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Dependency of The Exchange Rate with The Volume of Indonesian Aluminum Exports Using Copula

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Abstract

The downstreaming of bauxite, which is one of the raw materials for aluminum, indicates that the Indonesian government is serious about managing this mining resource. As one of the leading commodities, aluminum export activities not only influence investment but also the strengthening movement of the IDR-USD exchange rate. Increasing the circulation of the rupiah has had a positive impact on Indonesia in the international trade market. This research models the dependence between the IDR-USD exchange rate and the volume of Indonesian aluminum exports using copula. Copula does not require normality assumptions, so it is very good to use in measuring the dependence of economic data like this. The research results concluded that there was a positive correlation between the two variables, although it was not significant and was classified as very small. This positive correlation shows that the rupiah will strengthen as the volume of Indonesian aluminum exports increases. This research is very important to identify how one of the mining materials, namely aluminum, can be utilized as much as possible for export activities in maintaining the stability of the exchange rate and foreign exchange reserves.

Keywords: Aluminium; Copula; Correlation; Export Volume

1 Introduction

Indonesia's wealth of mining mineral resources is something that must be utilized properly. These resources can be a source of income, raw materials for processing goods, and can also be used to strengthen Indonesia's value in the international market. One of the mining resources that is the focus for management is alumina and bauxite. These two minerals are raw materials for making aluminum. Indonesia has bauxite reserves of 4% of the world's total reserves with a production level of 4.3% of the world's total bauxite production [1].

Currently, the government has banned bauxite exports and is focusing on domestic processing, such as exports in the form of aluminum [2]. This downstream program is very important to carry out considering that the national aluminum demand is very large with a total requirement of 1.024 million tons in various industries [3]. V Indonesia's aluminum export volume has reached more than 3 million tons from 2010 to August 2023 [4]. This large figure shows that there are large investment opportunities in Indonesian aluminum. One aspect that is affected by Indonesian export activities is fluctuations in the rupiah exchange rate.

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Export activities are generally directly proportional to changes in exchange rates [5]. Export activities increase the circulation of the rupiah on the international market and have a positive impact on strengthening the value of the rupiah. Recently, the rupiah has continued to weaken due to various market turmoil and global economic conditions [6]. Therefore, aluminum as a superior resource is seen in relation to changes in exchange rates (which in this study chose IDR-USD) and how the form of dependence between the two. The use of the IDR-USD exchange rate in this study was chosen based on technical and practical considerations relevant to Indonesia's aluminum export activities. As an exporting country, Indonesia sets production costs, taxes and export duties in domestic currency (IDR), while export proceeds are received in USD. Therefore, fluctuations in the IDR-USD exchange rate directly affect the amount of exporters' revenue in IDR after export proceeds are converted, which in turn has an impact on profit calculations, production costs, and export volume determination strategies. In addition, government regulations related to export proceeds require a portion of export revenues to be deposited in domestic banks, so the IDR-USD exchange rate is an important parameter in determining the effective exchange rate received by companies in the country. From a managerial perspective, the IDR-USD exchange rate is more representative because exporters must adjust export receipts in USD to IDR to fulfill production, operational, and tax obligations that are all paid in Rupiah. Thus, the selection of the IDR-USD rate as a variable in the copula model is considered more appropriate to represent the exchange rate relationship to the volume of Indonesian aluminum exports.

Dependency analysis in this research uses the copula model. Copula is very good at modeling dependence on economic data which is very volatile and tends to be asymmetrical and normal. Several previous literatures have also used copulas in modeling data dependency. Ignativea [7] modeled the dependence of spot prices in the Australian electricity market and the results gave rise to recommendations in the form of considering the price series for each region using the copula model. To expand the use of the model in describing dependency, a mixed copula model is used. Hu Research [8] shows the superiority of the mixed copula model in being able to construct various variations in market data dependency structure patterns empirically.

This study combines several copula concepts to find the best copula to describe the dependency of these variables. Where the selected variables are empirical distributions and can be utilized directly in Indonesia's import-export activities and can have a direct impact on exchange rate fluctuations. The selection of this variable is also based on the fact that no research has been conducted that describes the dependency using the copula model approach, which has many advantages in describing random distributions.

2 Methods

The volume of Indonesian aluminum exports is taken from the monthly Indonesian export foreign trade statistics bulletin published by the Indonesian Central Bureau of Statistics (BPS Indonesia). Exchange rate data uses monthly IDR-USD data taken from the Yahoo Finance website. Both data are monthly data for the period January 2010 to August 2023.

First, data on aluminum export volume and exchange rates were tested for autocorrelation effects using the Ljung-Box test. Data that does not contain autocorrelation effects will be the best distribution used for the distribution along with the copula. If the data has an autocorrelation effect, then the data will go through the time series analysis stage. Time series analysis begins with testing data stationarity for variance and means. Data whose variance is not stationary is subjected to Box-Cox transformation. Data were tested for stationarity against the mean using the Dickey Fuller test. Data whose average is not stationary is differentiated.

Then identify the order and estimate the parameters of the ARMA model. Next, the model error was tested for the white noise assumption and selected the best distribution using the Anderson-Darling test. After that, the error distribution will be transformed into a uniform distribution to form a copula model and the best model based on the Akaike's Information

Criterion (AIC) measure. Single copula parameter estimation uses the Maximum Likelihood method and mixed copula uses the Maximum Pseudo Likelihood method.

The copula used is the Archimedean copula family, which is defined as follows,

$$C_\theta^{Fr}(u,v) = -\frac{1}{\theta}\ln\left(1+\frac{(e^{-\theta u}-1)(e^{-\theta v}-1)}{e^{\theta}-1}\right), \qquad \theta > 0 \tag{1}$$

$$C_\theta^{Cl}(u,v) = (u^{-\theta}+v^{-\theta}-1)^{-\frac{1}{\theta}}, \qquad 0<\theta<\infty \tag{2}$$

$$C_\theta^{Gu}(u,v) = \exp\left(-\left[(-\ln u)^{\theta}+(-\ln v)^{\theta}\right]^{\frac{1}{\theta}}\right), \qquad 1\le\theta<\infty \tag{3}$$

Frank copula is defined by equation (1), while Clayton copula is defined by equation (2), while equation (3) defines Gumbel copula [9]–[11]. Variable u which states the volume of aluminum exports, while v is a variable that states the exchange rate. θ is a dependency parameter that expresses the strength and direction of dependency between two variables u and v. The copula models will be combined with each other to obtain a model that better captures the dependence of the data. Mixed copula in this study uses a linear combination of 2 copulas from the copula mentioned. The mixed copula model is defined as follows,

$$C(u;\theta) = \sum_{k=1}^{n}\lambda_k C_k(u;\theta_k) = \sum_{k=1}^{n}\lambda_k C_k(F_1(x_1;\alpha_1),\ldots,F_p(x_p;\alpha_p);\theta_k) \tag{4}$$

where lambda (λ) is the proportion of each copula and $F_1,\ldots,F_p$ is a function that expresses the set of ordered pairs of (u,v) of each copula.

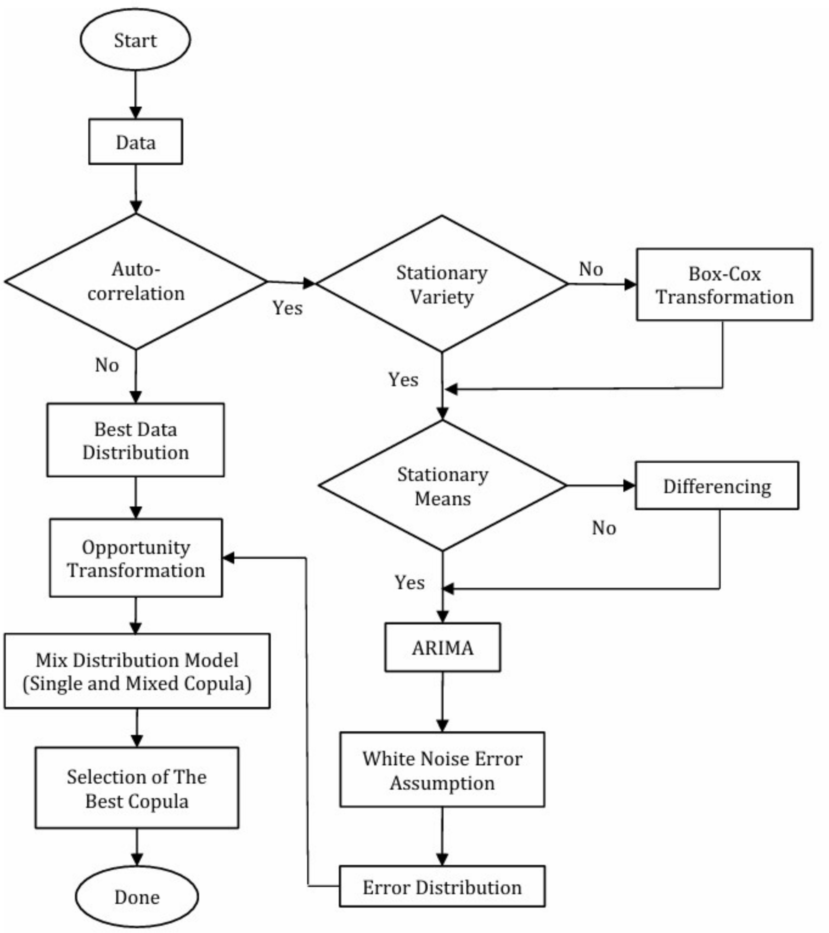


Figure 1: Research flowchart

3 Results and Discussion

This section presents a detailed analysis of the data and the results obtained from the various statistical and time series modeling techniques applied in the study. The discussion is structured into several subsections, beginning with the description of the dataset and followed by tests for autocorrelation and stationarity, model estimation, and dependence analysis using copula modeling. Each subsection is designed to guide the reader through the modeling process in a logical sequence, ultimately leading to the interpretation of the dependence structure between the aluminum export volume and the exchange rate.

3.1 Data Description

The data graphic presentation is designed to provide a comprehensive overview of the characteristics of the data collection used in this research. It also aims to provide a visual understanding of the data.

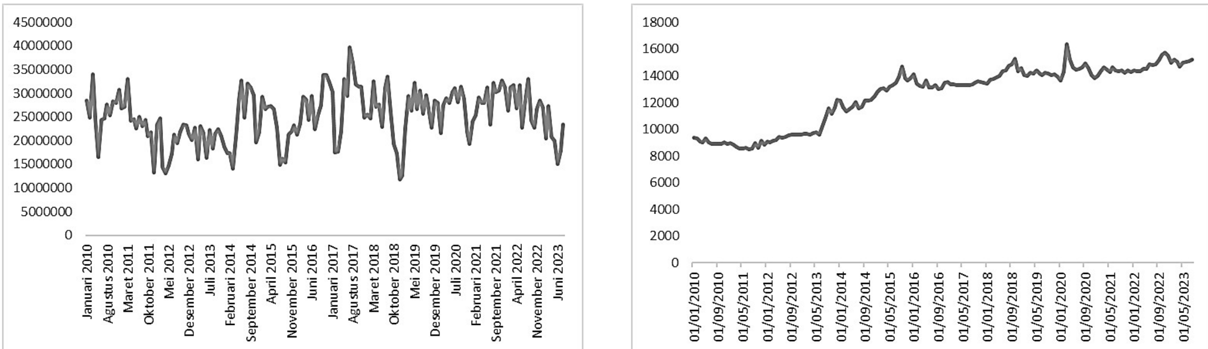


Figure 2: Graph of aluminum export volume (left) and IDR-USD exchange rate (right)

Figure 1 shows a graphic presentation of the two data used. From the visuals, the data is very fluctuating and tends to be asymmetrical and abnormal. Furthermore, you can see the descriptive statistics of the data presented in Table 1 below,

Skewness is a statistical measure used to describe the asymmetry of data distribution. Skewness indicates the degree to which data tends to skew to one side of the average [12]. Aluminum has a positive skewness value, meaning that most of the data is distributed to the left of the average. Meanwhile, the skewness value of the exchange rate is negative, meaning that most of the data is distributed to the right of the mean. The skewness value that stretches far from 0 also indicates that the data tends to be asymmetrical. Kurtosis is a statistical measure used to describe the shape of the peaks and weight of the tails of a data distribution [13]. Kurtosis values that range far from 3 indicate that the data tends to be non normal.

3.2 Autocorrelation Effect

Data freedom or checking the existence of autocorrelation effects is necessary to avoid errors in decision making [14]. The Ljung-Box test was chosen because it can check autocorrelation at several lags and is suitable for time series data [15]. Testing is carried out with a hypothesis:

$$H_0 : \text{There is no autocorrelation effect}$$
$$H_1 : \text{There in an autocorrelation effect}$$

with the H_0 rejection area being *p-value* $< \alpha = 0.05$.

Table 1: Ljung-Box Test Result

Data	χ^2 **Statistics**	***p-value***
Aluminum	55.89	1.23e-07
Exchange Rate	298.35	2.2e-16

The Ljung-Box test on both data obtained a *p-value* < 0.05 so that H_0 was rejected and this means there is an autocorrelation effect in the data. Therefore, aluminum export volume data and exchange rates must undergo time series analysis [16]. The first step in time series analysis is testing data stationarity.

3.3 Data Stationarity with Variability

This test uses the Box-Cox test by optimizing the lambda value. A lambda value that is close to 1 or a lambda confidence interval that contains a value of 1 indicates that the data has stationary variance.

Table 2: Box-Cox Test Result

Data	**Lambda** (λ_1)	**Confidence Interval**
Aluminum	0.93	0.65 – 1.21
Exchange Rate	2.00	1.88 – 2.00

The results of the Box-Cox test in Table 2 show that the aluminum data (A_t) has stationary variance. Exchange rate data (z) needs to be subjected to Box-Cox transformation to handle its non-stationarity with respect to variance. The transformation is carried out by the formula:

$$z^\lambda = \begin{cases} \frac{z^\lambda - 1}{\lambda}, & \lambda \neq 0 \\ \log z, & \lambda = 0 \end{cases} \tag{5}$$

Exchange rate data (z) is transformed $n-1$ times until it meets stationarity requirements. Table 4 shows the Box-Cox test after transformation.

Table 3: Box-Cox Transformation Result

Data	**Transformation Data**	**Lambda** (λ_n)	**Confidence Interval**
Exchange Rate (z)	z^λ	$\lambda_4 = 0.97$	0.78 – 1.51

The lambda value (λ_4) has approached 1 so it can be concluded that the variance of the z^λ transformed data is stationary. Another advantage of the box-cox transformation is that it uses the difference of the values of each previous distribution. Through box-cox transformation, outliers and possible missing data are addressed by checking the distribution curve and ensuring that there are no drastic changes and spikes in the new transformed distribution.

3.4 Data Stationarity with Means

Then we will test the stationarity of the data against the mean using the Dickey-Fuller test with a hypothesis:

$$H_0 : \text{Data is not stationary with respect to the mean}$$
$$H_1 : \text{Data is stationary with respect to the mean}$$

with the H_0 rejection area being *p-value* < 0.05.

Table 4: Dickey-Fuller Test Result

Data	***p-value***
A_t	0.03
z^{λ}	0.37

Based on Table 4, it shows that the A_t data has a *p-value* < 0.05 so that the data is stationary with respect to the mean. Meanwhile, the data z^{λ} has a *p-value* > 0.05, which means that the data average is not yet stationary, therefore it is necessary to differentiate using the formula:

$$Z_t = z_t^{\lambda_2} - z_{t-1}^{\lambda_2} \tag{6}$$

The data that has been differentiated is then checked again using the Dickey-Fuller test,

Table 5: Dickey-Fuller Test Result After Differencing

Differencing Data	***p-value***
Z_t	0.01

p-value from the Z_t data shows that the data mean is stationary. Figure 2 below shows a plot visualization of Z_t and supports that the data is stationary with respect to the mean.

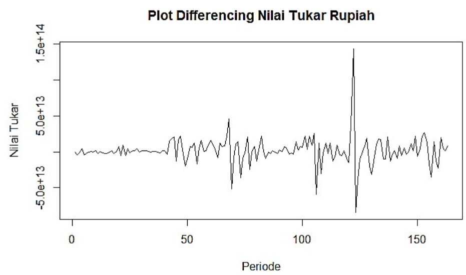
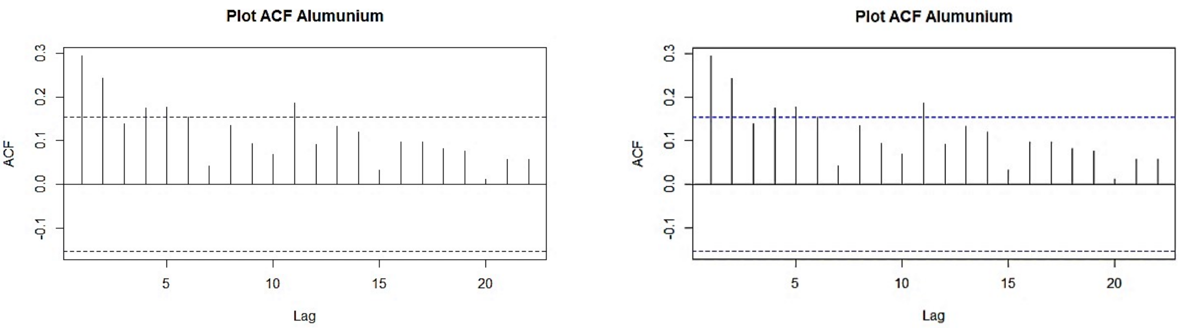


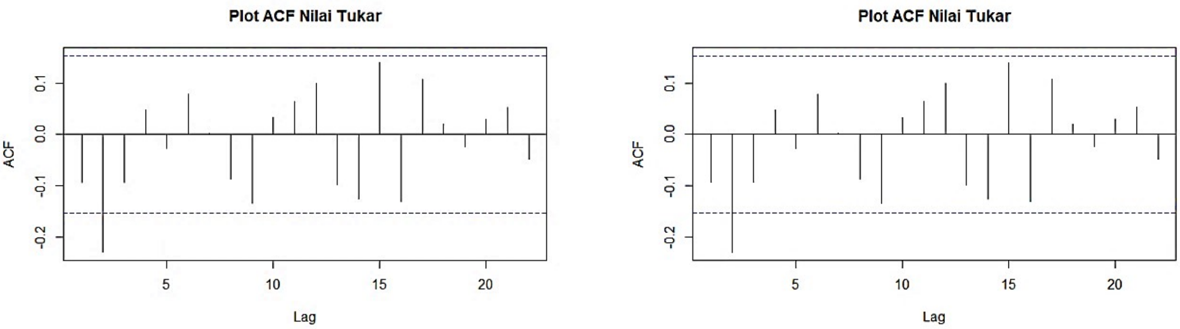
Figure 3: Plot Data of Z_t

3.5 Order Identification and ARIMA Model Estimation

Next, the best ARIMA (p,d,q) model will be selected from the two data. The orders p and q are obtained from initial estimates of the visualization results of the autocorrelation function plot (ACF) and partial autocorrelation function plot (PACF), while q defines the order of differencing in the data [17].



(a)



(b)

Figure 4: ACF and PACF Plot of A_t (a) and Z_t (b)

The initial estimation results obtained from Figure 3 are then overfitting to measure the minimum AIC value as the best model to be selected. Table 7 below shows the results of overfitting the selected ARIMA model.

Table 6: Overfitting Selected of ARIMA Model

Data	ARIMA Model	Parameter	Coefficient
A_t	ARIMA(1,0,1)	Intercept	20839970
		AR(1)	0.8780
		MA(1)	-0.6990
Z_t	ARIMA(1,1,2)	AR(1)	0.6232
		MA(1)	-1.8577
		MA(2)	0.8576

Based on Table 6, the selected ARIMA model is obtained as follows:

$$A_t = 20839970 + 0.8780A_{t-1} - 0.6990\varepsilon_{t-1} + \varepsilon_t \tag{7}$$

$$Z_t = 0.6232Z_{t-1} - 1.8577\varepsilon_{t-1} + 0.8576\varepsilon_{t-2} + \varepsilon_t \tag{8}$$

3.6 White Noise Error Assumption

Next, the model error will be checked for autocorrelation which is called the white noise error assumption using the Ljung-Box test with the hypothesis:

$$H_0 : \text{Independent errors}$$
$$H_1 : \text{Dependent errors}$$

with the H_0 rejection area being *p-value* < 0.05.

Table 7: Ljung-Box Test Result White Noise Assumption

Model Error	χ^2 **Statistics**	***p-value***
A_t	0.2308	0.6309
Z_t	0.4476	0.5034

Table 8 shows that the two model errors are independent of each other. The next step is to choose the best distribution for the copula model.

3.7 Selection of the Best Error Distribution

The best error distribution is selected on two grounds. There are many types of distributions observed such as normal, laplace, binomial, exponential, and others. Firstly, by matching the histogram visualization with the empirical distribution. After that, statistical testing to ensure the most fit distribution uses the Anderson-Darling test with the hypothesis:

$$H_0 : \text{The distribution fits empirical distribution}$$
$$H_1 : \text{The distribution doesn't fit the empirical distribution}$$

with the H_0 rejection area being *p-value* < 0.05.

Table 8: Appropriate Distribution

Data Error	**Distribution**	**Parameter**	***p-value***
Aluminum (A_t)	Normal	$\mu = -24677$ $\sigma = 6583070$	0.1848
Exchange Rate (Z_t)	Laplace	$\theta = 27927383355$ $s = 3.2649\text{e}+12$	0.7486

After fitting these two bases, it can be seen in Table 8 that the normal distribution is the most fit for the aluminum error distribution and the Laplace distribution for the exchange rate error distribution.

3.8 Copula

The definition area of a copula is a uniform distribution so that the two previous error distributions must be transformed into,

$$A \sim N(\mu, \sigma^2) \rightarrow U = F_A(a) \sim U(0,1) \tag{9}$$
$$Z \sim \text{Laplace}(\theta, s) \rightarrow V = F_Z(z) \sim U(0,1) \tag{10}$$

This transformation states that the data error has spread uniformly. Then, based on the results of the goodness-of-fit test for copula, it was found that the suitable copula to use to pair the dependencies of the two data was the Archimedean copula (Frank, Clayton, and Gumbel copula). This study tested many types of copulas, then the best 3 copulas with the most suitable distribution were Frank, Clayton, and Gumbel. This research was carried out by fitting single copula and mixed copula models. The best copula model was selected based on the lowest AIC value with single copula parameter estimation using the maximum likelihood method.

Table 9: Single Copula Fitting Model

Copula	**Parameter**	**Loglikelihood**	**AIC**
Frank	0.0015	2.7e-06	1.9999
Clayton	0.0324	-0.2043	2.4085
Gumbel	1.0000	-2.7e-07	2.0000

For the time being, the results of fitting the single copula model in Table 9 show that the best copula for modeling the dependence between aluminum export volume and the exchange rate is the Frank copula with an AIC value of 1.999. The Frank copula does not show dependence on the tails of any part of the distribution. Then a mixed copula model fitting is carried out to find a model that can better capture dependence with a minimum AIC.

Table 10: Mixed Copula Fitting Model

Parameter	**Frank – Clayton**	**Frank – Gumbel**	**Clayton – Gumbel**
$\hat{\theta}_1$	6.7168	0.0038	-0.6523
$\hat{\theta}_2$	-0.3504	1.0000	1.0000
w_1	0.3266	0.5725	0.0631
w_2	0.6734	0.4274	0.9368
loglikelihood	4.1790	3.7e-06	2.3260
AIC	-0.3578	7.9999	3.3489

Table 11 displays the results of fitting a mixed copula model using a combination of 3 Archimedean copulas. The results of the comparison between fitting the single copula and mixed copula models showed that the Frank - Clayton mixed copula model was the best model in measuring the dependence between aluminum export volume and the exchange rate with the minimum AIC value of -0.3578. When combining Frank and Clayton copulas, the result is usually a copula that can capture the dependency properties of both, considering both the symmetry of dependencies across the distribution (from Frank) and the sensitivity to dependencies in the lower tail (from Clayton). Based on the framework equation of the Frank copula model in equation (1), Clayton copula in equation (2) and the mixed copula model in equation (4), the best copula model is written as follows,

$$\begin{aligned}(A_t, Z_t) \sim C(u, v; \boldsymbol{\theta}) &= 0.3266\, C^{\mathrm{Fr}}(u, v; \hat{\theta}_1) + 0.6734\, C^{\mathrm{Cl}}(u, v; \hat{\theta}_2) \\ &= 0.3266 \left[-\frac{1}{6.7168} \ln \left(1 + \frac{(e^{-6.7168u} - 1)(e^{-6.7168v} - 1)}{e^{6.7168} - 1} \right) \right] + \\ &\quad 0.6734 \left[(u_4^{0.3504} + v^{0.3504} - 1)^{-\frac{1}{0.3504}} \right] \end{aligned} \tag{11}$$

where $\boldsymbol{\theta} = (\hat{\theta}_1, \hat{\theta}_2)$ is a vector of association parameters in the mixture that represents the level of dependence and (A_t, Z_t) is the Aluminum – Exchange Rate model. The accuracy of copula selection is very important so that the type of copula chosen can describe the dependency of the distribution based on the lowest AIC value. Each copula has its own characteristics that have been explained previously so that choosing the best copula does not mean choosing the copula with the most advantages, but choosing the copula that best suits the distribution. So that we can infer the dependence of the variables from the distribution according to the characteristics of the selected copula.

The level of dependence on copulas is measured using Kendall's Tau (τ) correlation. The results of the construction of the correlation value τ are presented in Table 12.

Table 11: Selected Copula

Data	**Copula**	**AIC**	τ
Aluminum – Exchange Rate	Frank – Clayton	-0.3578	0.04

A positive τ value interprets that when the IDR-USD exchange rate increases (the rupiah weakens) then the export volume also tends to increase and conversely when the IDR-USD exchange rate decreases (the rupiah strengthens) then the export volume also tends to decrease. Based on Table 12, a very small τ value of 0.04 and close to 0 indicates a very weak dependency of each pair of data. Therefore, the dependency for changes in each variable has no significant effect and even approaches no effect at all. With the finding of a weak dependency between the exchange rate and the volume of aluminum exports, it is suspected that there are several reasons. The export volume selected in this study may only be a small part of the total export volume that affects the exchange rate. Another reason is that the selected commodities may not cover all variations that can significantly affect the exchange rate. Then the exchange rate is influenced by many other economic factors besides export volume, such as monetary policy, inflation rate, interest rate, and global economic conditions. These factors may have a more dominant influence than export volume on exchange rate fluctuations. And interventions from the government or monetary institutions outside the discussion in this study that can directly affect the exchange rate may not be reflected in the export volume, thus disguising the existing relationship.

4 Conclusion

Based on the results of this research, it can be concluded that there is no significant relationship between the IDR-USD exchange rate and the volume of aluminum exports. Aluminum volume and exchange rate have a positive correlation with τ 0.04 and are classified as very weak for measuring the dependency between the two. The results of this copula model can be a recommendation regarding considerations for Indonesian export activities, especially for aluminum commodities, that there is no need to pay attention to the exchange rate as a key factor in export policy. On the other hand, fluctuations in the volume of aluminum exports also do not have a significant effect on the strengthening/weakening of the exchange rate.

Future research can consider other things such as commodity prices, inflation, taxes, and the influence of production availability in measuring dependency to find a significant influence on exchange rate changes.

CRediT Authorship Contribution Statement

Kurniadi Rizki: Conceptualization, Methodology, Software, Writing – Original Draft.
Retno Budiarti: Supervision, Formal Analysis, Writing – Review & Editing, Validation.
I Gusti Putu Purnaba: Data Curation, Visualization, Investigation, Resources.

Declaration of Generative AI and AI-assisted Technologies

The authors declare that no generative AI or AI-assisted technologies were used in the writing, editing, or content development of this manuscript.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data Availability

The data used in this study are publicly available. Monthly aluminum export volume data were obtained from the official bulletins of the Indonesian Central Bureau of Statistics (BPS Indonesia), and exchange rate data were retrieved from Yahoo Finance (https://finance.yahoo.com). Further details are available from the corresponding author on reasonable request.

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