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Mathematical Stability Analysis of Bullying's Impact on Student's Mental Health

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Abstract

Bullying is a problem that takes root from the previous generation to the next. The most common bullying cases occur in junior high school students or equivalent with an age range of 13-15 years. Bullying can have an impact on student's mental health. This study develops and analysis a SEIRS-type mathematical model to understand the dynamics of mental health disorders due to bullying among junior high school students. The model includes four subpopulations: subpopulations are vulnerable to mental health disorders due to bullying (S), subpopulations who experience bullying but have not shown any mental health disorders (E), subpopulations that experience mental health disorders due to bullying (I), and subpopulations who have recovered from mental health disorders due to bullying (R). Based on the results of the analysis, two equilibrium points were obtained, namely the mental disorder-free equilibrium point (P_1^*) and the endemic equilibrium point (P_2^*). Next, determine the basic reproduction number using the next matrix generation method. Based on the results of numerical simulation using Matlab R2013a software, it was obtained that if $\mathcal{R}_0 < 1$ then the mental disorder-free equilibrium point is stable local asymptotic and mental health disorders due to bullying cannot spread in the student population. Meanwhile, if $\mathcal{R}_0 > 1$ then the endemic equilibrium point is stable, local asymptotic and mental health disorders due to bullying can spread in the student population. The rate of interaction between subpopulations vulnerable to mental health disorders due to bullying and subpopulations that experience mental health disorders due to bullying (α) has a significant influence on the number of individuals in each subpopulation.

Keywords: bullying, mental health, SEIRS model, stability analysis, reproduction number

1 Introduction

Bullying is a problem that has taken root from the previous generation to the next generation [1]. The Indonesian Child Protection Commission reported data on bullying cases in Indonesia as many as 30 bullying cases in 2023 [2]. According to the Federation of Indonesian Teacher's Unions, in 2023, the most bullying cases in Indonesia will be experienced by students at the junior high school level or equivalent, which is 50 percent, while at the elementary level or equivalent by 30 percent, at the high school level or equivalent by 10 percent and at the vocational level by 10 percent [2]. In addition, bullying cases in Indonesia are mostly experienced by students aged 13 to 15 years [3]. In addition to the physical impact or death that can result from violence during

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bullying, victims also have the potential to experience mental health disorders, such as anxiety, fear, and depression [4]. If a person experiences a mental health disorder, negative emotions will arise so that they can cause negative behavior, including having suicidal thoughts [5]. Thus, the role of counseling guidance teachers in schools is very important in providing guidance and counseling services, so that it can help students who experience mental health disorders due to bullying [6].

Previous research has discussed the mathematical model [7], [8], [9]. There is a previous study that has discussed the problem of bullying in the school environment, namely research conducted by [4] that uses the SBP model (S = susceptible, B = bullying, P = post-bullying) to identify the dynamics of the spread of bullying among students. However, the model developed in the study by [4] has several limitations, namely the model only focuses on the process of bullying without paying attention to its impact on student's mental health. In addition, the model in the study conducted by [4] also did not pay attention to the possibility of being vulnerable to becoming a victim of bullying again and there was no compartment that represented post-bullying interventions.

Based on research conducted by [4], this study aims to develop a mathematical model that not only describes the dynamics of the spread of bullying in schools, but also pays attention to the impact of bullying on student's mental health. The model in this study was developed by adding a new compartment of R as a subpopulation that recovered from mental disorders due to the presence of counseling guidance factors denoted by γ. In addition, using compartment S as a subpopulation that is susceptible to mental disorders due to bullying, changing compartment B to E as a subpopulation that experiences bullying but has not shown mental disorders, and changes compartment P to I as a subpopulation that experiences mental health disorders due to bullying. In social problems influenced by the environment, this study pays attention to the possibility of vulnerability, by adding θ as the rate at which a subpopulation recovers from a mental disorder to a subpopulation that is susceptible to mental disorders, so that the mathematical model of students' mental health due to bullying becomes a SEIRS model. Thus, this model allows for a more realistic depiction of the dynamics of bullying and its effects on mental health.

2 Methods

This research is theoretical and analytical, which aims to develop and analyze a mathematical model of bullying on student's mental health. To analyze the model, pay attention to the following steps. First, constructing the SEIRS model. The SEIRS model in this study is constructed based on theoretical assumptions. This model involves four compartments representing the student population aged 13 to 15 years, namely subpopulations that are vulnerable to mental disorders due to bullying (S), subpopulations that have experienced bullying but have not shown mental disorders (E), subpopulations that have experienced mental disorders due to bullying (I), and subpopulations that have recovered from mental disorders due to bullying (R). The following assumptions are used for the construction of a mathematical model of student's mental health due to bullying, namely the population is limited to students who attend formal education at the junior high school level or equivalent with an age range of 13-15 years, the death rate in the subpopulation that experienced mental health disorders due to bullying was assumed to be two, namely natural death and death due to mental health disorders, in this study, namely death due to suicide, the death rate in the subpopulation that experiences bullying but does not have a mental health disorder is twofold, namely natural death and violent death at the time of bullying, the natural death rate is the death of an individual in any subpopulation caused in addition to suicide deaths and violent deaths while experiencing bullying, negative emotions due to mental health disorders can spread through social interactions, the spread of mental disorders is assumed to follow the standard rate of occurrence expressed in the form of $\frac{SI}{N}$, which indicates that a subpopulation is susceptible to mental disorders comparable to a subpopulation that experiences

a mental disorder. Based on these assumptions, the construction of a mathematical model of student mental health due to bullying can be described in the following compartment diagram

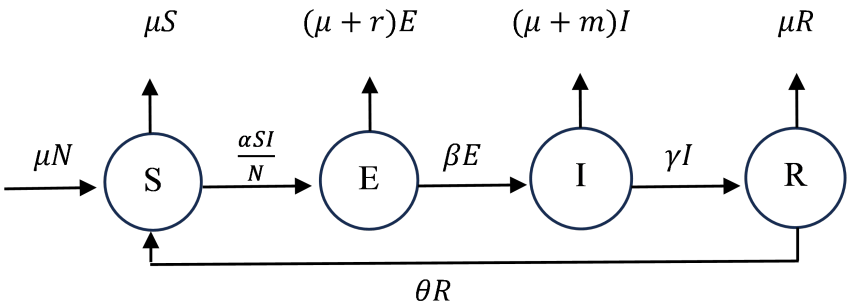

Figure 1: SEIRS model structure for mental health transitions due to bullying

Figure 1 can be formed in the following differential equation system

$$\begin{cases} \dot{S} &= \mu N + \theta R - \frac{\alpha SI}{N} - \mu S \\ \dot{E} &= \frac{\alpha SI}{N} - \beta E - (\mu + r)E \\ \dot{I} &= \beta E - \gamma I - (\mu + m)I \\ \dot{R} &= \gamma I - \theta R - \mu R \end{cases} \tag{1}$$

with, $N = S + E + I + R$.

Second, it proves that the system has positive and limited solutions. Third, conduct a stability analysis from the equilibrium point. At this stage, the equilibrium points and the basic reproduction number will be calculated. Then, to perform linearization to produce a Jacobi matrix, in analyzing stability, the Routh-Hurwitz criterion will be used to determine the local stability conditions. Fourth, perform numerical simulations using Matlab R2013a to describe the impact of various $\mathcal{R}_0$. The value of the parameters at this stage is taken from the literature or previous research and the assumptions of the researcher. The last, conclusions are drawn from the results of the analysis that has been carried out.

3 Results and Discussion

The parameters used for the construction of a mathematical model of student's mental health due to bullying are found in Table 1, including the following

Table 1: Parameters

Parameters	Descriptions	Value	Source	Unit
μ	Rate of population growth per individual pursuing formal education at junior high school level or equivalent at the age of 13 to 15 years / natural mortality rate	0,006	[4]	$\frac{1}{\text{Day}}$
α	The rate of interaction between vulnerable subpopulations and subpopulations that experience mental health disorders due to bullying	0,012	[4]	$\frac{1}{\text{Day}}$
β	The rate of movement from a subpopulation that experiences bullying but does not yet have a mental health disorder to a subpopulation that experiences a mental health disorder due to bullying	0,001	[10]	$\frac{1}{\text{Day}}$
γ	Counseling guidance factors	0,01	[4]	$\frac{1}{\text{Day}}$
m	Suicide death rate due to mental health disorders	0,001	[4]	$\frac{1}{\text{Day}}$
r	Death rate due to internal violence bullying	0,00018	assumption	$\frac{1}{\text{Day}}$
θ	The rate of return of subpopulations recovering from mental health disorders due to bullying to subpopulations that are vulnerable to mental health disorders due to bullying	0,0001	assumption	$\frac{1}{\text{Day}}$
N	The number of students attending formal education at the junior high school level or equivalent at the age of 13 to 15 years	13243704	[11] & [12]	Individual

3.1 Positive and Bounded Solutions

Since population states the number of student population (humans), it is assumed that $S, E, I, R \geq 0$. Therefore, all solutions from Eq. 1 must be non-negative.
Then,

$N(t) = S(t) + E(t) + I(t) + R(t)$

$\Leftrightarrow \dot{N} = \mu N + \theta R - \frac{\alpha SI}{N} - \mu S + \frac{\alpha SI}{N} - \beta E - (\mu + r)E + \beta E - \gamma I - (\mu + m)I + \gamma I - \theta R - \mu R$

$\Leftrightarrow \dot{N} = \mu N - \mu S - \mu E - \mu I - \mu R - rE - mI$

$\Leftrightarrow \dot{N} = \mu N - \mu N - rE - mI$

$\Leftrightarrow \dot{N} = -rE - mI$

Because rE and mI non-negatif, then

$\dot{N} \leq 0$

$\Leftrightarrow \int \frac{dN}{dt} \leq \int\ 0\ dt$

$\Leftrightarrow N(t) \leq c$, where c is a positive whole (the initial population number)

Because, $N(t) \geq 0$ dan $N(t) \leq c = N(0)$, then Eq. 1 is limited to $\Omega = \{(S(t), E(t), I(t), R(t)) \in \mathbb{R}^4_+ : 0 \leq N(t) \leq N(0)\}$. Thus, all solutions of system Eq. 1 are non-negative to $t > 0$ and are limited to $\Omega = \{(S(t), E(t), I(t), R(t)) \in \mathbb{R}^4_+ : 0 \leq N(t) \leq N(0)\}$.

3.2 Equilibrium Point

The mathematical model of student's mental health due to bullying meets the equilibrium state at the time $\dot{S} = \dot{E} = \dot{I} = \dot{R} = 0$. Thus, system Eq. 1 can be written as follows:

$$\mu N + \theta R - \frac{\alpha SI}{N} - \mu S = 0 \tag{1.1}$$

$$\frac{\alpha SI}{N} - \beta E - (\mu + r)E = 0 \tag{1.2}$$

$$\beta E - \gamma I - (\mu + m)I = 0 \tag{1.3}$$

$$\gamma I - \theta R - \mu R = 0 \tag{1.4}$$

Obtained,

1. The mental disorder-free equilibrium point, namely $P_1^* = (S_1^*, E_1^*, I_1^*, R_1^*) = (N, 0, 0, 0)$
2. The endemic equilibrium point, namely $P_2^* = (S_2^*, E_2^*, I_2^*, R_2^*)$

   $S_2^* = \frac{N(\beta+\mu+r)(\gamma+\mu+m)}{\alpha\beta}$,

   $E_2^* = \frac{(\gamma+\mu+m)I_2^*}{\beta}$,

   $I_2^* = \frac{(\mu N(\beta+\mu+r)(\gamma+\mu+m)-\alpha\beta\mu N)(\theta+\mu)}{\alpha(\theta\gamma\beta-(\beta+\mu+r)(\gamma+\mu+m)(\theta+\mu))}$,

   $R_2^* = \frac{\gamma I_2^*}{\theta+\mu}$

   with the condition $(\mu N(\beta+\mu+r)(\gamma+\mu+m) - \alpha\beta\mu N)(\theta+\mu) < 0$, because $\alpha(\theta\gamma\beta - (\beta + \mu + r)(\gamma + \mu + m)(\theta + \mu)) = -\alpha(\beta\mu\theta + \beta m\theta + \mu\gamma\theta + \mu^2\theta + \mu m\theta + r\gamma\theta + r\mu\theta + rm\theta + \beta\gamma\mu + \beta\mu^2 + \beta m\mu + \mu^2\gamma + \mu^3 + \mu^2 m + r\gamma\mu + r\mu^2 + rm\mu) < 0$

3.3 Basic Reproduction Number

The determination of basic reproductive numbers can be done using the next-generation matrix method [13]. The next-generation matrix method can be obtained from classes that have the potential to experience mental disorders due to bullying, namely E and I, so it can be written as follows:

$$\dot{E} = \frac{\alpha SI}{N} - \beta E - (\mu + r)E \tag{2}$$

$$\dot{I} = \beta E - \gamma I - (\mu + m)I \tag{3}$$

with

$$\mathcal{F} = \begin{pmatrix} \frac{\alpha SI}{N} \\ 0 \end{pmatrix} \rightarrow \mathbf{F} = \begin{pmatrix} 0 & \alpha \\ 0 & 0 \end{pmatrix} \tag{4}$$

and

$$\mathcal{V} = \begin{pmatrix} \beta E + (\mu + r)E \\ -\beta E + \gamma I + (\mu + m)I \end{pmatrix} \rightarrow \mathbf{V} = \begin{pmatrix} \beta + \mu + r & 0 \\ -\beta & \gamma + \mu + m \end{pmatrix} \tag{5}$$

$$\mathbf{V}^{-1} = \begin{pmatrix} \frac{1}{\beta+\mu+r} & 0 \\ \frac{\beta}{(\beta+\mu+r)(\gamma+\mu+m)} & \frac{1}{\gamma+\mu+m} \end{pmatrix} \tag{6}$$

so that

$$\mathbf{Z} = \mathbf{F}\mathbf{V}^{-1} = \begin{pmatrix} 0 & \alpha \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{\beta+\mu+r} & 0 \\ \frac{\beta}{(\beta+\mu+r)(\gamma+\mu+m)} & \frac{1}{\gamma+\mu+m} \end{pmatrix} \tag{7}$$

So, the basic reproductive number ($\mathcal{R}_0$) is

$$\mathcal{R}_0 = \frac{\alpha\beta}{(\beta+\mu+r)(\gamma+\mu+m)} \tag{8}$$

That is, if $(\beta+\mu+r)(\gamma+\mu+m)$ greater than $\alpha\beta$, then mental health disorders due to bullying cannot spread in the student population or $\mathcal{R}_0 < 1$, this means that mental disorders will disappear within a certain period of time, so it can be concluded that the existence of counseling guidance factors in schools is quite effective. However, if $(\beta+\mu+r)(\gamma+\mu+m)$ smaller than $\alpha\beta$, then mental health disorders due to bullying can spread in the student population or $\mathcal{R}_0 > 1$, meaning that the counseling guidance factor in schools is not effective enough.

3.4 Stability Analysis

Here is the Jacobi matrix obtained from the linearization of the Eq. 1.1 - Eq. 1.4

$$\mathbf{J} = \begin{pmatrix} -\left(\frac{\alpha I}{N}+\mu\right) & 0 & -\frac{\alpha S}{N} & \theta \\ \frac{\alpha I}{N} & -(\beta+\mu+r) & \frac{\alpha S}{N} & 0 \\ 0 & \beta & -(\gamma+\mu+m) & 0 \\ 0 & 0 & \gamma & -(\theta+\mu) \end{pmatrix} \tag{9}$$

1. Local stability at mental disorder-free equilibrium point
   Substitution of equilibrium points free of mental disorders due to bullying $P_1^* = (S_1^*, E_1^*, I_1^*, R_1^*) = (N, 0, 0, 0)$ on the Jacobi matrix, so that

$$\mathbf{J}(P_1^*) = \begin{pmatrix} -\mu & 0 & -\alpha & \theta \\ 0 & -(\beta+\mu+r) & \alpha & 0 \\ 0 & \beta & -(\gamma+\mu+m) & 0 \\ 0 & 0 & \gamma & -(\theta+\mu) \end{pmatrix} \tag{10}$$

   Based on the matrix $\mathbf{J}(P_1^*)$ in Eq. 10, a characteristic equation is obtained, namely

$$\det \begin{pmatrix} \lambda+\mu & 0 & \alpha & -\theta \\ 0 & \lambda+\beta+\mu+r & -\alpha & 0 \\ 0 & -\beta & \lambda+\gamma+\mu+m & 0 \\ 0 & 0 & -\gamma & \lambda+\theta+\mu \end{pmatrix} = 0 \tag{11}$$

   By using the cofactor expansion method [14] in Eq. 11, obtained

   $(\lambda+\mu)((\lambda+\beta+\mu+r)(\lambda+\gamma+\mu+m)(\lambda+\theta+\mu) - (\lambda+\theta+\mu)(-\beta)(-\alpha) = 0$

   Suppose,
   $a_1 = \beta+\mu+r$

$a_2 = \gamma + \mu + m$
$a_3 = \theta + \mu$
So that,

$$(\lambda+\mu)(\lambda^3 + (a_1+a_2+a_3)\lambda^2 + (a_1a_2 + a_1a_3 + a_2a_3 - \alpha\beta)\lambda + (a_1a_2a_3 - a_3\alpha\beta)) = 0 \tag{12}$$

Then, Eq. 12 simplified to

$$(\lambda+\mu)(\lambda^3 + q_1\lambda^2 + q_2\lambda + q_3) = 0 \tag{13}$$

So, one of the roots is obtained, namely $\lambda_1 = -\mu$ and the rest are the roots of the Eq. 13. Using the Routh-Hurwitz criterion [15] in Eq. 13, all parts of the real part of the eigenvalue are obtained with a negative value. Thus, the equilibrium point free of mental health disorders due to bullying is stable local asymptomatic or $\mathcal{R}_0 < 1$. This means that mental health disorders due to bullying cannot spread in the student population.

2. Local stability at endemic equilibrium points
From Eq. 9, the endemic equilibrium point of $P_2^* = (S_2^*, E_2^*, I_2^*, R_2^*) = \left(\frac{N(\beta+\mu+r)(\gamma+\mu+m)}{\alpha\beta}, \frac{(\gamma+\mu+m)I_2^*}{\beta}, \frac{(\mu N(\beta+\mu+r)(\gamma+\mu+m) - \alpha\beta\mu N)(\theta+\mu)}{\alpha(\theta\gamma\beta - (\beta+\mu+r)(\gamma+\mu+m)(\theta+\mu))}, \frac{\gamma I_2^*}{\theta+\mu}\right)$
with the condition $(\mu N(\beta+\mu+r)(\gamma+\mu+m) - \alpha\beta\mu N)(\theta+\mu) < 0$, because $\alpha(\theta\gamma\beta - (\beta+\mu+r)(\gamma+\mu+m)(\theta+\mu)) < 0$ will be substituted, so that

$$\mathbf{J}(P_2^*) = \begin{pmatrix} -\left(\frac{\alpha I_2^*}{N} + \mu\right) & 0 & -\frac{\alpha S_2^*}{N} & \theta \\ \frac{\alpha I_2^*}{N} & -(\beta+\mu+r) & \frac{\alpha S_2^*}{N} & 0 \\ 0 & \beta & -(\gamma+\mu+m) & 0 \\ 0 & 0 & \gamma & -(\theta+\mu) \end{pmatrix} \tag{14}$$

Based on the matrix $\mathbf{J}(P_2^*)$ in Eq. 14, a characteristic equation is obtained, namely

$$\det \begin{pmatrix} (\lambda + \frac{\alpha I_2^*}{N} + \mu) & 0 & \frac{\alpha S_2^*}{N} & -\theta \\ -\frac{\alpha I_2^*}{N} & (\lambda+\beta+\mu+r) & -\frac{\alpha S_2^*}{N} & 0 \\ 0 & -\beta & (\lambda+\gamma+\mu+m) & 0 \\ 0 & 0 & -\gamma & (\lambda+\theta+\mu) \end{pmatrix} = 0 \tag{15}$$

By using the cofactor expansion method [14] in Eq. 15, obtained

$$\left(\lambda + \tfrac{\alpha I_2^*}{N} + \mu\right)\left((\lambda+\beta+\mu+r)(\lambda+\gamma+\mu+m)(\lambda+\theta+\mu) - (\lambda+\theta+\mu)(-\beta)\left(-\tfrac{\alpha S_2^*}{N}\right)\right) +$$
$$\left(\tfrac{\alpha I_2^*}{N}\right)\left(\beta(\lambda+\theta+\mu)\left(\tfrac{\alpha S_2^*}{N}\right) - \beta(-\theta)(-\gamma)\right) = 0$$

Suppose,

$a_1 = \beta + \mu + r$
$a_2 = \gamma + \mu + m$
$a_3 = \theta + \mu$

So that,

$$\lambda^4 + \left(a_1+a_2+a_3+\mu+\tfrac{\alpha I_2^*}{N}\right)\lambda^3 + \Big(a_1a_2 + a_1a_3 + a_2a_3 + a_1\mu + a_2\mu + a_3\mu + \tfrac{\alpha I_2^*}{N}(a_1 + a_2 +$$

$$a_3) - \frac{\alpha\beta S_2^*}{N})\lambda^2 + \Big(a_1a_2a_3 + a_1a_2\mu + a_1a_3\mu + a_2a_3\mu + \frac{\alpha I_2^*}{N}(a_1a_2 + a_1a_3 + a_2a_3) - \frac{\alpha\beta S_2^*}{N}(a_3 + \mu)\Big)\lambda + \Big(a_1a_2a_3\mu + \frac{\alpha I_2^*}{N}a_1a_2a_3 - \frac{\alpha\beta S_2^*}{N}a_3\mu - \frac{\alpha\theta\beta\gamma I_2^*}{N}\Big) = 0 \tag{16}$$

Then, Eq. 16 simplified to

$$\lambda^4 + q_1\lambda^3 + q_2\lambda^2 + q_3\lambda + q_4 = 0 \tag{17}$$

By using Hurwitz's Routh Criterion [15] on Eq. 17, all real parts of the eigenvalue are obtained with a negative value. Thus, the endemic equilibrium point is stable local asymptotic or $\mathcal{R}_0 > 1$. This means that mental health disorders due to bullying can spread in the student population.

3.5 Numerical Simulation

The following are the results of the simulation of mental disorder-free equilibrium points with initial values $S(0) = 6.621.852, E(0) = 3.536.069, I(0) = 2.741.447$, dan $R(0) = 344.336$, time range to days 4200, and and parameter values in the Table 2:

Table 2: Parameters for the condition $\mathcal{R}_0 < 1$ and $\mathcal{R}_0 > 1$

Parameters	$\mathcal{R}_0 < 1$	Source	$\mathcal{R}_0 > 1$	Source
μ	0,006	[4]	0,006	[4]
α	0,012	[4]	0,5	[16]
β	0,001	[10]	0,001	[10]
γ	0,01	[4]	0,01	[4]
m	0,001	[4]	0,001	[4]
r	0,00018	assumption	0,00018	assumption
θ	0,0001	assumption	0,0001	assumption
N	13243704	[11] & [12]	13243704	[11] & [12]

The following is the result of the numerical simulation when $\mathcal{R}_0 < 1$ and $\mathcal{R}_0 > 1$:

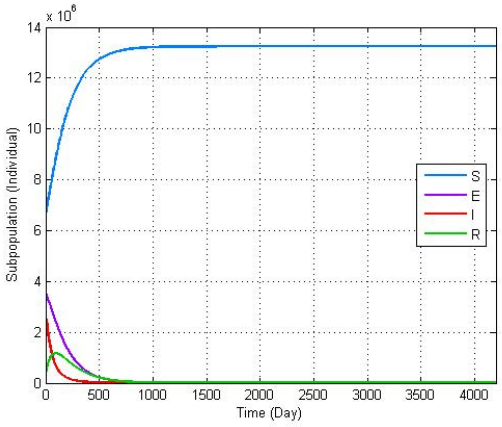

Figure 2: Dynamics of subpopulations when $\mathcal{R}_0 < 1$

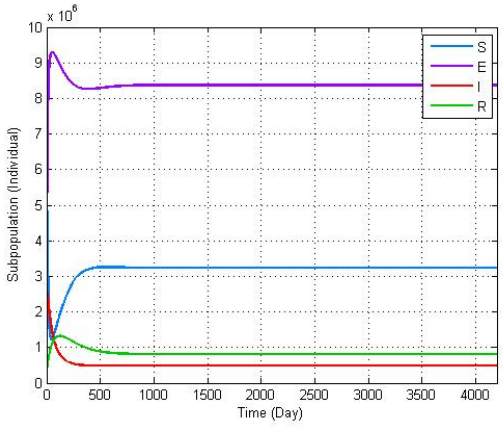

Figure 3: Dynamics of subpopulations when $\mathcal{R}_0 > 1$

Based on Figure 2, it can be concluded that when $\mathcal{R}_0 < 1$ the subpopulation S will increase and go to a stable point, while the subpopulations of E, I, and R decrease to 0 and stabilize. This means that mental disorders in students due to bullying will disappear within a certain period of time. Thus, the existence of counseling guidance factors in schools is effective enough

to prevent the spread of mental disorders due to bullying, so that the school environment is psychologically safe.

Based on Figure 3, it can be concluded that when $\mathcal{R}_0 > 1$ decreases in the S subpopulation and then increases significantly towards a stable point. Subpopulation E then increases significantly towards a stable point. Subpopulation I has declined significantly towards a stable point. The R subpopulation has increased and then declined significantly towards a stable point. That is, when $\mathcal{R}_0 > 1$ there is a spread of mental disorders due to bullying. Thus, students who experience mental disorders due to bullying have not received enough intervention from the school or counseling guidance factors at school have not been effective enough. Therefore, schools should take action to prevent the spread of mental disorders due to bullying.

4 Conclusion

This study developed a SEIRS-type mathematical model to examine the impact of bullying on student's mental health. In this model there are two equilibrium states identified, and their local stability is assessed through the analysis of the eigenvalue and the basic reproduction rate $\mathcal{R}_0$. The simulation confirms that when $\mathcal{R}_0 < 1$, mental health problems due to bullying do not spread to the population, whereas $\mathcal{R}_0 > 1$ causes the rate of mental disorders to become endemic. This model provides a valuable framework for evaluating the effectiveness of counseling guidance in addressing mental disorders due to bullying in the school environment. In addition, the rate of interaction between subpopulations susceptible to mental disorders due to bullying and subpopulations experiencing mental disorders (α) has an influence on the number of individuals in each subpopulation.

In the future, this model can be validated using empirical data obtained through surveys or direct observations in schools to improve its accuracy. In addition, the development of advanced models can include additional interventions such as peer support as well as secure and confidential reporting systems. The addition of stochastic elements is also recommended to describe the uncertainty of student behavior in a school environment that is not fully predictable in a deterministic manner.

CRediT Authorship Contribution Statement

Devi Marita Putri: Conceptualization, Methodology, Validation, Formal Analysis, Resources, Data Curation, Writing – Original Draft Preparation, Writing – Review & Editing, Project Administration, and Funding Acquisition. **Zulaikha Zulaikha**: Conceptualization, Methodology, Software, Investigation, Resources, Data Curation, Visualization, Conceptualization, Methodology, Validation, Formal Analysis, Writing – Review & Editing, and Supervision.

Declaration of Generative AI and AI-assisted technologies

The authors acknowledge the use of generative AI and AI-assisted technologies in the preparation of this manuscript. Specifically, Scholar AI 4o was employed to search for and cite peer-reviewed scientific literature; Gemini 2.5 Pro was utilized for idea organization, refinement of content structure, and exploratory drafting; and Quillbot Pro was used to assist in paraphrasing, grammar enhancement, and improving the fluency of academic writing.

All intellectual contributions, interpretations, and final decisions regarding the content were made by the authors. The use of these technologies complied with institutional, ethical, and publication standards, and all AI-generated content was critically reviewed and edited to ensure accuracy, originality, and scholarly integrity.

Declaration of Competing Interest

The authors declare no competing interests.

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Data and Code Availability

All data and source code used in this study are openly available and can be accessed via public repositories.

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