



Odd Harmonious Labeling of Lotus Flower Graphs

Fery Firmansah*, Tasari, and Muhammad Ridlo Yuwono

Widya Dharma University, Klaten, Indonesia

Abstract

The purpose of this research is to obtain a new graph class definition and to ensure that the new graph class satisfies the properties of odd harmonious labeling. This research method consists of several stages, namely identifying open problems, analyzing data, formulating definitions and theorems, and proving theorems. We define three new classes of graphs, namely the lotus flower graph, the lotus flower graph with pendant, and a variation of the lotus flower graph. Furthermore, the three new graph classes satisfy the odd harmonious labeling, making them odd harmonious graphs.

Keywords: Lotus flower graph; Odd harmonious graph; Odd harmonious labeling; Variation of lotus flower graph

Copyright © 2026 by Authors, Published by CAUCHY Group. This is an open access article under the CC BY-SA License (<https://creativecommons.org/licenses/by-sa/4.0>)

1. Introduction

Graph labeling has been extensively studied by researchers worldwide. Beyond its theoretical interest, various applications of graph labeling have been discovered, including its use in cryptography, coding theory, and data security systems [1]. Odd harmonious graphs were first introduced by Liang and Bai [2]. A graph $G = (V, E)$ of order m and size n with $m, n \in \mathbb{Z}^+$ is said to be an odd harmonious graph if there exists an injective vertex labeling $g : V \rightarrow \{0, 1, \dots, 2n - 1\}$ such that the induced edge labeling $g^*(ab) = g(a) + g(b)$ is a bijection from E to the set of odd integers $\{1, 3, \dots, 2n - 1\}$ [2].

Several classes of graphs have been proven to belong to the family of odd harmonious graphs. Abdel-Aal showed that cyclic snake graphs are odd harmonious [3]. Jeyanthi, Philo, and Sugeng identified multiple graph classes that fall into the odd harmonious category [4]. Renuka and Balaganesan showed that several cycle-related graph classes are also odd harmonious [5]. Jesintha and Stanley proved that the SSG(2) graph is both odd graceful and odd harmonious [6].

Sarasvati, Halikin, and Wijaya showed that graphs constructed by the edge comb product of a path P_n with either the cycle on four vertices C_4 or the shadow of a four-cycle $D_2(C_4)$ are odd harmonious [7]. Philo and Jeyanthi demonstrated that both line graphs and disjoint unions of graphs belong to the family of odd harmonious graphs [8]. Additional contributions by Jeyanthi and collaborators can be found in [9–16]. Zala *et al.* presented several graph classes having both odd and even harmonious properties [17].

Asumpta *et al.* proved that certain families of snake graphs are odd harmonious [18]. Hafez *et al.* demonstrated that the converse skew product of graphs is also an odd harmonious graph [19].

*Corresponding author. E-mail: firmanmath@gmail.com

Firmansah showed that the class of string graphs belongs to the category of odd harmonious graphs [20]. Other studies by Firmansah and collaborators are documented in [21–26].

Lasim, Halikin, and Wijaya investigated the relationships among harmonious, odd harmonious, and even harmonious graphs [27]. Venkataraman *et al.* identified additional structural properties of odd harmonious graphs [28]. Further related studies on odd harmonious and closely related labelings appear in [29–38].

Between 2020 and 2023, Firmansah developed a series of “flower graph” classes that are odd harmonious, including the double quadrilateral flower graph [39] and the zinnia flower graph [40]. It has also been shown that the union of these flower graph classes is itself odd harmonious [41]. Motivated by these developments, this work proposes a new class of graphs, referred to as the lotus flower graph. In line with this objective, we provide formal definitions of the lotus flower graph, the lotus flower graph with pendant, and their variations. We also prove that each of these classes is an odd harmonious graph.

2. Methods

This research consists of three stages, namely the preliminary stage, data analysis and verification of results. The preliminary stage is to find an open problem that will be tried to find a solution. The focus at this stage is to find as many new graph classes as possible that have not yet found their labeling, especially odd harmonious labeling. The data analysis stage is the formation of graph class definition construction and theorem formation construction. The focus at this stage is the construction of new graph class definitions with vertex, edge, order and size notations. The result verification stage is the mathematical proof of the theorem, at this stage the construction of injective vertex labeling that induces bijective edge labeling will be given.

This section formalizes the mathematical method used to

1. Define the proposed flower-type graph families,
2. Prove that each family admits an odd harmonious labeling.

We first fix notation and recall the target labeling property, then present a general labeling framework and a proof template that will be instantiated in Section 3 for the three graph classes introduced in this paper.

2.1. Preliminaries and Notation

Let $G = (V, E)$ be a finite, simple, connected graph of order $m = |V|$ and size $n = |E|$. A function $g : V \rightarrow \{0, 1, \dots, 2n - 1\}$ is injective if $g(u) \neq g(v)$ whenever $u \neq v$. The induced edge labeling $g^* : E \rightarrow \{1, 3, \dots, 2n - 1\}$ is defined by

$$g^*(uv) = g(u) + g(v), \forall u, v \in E$$

Graph G is odd harmonious if there exists an injective g such that g^* is a bijection from E onto the set of odd integers $\{1, 3, \dots, 2n - 1\}$.

Throughout, \mathbb{Z}^+ denotes the nonnegative integers, and for a finite set S we write \sqcup for a disjoint union. For an arithmetic progression $A = \{a_0 + a_1 t : t \in I\}$ with integer step a_1 , we write $step(A) = a_1$. We use that the sum of two integers is odd iff they have opposite parity.

2.2. Problem Setting

For each parameter tuple θ (e.g., $\theta = p$, or (p, k) , or (p, r)), we construct a flower-type graph $G_\theta = (V_\theta, E_\theta)$ by an explicit vertex partition and edge pattern (see the Definitions in Section 3). Let $n_\theta = |E_\theta|$. Our task is to design an injective vertex labeling

$$g_\theta : V_\theta \rightarrow \{0, 1, \dots, 2n_\theta - 1\}$$

such that the induced $g_\theta^* : E_\theta \rightarrow \{1, 3, \dots, 2n_\theta - 1\}$ is bijective.

2.3. Labeling Framework

The proposed constructions use the following framework.

(F1) Vertex partition. Write $V_\theta = \sqcup_{j=0}^J \mathbf{V}_j$ where each block \mathbf{V}_j is indexed by one or two discrete parameters (e.g., h or (h, t)) with prescribed index ranges coming from the graph definition. For instance, for $L(p)$ we use

$$\mathbf{V}_0 = \{a_0\}, \mathbf{V}_1 = \{a_h : 1 \leq h \leq p + 2\}, \mathbf{V}_2 = \{b_h : 1 \leq h \leq p + 1\}, \mathbf{V}_3 = \{c_h : 1 \leq h \leq p\}.$$

(F2) Affine label families with controlled parity. Assign to each block \mathbf{V}_j an affine arithmetic progression of the form

$$g_\theta(x) = \alpha_j + \beta_j$$

with index x and integer coefficients (α_j, β_j) chosen so that:

1. The families $\{g_\theta(\mathbf{V}_j)\}_{j=0}^J$ are pairwise disjoint (injectivity),
2. Each edge $uv \in E_\theta$ joins vertices whose labels have opposite parity (so $g_\theta(u) + g_\theta(v)$ is odd),
3. The image $g_\theta(V_\theta) \subseteq \{0, 1, \dots, 2n_\theta - 1\}$.

In practice we take $\beta_j \in \{2, 4, \dots\}$ and alternate the parity of α_j across blocks to force (2), while ranges and offsets are spaced to achieve (1) and (3).

(F3) Edge-sum stratification. Partition the edge set according to its defining pattern,

$$E_\theta = \sqcup_{l=1}^L \mathbf{E}_l$$

(e.g., star edges, rim edges, and petal/bridge edges). For each \mathbf{E}_l the induced sums $g_\theta^*(\mathbf{E}_l)$ form an arithmetic progression of odd integers with step 2, $gcd(\beta_j(u), \beta_j(v))$ determined by the meeting blocks. Carefully chosen (α_j, β_j) make these progressions pairwise disjoint and collectively exhaustive.

2.4. Proof Template

Fix θ and a labeling g_θ constructed by (F1)–(F3). The odd harmonious property follows from the next lemmas, applied verbatim to each graph class.

Lemma 1. (*Injectivity*). *If the label blocks $g_\theta(\mathbf{V}_j)$ are pairwise disjoint arithmetic progressions with distinct offsets modulo $\min_j \beta_j$, then g_θ is injective on V_θ .*

Lemma 2. (*Oddness of edge sums*). *Suppose that for every edge $uv \in E_\theta$ the incident blocks $\mathbf{V}_{j(u)}$ and $\mathbf{V}_{j(v)}$ are assigned opposite parities (i.e., $\alpha_{j(u)} \equiv \alpha_{j(v)} + 1 \pmod{2}$ and $\beta_{j(u)}, \beta_{j(v)}$ are even). Then $g_\theta(u) + g_\theta(v)$ is odd for all $uv \in E_\theta$.*

Lemma 3. (*Disjointness of edge-sum bands*). *Let $E_\theta = \sqcup_{l=1}^L \mathbf{E}_l$. If each \mathbf{E}_l induces an odd arithmetic progression $S_l = \{s_{l,0} + 2d_l t\}$ with distinct residues modulo $\text{lcm}(2d_1, \dots, 2d_L)$, then the sets S_1, \dots, S_L are pairwise disjoint.*

Lemma 4. (*Cardinality and coverage*). *If $\sqcup_{l=1}^L S_l \subseteq \{1, 3, \dots, 2n_\theta - 1\}$ and $\sum_{l=1}^L |S_l| = n_\theta$, then $\sqcup_{l=1}^L S_l = \{1, 3, \dots, 2n_\theta - 1\}$. Consequently, g_θ^* is a bijection onto the odd set.*

Verification protocol. For each class G_θ we

1. Compute $m_\theta = |V_\theta|$ and $n_\theta = |E_\theta|$ from the construction,
2. Specify (α_j, β_j) per block as in Section 3,
3. List the induced edge-sum bands S_l for the edge groups,
4. Check Lemmas 1–4.

This yields a complete, self-contained proof that G_θ is odd harmonious.

2.5. Instantiation Plan for the Three Classes

We will define three classes of lotus flower graphs: the lotus flower graph $L(p)$, the lotus flower graphs with pendant $L(p, k)$, and variations of lotus flower graphs $rL(p)$.

1. The lotus flower graph $L(p)$ (Definition 1), use four blocks $\mathbf{V}_0, \mathbf{V}_1, \mathbf{V}_2, \mathbf{V}_3$ with affine families of steps 2 and alternating parities, as already outlined in the proof of Theorem 1. Edge groups correspond to $a_0 - a_h, a_h - b_h, a_{h+1} - b_h, b_h - c_h, b_{h+1} - c_h$.
2. The lotus flower graph with pendant $L(p, k)$ (Definition 2), append a pendant block $\mathbf{V}_4 = \{d_s\}$ with labels shifted beyond the previous maxima so that $g_\theta(\mathbf{V}_4)$ stays disjoint and preserves opposite parity relative to a_0 . The new edge group $a_0 - d_s$ produces a terminal odd progression that fills the topmost odd integers.
3. The variations of lotus flower graph $rL(p)$ (Definition 3), replace the single c -block by r replicated c^t -blocks with controlled offsets (α_{c^t}) increasing in t . Each replicated petal induces two edge-sum bands; the step and residues are arranged so that all bands across $t = 1, \dots, r$ are disjoint and the total cardinality equals n_θ .

2.6. Remark on template consistency

The concrete formulas given in Section 3 (Equations (1)-(9), (10)-(20), and (21)-(29) are specific instances of (F2)-(F3). Their offsets and steps are chosen to satisfy Lemmas 1-4 by construction:

1. Steps are even,
2. Parities alternate across adjacent blocks,
3. Ranges are nonoverlapping and cover the required index spans,
4. The resulting edge-sum bands partition $\{1, 3, \dots, 2n_\theta - 1\}$ exactly.

3. Results and Discussion

In this section, three discussions are given, namely lotus flower graph, lotus flower graph with pendant, and variations of lotus flower graph.

3.1. Lotus Flower Graph

Definition 1 (Lotus Flower Graph). *The lotus flower graph $L(p)$ with $p \geq 1, p \in \mathbb{Z}^+$ is a graph defined by $V(L(p))$ as the set of vertices*

$$V(L(p)) = \{a_0\} \cup \{a_h \mid 1 \leq h \leq p + 2\} \cup \{b_h \mid 1 \leq h \leq p + 1\} \cup \{c_h \mid 1 \leq h \leq p\}$$

and $E(L(p))$ as the set of edges

$$E(L(p)) = \{a_0 a_h \mid 1 \leq h \leq p + 2\} \cup \{a_h b_h \mid 1 \leq h \leq p + 1\} \\ \cup \{a_{h+1} b_h \mid 1 \leq h \leq p + 1\} \cup \{b_h c_h \mid 1 \leq h \leq p\} \cup \{b_{h+1} c_h \mid 1 \leq h \leq p\}.$$

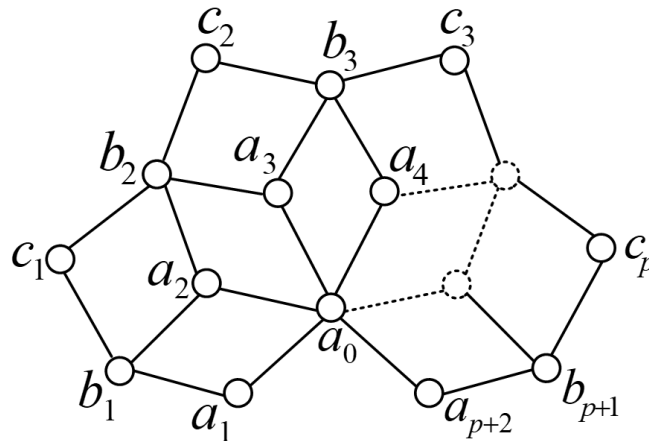


Fig. 1: Lotus flower graph $L(p)$

From the construction, we obtain:

$$m = |V(L(p))| = 3p + 4 \quad \text{and} \quad n = |E(L(p))| = 5p + 4.$$

Theorem 1. Lotus flower graph $L(p)$ with $p \geq 1, p \in \mathbb{Z}^+$ is an odd harmonious graph.

Proof. Defined functions $g : V(L(p)) \rightarrow \{0, 1, 2, \dots, 10p + 7\}$ of the lotus flower graph $L(p)$.

$$g(a_0) = 0 \tag{1}$$

$$g(a_h) = 2h - 1, \quad 1 \leq h \leq p + 2 \tag{2}$$

$$g(b_h) = 2p + 2h + 2, \quad 1 \leq h \leq p + 1 \tag{3}$$

$$g(c_h) = 4p + 2h + 3, \quad 1 \leq h \leq p \tag{4}$$

Based on equation (1), (2), (3) and (4)

$$g(V(L(p))) = \{0\} \cup \{1, 3, 5, \dots, 2p + 3\} \cup \{2p + 4, 2p + 6, 2p + 8, \dots, 4p + 4\} \\ \cup \{4p + 5, 4p + 7, 4p + 9, \dots, 6p + 3\}$$

it is obtained that the function g is injective because it gives different labels to each vertex and $g(V(L(p))) \subseteq \{0, 1, 2, 3, \dots, 10p + 7\}$.

Defined functions $g^* : E(L(p)) \rightarrow \{1, 3, 5, \dots, 10p + 7\}$ of the lotus flower graph $L(p)$.

$$g^*(a_0a_h) = 2h - 1, \quad 1 \leq h \leq p + 2 \tag{5}$$

$$g^*(a_hb_h) = 2p + 4h + 1, \quad 1 \leq h \leq p + 1 \tag{6}$$

$$g^*(a_{h+1}b_h) = 2p + 4h + 3, \quad 1 \leq h \leq p + 1 \tag{7}$$

$$g^*(b_hc_h) = 6p + 4h + 5, \quad 1 \leq h \leq p \tag{8}$$

$$g^*(b_{h+1}c_h) = 6p + 4h + 7, \quad 1 \leq h \leq p \tag{9}$$

Based on equation (5), (6), (7), (8), and (9)

$$g^*(E(L(p))) = \{1, 3, 5, \dots, 2p + 3\} \cup \{2p + 5, 2p + 9, 2p + 13, \dots, 6p + 5\} \\ \cup \{2p + 7, 2p + 11, 2p + 15, \dots, 6p + 7\} \cup \{6p + 9, 6p + 13, 6p + 17, \dots, 10p + 5\} \\ \cup \{6p + 11, 6p + 15, 6p + 19, \dots, 10p + 7\}$$

it is obtained that the function g^* is bijective because it gives different labels to each edge and $g^*(E(L(p))) = \{1, 3, 5, \dots, 10p + 7\}$. Such that lotus flower graph $L(p)$ is an odd harmonious. \square

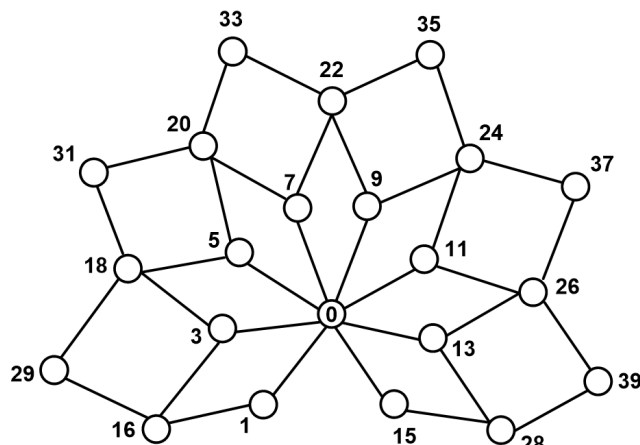


Fig. 2: Lotus flower graph $L(6)$

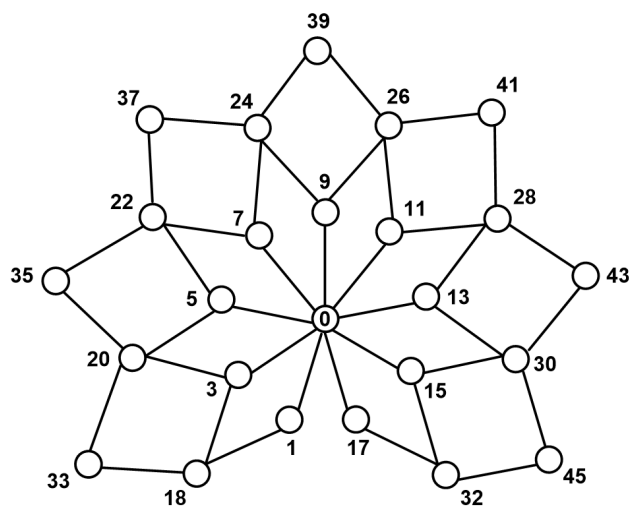


Fig. 3: Lotus flower graph $L(7)$

3.2. Lotus Flower Graph with Pendant

Definition 2. Lotus flower graph with pendant $L(p, k)$ with $p \geq 1, p \in \mathbb{Z}^+$ and $k \geq 1, k \in \mathbb{Z}^+$ is a graph defined by $V(L(p, k))$ as the set of vertices

$$V(L(p, k)) = \{a_0\} \cup \{a_h \mid 1 \leq h \leq p+2\} \cup \{b_h \mid 1 \leq h \leq p+1\} \cup \{c_h \mid 1 \leq h \leq p\} \cup \{d_s \mid 1 \leq s \leq k\}$$

and by $E(L(p, k))$ as the set of edges

$$E(L(p, k)) = \{a_0 a_h \mid 1 \leq h \leq p+2\} \cup \{a_h b_h \mid 1 \leq h \leq p+1\} \cup \{a_{h+1} b_h \mid 1 \leq h \leq p+1\} \\ \cup \{b_h c_h \mid 1 \leq h \leq p\} \cup \{b_{h+1} c_h \mid 1 \leq h \leq p\} \cup \{a_0 d_s \mid 1 \leq s \leq k\}.$$

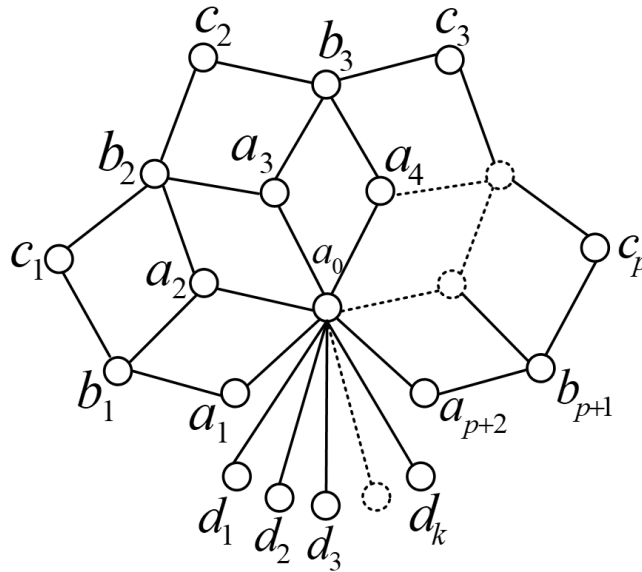


Fig. 4: Lotus flower graph with pendant $L(p, k)$

Obtained $m = |V(L(p, k))| = 3p + k + 4$ and $n = |E(L(p, k))| = 5p + k + 4$.

Theorem 2. Lotus flower graph with pendant $L(p, k)$ with $p \geq 1, p \in \mathbb{Z}^+$ and $k \geq 1, k \in \mathbb{Z}^+$ is an odd harmonious graph.

Proof. Defined functions $g : V(L(p, k)) \rightarrow \{0, 1, 2, \dots, 10p + 2k + 7\}$ of the lotus flower graph with pendant $L(p, k)$.

$$g(a_0) = 0 \tag{10}$$

$$g(a_h) = 2h - 1, \quad 1 \leq h \leq p + 2 \tag{11}$$

$$g(b_h) = 2p + 2h + 2, \quad 1 \leq h \leq p + 1 \tag{12}$$

$$g(c_h) = 4p + 2h + 3, \quad 1 \leq h \leq p \tag{13}$$

$$g(d_s) = 10p + 2s + 7, \quad 1 \leq s \leq k \tag{14}$$

Based on equation (10), (11), (12), (13) and (14)

$$g(V(L(p, k))) = \{0\} \cup \{1, 3, 5, \dots, 2p + 3\} \cup \{2p + 4, 2p + 6, 2p + 8, \dots, 4p + 4\} \\ \cup \{4p + 5, 4p + 7, 4p + 9, \dots, 6p + 3\} \cup \{10p + 9, 10p + 11, \dots, 10p + 2k + 7\}$$

it is obtained that the function g is injective because it gives different labels to each vertex and $g(V(L(p, k))) \subseteq \{0, 1, 2, \dots, 10p + 2k + 7\}$.

Defined functions $g^* : E(L(p, k)) \rightarrow \{1, 3, 5, \dots, 10p + 2k + 7\}$ of the lotus flower graph with pendant $L(p, k)$.

$$g^*(a_0a_h) = 2h - 1, \quad 1 \leq h \leq p + 2 \tag{15}$$

$$g^*(a_hb_h) = 2p + 4h + 1, \quad 1 \leq h \leq p + 1 \tag{16}$$

$$g^*(a_{h+1}b_h) = 2p + 4h + 3, \quad 1 \leq h \leq p + 1 \tag{17}$$

$$g^*(b_hc_h) = 6p + 4h + 5, \quad 1 \leq h \leq p \tag{18}$$

$$g^*(b_{h+1}c_h) = 6p + 4h + 7, \quad 1 \leq h \leq p \tag{19}$$

$$g^*(a_0d_s) = 10p + 2s + 7, \quad 1 \leq s \leq k \tag{20}$$

Based on equation (15), (16), (17), (18), (19) and (20)

$$g^*(E(L(p, k))) = \{1, 3, 5, \dots, 2p + 3\} \cup \{2p + 5, 2p + 9, 2p + 13, \dots, 6p + 5\} \\ \cup \{2p + 7, 2p + 11, 2p + 15, \dots, 6p + 7\} \cup \{6p + 9, 6p + 13, 6p + 17, \dots, 10p + 5\} \\ \cup \{6p + 11, 6p + 15, 6p + 19, \dots, 10p + 7\} \cup \{10p + 9, 10p + 11, \dots, 10p + 2k + 7\}$$

it is obtained that the function g^* is bijective because it gives different labels to each edge and $g^*(E(L(p, k))) = \{1, 3, 5, \dots, 10p + 2k + 7\}$. Such that lotus flower graph with pendant $L(p, k)$ is an odd harmonious. \square

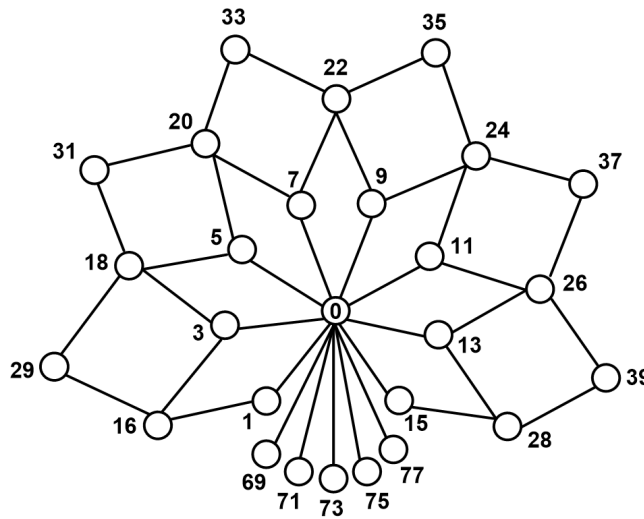


Fig. 5: Lotus flower graph with pendant $L(6, 5)$

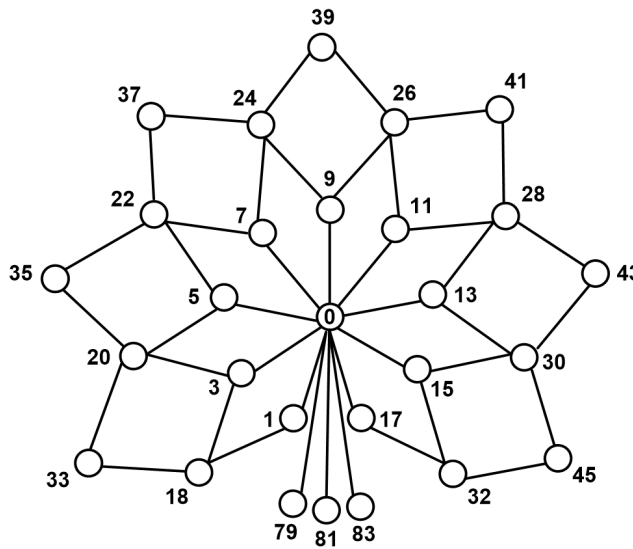


Fig. 6: Lotus flower graph with pendant $L(7, 3)$

3.3. Variation of Lotus Flower Graph

Definition 3. Variation of lotus flower graph $rL(p)$ with $p \geq 1, p \in \mathbb{Z}^+$ and $r \geq 1, r \in \mathbb{Z}^+$ is a graph defined by $V(rL(p))$ as the set of vertices

$$V(rL(p)) = \{a_0\} \cup \{a_h \mid 1 \leq h \leq p + 2\} \cup \{b_h \mid 1 \leq h \leq p + 1\} \cup \{c_h^t \mid 1 \leq h \leq p, 1 \leq t \leq r\}$$

and $E(rL(p))$ as the set of edges

$$E(rL(p)) = \{a_0 a_h \mid 1 \leq h \leq p + 2\} \cup \{a_h b_h \mid 1 \leq h \leq p + 1\} \cup \{a_{h+1} b_h \mid 1 \leq h \leq p + 1\} \cup \{b_h c_h^t \mid 1 \leq h \leq p, 1 \leq t \leq r\} \cup \{b_{h+1} c_h^t \mid 1 \leq h \leq p, 1 \leq t \leq r\}.$$

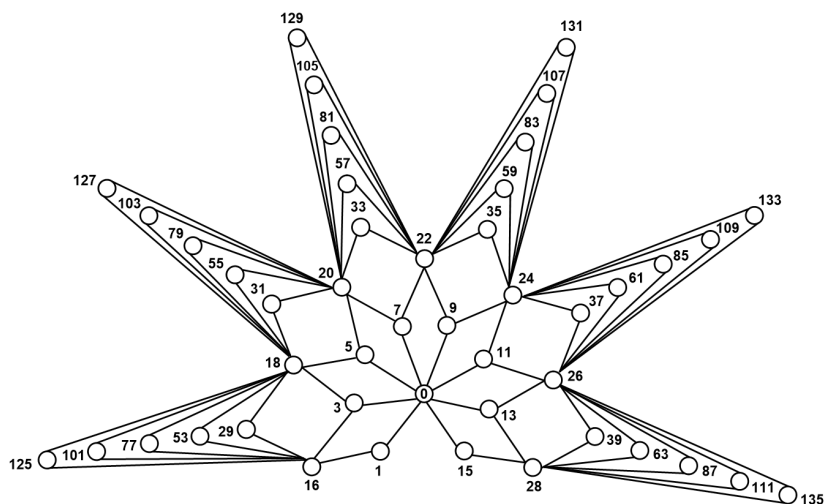


Fig. 8: Variation of lotus flower graph $5L(6)$

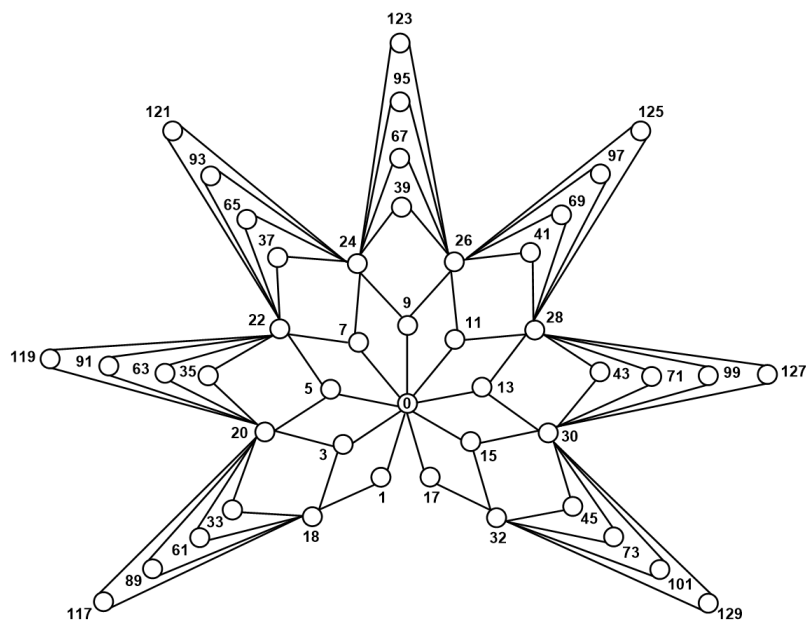


Fig. 9: Variation of lotus flower graph $4L(7)$

Based on the results of Definition 1, Definition 2, and Definition 3, novelty is obtained in the form of flower graph classes, namely lotus flower graph, lotus flower graph with pendant and variations of lotus flower graph. Based on Theorem 1, Theorem 2, and Theorem 3, it has been proven that lotus flower graph, lotus flower graph with pendant, and variations of lotus flower graph are odd harmonious graphs. This result is in line with the research of [39], [40], and [41].

4. Conclusion and Suggestion

The definition of lotus flower graph, lotus flower graph with pendant, and variations of lotus flower graph are obtained. Furthermore, it has also been proven that lotus flower graphs, lotus flower graphs with pendant, and variations of lotus flower graphs are odd harmonious graphs. A suggestion for future researchers is to find new graph classes along with their labeling types.

CRediT Authorship Contribution Statement

Fery Firmansah: Conceptualization, Methodology, Writing–Original Draft, Supervision, Project Administration, Funding Acquisition. **Tasari:** Data Curation, Formal Analysis, Writing–Review & Editing. **Muhammad Ridlo Yuwono:** Software, Validation, Visualization.

Declaration of Generative AI and AI-assisted technologies

Authors must explicitly disclose if generative AI or AI-assisted technologies were used during the preparation of the manuscript. If such tools were utilized, clearly state their role and purpose, for example, in data analysis, writing assistance, proofreading, or generating graphical outputs. Provide the names and versions of the specific tools (e.g., ChatGPT version 4, Grammarly, etc.). If no AI or AI-assisted technologies were involved in the preparation of your manuscript, state clearly: “No generative AI or AI-assisted technologies were used during the preparation of this manuscript.”

Declaration of Competing Interest

The authors declare no conflict of interest.

Funding and Acknowledgments

The researcher thanks LPPM Widya Dharma University Klaten for helping to finance this research.

Data and Code Availability

The data and code supporting the findings of this study are available from the corresponding author upon reasonable request and subject to confidentiality agreements.

References

- [1] J. A. Gallian. “A Dynamic Survey of Graph Labeling”. In: *Electronic Journal of Combinatorics* 6.28 (2025), pp. 1–805. URL: <https://www.combinatorics.org/ojs/index.php/e1jc/article/view/DS6/pdf>.
- [2] Z. H. Liang and Z. L. Bai. “On the odd harmonious graphs with applications”. In: *Journal of Applied Mathematics and Computing* 29.1 (2009), pp. 105–116. DOI: [10.1007/s12190-008-0101-0](https://doi.org/10.1007/s12190-008-0101-0).
- [3] M. E. Abdel-Aal. “Odd Harmonious Labelings of Cyclic Snakes”. In: *International Journal on Applications of Graph Theory In Wireless Ad Hoc Networks And Sensor Networks* 5.3 (2013), pp. 1–11. DOI: [10.5121/jgraphoc.2013.5301](https://doi.org/10.5121/jgraphoc.2013.5301).
- [4] P. Jeyanthi, S. Philo, and K. A. Sugeng. “Odd harmonious labeling of some new families of graphs”. In: *SUT Journal of Mathematics* 51.2 (2015), pp. 181–193. DOI: [10.55937/sut/1454685507](https://doi.org/10.55937/sut/1454685507).
- [5] J. Renuka and P. Balaganesan. “Odd harmonious labeling of some classes of cycle related graphs”. In: *Indian Journal of Public Health Research and Development* 9.9 (2018), pp. 403–408. DOI: [10.5958/0976-5506.2018.01032.X](https://doi.org/10.5958/0976-5506.2018.01032.X).
- [6] J. J. Jesintha and K. E. H. Stanley. “The graph $SSG(2)$ is odd graceful and odd harmonious”. In: *International Journal of Computer Aided Engineering and Technology* 14.1 (2021), pp. 93–101. DOI: [10.1504/IJCAET.2021.111639](https://doi.org/10.1504/IJCAET.2021.111639).

- [7] S. S. Sarasvati, I. Halikin, and K. Wijaya. “Odd Harmonious Labeling of $P_n C_4$ and $P_n D_2(C_4)$ ”. In: *Indonesian Journal of Combinatorics* 5.2 (2021), pp. 94–101. URL: https://www.researchgate.net/publication/357533846_Odd_Harmonious_Labeling_of_P_n_C_4_and_P_n_D_2_C_4.
- [8] S. Philo and P. Jeyanthi. “Odd Harmonious Labeling of Line and Disjoint Union of Graphs”. In: *Chinese Journal of Mathematical Sciences* 1.1 (2021), pp. 61–68. URL: <https://romanpub.com/resources/cjms%20v1-1-paper6tex.pdf>.
- [9] P. Jeyanthi and S. Philo. “Odd Harmonious Labeling of Some New Families of Graphs”. In: *Electronic Notes in Discrete Mathematics* 48 (2015), pp. 165–168. DOI: [10.1016/j.endm.2015.05.024](https://doi.org/10.1016/j.endm.2015.05.024).
- [10] P. Jeyanthi and S. Philo. “Odd harmonious labeling of some cycle related graphs”. In: *Proyecciones* 35.1 (2016), pp. 85–98. DOI: [10.4067/S0716-09172016000100006](https://doi.org/10.4067/S0716-09172016000100006).
- [11] P. Jeyanthi and S. Philo. “Odd Harmonious Labeling of Subdivided Shell Graphs”. In: *International Journal of Computer Sciences and Engineering Open Access Research Paper* 7.5 (2019), pp. 77–80. URL: <https://ijcseonline.org/index.php/j/article/view/4830/4822>.
- [12] P. Jeyanthi and S. Philo. “Some Results On Odd Harmonious Labeling Of Graphs”. In: *Bulletin Of The International Mathematical Virtual Institute* 9 (2019), pp. 567–576. URL: http://www.imvibl.org/buletin/bulletin_imvi_9_3_2019/bulletin_imvi_9_3_2019_567_576.pdf.
- [13] P. Jeyanthi, S. Philo, and M. K. Siddiqui. “Odd harmonious labeling of super subdivision graphs”. In: *Proyecciones* 38.1 (2019), pp. 1–11. DOI: [10.4067/S0716-09172019000100001](https://doi.org/10.4067/S0716-09172019000100001).
- [14] P. Jeyanthi, S. Philo, and M. Z. Youssef. “Odd harmonious labeling of grid graphs”. In: *Proyecciones* 38.3 (2019), pp. 411–428. DOI: [10.22199/issn.0717-6279-2019-03-0027](https://doi.org/10.22199/issn.0717-6279-2019-03-0027).
- [15] P. Jeyanthi and S. Philo. “Odd harmonious labeling of step ladder graphs”. In: *Utilitas Mathematica* 115 (2020), pp. 89–103. URL: https://www.researchgate.net/publication/352284863_Odd_Harmonious_Labeling_of_Step_Ladder_Graphs.
- [16] P. Jeyanthi and S. Philo. “New Results on Odd Harmonious Labeling Of Graphs”. In: *Turkish World Mathematical Society Journal of Applied and Engineering Mathematics* 12.4 (2022), pp. 1301–1310. URL: <https://jaem.isikun.edu.tr/web/images/articles/vol.12.no.4/14.pdf>.
- [17] D. H. Zala, Narendra T. Chotaliya Chotaliya, and M. A. Chaurasiya. “Even odd Harmonious Labeling of Some Graphs”. In: *International Journal of Innovative Technology and Exploring Engineering* 10.4 (2021), pp. 149–151. DOI: [10.35940/ijitee.d8513.0210421](https://doi.org/10.35940/ijitee.d8513.0210421).
- [18] E. Asumpta, Purwanto, and T. D. Chandra. “Odd Harmonious Labeling of Some Family of Snake Graphs”. In: *AIP Conference Proceedings*. Vol. 2639. 1. 2022. DOI: [10.1063/5.0111278](https://doi.org/10.1063/5.0111278).
- [19] H. M. Hafez, R. El-Shanawany, and A. A. E. Atik. “Odd harmonious labeling of the converse skew product of graphs”. In: *Bulletin of the Institute of Combinatorics and Its Applications* 98 (2023), pp. 90–110. URL: <https://luca-giuzzi.unibs.it/ICA/Volumes/98/Reprints/BICA2022-39-Reprint.pdf>.
- [20] F. Firmansah. “Odd Harmonious Labeling on Some String Graph Classes”. In: *BAREKENG: Jurnal Ilmu Matematika Dan Terapan* 16.1 (2022), pp. 315–322. DOI: [10.30598/barekengvol16iss1pp313-320](https://doi.org/10.30598/barekengvol16iss1pp313-320).
- [21] F. Firmansah. “The Odd Harmonious Labeling on Variation of the Double Quadrilateral Windmill Graphs”. In: *Jurnal Ilmu Dasar* 18.2 (2017), pp. 109–118. URL: https://www.researchgate.net/publication/321493758_The_Odd_Harmonious_Labeling_on_Variation_of_the_Double_Quadrilateral_Windmill_Graphs.

- [22] F. Firmansah and M. R. Yuwono. “Odd Harmonious Labeling on Pleated of the Dutch Windmill Graphs”. In: *Cauchy Jurnal Matematika Murni Dan Aplikasi* 4.4 (2017), pp. 161–166. DOI: [10.18860/ca.v4i4.4043](https://doi.org/10.18860/ca.v4i4.4043).
- [23] F. Firmansah and Tasari. “Odd Harmonious Labeling on Edge Amalgamation from Double Quadrilateral Graphs”. In: *Desimal: Jurnal Matematika* 3.1 (2020), pp. 65–72. URL: <https://ejournal.radenintan.ac.id/index.php/desimal/article/view/5712/3324>.
- [24] F. Firmansah. “Odd harmonious labelling on pleated net snake graph (in Indonesian)”. In: *Sainmatika: Jurnal Ilmiah Matematika Dan Ilmu Pengetahuan Alam* 17.1 (2020), pp. 1–8. DOI: [10.31851/sainmatika.v17i1.3182](https://doi.org/10.31851/sainmatika.v17i1.3182).
- [25] F. Firmansah and W. Giyarti. “Odd harmonious labeling on the amalgamation of the generalized double quadrilateral windmill graph”. In: *Desimal: Jurnal Matematika* 4.3 (2021), pp. 373–378. DOI: <https://doi.org/10.24042/djm.v4i3.10823>.
- [26] F. Firmansah. “The Odd Harmonious Labeling of Layered Graphs”. In: *JTAM (Jurnal Teori Dan Aplikasi Matematika)* 7.2 (2023), pp. 339–348. DOI: [10.31764/jtam.v7i2.12506](https://doi.org/10.31764/jtam.v7i2.12506).
- [27] A. Lasim, I. Halikin, and K. Wijaya. “The Harmonious, Odd Harmonious, and Even Harmonious Labeling”. In: *BAREKENG: Jurnal Ilmu Matematika Dan Terapan* 16.4 (2022), pp. 1131–1138. DOI: [10.30598/barekengvol16iss4pp1131-1138](https://doi.org/10.30598/barekengvol16iss4pp1131-1138).
- [28] Y. Venkataraman, J. Dhamayanthi Hayyath, I. Krishnaperumal, and V. E. Balas. “On Properly Odd Harmonious Labeling of Graphs”. In: *SISY 2022 - IEEE 20th Jubilee International Symposium on Intelligent Systems and Informatics, Proceedings*. 2022. DOI: [10.1109/SISY56759.2022.10036285](https://doi.org/10.1109/SISY56759.2022.10036285).
- [29] Abdel-Aal and Seoud. “Further Results on Odd Harmonious Graphs”. In: *International Journal on Applications of Graph Theory In Wireless Ad Hoc Networks And Sensor Networks* 8.3/4 (2016), pp. 1–14. DOI: [10.5121/jgraphoc.2016.8401](https://doi.org/10.5121/jgraphoc.2016.8401).
- [30] G. A. Saputri, K. A. Sugeng, and D. Froncek. “The odd harmonious labeling of dumbbell and generalized prism graphs”. In: *AKCE International Journal of Graphs and Combinatorics* 10.2 (2013), pp. 221–228. URL: <https://www.tandfonline.com/doi/abs/10.1080/09728600.2013.12088738>.
- [31] M. A. A. Seoud and H. M. Hafez. “Odd harmonious and strongly odd harmonious graphs”. In: *Kyungpook Mathematical Journal* 58.4 (2018), pp. 747–759. DOI: [10.5666/KMJ.2018.58.4.747](https://doi.org/10.5666/KMJ.2018.58.4.747).
- [32] K. A. Sugeng, S. Surip, and R. Rismayati. “On odd harmonious labeling of m-shadow of cycle, gear with pendant and Shuriken graphs”. In: *AIP Conference Proceedings*. Vol. 2192. 1. 2019. DOI: [10.1063/1.5139141](https://doi.org/10.1063/1.5139141).
- [33] R. Govindarajan and V. Srividya. “On odd harmonious labelling of even cycles with parallel chords and dragons with parallel chords”. In: *International Journal of Computer Aided Engineering and Technology* 13.4 (2020), pp. 409–424. DOI: [10.1504/ijcaet.2020.10029299](https://doi.org/10.1504/ijcaet.2020.10029299).
- [34] F. Febriana and K. A. Sugeng. “Odd harmonious labeling on squid graph and double squid graph”. In: *Journal of Physics: Conference Series* 1538.1 (2020), pp. 1–5. DOI: [10.1088/1742-6596/1538/1/012015](https://doi.org/10.1088/1742-6596/1538/1/012015).
- [35] K. Mumtaz, P. John, and D. R. Silaban. “The odd harmonious labeling of matting graph”. In: *Journal of Physics: Conference Series* 1722.1 (2021), pp. 1–4. DOI: [10.1088/1742-6596/1722/1/012050](https://doi.org/10.1088/1742-6596/1722/1/012050).
- [36] E. A. Pramesti and Purwanto. “Odd harmonious labeling of $S_n(m, r)$ graph”. In: *Journal of Physics: Conference Series* 1872.1 (2021), pp. 1–8. DOI: [10.1088/1742-6596/1872/1/012006](https://doi.org/10.1088/1742-6596/1872/1/012006).

- [37] D. A. Pujiwati, I. Halikin, and K. Wijaya. “Odd harmonious labeling of two graphs containing star”. In: *AIP Conference Proceedings*. Vol. 2326. 1. 2021. DOI: [10.1063/5.0039644](https://doi.org/10.1063/5.0039644).
- [38] D. Kolo, K. B. Ginting, and G. L. Putra. “Odd Harmonic Labeling on $C_{m,n}$ e C_4 Graph”. In: *Jurnal Diferensial* 5.1 (2023), pp. 22–28. DOI: [10.35508/jd.v5i1.9824](https://doi.org/10.35508/jd.v5i1.9824).
- [39] F. Firmansah. “Odd Harmonious Labeling on Double Quadrilateral Flower Graphs (in Indonesian)”. In: *Jurnal Ilmiah Sains* 20.1 (2020), pp. 12–17. DOI: [10.35799/jis.20.1.2020.27278](https://doi.org/10.35799/jis.20.1.2020.27278).
- [40] F. Firmansah, T. Tasari, and M. R. Yuwono. “Odd Harmonious Labeling of the Zinnia Flower Graphs”. In: *Jurnal Ilmiah Sains* 23.1 (2023), pp. 40–46. DOI: [10.35799/jis.v23i1.46771](https://doi.org/10.35799/jis.v23i1.46771).
- [41] F. Firmansah, T. Tasari, and J. Sungkono. “Odd harmonious labeling on the union of flower graphs”. In: *Desimal: Jurnal Matematika* 7.3 (2024), pp. 567–582. URL: <https://ejournal.radenintan.ac.id/index.php/desimal/article/view/24612>.