CAUCHY: Jurnal Matematika Murni dan Aplikasi
Volume 10(1) (2025), Pages 345-361
p-ISSN: 2086-0382; e-ISSN: 2477-3344

Mean-Variance Portfolio Optimization with Lot Size Constraints in Energy Stocks: A Monte Carlo Approach

Willen Vimelia*, Riaman, Sukono

Department of Mathematics, Faculty of Mathematics and Natural Science, Universitas Padjadjaran, Sumedang 45363, Indonesia

Email: willen20001@mail.unpad.ac.id

ABSTRACT

Stock investment requires portfolio optimization strategies that maximize returns while considering risks and practical constraints, such as target lot sizes. Ensuring realistic portfolio implementation in compliance with market regulations is essential, particularly in Indonesia, where 1 lot equals 100 shares. However, existing research on the Mean-Variance model and Monte Carlo simulation has rarely incorporated target lot constraints, limiting their practical applicability. This study conducts a systematic literature review (SLR) on portfolio optimization in Indonesia's energy sector stocks, focusing on the Mean-Variance model, risk aversion, Monte Carlo simulation, and target lot constraints. The PRISMA framework guides the SLR, with bibliometric analysis performed using RStudio. A selection process from Scopus and ScienceDirect databases yielded 13 relevant articles for analysis. Results indicate that while the Mean-Variance model remains fundamental, no research explicitly integrates target lot constraints. Monte Carlo simulation is widely applied for risk assessment, but its combination with target lot constraints is largely unexplored. This study highlights a critical research gap and the need for further investigation. The findings contribute to improving investment strategies in Indonesia's energy sector by demonstrating the importance of incorporating target lot constraints for practical portfolio optimization.

Keywords: Mean-Variance Model; Monte Carlo Simulation; Lot constraints; Energy Sector Investment; Stock Market Constraints

INTRODUCTION

Investment is one form of capital allocation by investors to achieve high returns. Typically, investors engage in investment activities to enhance their quality of life and well-being through the profits obtained [1]. Decision-making in investment is inseparable from the consideration of two crucial factors: return and risk [2]. The relationship between expected return and risk is linear, meaning that the higher the expected return from an investment, the higher the associated risk, and the opposite holds [3].

To achieve optimal investment objectives, investors must consider various strategies to minimize risk while attaining the expected return. The most recognized

Submitted: December 18, 2024 Reviewed: January 21, 2025 Accepted: March 20, 2025
DOI: https://doi.org/10.18860/cauchy.v10i1.32159

approach in risk management is diversification [4]. This strategy involves spreading investment funds across various assets or financial instruments. Diversification aims to reduce specific risks associated with a single asset, as the decline in one asset's value can be offset by the increase in another's [5]. Combining several assets into one investment through diversification is an investment portfolio [6].

After asset diversification, the next step is to optimize the portfolio. Portfolio optimization aims to find the combination of assets that offers maximum return for a certain level of risk or minimal risk for a certain level of return [7]. In this context, the Mean-Variance model introduced by Harry Markowitz in 1952 forms the foundation of modern portfolio theory. This model evaluates portfolios based on two main parameters: expected return and volatility or risk [8]. To obtain an efficient portfolio, the objective function usually maximizes

$$\mu_P - \frac{\rho}{2}\sigma_P^2, \rho \geq 0 \tag{1}$$

where the investor's risk aversion is represented by the parameter ρ. For investors with a certain risk aversion level ρ ($\rho \geq 0$), the portfolio problem is solved using the following equation:

$$\begin{gathered} \text{Maximize } \left\{\mu_P - \frac{\rho}{2}\sigma_P^2\right\} \\ \text{subject to constraints } \sum_{i=1}^{N} w_i = 1 \\ w_i \geq 0, \\ \text{Or} \\ \text{Maximize } \left\{\mathbf{w}^{\mathrm{T}}\boldsymbol{\mu} - \frac{\rho}{2}\mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w}\right\} \\ \text{Subject to constraints } \mathbf{w}^T\mathbf{e} = 1 \\ w_i \geq 0, \end{gathered} \tag{2}$$

Where $\mathbf{e}^T = (1, 1, \dots, 1) \in \Re^N$ [9]. Using this model, investors can create portfolios that balance risk and return according to their risk preferences.

Every investor in the stock market naturally expects their investments to yield optimal returns while minimizing potential risks. Investors exhibit varying attitudes toward risk, including risk-seekers individuals, risk-neutral individuals, and risk-averse individuals. Among these, risk-averse investors are the most common, as they naturally tend to avoid risk [10]. Risk aversion describes the extent to which an investor is reluctant to take additional risks for higher returns [11]. Investors with a high level of risk aversion tend to choose portfolios with lower risks, even if the potential returns are also low. Conversely, investors with lower risk aversion levels are more willing to take on greater risks in pursuit of higher returns.

In practice, there are several challenges that investors must face when investing. Certain regulations require investors to purchase a minimum number of shares. In Indonesia, the minimum purchase amount for a company's publicly traded shares is 1 lot, equivalent to 100 shares [12]. This regulation is important for Indonesia as a developing country to maintain market liquidity and stability, prevent excessive volatility, and reduce operational burdens for the stock exchange and securities firms. The Mean-Variance model described in the previous paragraph has not yet incorporated realistic constraints, such as the minimum share purchase requirement. The inclusion of this constraint is considered important to ensure that optimization results can be practically implemented in stock market transactions [3]. Incorporating target lot constraints allows investors to design more realistic investment strategies aligned with market conditions.

To support more accurate optimization, Monte Carlo simulation can be employed to simulate various market scenarios that may occur in the future [13]. This method enables precise calculations to estimate portfolio return distributions while accounting for input variations such as stock prices, interest rates, and other market variables [14]. Portfolio performance can be measured using the Sharpe ratio, which incorporates two indicators: expected return and standard deviation. A higher Sharpe ratio indicates better portfolio performance, as the average return exceeds the risk-free rate while maintaining a relatively low standard deviation [15].

Various methods have been used in research to implement and evaluate the performance of portfolio optimization models, including the combination of the Mean-Variance model and Monte Carlo simulation. This approach has been applied across various sectors to identify optimal investment strategies and manage risk effectively. The following studies highlight how this model has been utilized in different contexts. Mari [16] used Mean-Variance and Monte Carlo simulation to analyze how nuclear energy could be a hedging asset to reduce electricity price volatility. Mean-Variance was applied to optimize a portfolio of electricity generation technologies. The findings revealed that nuclear energy could reduce emissions and stabilize electricity prices, providing a reliable baseload resource. Monte Carlo analysis simulated various energy market scenarios, strengthening the portfolio model's reliability. This study concluded that a diversified electricity generation portfolio, including nuclear energy, effectively reduces electricity price volatility and offers strategic advantages for energy policy and investment.

Petropoulos et al. [17] explored portfolio optimization in real estate investment, aiming to maximize Economic Value Added (EVA) and portfolio returns while managing risk at a predetermined level using the Mean-Variance model. Monte Carlo simulations were employed to account for uncertainties in the model by simulating various market scenarios, aiding in portfolio risk and return calculations. The researchers found that maximizing EVA and expected returns can occur simultaneously, although minimizing EVA risk may conflict with minimizing overall portfolio risk.

Wang et al. [18] aimed to develop a Mean-Variance optimization method specifically for portfolios including nonlinear derivative securities. Their study demonstrated that the proposed optimization method effectively optimized portfolios with nonlinear derivative securities using the Mean-Variance model and Monte Carlo simulations. Mallieswari et al. [19] researched optimizing investment portfolios by integrating Monte Carlo simulations and Markowitz's Portfolio Theory (Mean-Variance), focusing on the NIFTY Pharma index, which includes eight pharmaceutical companies, to evaluate their performance from 2020 to 2023. The study concluded that the NIFTY Pharma portfolio is more suitable for investors willing to take higher risks.

Most previous research on Mean-Variance and Monte Carlo simulations has focused on applications in various sectors such as energy, real estate, and derivative securities-based portfolios. Although some studies have utilized this approach in stock investments, research considering target lot constraints remains limited. No studies have specifically addressed the optimization of stock investment portfolios in Indonesia's energy sector, considering risk aversion and target lot constraints using Monte Carlo simulations. Therefore, a gap in the literature can be filled by investigating how integrating the Mean-Variance model, risk aversion, and target lot constraints can provide more realistic and practical portfolio optimization solutions in the Indonesian market. This research is expected to contribute to the development of more effective investment strategies for investors aiming to maximize returns while accounting for practical constraints in stock transactions.

This article conducts a Systematic Literature Review (SLR) focusing on the Mean-Variance portfolio optimization model, incorporating risk aversion and target lot constraints, specifically in the context of stock investments in Indonesia's energy sector. The primary objective of this SLR is to provide a comprehensive understanding and critical analysis of previous studies relevant to this topic, identifying research trends, gaps in the literature, and potential model developments for future research.

The study adopts a structured and systematic methodology using the PRISMA (Preferred Reporting Items for Systematic Reviews and Meta-Analyses) approach. This approach provides a clear framework for article selection, literature search strategies, data extraction, and rigorous and transparent data analysis procedures. PRISMA is applied in bibliometric analysis using RStudio software with the “R-bibliometrix” package, enabling visualization of bibliometric maps, analysis of research trends, and identification of interrelated topics in this field.

The article structure comprises several main sections. The second section briefly reviews the PRISMA method, including article selection stages and data analysis strategies. The third section presents the results and discussion, covering bibliometric visualization, analysis of interrelated research using RStudio, and developments in research themes related to investment portfolio optimization. Finally, the fourth section concludes the key findings of this study and offers recommendations for future research in stock portfolio optimization, considering practical constraints in Indonesia’s capital market.

METHODS

This study adopts a bibliometric analysis approach combined with PRISMA (Preferred Reporting Items for Systematic Reviews and Meta-Analysis). PRISMA is a systematic review method used to obtain objective results about what authors have done and the findings they produced, which are then analyzed for their relevance to the research topic. PRISMA helps minimize bias and enhance transparency in systematic reviews and meta-analyses by ensuring that every research stage has clear and meticulously documented procedures [20].

PRISMA is a systematic method comprising two main components: a flow diagram and a checklist table, designed to improve transparency and accuracy in the systematic review process. The PRISMA flow diagram consists of four main steps: identification, screening, eligibility, and inclusion. During the identification stage, all relevant articles are collected from various sources, such as scientific databases, article references, and manual searches. Duplicate articles are then removed to ensure process efficiency. After that, the screening process is conducted in several stages to evaluate whether the articles meet the pre-established inclusion and exclusion criteria. Articles that do not meet these criteria, for example, due to irrelevance to the topic or inappropriate research design, are excluded from further review.

The eligibility stage involves an in-depth review of articles that passed the screening to ensure that their content, methods, or research objects align with the study's focus and objectives. Articles that meet the eligibility criteria proceed to the inclusion stage, where they are further verified for relevance to the research question and compliance with the inclusion criteria. Additionally, a thorough analysis is conducted to ensure that the included articles significantly contribute to the development of the research. The results of this entire process are then systematically organized into a checklist table, which includes key data such as methodological information, main findings, and relevance to the research. By using this approach, PRISMA enables researchers to ensure that only high-quality and relevant articles are included in the systematic review, providing a strong foundation for further interpretation and analysis [20].

In addition to PRISMA, this study also employs bibliometric analysis to provide additional insights into research trends in the field of investment portfolio optimization.

Bibliometric analysis is used to identify research developments, collaboration patterns among researchers, and interrelated concepts frequently appearing in the literature. This method complements the PRISMA approach by visually representing key topics commonly discussed in related studies, such as keyword network analysis, thematic map analysis, word cloud analysis, and tree map analysis. Keyword Network Analysis is used to identify frequently occurring keywords and their relationships within the literature, allowing for the mapping of the main focus areas in this research. Thematic Map Analysis is employed to categorize research themes based on their level of importance and development, helping to determine trending topics or those that are declining in relevance. Word Cloud Analysis aids in visualizing the distribution of keywords, with their size indicating the frequency of occurrence in the literature. Additionally, Tree Map Analysis is utilized to group key concepts in the research and assess the proportional contribution of each topic within the overall study. By integrating PRISMA and bibliometric analysis, this study not only presents a systematic review of relevant literature but also provides a comprehensive mapping of research developments.

In this study, the identification stage began with searching various articles in databases based on predetermined keywords. The databases used in this research are Scopus and Science Direct. These databases were chosen as the primary sources for this study because they are highly reputable academic databases widely used in scientific research. Scopus, managed by Elsevier, indexes high-impact journals and conferences across various disciplines, including portfolio optimization and finance. This database offers extensive indexing features, covering millions of articles from leading international journals, ensuring that the reviewed research comes from credible sources. Meanwhile, ScienceDirect provides access to numerous journals published by Elsevier, which are extensively utilized in quantitative research and financial economics. Both databases offer advanced search and filtering capabilities, allowing for a more systematic selection of articles based on keywords, publication years, and document types. The selection of these databases aims to ensure that the articles used in this study originate from valid, up-to-date, and relevant sources. Keyword determination is crucial during the article search stage. In this study, five keywords were used:

A) "Optimization"
B) "Investment" OR "Portfolio"
C) "Mean-Variance" OR "Mean-Var" OR "Mean Variance" OR "Markowitz"
D) "Monte Carlo"
E) "Target Lot" OR "Lot Target" OR "Lot Constraint"

Article searches in both databases started with keyword **A**, followed by **A** and **B** using the boolean "AND". The search continued until the keyword **D** was included. To incorporate all keywords, the boolean "OR" was used to combine keyword **E** with the other four keywords. This approach was taken because searching with all keywords combined using "AND" would not yield papers relevant to all keywords.

In addition to determining keywords, several limitations were applied during the article search stage. Articles reviewed were limited to 10 years starting from 2014, ensuring that the resulting articles represent the most recent research. Articles were also restricted to English-language publications. In the Scopus database, keywords were searched in the title, abstract, and keywords sections. Other limitations included source type (journals and conference papers), document type (articles), and final publication stage. In the Science Direct database, keywords were searched in the title, abstract, or author-specified keywords. Articles selected were full-text, written in English, and derived from either research articles or conference papers. The results of the search based on keywords and limitations in both databases are presented in **Table 1**.

Table 1. Results of Article Searches from Both Databases

Variable	Number of Articles Found in Databases	
	Scopus	**Science Direct**
A	805.571	356.989
A and B	16.573	8.038
A and B and C	1.236	380
A and B and C and D	47	15
A and B and C and D or E	57	15

Based on **Table 1**, searching for articles with the keyword "Optimization" yielded 805,571 articles in the Scopus database and 356,989 articles in the Science Direct database. Articles searched using "Optimization" AND ("Portfolio" OR "Investment") resulted in 16,573 articles in Scopus and 8,038 articles in Science Direct. Further searches with "Optimization" AND ("Portfolio" OR "Investment") AND ("Mean-Variance" OR "Mean-Var" OR "Mean Variance" OR "Markowitz") yielded 1,236 articles in Scopus and 380 articles in Science Direct. Including the keyword "Monte Carlo" in the search resulted in 47 and 15 articles in Scopus and Science Direct, respectively. Searching with the keywords "Optimization" AND ("Portfolio" OR "Investment") AND ("Mean-Variance" OR “Mean-Var” OR “Mean Variance” OR “Markowitz”) AND “Monte Carlo” OR (“Target Lot” OR "Lot Target" OR "Lot Constraint") yielded 57 and 15 articles in Scopus and Science Direct, respectively.

Once relevant articles were obtained through the keyword search stage, the next step was article selection. In this stage, duplicate articles were identified and removed to ensure data integrity. Subsequently, the selection proceeded with title and abstract screening, where the title and abstract of each article were reviewed individually to assess their relevance to the research topic. Articles deemed irrelevant, such as those focusing on general financial strategies without specific portfolio optimization techniques, were excluded during this screening process. Articles that passed the title and abstract screening were then reviewed in full text to ensure they contained substantial discussions on relevant methodologies.

The inclusion criteria for article selection focused on studies discussing portfolio formation using the Mean-Variance method, incorporating target lot constraints, or applying the Monte Carlo method in investment optimization. Articles were considered relevant if they explicitly addressed portfolio optimization strategies using mean-variance analysis, explored constraints related to target lot allocation in stock investments, or implemented Monte Carlo simulations to enhance investment decision-making.

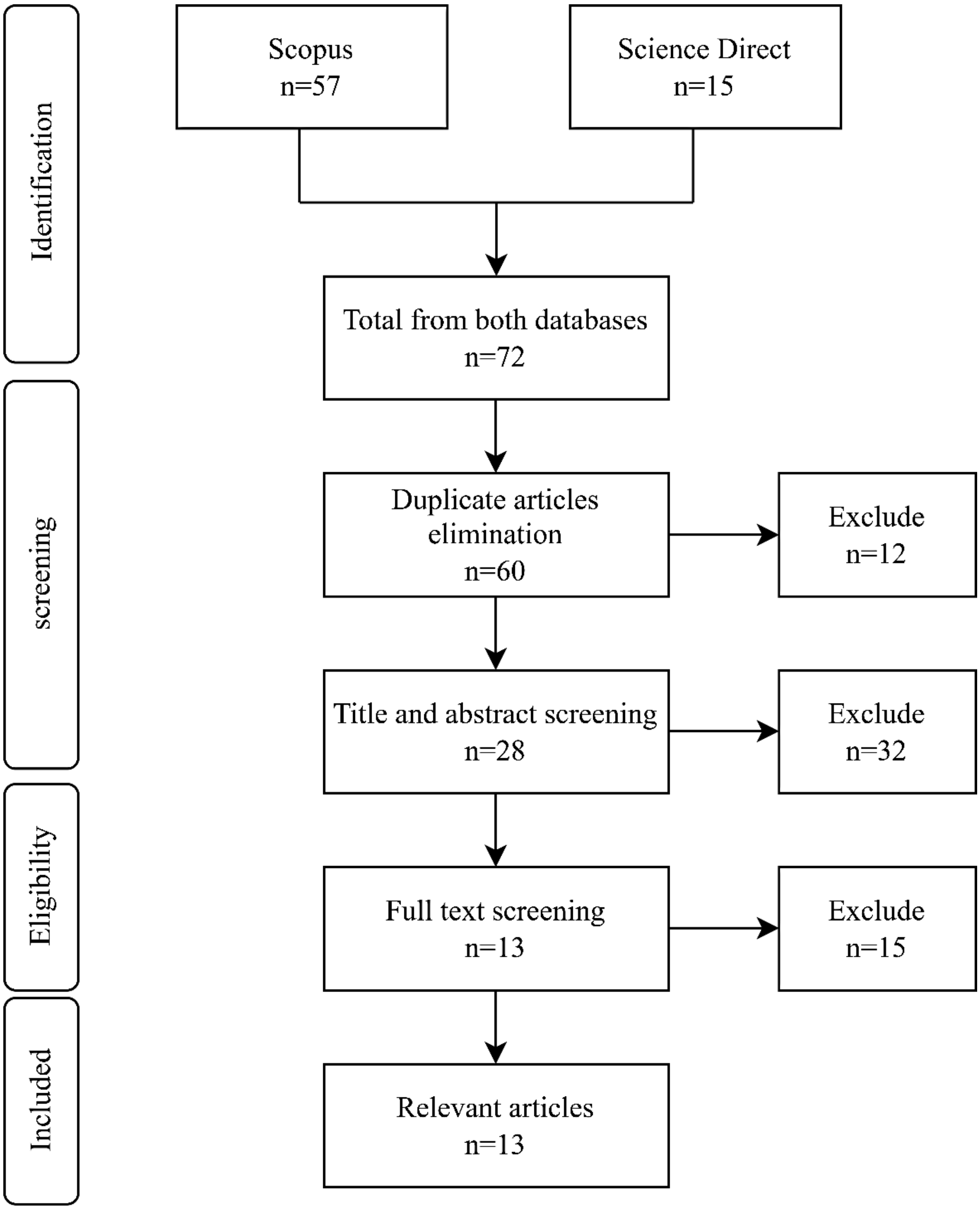

Figure 1. PRISMA Diagram

Figure 1 illustrates the systematic process of article selection using the PRISMA (Preferred Reporting Items for Systematic Reviews and Meta-Analyses) framework, comprising four key stages: Identification, Screening, Eligibility, and Inclusion. Each stage was meticulously designed to ensure that only relevant and high-quality articles were selected for this study.

In the Identification stage, articles were retrieved from two major databases: Scopus and ScienceDirect. The search yielded 57 articles from Scopus and 15 articles from ScienceDirect, resulting in a total of 72 articles. The search process utilized specific keywords related to portfolio optimization using the Mean-Variance model and Monte Carlo simulation to identify studies relevant to the research objectives.

The process proceeded to the Screening stage, which involved two steps. First, duplicate articles were removed, eliminating 12 duplicates and leaving 60 unique articles for further evaluation. Next, a title and abstract screening was conducted to assess the relevance of the articles based on pre-established inclusion and exclusion criteria. During this step, 32 articles were excluded due to their lack of alignment with the research scope, leaving 28 articles eligible for the subsequent phase.

In the Eligibility stage, the remaining articles underwent a full-text review to ensure their compliance with the research requirements. This thorough evaluation led to the exclusion of 15 articles that either lacked methodological rigor, contained insufficient data or were outside the scope of the study. As a result, 13 articles were deemed suitable for inclusion in the final analysis.

Finally, in the Inclusion stage, the selected 13 articles were considered highly relevant and of sufficient quality to contribute to the study. These articles were carefully analyzed to provide valuable insights into the integration of the Mean-Variance model, Monte Carlo simulation, and target lot constraints in portfolio optimization, particularly within the energy sector in Indonesia.

This systematic selection process underscores the rigor and transparency of the methodology, ensuring that the findings are based on credible and relevant sources. By narrowing down the articles through a well-defined framework, the research effectively addresses existing gaps and lays a solid foundation for future advancements in the field.

RESULTS AND DISCUSSION

Results

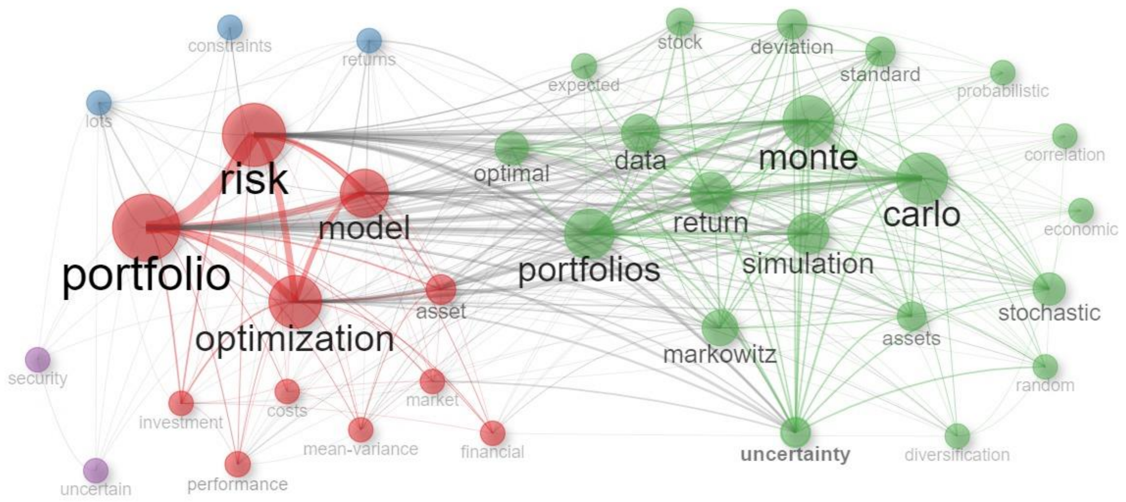

Figure 2. Bibliometric Visualization of Relevant Articles

Figure 2 illustrates the network of interconnections between research topics derived from the keywords used. The visualization identifies four main clusters, represented by different colors: red, green, blue, and purple. Each cluster represents a specific research theme that is interconnected with others.

The red cluster focuses on portfolio optimization and risk management. Dominant keywords such as "portfolio", "risk", "optimization", and "model" highlight research focused on strategies to maximize returns while considering risk management. In this context, the Mean-Variance method also emerges as a commonly used approach to achieve optimal portfolios. Meanwhile, the green cluster emphasizes Monte Carlo simulations to address market uncertainty. Keywords such as "Monte", "Carlo", "simulation", and "uncertainty" dominate this cluster, indicating that Monte Carlo simulations are frequently employed to model uncertainty and asset variability in investments. These simulations enable scenario analysis involving probabilistic factors that are difficult to predict deterministically.

In addition to the dominant red and green clusters, there are two smaller clusters: blue and purple, which add specific layers to this analysis. The blue cluster emphasizes portfolio constraints and returns. Keywords like "constraints" and "returns" highlight the focus on restrictions, such as asset allocation or target lot size, that must be considered in investment decision-making. Although smaller in size, this topic remains important in the context of optimization, as it underscores the need for realistic constraint design to ensure that optimal portfolios can be implemented in the market. On the other hand, the purple cluster focuses on financial securities and investment uncertainty. Keywords like "security" and "uncertain" describe how financial instruments and elements of uncertainty are integral components of investment decisions.

The clusters are interconnected through linking lines that indicate keyword co-occurrence. Strong ties between "portfolio" and "risk" (red cluster) and "simulation" and "Monte Carlo" (green cluster) highlight the frequent use of simulations in portfolio optimization. The blue cluster's "constraints" relate to "optimization" in the red cluster, while "uncertainty" links the green and purple clusters, emphasizing its critical role in investments. Larger nodes, such as "portfolio", "risk", "Monte Carlo", and "simulation", represent frequently discussed topics, with thicker lines indicating stronger relationships. This visualization illustrates the interconnectedness of portfolio optimization, risk management, simulations, constraints, and uncertainty in investment research.

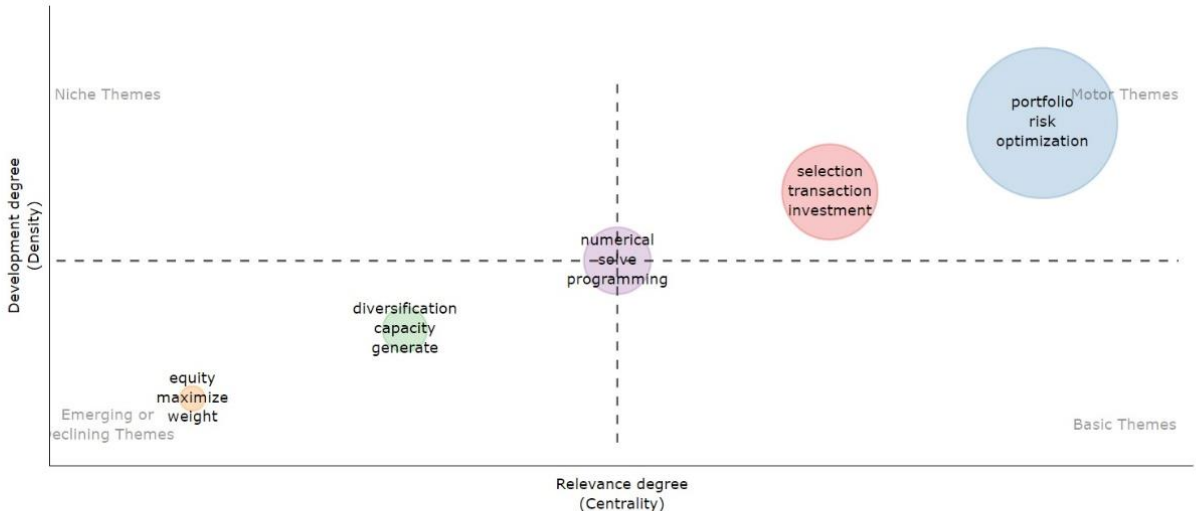

Figure 3. Thematic Map

Figure 3 presents a thematic map illustrating the relevance and development of research themes in portfolio and investment studies. The vertical axis represents development (density) which measures how well-developed a theme is, while the horizontal axis indicates relevance (centrality) which measures the theme's importance within the overall research context. The map is divided into four quadrants, though only two contain identified themes.

In the Motor Themes quadrant (high relevance and development), "Portfolio Risk Optimization" and "Selection Transaction Investment" emerge as dominant topics. Research on portfolio risk optimization is rapidly evolving, employing methods such as mean-variance optimization, Monte Carlo simulation, and machine learning. Meanwhile, "Selection Transaction Investment" highlights asset selection, diversification, and transaction risk management strategies to maximize returns.

The Emerging or Declining Themes quadrant (low relevance and development) includes "Diversification Capacity Generate" and "Equity Maximize Weight." The former relates to portfolio diversification strategies but appears to be losing prominence as research shifts to modern approaches. Similarly, "Equity Maximize Weight," which focuses on weight allocation in equity portfolios, seems to be giving way to more advanced methodologies.

Notably, no topics fall into the Basic Themes (fundamental but underdeveloped) or Niche Themes (highly developed but narrowly focused) quadrants, suggesting that research is currently centered on major evolving trends. One interesting element in the map is "Numerical Solve Programming," positioned at the intersection of all quadrants, indicating moderate relevance and development. This theme covers computational techniques like Monte Carlo simulations, quadratic programming, and linear programming for portfolio optimization. While essential to various studies, it has yet to develop enough to be classified as a dominant research theme.

Figure 4. Word Cloud

Figure 4 presents a bibliometric word cloud visualization, highlighting the most frequently occurring keywords in portfolio optimization research, particularly those employing the Mean-Variance approach and Monte Carlo simulation. The larger and bolder the word, the more frequently it appears in the analyzed literature, offering insights into key research trends.

Prominent terms such as Monte Carlo Methods, Portfolio Optimization, Optimization, and Financial Data Processing indicate that Monte Carlo simulation is widely used to address financial market uncertainties, assisting in evaluating investment risks and returns. The presence of Risk Assessment underscores the focus on risk analysis, while keywords like Hedging and Cardinalities suggest studies on hedging strategies and cardinality constraints in portfolio selection. Additionally, Covariance Matrices and Cholesky Decomposition highlight the importance of asset correlation analysis in risk modeling.

Beyond financial aspects, the inclusion of terms like Electricity Price Volatilities, Energy Resources, Fossil Fuel, and Environmental Policy suggests that portfolio optimization research also extends to the energy sector, particularly in managing price volatility and incorporating sustainability considerations such as Carbon Emission and Renewable Energy Investments.

In terms of methodologies, keywords like Genetic Algorithms, Cross Entropy, and Constrained Optimization indicate the use of evolutionary algorithms and optimization techniques beyond Monte Carlo methods. The presence of Cardinality Constraints further suggests research on limiting the number of assets in a portfolio to enhance practical applicability. This word cloud visualization provides a comprehensive overview of the dominant themes and evolving methodologies in portfolio optimization research.

Figure 5. Tree Map

Figure 5 presents another visualization of the keywords that appear in the analyzed research. The tree map in Figure 5 illustrates the distribution of key terms related to portfolio optimization based on bibliometric analysis. In general, this tree map provides an overview of the broader research landscape, serving as a foundation for understanding dominant research areas while identifying potential research gaps. Each rectangle represents a keyword, with its size and color indicating the frequency of occurrence within the analyzed literature.

The term "Monte Carlo Methods" emerges as the dominant keyword with the highest frequency (6%), highlighting the significance of Monte Carlo simulation in supporting investment decision-making, particularly in addressing market uncertainties. The terms "Optimization" and "Portfolio Optimization" appear with frequencies of 6% and 4%, respectively, reflecting the primary focus of research on optimization strategies aimed at maximizing returns or minimizing investment risks. Additionally, the keywords "Portfolio Selection" and "Financial Data Processing", each with a 4% frequency, underscore the importance of portfolio selection processes and financial data processing in developing more sophisticated and market-relevant models.

Furthermore, Figure 5 also reveals the presence of keywords with lower frequencies, providing insights into emerging trends or specialized research focuses. Terms such as "Carbon Emission", "Electricity Price Volatilities", and "Energy Resources" indicate the connection between portfolio research and sustainability issues within the energy sector. The presence of "Genetic Algorithms", "Covariance Matrices", and "Constrained Optimization" highlights the technical and mathematical approaches employed to solve complex optimization problems. More specifically, terms like "Cardinality Constraints" and "Constrained Portfolios" reflect ongoing explorations in integrating specific constraints, such as the number of assets in a portfolio. The inclusion of other keywords, such as "Electronic Trading", "Risk Assessment", and "Hedging", demonstrates the diverse aspects considered in this research field, ranging from technology implementation to risk evaluation and mitigation strategies.

Table 2 presents various previous studies relevant to portfolio optimization using the Mean-Variance model and Monte Carlo simulation. These studies have significantly contributed to the development of portfolio optimization theory and applications, particularly in the energy sector. For example, Mari [16] and Neto et al. [22] focused their research on risk management and investment diversification in the nuclear and renewable energy sectors. Meanwhile, Zhang et al. [23] emphasized reducing transaction costs and improving lot size efficiency, although they did not explicitly incorporate target lot constraints into their optimization models.

A critical analysis of these findings reveals that while Monte Carlo simulations have been widely used to address market uncertainties, as seen in studies by Shadabfar and Cheng [24] and Mukherjee et al. [27], their implementation remains limited to return and risk optimization. Most studies, such as Lubis et al. [25] and Leung & Wang [26], introduced transaction cost minimization and cardinality constraints to enhance practicality, yet they overlooked share lot size constraints. This gap is particularly significant because target lot constraints are essential in real-world trading, especially in stock markets like Indonesia, where one lot equals 100 shares.

Table 2. State of The Art

Author(s)	Research Focus	Energy Stocks	Monte Carlo	Lot Target	Mean-Variance
[16]	Combining Mean-Variance portfolio optimization with Monte Carlo simulation to evaluate the role of nuclear power in reducing electricity price volatility.	×	✓	×	✓
[17]	Examining portfolio optimization in the context of the Greek real estate market to maximize the economic value added (EVA) of a property portfolio while minimizing risk using a stochastic approach that accounts for uncertainty in market data.	×	✓	×	✓
[18]	Proposing a simulation approach for Mean-Variance portfolio optimization consisting of derivative securities to effectively solve the Mean-Variance optimization problem.	×	✓	×	✓
[19]	Exploring stochastic methods in optimizing investment portfolios by integrating Monte Carlo simulation and Markowitz Portfolio Theory.	×	✓	×	✓
[21]	Analyzing the impact of structured securities on investment management, particularly in enhancing portfolio diversification and investment returns.	×	✓	×	✓
[22]	Managing the physical and financial risks associated with electricity sector operations by diversifying investments in renewable energy technologies.	×	✓	×	✓
[23]	Developing a robust portfolio selection model that takes into account transaction costs and minimum transaction lot size, using a Mean-Variance approach and genetic algorithms for optimization.	×	×	✓	✓
[24]	Applying a probabilistic approach to optimal portfolio selection, using Mean-Variance for evaluation and Monte Carlo simulation for optimization.	×	✓	×	✓
[25]	Developing an active constraint-based search algorithm to minimize Value at Risk (VaR) in portfolio optimization, considering minimum transaction lot size and using the Mean-Variance model to maximize returns while reducing risk.	×	×	✓	×
[26]	Optimizing asset selection while adhering to cardinality constraints is crucial to avoid odd lots and minimize transaction costs, especially for high-frequency traders.	×	×	✓	✓
[27]	Optimizing a portfolio according to modern portfolio theory for equity instruments based in the U.S. using Monte Carlo simulation.	×	✓	×	✓
[28]	Proposing a bi-objective mean-entropy portfolio model that accounts for uncertainty in investments with minimum lot size, dividends, and taxes. Using a hybrid intelligent algorithm to optimize portfolio selection while enhancing efficiency and risk management.	×	×	✓	✓
[29]	Optimizing a multi-period portfolio using Monte Carlo simulation and the quasi-Newton method, considering risk through Mean-Variance and Mean-Semivariance approaches.	×	✓	×	✓

Discussion

The analysis of keyword clusters (**Figure 2**) indicates a strong research focus on portfolio optimization strategies, risk management, and Monte Carlo simulations. These findings confirm that the Mean-Variance model remains a fundamental approach in portfolio optimization. However, the limited focus on target lot constraints, as observed in the bibliometric clusters, suggests a research gap in practical implementation. The thematic map (**Figure 3**) further supports this observation. The presence of "Portfolio Risk Optimization" and "Selection Transaction Investment" as dominant themes highlights the growing interest in portfolio risk management and asset selection strategies. However, the absence of themes explicitly addressing lot size constraints underscores the need for more research integrating these practical considerations into portfolio optimization models. The word cloud (**Figure 4**) reinforces the bibliometric analysis, emphasizing key terms like "Monte Carlo Methods," "Portfolio Optimization," and "Risk Assessment." These terms reflect the primary focus of research on risk evaluation and optimization techniques. However, the relatively lower frequency of terms related to "Lot Constraints" or "Lot Size" suggests that existing studies have not fully explored this aspect. The tree map (**Figure 5**) provides additional insights into keyword distribution, revealing the relative importance of terms related to portfolio optimization. The dominance of "Monte Carlo Methods," "Optimization," and "Portfolio Optimization" underscores their critical role in existing literature. However, keywords related to "Lot Constraints" appear less frequently, reinforcing the gap in research addressing this specific practical constraint.

Table 2 provides further context by comparing previous studies on portfolio optimization. While many studies incorporate Monte Carlo simulations and risk management strategies, few explicitly consider target lot constraints. This highlights the contribution of the current study in bridging this gap by integrating practical constraints into Mean-Variance optimization models.

The implications of these findings extend to both theoretical advancements and practical applications. For investors and market practitioners, incorporating target lot constraints into portfolio optimization allows for more accurate investment decision-making. By explicitly considering these constraints, investors can better balance risk diversification, return maximization, and transaction feasibility. Additionally, Monte Carlo simulation enhances investment strategies by generating multiple market scenarios, enabling investors to assess portfolio performance under different conditions. This makes the model more adaptable to market volatility and regulatory frameworks.

One reason why target lot constraints have been underexplored in prior research may be the complexity involved in integrating them into traditional optimization models. The discrete nature of lot size adds computational challenges, requiring more advanced numerical techniques such as integer programming. Furthermore, limited availability of granular trading data may have hindered empirical studies in this area. Another factor is the predominant focus on Western financial markets, where fractional trading and flexible lot sizes are more common, reducing the urgency of incorporating rigid lot constraints in optimization models.

This study's contribution is particularly relevant to the Indonesian market, where lot size regulations directly impact trading strategies. By adapting the Mean-Variance model to reflect local market constraints, this research offers a more practical and implementable approach for domestic investors. Unlike previous models that assume continuous asset allocation, this study ensures compliance with market rules while maintaining portfolio efficiency.

CONCLUSIONS

This study conducts a systematic literature review (SLR) on portfolio optimization using the Mean-Variance model with target lot constraints in Indonesia's energy stock sector, employing Monte Carlo simulation. Through a rigorous selection process based on the PRISMA framework and bibliometric analysis, this research identifies a key gap in the literature—namely, the limited studies incorporating target lot constraints into portfolio optimization.

The analysis reveals that while the Mean-Variance model and Monte Carlo simulation have been widely applied in various financial contexts, the integration of target lot constraints remains underexplored. This constraint is particularly relevant in the Indonesian stock market, where the minimum tradable unit is 1 lot (100 shares). By addressing this gap, this study provides both a theoretical and practical foundation for investors to develop more realistic investment strategies that balance risk aversion, return maximization, and compliance with market regulations.

Based on these findings, future research opportunities exist to further enrich the literature and applications of portfolio optimization. These include modifying the Mean-Variance model by incorporating target lot constraints while adhering to optimization principles and theorems. Furthermore, empirical validation using historical stock transaction data from Indonesia would be valuable in assessing the performance of the optimized portfolio compared to conventional investment strategies.

ACKNOWLEDGMENT

The authors express their sincere gratitude to Universitas Padjadjaran and the Ministry of Primary and Secondary Education for their support in this research. This study is conducted as part of the Fast Track Program from Bachelor's to Master's Degree in Mathematics at Universitas Padjadjaran and is supported by the “Beasiswa Unggulan” from the Ministry of Primary and Secondary Education, as stated in Decree No.2346/J5.2.2/BP/PKS/X/2024.

REFERENCES

[1] M. Purwantini, R. H. Yustrianthe, E. Grediani, and Handayani, “Determinan faktor yang mempengaruhi minat investasi,” *Fair Value Jurnal Ilmiah Akuntansi dan Keuangan*, vol. 5, no. 4, pp. 1577–1585, 2022, doi: 10.32670/fairvalue.v5i4.2623.

[2] S. Senthilnathan, “Risk, Return and Portfolio Theory A Contextual Note,” *SSRN Electronic Journal*, 2015, doi: 10.2139/ssrn.2627423.

[3] Z. Puspitaningtyas, *Prediksi Risiko Investasi Saham*. Yogyakarta: Griya Pandiva, 2015, isbn: 978-602-14113-6-0.

[4] Liestyowati, L. M. Possumah, R. M. Yadasang, and H. Ramadhani, “Pengaruh Diversifikasi Portofolio terhadap Pengelolaan Risiko dan Kinerja Investasi: Analisis pada Investor Individu,” *Jurnal Akuntansi Dan Keuangan West Science*, vol. 2, no. 03, pp. 187–194, 2023, doi: 10.58812/jakws.v2i03.642.

[5] M. Ramadhan, T. Suharti, and I. Nurhayati, “Diversifikasi Saham Dalam Pembentukan Portofolio Untuk Meminimumkan Risiko,” *Manager: Jurnal Ilmu Manajemen*, vol. 3, no. 4, p. 450, 2020, doi: 10.32832/manager.v3i4.3914.

[6] D. Jayeola, Z. Ismail, and S. F. Sufahani, "Effects of diversification of assets in optimizing risk of portfolio," *Malaysian Journal of Fundamental Applied Sciences*, vol. 13, no. 4, pp. 584–587, 2017, doi: 10.11113/mjfas.v0n0.567.

[7] I. G. A. G. N. Raditya, D. Saepudin, and I. Kurniawan, "Optimasi Portofolio Saham dengan Pendekatan Box Uncertainty Set Studi Kasus : IDX 30," *e-Proceeding of Engineering*, vol. 8, no. 1, pp. 918–929, 2021, issn: 2355-9365.

[8] Z. Maf'ula, S. R. Handayani, and Z. Z. A., "Portofolio Optimal dengan Penerapan Model Markowitz sebagai Dasar Keputusan Investasi (Studi pada Perusahaan yang Tergabung dalam Indeks LG-45 Tahun 2014)," *Jurnal Administrasi Bisnis*, vol. 63, no. 1, 2018.

[9] M. Ivanova and L. Dospatliev, "Application of Markowitz Portfolio Optimization on Bulgarian Stock Market From 2013 To 2016," *International Journal of Pure and Apllied Mathematics*, vol. 117, no. 2, 2018, doi: 10.12732/ijpam.v117i2.5.

[10] R. Robiyanto, "Performance Evaluation and Risk Aversion Rate for Several Stock Indices in Indonesia Stock Exchange," *Jurnal Manajemen dan Kewirausahaan*, vol. 19, no. 1, pp. 60–64, 2017, doi: 10.9744/jmk.19.1.60-64.

[11] A. Díaz and C. Esparcia, "Assessing Risk Aversion From the Investor's Point of View," *Frontiers in Psychology*, vol. 10, 2019, doi: 10.3389/fpsyg.2019.01490.

[12] D. S. Lestari, "Analisis Pengaruh Program 'Yuk Nabung Saham' oleh PT Bursa Efek Indonesia terhadap Minat Mahasiswa untuk Berinvestasi di Pasar Modal," *Jurnal Ilmu Sososial, Manajemen, Akuntansi, dan Bisnis*, vol. 2, no. 3, pp. 1–13, 2021, doi: 10.47747/jismab.v2i3.363.

[13] Megawati, Resmawan, B. R. Payu, and A. Adityaningrum, "Prediksi Pergerakan Saham Menggunakan Metode Simulasi Monte Carlo untuk Pembentukan Portofolio Optimal dengan Pendekatan Model Markowitz," *Jurnal Statistika dan Aplikasinya*, vol. 6, no. 1, pp. 86–95, 2022, doi: 10.21009/jsa.06108.

[14] R. D. Putra, Y. Apridiansyah, and E. Sahputra, "Penerapan Metode Monte Carlo pada Simulasi Prediksi Jumlah Calon Mahasiswa Baru Universitas Muhammadiyah Bengkulu," Jurnal *PROCESSOR*, vol. 17, no. 2, pp. 74–81, 2022, doi: 10.33998/processor.2022.17.2.1224.

[15] H. Manurung, "Analisis Kinerja Portofolio Saham dengan Menggunakan Metode Sharpe, Jensen dan Treyno," *Journal of Business Studies*, vol. 04, no. 1, pp. 1–16, 2019, issn: 2443-3837.

[16] C. Mari, "Hedging electricity price volatility using nuclear power," *Applied Energy*, vol. 113, pp. 615–621, 2014, doi: 10.1016/j.apenergy.2013.08.016.

[17] T. Petropoulos, K. Liapis, and E. Thalassinos, "Optimal Structure of Real Estate Portfolio Using EVA: A Stochastic Markowitz Model Using Data from Greek Real Estate Market," *Risks*, vol. 11, no. 2, 2023, doi: 10.3390/risks11020043.

[18] S. Wang, G. Cai, P. Yu, G. Liu, and J. Luo, "Mean-Variance Portfolio Optimization with Nonlinear Derivative Securities," *Proceedings - Winter Simulation Conference*, pp. 576–587, 2023, doi: 10.1109/WSC60868.2023.10407819.

[19] R. Mallieswari, V. Palanisamy, A. T. Senthilnathan, S. Gurumurthy, J. J. Selvakumar, and S. Pachiyappan, "A Stochastic Method for Optimizing Portfolios Using a Combined Monte Carlo and Markowitz Model: Approach on Python," *ECONOMICS - Innovative Economics Research Journal*, vol. 12, no. 2, pp. 113–127, 2024, doi: 10.2478/eoik-2024-0014.

[20] M. J. Page *et al.*, "The PRISMA 2020 statement: An updated guideline for reporting systematic reviews," The *British Medical Journal*, vol. 372, 2021, doi: 10.1136/bmj.n71.

[21] E. Bikas and E. Bikas, "Towards sustainable financial markets: Impact of structured securities on portfolio management," *Journal Security and Sustainability Issues*, vol. 6, no. 2, pp. 275–288, 2016, doi: 10.9770/jssi.2016.6.2(7).
[22] D. P. Neto, E. G. Domingues, A. P. Coimbra, A. T. de Almeida, A. J. Alves, and W. P. Calixto, "Portfolio optimization of renewable energy assets: Hydro, wind, and photovoltaic energy in the regulated market in Brazil," *Energy Economics*, vol. 64, pp. 238–250, 2017, doi: 10.1016/j.eneco.2017.03.020.
[23] C. Zhang, R. Hu, and L. Wei, "Uncertain portfolio selection model considering transaction costs and minimum transaction lots requirement," *Journal of Intelligent and Fuzzy Systems*, vol. 32, no. 6, pp. 4543–4554, 2017, doi: 10.3233/JIFS-169218.
[24] M. Shadabfar and L. Cheng, "Probabilistic approach for optimal portfolio selection using a hybrid Monte Carlo simulation and Markowitz model," *Alexandria Engineering Journal*, vol. 59, no. 5, pp. 3381–3393, 2020, doi: 10.1016/j.aej.2020.05.006.
[25] R. S. Lubis, H. Mawengkang, O. Darnius, and Mardiningsih, "A novel approach based on active constraint for minimizing var in the portofolio optimization problem," *Journal of Theoritical and Applied Information Technology*, vol. 99, no. 15, pp. 3703–3712, 2021.
[26] M.-F. Leung and J. Wang, "Cardinality-constrained portfolio selection based on collaborative neurodynamic optimization," *Neural Networks*, vol. 145, pp. 68–79, 2022, doi: 10.1016/j.neunet.2021.10.007.
[27] A. Mukherjee, A. K. Singh, P. K. Mallick, and S. R. Samanta, "Portfolio Optimization for US-Based Equity Instruments Using Monte-Carlo Simulation," 2022, pp. 691–701. doi: 10.1007/978-981-16-8763-1_57.
[28] L. Lv, B. Zhang, and H. Li, "An uncertain bi-objective mean-entropy model for portfolio selection with realistic factors," *Mathematics and Computers in Simulation*, vol. 225, pp. 216–231, 2024, doi: 10.1016/j.matcom.2024.05.013.
[29] R. A. E. Mäkinen and J. Toivanen, "Short Communication: Monte Carlo Expected Wealth and Risk Measure Trade-Off Portfolio Optimization," *SIAM Journal Financial Mathematics*, vol. 15, no. 2, pp. SC41–SC53, 2024, doi: 10.1137/23M1624439.