table 1. Parameters and Initial Value of Diabetes Mellitus Insurance

Symbol	Description	Value	Source
i	The interest rate	0.06	[20]
β_1	The transition rate from the pre-diabetes with genetic factors to diabetes without complication	0.2	[13]
β_2	The transition rate from diabetes without complications to diabetes with complications	0.5	[13]
β_3	The transition rate from pre-diabetes with genetic factors to diabetes with complications	0.1	[13]
γ	The transition rate from pre-diabetes due to social factors to diabetes without complications	0.8	[13]
α	The influence of social factors on diabetes	0.8	[13]
ϕ	The death rate for individuals with diabetes and complications	0.001	[13]
N	Total population in Indonesia	273782000	[6]
$P(0)$	Initial value of class pre-diabetes with genetic factors	4700	[6]
$D(0)$	Initial value of class diabetes without complication	14341900	[6]
$C(0)$	Initial value of class diabetes with complication	19465100	[6]
$L(0)$	Initial value of class death because of diabetes	236711	[6]

Based on the parameter's data in table 1, it can be determined the continuous interest rate $(\delta) = \ln(1 + i) \approx 0.0582$, and initial value of class pre-diabetes due to social factors $E(0) = N - (P(0) + D(0) + C(0) + L(0)) = 239733589$. The numerical simulation stage itself begins with the calculation of the scores of each class for the next few years using the fourth-order Runge Kutta method, the results of which can be seen in Figure 3 below.

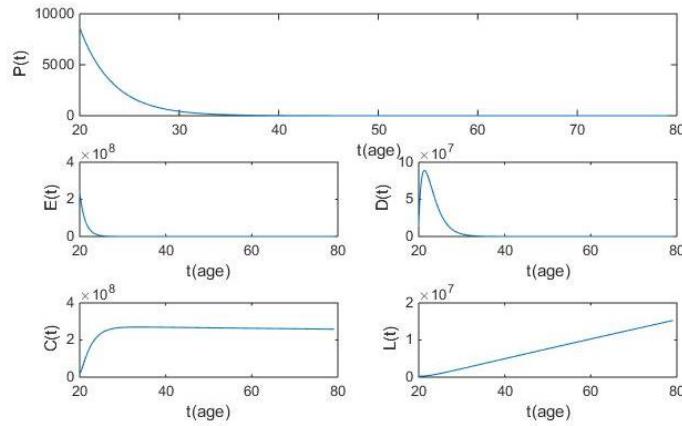


Figure 3. Numerical solution for actuarial dynamic system

Based on Figure 3, you can see the population size of each class from age 20 to age 79. It can be seen that the class of deaths due to diabetes will always increase every year (L), and other classes will decrease every year because the natural birth rate is ignored. After getting the value for each class for all periods, actuarial calculations can be carried out according to the model obtained in Equation (16-22). The integration calculation for each model is calculated using the approximation of the trapezoidal numerical integration method in Equation (6), and the following results are obtained.

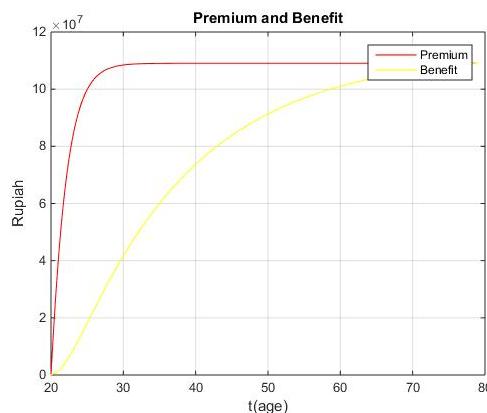


Figure 4. Premium and benefit

Based on Figure 4, it can be seen that the premium payment will be greater at the beginning compared to the benefit payment. This is because the older a person is, the greater the chance of dying from diabetes. Based on the principle of retrospective reserves, it can be determined that the premium reserves that must be prepared are larger at the beginning of the insurance period, this is because the premium is much higher than the benefits paid by the company.

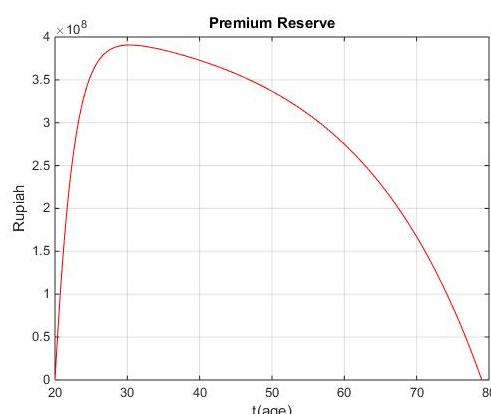


Figure 5. Premium reserve

Based on Figure 5 can be seen that the premium reserve will be larger at the beginning. This shows that the company must prepare larger funds at the start of the insurance period so that it can pay hospital benefits every year and prepare insurance claims if the policyholder dies at the beginning of the insurance period.

CONCLUSIONS

Diabetes mellitus insurance using the PEDC compartment model indicates that premium payments are allocated to classes $P(t)$, $E(t)$, and $D(t)$. Meanwhile, the benefits provided by the insurance company are of two types: hospital benefits paid annually for class $C(t)$ and death benefits paid when an individual enters class $L(t)$ or loss of life due to diabetes mellitus. Based on the equivalence principle, there is no fixed net level premiums of diabetes mellitus, as the benefits paid by the insurance company involve two different payment times. Based on the numerical simulation that has been carried out, it is shown that the company must prepare more premium reserves at the beginning of the insurance period, so that it can pay claims if the policyholder dies at the beginning of the insurance period.

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