



# A Sinusoidal-Based Mathematical Model for Psychotherapy Effects in Bipolar Disorder Type 2 Patients

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## Abstract

Bipolar Disorder (BD) is a mental disorder characterized by recurrent manic and depressive episodes. This study aims to build a mathematical model that models mood changes in BD type 2 before and after psychotherapy. Daily mood data were collected for more than 3 months from one BD patient, then divided into seven terms of 14 days each. The analysis was carried out through a sinusoidal function fitting process and numerical simulation based on the Van der Pol differential equation. The results showed that before psychotherapy, the mood amplitude reached 1.99632, the frequency was 0.4926, and the moment of inertia was 4.121081. After undergoing routine psychotherapy 9 times, the amplitude decreased to 0.635, the frequency increased to 1.052, and the moment of inertia decreased to 0.903584. The average mood was controlled at 6.492, within the normal mood range. The decrease in amplitude and moment of inertia indicated that BD mood became more stable and less easily affected by the environment, while the increase in frequency indicated a faster recovery of emotional rhythm. Conclusion: Routine psychotherapy is effective in quantitatively stabilizing the mood of BD type 2.

**Keywords:** Bipolar Disorder Type 2; Psychotherapy Modeling; Sinusoidal Approximation; Mood Dynamics; Van der Pol Equation.

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## 1 Introduction

Bipolar disorder (BD) is a psychological disorder that causes extreme mood swings, but its causes and mechanisms are still widely unknown. Mood swings experienced by bipolar patients are more extreme, and tend to occur more rapidly [1]. The mood swings of BD, which vary in severity from extreme depression to extreme mania, irritability, and anger, are often accompanied by difficulties in socializing [2]. Although BD have higher levels of education, they are still unemployed at a rate of around 65% [3]. BD have many creative talents [4] and some extraordinary people such as Ernest Hemingway and Charles Dickens [5]. This study is expected to attract more researchers because people with BD have a lot of potential and high Intelligence Quotient (IQ) [6][7]. A BD with a higher IQ has a greater risk of injury. High IQ can cause psychological stimulation in someone who has difficulty controlling their mood [8].

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BD must take medication and therapy regularly to achieve a stable condition. If not regular, the condition will get worse over time [9][10]. More than 20% of BD (most of whom do not receive treatment) commit suicide [9]. In addition, 18% of BD feel the need to harm themselves at night using poison or sharp weapons [10]. BD mental health can also affect their family members or those living with them [11]. The mood phenomena that occur in BD need to be studied in depth. This can minimize the instability of BD. Treatments that can be done for BD include psychotherapy [12], ISRT [13], food [14], drugs [15], and others. With the right therapy, it will greatly help the mental stability and safety of BD.

Previous studies have shown that BD type 2 is characterized by episodes of hypomania and depression [16][17]. In this study, the Van der Pol equation as applied by Daugherty *et al.*, [6], was used to model the dynamics of mood in BD patients. The Daugherty model uses the Van der Pol equation with negatively damped oscillations to represent mood changes in individuals with BD who are not receiving treatment. This study also emphasizes the importance of examining the interaction model between therapists and BD patients, considering that this topic is still ongoing but has not been widely published. Some of the assumptions underlying this model include dysregulation of dopaminergic neurotransmission [18], dysfunction of mood-stabilizing neurons [16], and circadian rhythm disturbances [17], all of which play an important role in triggering bipolar episodes. Based on the literature, mathematical models of BD can follow a systematic approach that varies, depending on the type of mood episode, type of BD, type of therapy applied, and the patient's environmental and family conditions.

Treatment for BD includes at least two types of therapy, namely medication and psychotherapy with a psychologist or psychiatrist. Psychotherapy is the second most common treatment after pharmacotherapy given to BD [19]. This type of research uses quantitative methods, namely by utilizing mathematics in completing the model and studying the literature. The quantitative method used is the mathematical model of Daugherty, *et al.*, [6] From the main model, a BD mood model is sought with a sinusoidal approach through psychotherapy. From this model, a simulation is obtained that describes the stability of BD mood.

This research was initially motivated by the development of new types of therapy and interactions between therapists and BD patients to stabilize BD faster. This knowledge will help us understand how bipolar patients and therapists interact, and which therapies are most interesting and useful for future research. This research refers to a number of articles that discuss mathematical models of BD, including treatment approaches to BD [7], the possibility of chronic depression in BD patients [20], and the role of therapy in improving mental stability in BD patients [21].

Although several mathematical models, including those based on the Van der Pol equation have been developed to describe mood variations in BD, most have focused on untreated individuals or ignored the measurable effects of therapeutic interventions. In addition, limited research integrates real-world mood tracking data into model development, especially in the context of psychotherapy outcomes. The data used as a model is the mood level of a BD taken every day for more than 3 months. In addition to these data, the BD mood level is also taken before and after undergoing psychotherapy. From the results of these recordings, it is hoped that the mood level can be determined. Data processing is carried out with the help of software to simulate mathematical models.

Most mathematical models of BD focus on untreated cases and ignore the measurable effects of therapy. Furthermore, few studies have incorporated real-world daily mood data into modeling, particularly in the context of psychotherapy outcomes. To address this gap, the current study aims to build a sinusoidal-based mathematical model that describes the mood dynamics in Type 2 BD patients before and after psychotherapy. By analyzing daily mood data over three months, key parameters such as frequency, amplitude, and moment of inertia were extracted and examined to evaluate the quantitative effects of psychotherapy on emotional stability. Combining multiple treatments may help BD patients achieve more optimal mental stability. Intervention

and protection [22] are often used in conjunction with pharmacological therapy. Intervention can be carried out by therapists such as psychologists or psychiatrists, while protection is provided by family members, friends, or the social environment. This study only examined the impact of interactions between therapists and BD patients including the duration and therapy provided, not with medication. The results of this study are expected to be the basis for developing a more accurate therapist-BD interaction model by researchers in the future.

This research contributes to the advancement of quantitative approaches in evaluating psychotherapy outcomes for BD patients. The novelty of this research lies in the development of a sinusoidal estimate of bipolar mood patterns in psychotherapy sessions, integrating daily mood tracking with Van der Pol-based modeling. This provides a new perspective in measuring therapy efficacy mathematically.

This article is structured as follows: the urgency and impact of therapy are presented first, followed by an overview of previous studies and mathematical approaches. The next section explains the methodology, including data collection and analysis techniques. Subsequently, the core results of the mathematical modeling are presented, along with graphical interpretation and simulations. The identified research gap and study objectives are then described. Finally, the article concludes with the main findings and outlines directions for future research.

## 2 Methods

This study used a quantitative approach through mathematical modeling to analyze mood fluctuations in patients with BD Type 2. The modeling is based on differential equations inspired by the Van der Pol oscillator, which has been widely used to describe self-organizing dynamic systems, including mood variations in BD. The sinusoidal approximation was chosen to reflect the observed periodicity and oscillatory nature of mood patterns, especially under the influence of psychotherapy.

### 2.1 Data Collection Techniques

In Daugherty's research [6], the Van Der Pol equation is written in application form with BD. In modeling changes in mood, it does not use a compartment diagram like disease modeling in general, but uses an adaptation modeling approach. This adaptation model can be described as a phenomenon that is approached with the chosen formulation of the rate of change [22]. This adaptation model can also be seen in the concept of pendulum dynamics which has been used to view mood regulation in BD [23]. As for the mathematical model of BD mood formed by Daugherty is

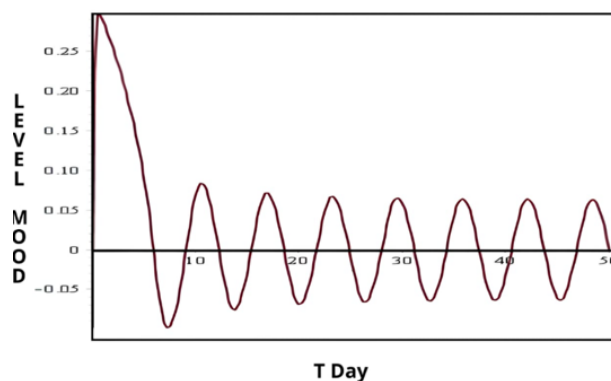
$$x'' - (\alpha - (\beta + \delta)x^2)x' + \omega^2x = 0 \quad (1)$$

Let  $x$  denote the level of mood,  $x'$  the change in mood, and  $x''$  the rate of change in mood. The parameter  $\alpha$  indicates the level of dysregulation in the dopaminergic neurotransmission system,  $\beta$  represents the level of mood-stabilizing neural activity,  $\delta$  is the effect of treatment or psychotherapy, and  $\omega^2$  denotes the degree of circadian rhythm abnormality [18].

The parameter  $\alpha$  represents the degree of dysregulation or imbalance in the brain's dopamine system in bipolar disorder (BD) patients. A higher value of  $\alpha$  implies that the patient's mood becomes more unstable due to a more disrupted dopaminergic system. The parameter  $\beta$  indicates the effectiveness of neurons in stabilizing mood. A larger  $\beta$  suggests a better ability of the brain to dampen emotional fluctuations, resulting in a more stable mood condition.

The parameter  $\delta$  reflects the effectiveness of treatments or psychotherapy provided to the patient. A higher  $\delta$  value indicates greater therapeutic influence in managing mood swings. Meanwhile,  $\omega^2$  captures the degree of circadian rhythm abnormality, which significantly affects the frequency of mood variation. In this model,  $\omega$  also describes the speed of the mood cycle

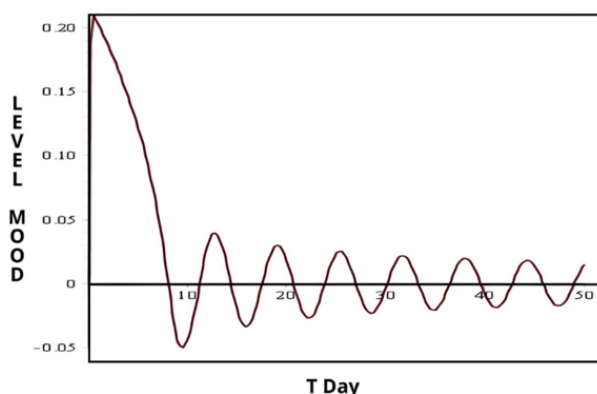
per day. Circadian rhythm disruptions can severely impact the stability of a patient's emotional state.



**Figure 1:** Mood of bipolar disorder patients without treatment  $\alpha = 0.1$ ,  $\beta = 100$ ,  $\omega^2 = 1$ , and initial value  $x(0) = 0$

Figure 1 is Daugherty's mathematical model, which is a graph of the mood of BD patients without treatment with  $\alpha = 0.1$ ,  $\beta = 100$ ,  $\omega^2 = 1$ , and initial value  $x(0) = 0$ . The figure shows that the mood of the BD patient follows a sinusoidal function.

By enlarging  $\beta = 300$  and reducing the value of  $\alpha = 0.001$ , it can be seen that the amplitude of mood changes decreases as shown in Figure 2. It can be concluded that there is a need for an increase in mood stabilizer in medication so that the dysregulation of the dopaminergic neurotransmission system decreases. For this reason, a treatment is expected to help control the mood of BD patients.



**Figure 2:** Mood of bipolar disorder patients without treatment  $\alpha = 0.001$ ,  $\beta = 300$ ,  $\omega^2 = 1$ , and initial value  $x(0) = 0$

In addition, data collection techniques were carried out when BD patients were undergoing psychotherapy. This is done before and after psychotherapy with the aim of measuring the level of mood. Recording was carried out 9 times with psychiatrists and psychologists. In addition, data was also collected independently from a BD who journaled moods every day. This data was taken for more 3 months from June to August 2021. From the results of these recordings it is hoped that the level of BD mood can be known. Data processing is done with the help of software to simulate a mathematical model. The results of the data are used as a model in this research. In this study, the research variable is the mood of a BD whose mood level will be measured so that the mood level can be controlled to be normal. The mood scale used is a mood scale from 1 to 10 with reference from Bipolaruk.org [24].

## 2.2 Data Analysis and Methods

The Van der Pol equation used in the Daugherty model is a nonlinear differential equation of order 2 with damping. After the therapeutic effect is given (small  $\alpha$  value, large  $\beta$ ,  $\delta \neq 0$ ), the oscillation becomes more regular and resembles a sinusoid. The use of the sinusoidal function in this study is based on the natural characteristics of the mood change pattern of BD Type 2 patients, which shows periodic fluctuations similar to a sine wave. In addition, this approach is realistic to patient mood data that appears periodic and stable, especially after therapeutic intervention. So that the sinusoidal function is used as an approximation to the numerical solution of the Van der Pol equation in psychotherapy.

In this study, the data fitting process to the sinusoidal function was carried out through several stages. First, daily mood data were collected from a BD patient more than 3 months. Then the BD was seen for its mood level during the psychotherapy treatment before and after it. After the data was collected, the next stage was data preprocessing with the smoothing method using the moving average technique, aiming to reduce random fluctuations so that the mood pattern was more clearly visible. The smoothed data was then divided into 7 segments, each representing 14 days, so that the short-term fluctuation pattern could be analyzed more accurately.

The mathematical model used for fitting is a sinusoidal function in the form of

$$y(t) = A \sin(\omega t + \varnothing) + c \quad (2)$$

with four main parameters that need to be found, namely:  $A$  as the height of the mood fluctuation or amplitude,  $\omega$  as the frequency or speed of mood changes,  $\varnothing$  as the phase or shift of the mood cycle, and  $c$  as the average mood level. The fitting process was carried out using nonlinear regression techniques through Maple software, especially with the help of curve fitting tools to obtain optimal parameter values that minimize the difference between actual data and the model.

After fitting is done, the results are validated visually by comparing the model result curve with the original data through a graph. If the fitting results are not appropriate, then the process is repeated by providing different initial parameter limits (initial guess). The final stage of this procedure is the interpretation of the fitting result parameters to explain the patient's mood dynamics quantitatively.

After getting the model, then the model is simulated so that it can be interpreted graphically, so that the next level of mood can be predicted. By adding treatment to BD, it can be seen the stability of the graph which describes the stability of the mood of a BD.

## 3 Results and Discussion

Before BD patients undergo therapy, data is collected on daily mood levels for a full three-month period. This observation is carried out directly on a type 2 BD patient, where every day the patient records or tracks BD mood level based on a numerical scale. The scale used refers to the Bipolaruk.org Mood Scale [24], which describes various mood conditions ranging from severe depression to hypomania. During the three months, data is collected consistently to obtain a picture of the patient's mood fluctuations in a state where they have not received any therapeutic intervention, so that it can be used as initial data in the analysis and mathematical modeling of the effects of psychological therapy on the patient's mood condition.

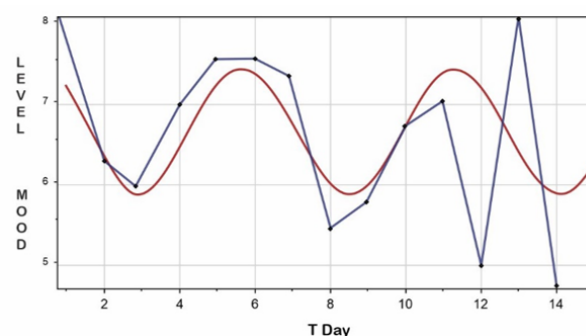
The Bipolar Mood Scale is a numeric rating tool from 1 to 10 used to monitor mood swings in BD, including Type 2. The scale describes a range of mood states, from severe depression at the lowest score to extreme mania at the highest score. In people with Type 2 BD, the two main phases that occur frequently are hypomania and depression, with extreme mania (scores 9–10) generally absent. Scores of 7 to 8 indicate a hypomanic phase. Scores of 1 to 4 indicate a

depressive phase of varying severity. Scores of 5 and 6 indicate a balanced mood state, which is the target phase of therapy.

**Table 1:** BD Mood Descriptive Statistics for 3 months

	June	July	August
Mean	6.41	6.66	5.86
Median	6.3	7	6
Mode	7	7	6
Variance	1.035	1.28	1.83
Std dev	1.017	1.13	1.35

Descriptive statistics from the Mood Tracker above can be seen from Table 1. From the table it can be seen that the average mood of the BD is 6.31, which means it is at a normal mood level [24]. However, the mode or value that often comes out is at mood level 7 which states that the BD often experiences a level 7 mood phase, namely hypomanic. Here it can be said that the BD is Bipolar Type 2. For the variance and standard deviation values, both state that the BD is Bipolar Type 2. Figure 3 shows the results of the mood tracker plot of BD patients for the first 14 days of the total observation period of more than 3 months. This mood data is then divided into 7 terms, each covering a timespan of 14 days, so that the entire period is represented in 7 consecutive terms. This division is done to facilitate the modeling process and fitting of sinusoidal functions in each time segment. Figure 3 specifically shows the first term, namely the observation of the patient's mood from day 1 to day 14. This term is the initial basis for evaluating mood dynamics before being compared with the following terms or with conditions after therapy.



**Figure 3:** Mood Tracker on days 1 to 14 which is fitted to the sinusoidal function

Based on Figure 3, the blue line represents the mood tracker data of BD patients recorded daily, while the red line shows the results of the sinusoidal function approach to the data. This approach is carried out by fitting the daily mood data, so that a mathematical model can be formed that describes the dynamics of the BD patient's mood quantitatively.

The mathematical model with the sinusoidal function approach obtained from the data fitting process in the first term (day 1 to day 14), as shown in Figure 3, is expressed in the following equation:

$$y = 0.7368 \sin(1.13768x + 1.2653) + 6.6355 \quad (3)$$

with the following sinusoidal function parameters were obtained: frequency  $\omega = 1.13768$ , which represents the speed of the patient's mood change; phase angle or shift of the mood cycle  $\phi = 1.2653$ ; amplitude  $A = 0.7368$ , which shows the difference between the highest and lowest mood levels; and the average mood  $c = 6.6355$ . In addition, the moment of inertia value was also obtained as  $m = 0.77$ , which describes the strength or resistance of the patient's mood to environmental influences. This relatively low moment of inertia value indicates that the patient's mood is still quite easily influenced by external factors.



Based on Figure 3 and the mathematical model obtained, it can be seen that the average mood level of patients is in the range of 6.635. This value indicates that the patient's mood condition is slightly above the normal limit, and is close to 7, which on the Bipolar UK scale indicates a hypomanic phase. Thus, it can be concluded that in this first term, patients tend to be in a high mood condition but are still within the limits of mild symptoms.

In the same way for term 2 to term 7, the following table is obtained:

**Table 2:** The value of the results of fitting the mood tracker to the sinusoidal function

Term	$ A $	$\omega$	$\phi$	$C$	$m$ for $k = 1$
1	0.7368	1.13768	1.26529	6.6355	0.772609
2	0.59626	1.1179	-0.91938	5.9328	0.800192
3	0.8289	1.3067	-1.15158	7.36718	0.585664
4	0.90038	0.7737	2.25106	6.1131	1.670532
5	0.74077	1.6554	-11.2969	6.439	0.364917
6	1.99632	0.4926	-7.613543	5.788	4.121081
7	1.51579	0.5989	-4.8818	5.4567	2.787991

Based on Table 2, in Term 1, the frequency value was  $\omega = 1.13768$  and amplitude  $A = 0.7368$ , indicating that the patient experienced fairly rapid mood changes and moderate fluctuations, with an average mood of  $c = 6.6355$ , approaching the hypomanic limit. The moment of inertia of  $m = 0.77$  indicates a moderate level of mood resilience to environmental influences.

Term 2 shows a decrease in frequency  $\omega = 1.1179$  and a lower amplitude  $A = 0.59626$ , indicating a flatter mood, but the moment of inertia value actually increased slightly to  $m = 0.80$ , meaning that the mood is more resistant to external disturbances even though it tends to be flat. Interestingly, Term 3 recorded a significant increase in the average mood  $c = 7.36718$ , indicating that the patient entered the hypomanic zone. Although the amplitude is quite high  $A = 0.8289$ , the moment of inertia is low  $m = 0.585664$ , indicating that mood is very easily influenced by the environment at this stage.

In Term 4, the average mood returned to normal, but the amplitude remained high  $A = 0.90038$  and the moment of inertia jumped sharply to  $m = 1.6705$ , reflecting stronger emotional stability even though mood remained fluctuating. Term 5 showed the highest frequency  $\omega = 1.6554$ , indicating extreme acceleration of mood changes, but accompanied by moderate amplitude  $A = 0.74077$ . The lowest moment of inertia was recorded in this term at  $m = 0.3649$ , indicating that in this acceleration phase, the patient's mood was very easily disturbed by external factors.

In contrast, Term 6 showed a very contrasting condition. The amplitude reached the highest number  $A = 1.99632$ , but the frequency was the lowest  $\omega = 0.4926$ , which means that mood changes occur slowly but with very large fluctuations. The moment of inertia also reached the highest value  $m = 4.121$ , indicating a very high level of mood resilience, even though the mood changes themselves were very drastic.

Finally, in Term 7, both the frequency  $\omega = 1.51579$  and the moment of inertia  $m = 2.78799$  were in the medium-high category. This shows that the patient's mood again experienced rapid changes but with resilience that began to decrease compared to the previous term. The amplitude  $A = 1.51579$  was also relatively high, indicating that mood fluctuations were still quite strong towards the end of the observation period.

From this description, it is known that the frequency value ( $\omega$ ), which describes the speed of mood changes, reaches the highest value in the 5th term, which is 1.6554, and the lowest in the 6th term, which is 0.4926. Meanwhile, the amplitude value ( $A$ ), which shows the difference between the highest and lowest mood levels, is highest in the 6th term with a value of 1.99632 and lowest in the 2nd term with a value of 0.59626. The moment of inertia value ( $m$ ), which describes the strength of the patient's mood in resisting the influence of the surrounding environment, is highest recorded in the 6th term, which is 4.121081 and lowest in the 5th term, which is 0.364917.

From these results, it can be concluded that when the speed of mood changes ( $\omega$ ) of BD patients increases, the strength of the mood to withstand environmental influences tends to decrease. Conversely, when mood changes are slow (low frequency), patients show high mood resilience, even though the amplitude or difference in mood remains large. This indicates an inverse relationship between the speed of mood fluctuations and emotional stability of BD patients.

### 3.1 The Mood Phenomenon of a Bipolar Disorder

The mood phenomenon is measured based on the mood level obtained by scaling it from the Bipolaruk.org website [24]. Mood tracers taken for more than 3 months from a person with BD describe how their mood changes over time and there is a tendency to be in a depressive phase compared to a manic phase. From this phenomenon it can also be described that he is a Bipolar Type 2 who only experiences hypomanic phases but does not experience manic phases.

### 3.2 Sinusoidal Function Approach to BD Mood Model at the moment Psychotherapy

After BD did psychotherapy nine times in the same period of about half a month, mood levels were 7, 7, 6.25, 6.5, 5.75, 6.3, 7, 7.5, and 6.3, with an average mood level during psychotherapy of 6.62. Based on Equation (2) and in the same way as in Equation (3), a mathematical model of BD during psychotherapy is obtained, namely:

$$y(t) = 0.635 \sin(1.052t - 0.16) + 6.492 \quad (4)$$

Equation (4) is a sinusoidal function approach of BD mood during psychotherapy, with amplitude  $A = 0.635$ , frequency  $\omega = 1.052$ , angle of refraction  $\varnothing = -0.16$ , mood level  $c = 6.492$ , and moment of inertia  $m = 0.903584$ .

Amplitude  $A = 0.635$  indicates that the difference in mood levels from the highest to the lowest point is relatively small, meaning that emotional fluctuations are no longer extreme. This is an early indication of emotional stabilization. Frequency  $\omega = 1.052$  indicates that mood changes occur more quickly and rhythmically than several terms before therapy. This increase in frequency indicates that the patient's emotional rhythm is becoming more regular and responsive, not slow or erratic as before.

Phase angle  $\varnothing = -0.16$  describes an initial shift in the mood cycle, but the value is relatively small and does not significantly affect stability. This can be associated with the emotional adaptation phase during therapy. Average mood  $c = 6.492$ , within the normal range (5–6.5), only slightly above, but not reaching the hypomanic threshold (7). This indicates that the patient's mood during therapy is at a clinically stable level.

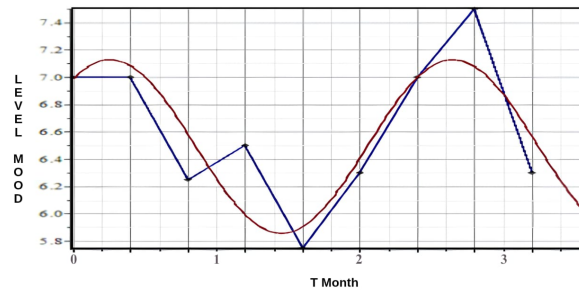
Moment of inertia  $m = 0.903584$  indicates that the patient's mood is quite stable against environmental disturbances, because the value is close to 1. This is a significant improvement compared to the period before therapy, where the moment of inertia was very fluctuating (up to 4.12).

The graphical form of the sinusoidal function approach simulation for BD psychiatry and psychotherapy can be seen in Figure 4.

If the values of amplitude  $A$ , frequency  $\omega$ , and moment of inertia  $m$  during psychotherapy are compared with the values in Table 2, it can be seen that the values of amplitude  $A$  tend to be low, while the values of frequency  $\omega$  and moment of inertia  $m$  tend to be moderate. This shows that the psychotherapy process is successful. In other words, BD mood can be stable with the psychotherapy process routinely for 9 times.

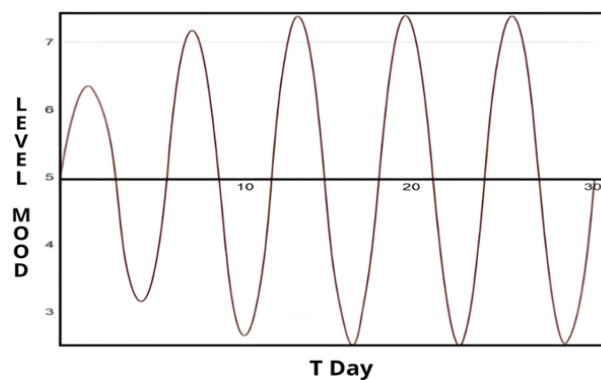
Using the frequency  $\omega = 1.052$ , the result of the sinusoidal function approach, it is seen how the mathematical model of BD with the Van der Pol approach is simulated during psychotherapy.





**Figure 4:** Simulation of a sinusoidal function approach for psychiatrist and BD psychotherapy

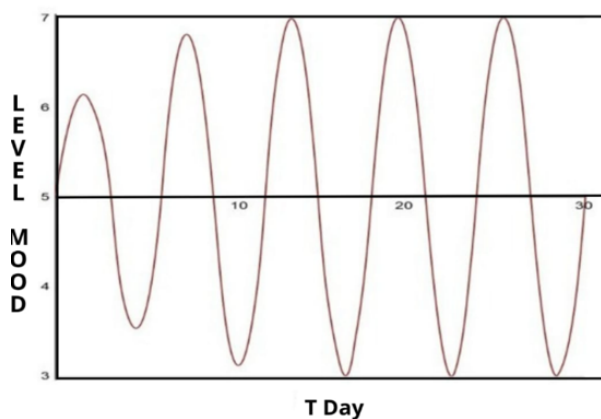
The simulation describes the phenomenon of BD mood with amplitude values, above and below the threshold of BD type 2 mood.



**Figure 5:** Mood of bipolar disorder patients with treatment  $\delta = 0.1$ ,  $\alpha = 0.36$ ,  $\beta = 0.1$ ,  $\omega = 1.052$ , and initial value  $x(0) = 0$

After BD patients undergo psychotherapy, there is a change in mood caused by the effects of the treatment. In Figure 5, when the value of the therapy parameter is set at  $\delta = 0.1$ , the highest amplitude of the mood graph exceeds level 7. This indicates that the BD mood is above the hypomanic threshold and approaching the manic phase. In other words, low-intensity therapy ( $\delta = 0.1$ ) has not been able to control the mood of Type 2 BD patients effectively.

On the contrary, in Figure 6, when the value of  $\delta$  is increased to 0.25, the highest amplitude is exactly at level 7. This indicates that higher-intensity therapy is able to control BD mood within reasonable limits, namely remaining at the hypomanic threshold without crossing it.



**Figure 6:** Mood of bipolar disorder patients with treatment  $\delta = 0.25$ ,  $\alpha = 0.36$ ,  $\beta = 0.1$ ,  $\omega = 1.052$ , and initial value  $x(0) = 0$

Table 3 shows significant differences between before and after psychotherapy. The results showed that before psychotherapy, the mood of BD Type 2 sufferers was unstable, with high amplitude (up to 1.99), irregular frequency (0.49–1.65), and a very variable moment of inertia (0.36–4.12). The average mood in some respects was also close to 7, indicating a tendency towards the hypomanic phase. This reflects a fluctuating emotional condition that is easily influenced by the environment. After undergoing 9 psychotherapy sessions, there was a decrease in amplitude to 0.635, an increase in frequency to 1.052, and a decrease in moment of inertia to 0.903. The average mood was also at 6.492, within the normal and stable range. These changes indicate that psychotherapy is effective in reducing emotional fluctuations, stabilizing mood rhythms, and increasing emotional resilience in BD Type 2.

**Table 3:** Comparison of BD mood before and after therapy

	Before	After	Description
Amplitude ( $A$ )	high and variable (up to 1.99)	decreased (0.635)	mood no longer extreme
Frequency ( $\omega$ )	Unstable (0.49–1.65)	Stable and moderate (1.052)	regular rhythm
Moment of Inertia ( $m$ )	Inconsistent (0.36–4.12)	moderate (0.903)	more resistant to stress
Average mood	Sometimes approaching hypomanic ( $> 7$ )	stable (6.492)	approaching normal mood

## 4 Conclusion

The results showed that after undergoing regular psychotherapy for nine sessions, Bipolar Disorder Type 2 patients experienced a significant increase in mood stability. Mood amplitude decreased from 1.99 to 0.635, meaning that emotional fluctuations became lighter. The frequency of mood changes increased from 0.49 to 1.052, indicating a more regular emotional rhythm. Meanwhile, the moment of inertia decreased from 4.12 to 0.903, indicating that the patient's mood became more resistant to external environmental influences. The average patient mood was also controlled at 6.492, within the normal range and no longer entering the hypomanic phase consistently. These findings quantitatively prove that psychotherapy is effective in stabilizing the mood of BD Type 2. Multi-year research is needed to discuss BD because this topic has a high level of difficulty and novelty. In addition, by knowing the topic that is the main discussion, it is hoped that future researchers can conduct research by developing appropriate therapeutic techniques and diagnosing BD so that their quality of life improves and their safety is guaranteed.

## CRedit Authorship Contribution Statement

**Indah Nursuprianah:** Conceptualization, Methodology, Visualization, Writing–Original Draft, Software. **Nursanti Anggriani:** Validation, Investigation, Supervision, Project Administration. **Nuning Nuraini:** Conceptualization, Validation, Investigation, Data curation, Supervision. **Yudi Rosandi:** Validation, Data curation, Supervision.

## Declaration of Generative AI and AI-assisted technologies

The authors declare that only Grammarly was used to assist in language editing. No generative AI tools were used for data analysis, content generation, or interpretation.

## Declaration of Competing Interest

The authors declare no competing interests

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## Data and Code Availability

The data supporting the findings of this study are available from the corresponding author upon reasonable request and subject to confidentiality agreements

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