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Systematic Literature Review: Optimal Stopping and Investment Optimization for Bankruptcy Risk Management in Sharia Insurance

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Abstract

The increasing demand for Sharia-compliant financial services in Muslim-majority countries, particularly Indonesia, has driven the notable development of Sharia insurance (Takaful). Despite this growth, the industry continues to face substantial challenges, primarily in optimizing investment strategies and managing bankruptcy risk. Addressing these challenges requires a solid understanding of models aligned with Islamic principles. Several studies have proposed stochastic approaches to model surplus processes, investment returns, and risk probabilities. The Cramér–Lundberg model estimates surplus dynamics and bankruptcy risk, while the Vasicek model offers a stochastic framework for investment returns. Quadratic programming has also been applied to optimize asset allocation. However, these methods are often examined in isolation, limiting their ability to form an integrated and effective framework for managing both bankruptcy risk and Sharia-compliant investments. This methodological gap constrains the development of comprehensive, practical, and theoretically sound solutions aligned with the specific operational characteristics of Sharia insurance. This systematic review critically evaluates existing models and assesses their potential for integration into a cohesive decision-making framework. The findings offer a structured research agenda for future model development and present implications for academic inquiry and practical implementation. Ultimately, this review contributes to the advancement of Sharia insurance by proposing a unified, theoretically sound, and practically applicable approach to risk and investment management in the Islamic finance context.

Keywords: Optimization, Sharia Investment, Cramér-Lundberg Model, Optimal Stopping, Ruin Probability.

1 Introduction

Based on the 2022 National Survey on Financial Literacy and Inclusion (SNLIK) conducted by the Financial Services Authority (OJK), the inclusion rate of Sharia-compliant financial services in Indonesia increased from 9.10% in 2019 to 12.12% in 2022 [1]. This growth reflects the rising public interest and demand for Sharia-based financial services. One of the sectors

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showing significant development is the Sharia insurance industry. According to OJK data in 2023, the total assets of the Islamic financial industry reached IDR 2,375 trillion by the end of 2022, with the Sharia insurance sector making a substantial contribution. The total gross contribution collected by the Sharia insurance industry during the same period was recorded at IDR 44.85 trillion [2]. This figure demonstrates not only the sector's quantitative progress but also its increasing role in the broader Islamic financial ecosystem. Furthermore, the Indonesian Sharia Insurance Association (AASI) reported that the total assets of Sharia insurance companies increased by 56.89%, from IDR 29.14 trillion in 2016 to IDR 45.72 trillion in April 2023 [3]. This indicates a long-term upward trend in asset accumulation that aligns with the general trajectory of Islamic financial development in Indonesia. Nevertheless, growth in asset size alone does not necessarily reflect the overall health of the industry.

Despite this positive trend in asset growth, several key performance indicators reveal persistent challenges within the Sharia insurance industry. Gross contributions decreased by 6.91%, while gross claims rose by 14.78%. Investment returns also experienced a sharp decline of 45.33%, and the penetration rate of Sharia insurance stood at only 0.130% [2]. This imbalance between asset growth and operational performance suggests that quantitative expansion has not been accompanied by a proportional enhancement in financial resilience. The Sharia insurance industry continues to face structural challenges, including economic uncertainty, financial market volatility, and suboptimal cash and reserve management [4]. Limited diversification in Sharia-compliant investment instruments may further exacerbate these vulnerabilities. These financial pressures expose Sharia insurance companies to potential losses and, in extreme cases, an increased risk of bankruptcy [5]. Consequently, effective bankruptcy risk management has emerged as a strategic issue that requires further scholarly exploration, particularly within the framework of Sharia-compliant principles that underpin the industry.

Various quantitative approaches have been developed in previous studies to measure and manage risk in the insurance sector. These approaches include the Cramér–Lundberg surplus model, the Vasicek model for investment return estimation, loss distribution models for large claims, and ruin probability theory [6]–[9]. In addition, optimal stopping methods have been widely applied in investment decision-making and risk management [10], [11], while quadratic programming has been frequently utilized in investment portfolio optimization [12]. These techniques have provided important insights into financial risk control in conventional settings. However, research specifically applying these approaches in the context of Sharia insurance remains limited. In particular, studies that integrate bankruptcy risk management, optimal stopping strategies, and Sharia-compliant investment optimization within a comprehensive framework are still scarce. Moreover, the application of quadratic programming in Islamic investment management that simultaneously considers bankruptcy risk and optimal stopping strategies has yet to be extensively explored.

Despite the growing literature on financial modeling and risk mitigation, an integrated framework that simultaneously addresses surplus modeling, optimal investment, and bankruptcy risk within the specific context of Sharia-compliant insurance remains underdeveloped. This study seeks to fill that gap by systematically reviewing the existing models and identifying opportunities for their integration. The scientific contribution of this study lies in its effort to map, synthesize, and analyze the intersection between stochastic surplus modeling, optimal stopping theory, and investment optimization through quadratic programming—specifically tailored to the operational and ethical requirements of Sharia insurance. By doing so, the study offers a novel conceptual foundation that bridges theoretical advancements with practical applications, aiming to support more resilient and compliant risk management strategies for Sharia insurance institutions.

In response to these gaps, this study aims to examine the development of academic research related to bankruptcy risk management and investment optimization within the Sharia insurance industry. It also seeks to map the approaches, methods, and variables commonly employed in the literature on bankruptcy risk, surplus models, Sharia-compliant investment strategies, optimal stopping methods, and quadratic programming in the context of Sharia insurance. Through a systematic mapping, the study is expected to identify research gaps that may serve as the foundation for future empirical and applied investigations in the field of Sharia insurance risk management. The primary contribution of this study lies in its effort to provide a comprehensive and systematic review of methods and models for bankruptcy risk management in the Sharia insurance industry. The study also aims to identify underexplored research areas, particularly regarding the integration of optimal stopping methods and quadratic programming within a unified Sharia-compliant investment framework. By doing so, it contributes to bridging theoretical advancements with practical implications for Sharia insurance operations. Accordingly, the findings of this study are expected to serve as a solid foundation for future research aimed at designing integrated and practical models to enhance the financial resilience of Sharia insurance companies.

2 Methods

This chapter presents the methodological foundation of the study, integrating both bibliometric and analytical approaches. The methods are designed to explore research patterns and develop quantitative models relevant to risk management in Sharia insurance. The approach ensures methodological rigor and coherence in linking literature analysis with mathematical formulation and simulation.

2.1 Research Design

This study adopts a Systematic Literature Review (SLR) approach combined with bibliometric analysis using the VOSviewer software and the PRISMA (Preferred Reporting Items for Systematic Reviews and Meta-Analyses) method as a systematic guideline. The integration of these approaches is intended to ensure a comprehensive, replicable, and transparent review process of previous studies related to the management of bankruptcy risk, optimal stopping, and investment optimization in the context of Sharia insurance. Furthermore, this review lays the groundwork for the formulation of an integrated modeling framework to be developed in future empirical research.

2.2 Bibliometric Analysis

Bibliometric analysis is a quantitative study of scientific publications that uses statistical and informatic methods to measure citations, research productivity, and knowledge dissemination. According to Hamidah et al. [13], bibliometric analysis is widely used to evaluate the contribution of individual researchers, institutions, and scientific disciplines while providing a clear overview of the relationships between different research topics. This method is particularly useful for identifying the main trends and research gaps, which are essential to formulating research priorities and policies, as emphasized by Shah et al.[14] and Dubyna et al.[15].

In this study, bibliometric analysis is supported by the VOSviewer software, which enables visualization and mapping of bibliometric networks such as citation links, co-citation, and keyword co-occurrence. VOSviewer, as developed by Van Eck and Waltman[16], provides distance-based visualizations that effectively illustrate the relationships between authors, journals, and keywords. Its intuitive interface allows researchers to explore the knowledge structure and track the development of research themes more easily. Furthermore, VOSviewer supports comprehensive bibliometric analysis in several major databases, including Scopus, ScienceDirect, and Dimensions. The literature review process follows the PRISMA methodology to ensure methodological rigor and transparent reporting, as outlined by Moher et al. [17]. The PRISMA protocol will be implemented in the screening process, including the following:

1. Identification: Retrieving all articles based on the defined search strategy.

2. Screening: Removing duplicates and screening titles and abstracts based on inclusion and exclusion criteria.
3. Eligibility: Conducting full-text reviews to ensure the articles meet the required relevance.
4. Inclusion: Final selection of articles that will be included in the analysis.

The search of the literature in this study focused on four main topics, namely *'surplus model'* or *'Cramér-Lundberg'*, *'ruin probability'*, *'optimal stopping'* and *'quadratic programming'*. These keywords were selected to capture the relevant intersections of risk modeling, stochastic control theory, and Sharia-compliant investment optimization in the context of Sharia insurance research.vTo ensure the relevance and quality of the selected literature, several inclusion criteria were applied. Only articles published between 2014 and 2025 were considered, and they had to be written in English. Furthermore, the study included only works that were classified as original research articles or conference papers and had reached final publication status. Conversely, certain exclusion criteria were established to refine the search. Articles not written in English were excluded, as were those categorized as editorials, reviews, or short communications. Additionally, any articles deemed irrelevant to the domains of insurance, finance, or stochastic decision-making were also excluded from the review.

2.3 Mathematical Models and Formulations

This subsection presents a set of mathematical models identified from the literature and conceptually integrated in this study to propose a comprehensive framework for Sharia-compliant risk and investment management. These models have been empirically implemented in various previous studies. However, they have been applied in a fragmented and unintegrated manner, resulting in limited analytical coherence. This lack of coherence leads to a partial analysis that may introduce biases in the financial management of Sharia insurance. Therefore, the integration of these models is essential to establish a comprehensive analytical framework. By linking surplus modeling with return forecasting, claim dynamics, and portfolio optimization under Sharia principles, the proposed framework enhances decision-making capabilities and promotes financial sustainability in Islamic insurance institutions.

1. **Cramér-Lundberg Surplus Model**

   The classical risk model is chosen for its effectiveness in representing insurance company surplus dynamics based on premium income and claim processes. It directly supports the core objective of evaluating bankruptcy risk under stochastic claim occurrences. The surplus process of an insurance company is modeled using the classical Cramér-Lundberg model:

   $$U(t) = u + ct - \sum_{i=1}^{N(t)} X_i. \tag{1}$$

   In this classical Cramér–Lundberg surplus model, $U(t)$ denotes the surplus of an insurance company at time t, which is calculated as the sum of the initial surplus u, the accumulated premium income ct received up to time t, and the total claims paid up to that time. The number of claims that occur up to time t is modeled by a Poisson process $N(t)$, while X_i represents the individual claim sizes, which are assumed to be independent and identically distributed (i.i.d.) random variables. Therefore, the term $\sum_{i=1}^{N(t)} X_i$ represents the aggregate amount of claims paid by the insurer up to time t.

2. **Revenue Model with Pareto Distribution**

   The Pareto distribution is widely used to model income and revenue in skewed financial systems. Its heavy-tailed nature captures the volatility and inequality often present in Sharia insurance revenue streams. The average revenue of the company per period is modeled using a Pareto distribution:

   $$f(x; \alpha, x_m) = \frac{\alpha x_m^{\alpha}}{x^{\alpha+1}}, \quad x \geq x_m > 0. \tag{2}$$

Where α is the shape parameter and x_m is the scale parameter. Both parameters are estimated using the Maximum Likelihood Estimation (MLE) method.
The MLE estimators for the Pareto distribution are:

$$\hat{x}_m = \min(x_1, x_2, \ldots, x_n), \quad \hat{\alpha} = \frac{n}{\sum_{i=1}^{n} \ln\left(\frac{x_i}{\hat{x}_m}\right)}. \tag{3}$$

3. **Investment Return Model with Vasicek Process**
The Vasicek model is suitable for capturing mean-reverting behavior in investment returns, making it appropriate for modeling the long-term performance of Sharia-compliant assets under uncertainty. Investment returns are modeled using the Vasicek interest rate model:

$$dr(t) = k\left(\eta - r(t)\right) dt + \sigma dW(t). \tag{4}$$

This equation represents the Vasicek model, a stochastic differential equation used to describe the evolution of the instantaneous interest or return rate $r(t)$ over time. In this model, the parameter k denotes the speed at which $r(t)$ reverts to its long-term mean η, while σ represents the volatility of the process. The term $W(t)$ denotes a standard Brownian motion, introducing randomness into the system. The model captures the tendency of the return rate to fluctuate around a stable long-run mean, which makes it suitable for modeling interest rates or investment returns in financial and actuarial contexts. The Vasicek model captures the stochastic nature of investment returns and is appropriate for modeling fluctuating Sharia-compliant assets.

4. **Total Claims Model with Log-Logistic Distribution**
The Log-Logistic distribution effectively models insurance claim amounts with varying severity and frequency. Its flexibility in modeling long-tailed data aligns with the real-world nature of insurance claims. The total claims per period are modeled using a log-logistic distribution:

$$f(x; \alpha, \beta) = \frac{(\alpha/\beta)(x/\beta)^{\alpha-1}}{\left[1 + (x/\beta)^{\alpha}\right]^2}. \tag{5}$$

Where α and β are shape and scale parameters, respectively. The parameter estimation is performed using the Maximum Likelihood Estimation (MLE) method.
The MLE for the log-logistic parameters is solved numerically to maximize the log-likelihood function:

$$\ell(\alpha, \beta) = \sum_{i=1}^{n} \ln\left[\frac{(\alpha/\beta)(x_i/\beta)^{\alpha-1}}{\left[1 + (x_i/\beta)^{\alpha}\right]^2}\right]. \tag{6}$$

5. **Ruin Probability and Severity**
The ruin probability models are critical in evaluating not only the probability of bankruptcy but also its financial consequences. They support the central aim of risk quantification and resilience assessment. The probability of ruin is defined as:

$$\varphi(u) = \Pr(U(t) < 0), \quad t \geq 0. \tag{7}$$

The severity of ruin is described by:

$$G(u, y) = \Pr(-y < U(t) < 0), \quad t > 0,\ y \geq 0. \tag{8}$$

These probabilities quantify the likelihood and impact of insolvency events.

6. **Optimal Stopping Theory**
The optimal stopping method is incorporated to identify strategic decision points under uncertainty, especially relevant for dynamic asset management in fluctuating markets while preserving Sharia compliance. Optimal stopping problems are formulated as:

$$V(U_t, t) = \sup_{\tau \in \mathcal{T}_t} \mathbb{E}\left[g(U_\tau, \tau) \mid U_t = u\right]. \tag{9}$$

Where $V(U_t, t)$ is the value function, $\mathcal{T}_t$ is the set of admissible stopping times, and $g(U_\tau, \tau)$ is the payoff function.
This study refines the stopping time τ to represent the point at which the asset surplus approaches zero, considering both Sharia compliance and market dynamics.

7. **Quadratic Programming for Investment Optimization**
Quadratic programming is used for its robustness in managing portfolio allocation under constraints. It aligns with Islamic finance principles by excluding short-selling and interest-bearing instruments. Investment portfolio optimization is addressed using quadratic programming:
$$\min_X f(X) = C^T X + \frac{1}{2} X^T D X. \tag{10}$$
subject to:
$$\sum_{i=1}^{n} x_i = 1, \quad x_i \geq 0 \quad \forall i. \tag{11}$$
This quadratic programming formulation represents an optimization problem commonly used in portfolio selection. The objective is to minimize the function $f(X) = C^T X + \frac{1}{2} X^T D X$, where X is the decision variable vector representing the allocation weights of Sharia-compliant assets. The vector C denotes the expected returns of the assets, and D is the covariance matrix of asset returns, reflecting the associated risks and correlations among the assets. The constraints require that the sum of all asset weights equals one, ensuring full capital allocation, and that each weight x_i is non-negative, in accordance with Sharia principles which prohibit short-selling. This formulation is particularly suitable for optimizing Sharia-compliant portfolios under risk-return trade-offs.

2.4 Bibliometric Mapping Methodology

To explore the research landscape and identify themes related to optimal stopping, surplus modeling, and risk mitigation in Sharia-compliant insurance, this study applies a structured bibliometric mapping methodology:

1. **Defining Relevant Scientific Domains:**
The initial phase involved the identification of key scientific domains that underpin the theoretical and methodological framework of the study. These domains include actuarial science, financial mathematics, stochastic control, Islamic finance, risk theory, and operations research. Particular attention is devoted to studies that focus on surplus dynamics, ruin probability, optimal stopping rules, investment optimization. This comprehensive scope enables a multidimensional analysis of financial sustainability in Sharia Insurance operations.
2. **Selecting and Accessing Academic Databases:**
Scopus, ScienceDirect, and Dimension were selected for their broad coverage and compatibility with bibliometric tools, providing a reliable base for literature extraction.
3. **Formulating Search Strategy and Mining Bibliometric Data:**
A search strategy using Boolean operators and keywords was formulated, with screening based on PRISMA (Preferred Reporting Items for Systematic Reviews and Meta-Analyses) guidelines to ensure transparency and replicability.
4. **Exporting and Preparing Data for Analysis:**
Bibliographic metadata comprising article titles, abstracts, keywords, publication years, author names, and source titles were exported in formats compatible with bibliometric software. Data cleaning was conducted to remove duplicates, irrelevant records, and inconsistencies.
5. **Data Importation and Configuration in VOSviewer:**
The curated dataset was imported into VOSviewer, a widely used tool for bibliometric

visualization. Parameters were configured to construct term co-occurrence maps, author collaboration networks, and keyword co-authorship analyses.

6. **Network Construction and Parameter Setting:**
   A co-occurrence network of terms was generated based on the frequency of shared keywords, abstract phrases, and title terms. A minimum threshold of two occurrences was set to ensure meaningful node representation while avoiding noise from isolated terms.
7. **Visualization of Bibliometric Maps:**
   VOSviewer's visualization features were employed to generate cluster maps and overlay maps. These visualizations provide insights into thematic groupings, research density, temporal evolution of topics, and the relative prominence of different themes across time.
8. **Interpretation and Thematic Synthesis:**
   The visual and quantitative bibliometric results were analyzed to identify dominant research trends, knowledge structures, and underexplored areas. The resulting thematic clusters serve as the empirical basis for defining the scope of the present study and guiding future model development.

3 Results and Discussion

This chapter presents the results and detailed discussions derived from the systematic literature review. Several key aspects are covered, including the research flow, publication trends based on keyword analysis, keyword combination search outcomes, the PRISMA flow diagram for article selection, and bibliometric visualizations. The bibliometric analysis encompasses network and overlay mapping to identify collaboration patterns, research hotspots, and thematic evolution over time. Each section is discussed to provide comprehensive insights into the development, current state, and potential future directions of research related to surplus models, ruin probability, optimal stopping, and investment strategies in Sharia insurance.

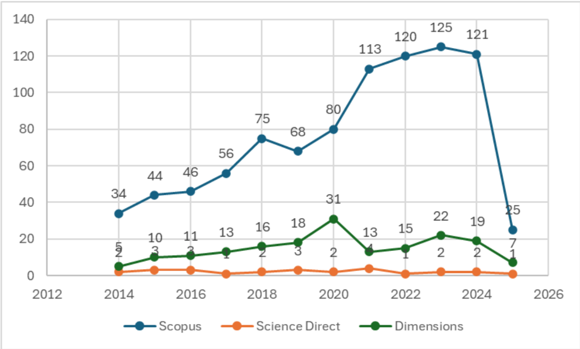
3.1 Publication Trends by Keyword

The trend of research publications from 2014 to 2025 for each keyword is shown in Figure 1, panels (a)–(d) corresponding to "surplus model Cramér-Lundberg", "ruin probability", "optimal stopping", and "quadratic programming", respectively. These plots illustrate the growth of interest in each topic over time across the three databases.

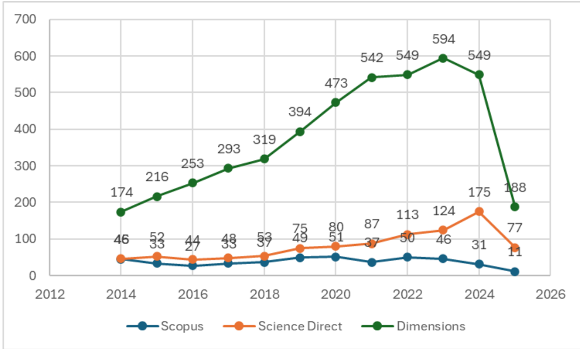
The "*surplus model Cramér-Lundberg*" trend indicates a relatively stable increase, particularly within the Scopus database. Although the volume of publications remains lower compared to other keywords. Scopus consistently shows the most notable growth in this area from 2014 to 2024. Science Direct and Dimensions also record publications on this topic but at a more modest and less fluctuating rate. This suggests that the surplus model, especially in the Cramér-Lundberg context, has gained steady academic attention within specialized research communities.

For the "*ruin probability*" keyword, a similar steady growth trend is observed. Publications in Dimensions have consistently increased from 2014, peaking at 594 articles in 2023. Science Direct and Scopus also display stable publication trends with moderate growth, especially in the last five years. The persistent interest in the probability of ruin is probably driven by its critical role in insurance risk analysis and financial stability assessment.

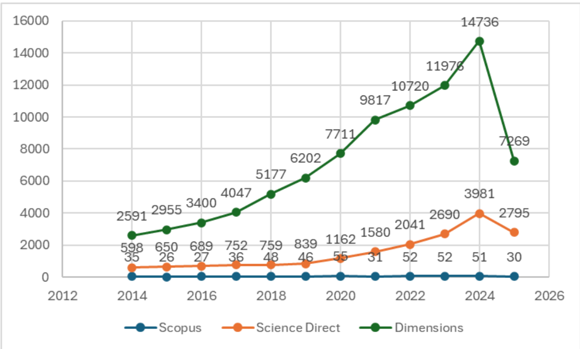
Moving to "*optimal stopping*", the publication trend shows a sharp increase, particularly in the Dimensions database. Publications started to rise significantly from 2018, reaching a peak of 14,736 articles in 2024. Science Direct and Scopus also demonstrate consistent growth, though at a considerably lower volume compared to Dimensions. This strong upward trend indicates that optimal stopping has become an increasingly relevant and actively explored topic in risk management and decision-making strategies.



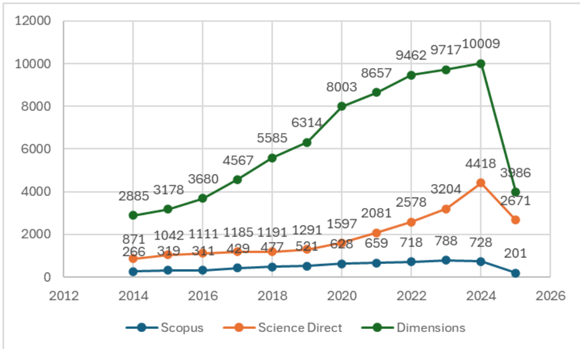
(a) Search results for the keyword "surplus model Cramér-Lundberg"



(b) Search results for the keyword "ruin probability"



(c) Search results for the keyword "optimal stopping"



(d) Search results for the keyword "quadratic programming"

Figure 1: Comparison of search results for the keywords "surplus model Cramér-Lundberg", "ruin probability", "optimal stopping", and "quadratic programming".

Finally, for the "*quadratic programming*" keyword, the publication volume has experienced consistent growth, especially in the Dimensions database. Since 2014, there has been a steady rise, reaching a peak of 10,009 articles in 2024. Science Direct also shows a marked increase in recent years, while Scopus maintains a steady but smaller contribution. These patterns reflect a growing academic interest in quadratic programming as a mathematical approach to solving optimization problems, particularly in the contexts of investment and financial decision-making.

In summary, publication trends related to surplus model, ruin probability, optimal stopping, and investment optimization show distinct patterns across academic databases. Dimensions emerges as the most responsive in capturing multidisciplinary and emerging research intersecting actuarial science, stochastic control, and Islamic finance. Scopus, though more selective, demonstrates steady growth, particularly in surplus and ruin modeling. These trends highlight rising scholarly interest in Sharia insurance risk management and support the integration of advanced quantitative methods—such as optimal stopping and portfolio optimization into solvency frameworks. The findings provide a solid foundation for this study's theoretical synthesis in Sharia-compliant insurance and financial sustainability.

3.2 Keyword Combination Analysis

The keyword combination searches conducted across Scopus, Science Direct and Dimension are summarized in Table 1.

Table 1 summarizes the search results from Scopus, Science Direct, and Dimension based on relevant keyword combinations. The combination keywords of "*surplus model*" or "*Cramér-Lundberg*" with "*ruin probability*" yielded the highest number of publications with 354 articles, highlighting its dominance in risk management and actuarial research. This indicates that the

Table 1: Search Results Based on Keyword Combinations

Keyword Combination	Scopus	Science Direct	Dimension	Total
("surplus model" OR "Cramér-Lundberg") AND ("ruin probability")	38	145	171	354
("surplus model" OR "Cramér-Lundberg") AND ("optimal stopping")	2	10	14	26
("surplus model" OR "Cramér-Lundberg") AND ("quadratic programming")	-	1	1	2
("optimal stopping") AND ("quadratic programming")	2	29	43	74
("surplus model" OR "Cramér-Lundberg") AND ("ruin probability") AND ("optimal stopping")	-	5	8	13
("ruin probability") AND ("optimal stopping") AND ("quadratic programming")	-	-	-	0
Total	**42**	**190**	**237**	**469**

analysis of surplus processes and ruin probabilities remains a well-established and extensively studied area, providing a solid theoretical foundation for financial stability and risk assessment in insurance and related sectors. The sustained research interest in this combination suggests that these models are central to both academic exploration and practical applications, particularly in conventional insurance and reinsurance systems.

In contrast, studies combining "*surplus model*" or "*Cramér-Lundberg*" with "*optimal stopping*" or "*quadratic programming*" remain scarce, with only 26 and 2 articles, respectively. This limited number points to an underdeveloped intersection that offers significant opportunities for future research, especially in dynamic decision-making under uncertainty and optimization-based surplus management. The lack of integration between "*surplus processes*" and "*optimal stopping*" rules suggests that the potential benefits of timing strategies in surplus optimization have not been fully explored. Similarly, the minimal connection with "*quadratic programming*" indicates that advanced optimization techniques have not yet been widely applied to surplus-related risk management problems.

The combination of "*optimal stopping*" and "*quadratic programming*" produced 74 articles, showing moderate research interest. This reflects the relevance of decision-making and optimization techniques across broader financial and operational contexts. However, the literature integrating these topics with "*surplus models*" is extremely limited, suggesting that cross-disciplinary approaches incorporating surplus modeling, optimal decision timing, and optimization algorithms remain underrepresented in existing studies.

Notably, no articles were found that simultaneously address "*ruin probability*", "*optimal stopping*", and "*quadratic programming*" within a unified framework. This significant research gap highlights the absence of comprehensive models that can holistically manage risk, optimize investment strategies, and determine the best timing decisions within the surplus process context. The lack of such integrative studies is particularly critical in Sharia insurance, where tailored financial models that comply with Islamic principles are essential. Developing such integrated approaches could not only advance the theoretical landscape but also provide practical tools to improve financial stability, enhance investment outcomes, and effectively mitigate bankruptcy risks in Sharia-compliant financial institutions. This emphasizes the importance of future research efforts to fill this gap by bridging actuarial science, decision analysis and optimization theoryin a Sharia-based framework.

3.3 PRISMA flow Diagram

The article selection process, illustrated in the PRISMA flow diagram, shows the number of articles included and excluded at each stage.

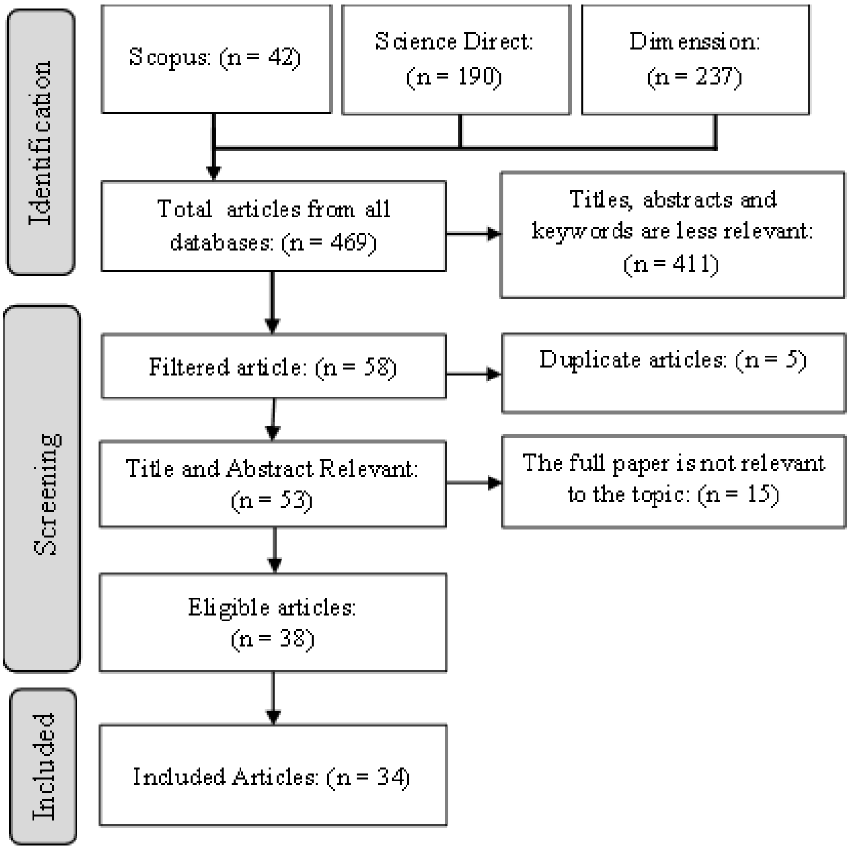


Figure 2: PRISMA Flow Diagram of the Article Selection Process

An initial search conducted across the Scopus, Science Direct, and Dimensions databases identified a total of 469 articles. Subsequent screening based on titles, abstracts, and keywords narrowed the selection to 58 articles, comprising 52 studies on surplus models and ruin probability, 3 studies integrating surplus models with optimal stopping, and 3 studies addressing the three core topics simultaneously, surplus model, ruin probability and optimal stopping. After the removal of 5 duplicate entries and the exclusion of 14 articles deemed irrelevant upon detailed evaluation, a final set of 34 articles was selected for comprehensive analysis. Notably, no studies were found that comprehensively integrate surplus models, ruin probability, optimal stopping, and quadratic programming in a unified framework. Furthermore, existing literature largely focuses on conventional insurance models, with none application within the Sharia insurance context. This indicates a significant research gap and opens potential avenues for future studies to develop integrated, Sharia-compliant risk management and investment optimization models. Addressing this gap could contribute to enhancing financial stability, improving surplus management, and formulating optimal asset allocation strategies in Sharia insurance, which remain underexplored in current academic discourse.

3.4 State of the Art

The state of the art of the reviewed literature is summarized in Table 2. A comprehensive mapping of recent advancements in insurance risk modeling was conducted through a systematic review of the selected articles. The evaluation focused on the extent to which each study contributes to four key components: surplus modeling, ruin probability estimation, optimal stopping strategies, and the implementation of quadratic programming techniques.

Further analysis of the selected articles was conducted using VOSviewer to perform bibliometric mapping based on keyword co-occurrence. This analysis aimed to identify dominant topics, research clusters, and thematic relationships within the literature. The co-occurrence analysis was performed using VOSviewer with a threshold of at least two keyword occurrences to identify core themes relevant to bankruptcy risk management, optimal stopping strategies, and investment optimization in insurance applications.

Table 2: State of the Art of Reviewed Studies

No	Reference	Surplus Model	Ruin Probability	Optimal Stopping	Quadratic Programming
1	Adékambi and Essiomle (2020)[18]	✓	✓	-	-
2	Aurzada and Buck (2020)[19]	✓	✓	-	-
3	Azcue et al. (2023)[20]	✓	✓	-	-
4	Bäuerle and Leimcke (2020)[21]	✓	-	✓	-
5	Bäuerle and Leimcke (2022)[22]	✓	-	✓	-
6	Belkina et al. (2014)[23]	✓	-	✓	-
7	Brachetta and Ceci (2021)[24]	✓	-	✓	-
8	Burnecki et al. (2021)[25]	✓	✓	-	-
9	Christensen et al. (2021)[26]	✓	-	✓	-
10	Cohen and Young (2020)[27]	✓	✓	-	-
11	Delsing et al. (2019)[28]	-	-	-	✓
12	Delsing et al. (2022)[29]	✓	✓	-	✓
13	Denisov et al. (2024)[30]	✓	✓	-	-
14	Euphasio and Carvalho (2022)[31]	✓	✓	-	-
15	Fan et al. (2017)[32]	✓	✓	-	-
16	Federico et al. (2024)[33]	✓	-	✓	-
17	Gauchon et al. (2020)[34]	✓	✓	✓	-
18	Guan et al. (2025)[35]	-	-	-	✓
19	Ieosanurak and Moumeesri (2025)[36]	✓	✓	-	-
20	Jin et al. (2021)[37]	✓	-	✓	-
21	Kim and Drekic (2016)[38]	✓	✓	-	-
22	Korzeniowski (2023)[39]	✓	✓	-	-
23	Li and He (2020)[40]	-	-	-	✓
24	Li and Qiu (2021)[41]	-	-	-	✓
25	Li et al. (2023)[42]	-	-	-	✓
26	Miao et al. (2023)[43]	✓	-	-	-
27	Mishura et al. (2015)[44]	✓	✓	-	-
28	Mitric and Trufin (2015)[45]	✓	✓	-	-
29	Motanya et al. (2025)[46]	✓	✓	-	-
30	Nie et al. (2020)[47]	✓	✓	-	-
31	Ragulina (2017)[48]	✓	✓	✓	-
32	Tian et al. (2021)[49]	-	-	✓	✓
33	Vidmar (2017)[50]	✓	✓	-	-
34	Yuan and Hu (2021)[51]	-	-	-	✓
35	**This Study**	✓	✓	✓	✓

3.5 Bibliometric Visualization

This subsection presents the bibliometric network generated via VOSviewer, based on keyword co-occurrence with a minimum threshold of two. The mapping reveals thematic clusters and conceptual linkages within the domains of surplus models, ruin probability, optimal stopping, and quadratic programming.

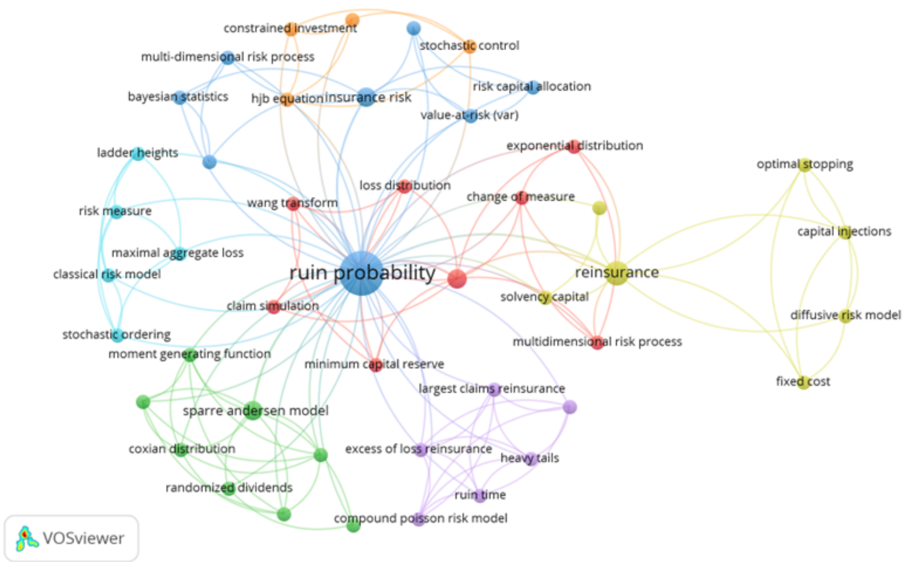


Figure 3: The VOSviewer Bibliometric Visualization

The visualization highlights ruin probability as the most central and frequently occurring keyword, underscoring its foundational role in the field of insurance risk modeling. The network reveals seven thematic clusters, each representing a distinct research focus in surplus modeling and financial risk management. The first cluster (light blue) centers on fundamental surplus modeling using analytical and stochastic approaches involving risk measures and classical models. The second cluster (blue) reflects more adaptive and quantitative risk management, incorporating Bayesian statistics, multidimensional risk processes, and value-at-risk (VaR). The third cluster (green) focuses on modeling inter-claim times using tools like moment generating functions and Sparre Andersen models to better represent surplus fluctuations. The fourth cluster (red) advances risk management through mathematical transformation and simulation methods such as claim simulation, loss distributions, and the Wang transform. The fifth cluster (purple) addresses extreme risks by examining large claims, reinsurance strategies, and ruin time using compound Poisson models and heavy-tailed distributions. The sixth cluster (yellow) emphasizes optimal timing of financial interventions, incorporating optimal stopping, capital injections, and diffusive risk models to mitigate bankruptcy. Lastly, the seventh cluster (orange) explores complex surplus and investment optimization using constrained investment, stochastic control, and the Hamilton–Jacobi–Bellman (HJB) equation, reflecting the increasing sophistication of decision-making models in modern insurance systems.

The overlay visualization in Figure 4 illustrates the evolution of research focus over time. In this mapping, the color gradient from dark blue to light green represents the progression of publication years, where blue nodes indicate earlier research focus, while light green nodes reflect more recent developments. This visual pattern demonstrates how the interest in certain topics has advanced over time, offering insights into the emerging trends and potential directions for future studies in this field. The VOSviewer-based bibliometric visualization of the selected articles is shown in Figure 4.

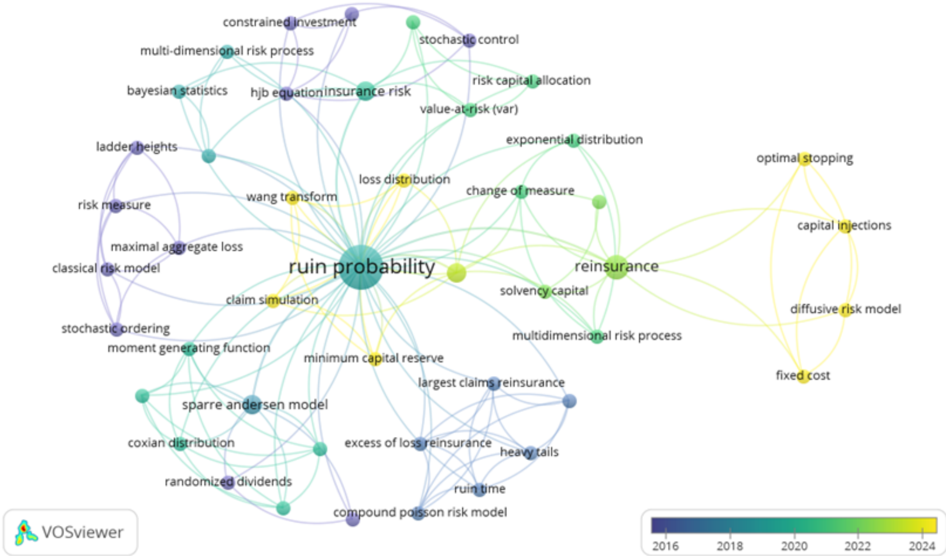


Figure 4: The VOSviewer Overlay Visualization

Based on the bibliometric visualization map, the development of research in insurance risk management exhibits a clear transition from classical frameworks to more advanced and dynamic methodologies. In the early period (2016–2019), studies concentrated on foundational themes such as classical risk model, stochastic ordering, risk measure, and maximal aggregate loss, which established the basis for analytical surplus modeling and systematic risk assessment. During this stage, the emphasis was on relatively simple yet structured approaches. Over time, research advanced toward stochastic control and constrained investment, indicating a growing interest in modeling uncertainty and investment regulation. This phase also saw increased use of Bayesian statistics and multi-dimensional risk processes, reflecting the rising complexity of capital allocation and risk quantification. In the intermediate period (2020–2022), attention shifted to simulation-based methods and non-linear risk transformations. Studies began exploring claim simulation, Wang transform, and techniques involving exponential distribution, minimum capital reserves, and change of measure, aiming to enhance risk estimation accuracy and capital adequacy. The most recent period (2022–2024) marks a pronounced focus on optimal decision-making in stochastic environments, with optimal stopping, capital injections, and diffusive risk models emerging as key themes. These approaches stress the importance of strategic intervention to maintain financial stability and prevent insolvency. Additionally, fixed cost considerations have become increasingly relevant in cost-aware decision processes. Overall, the bibliometric trends illustrate a trajectory from deterministic and analytical models to stochastic, multi-dimensional, and simulation-based frameworks. This evolution underscores a growing emphasis on adaptive, realistic, and sophisticated approaches that can better address extreme risks and systemic vulnerabilities. The overlay visualization further reveals that contemporary research prioritizes not only risk minimization but also optimal capital intervention and Sharia-compliant investment optimization as integral components of modern insurance risk management.

3.6 Future Research Design

Drawing upon insights derived from the bibliometric analysis, which underscore the increasing integration of optimal stopping theory and risk-based surplus modeling in Sharia insurance, the subsequent phase of this study will be focused on the construction of a comprehensive stochastic framework. In this framework, surplus dynamics, bankruptcy risk, optimal stopping, and investment optimization will be incorporated in accordance with the distinctive principles of Islamic finance.

The proposed research design consists of the following interrelated stages:

1. **Model Development**
   A stochastic surplus process will be formulated by incorporating the log-logistic distribution for aggregated total claims, along with mean-reverting investment returns, and continuous premium inflows. The use of the log-logistic distribution allows for a more realistic representation of heavy-tailed claim behavior, as supported by recent empirical studies.
2. **Parameter Estimation**
   Historical data on participant contributions (revenue), claim payouts, and investment performance will be used to estimate model parameters. The log-logistic claim size and Pareto-based revenue distributions will be fitted using Maximum Likelihood Estimation (MLE), ensuring that the model captures real-world risk behavior.
3. **Ruin Probability Measurement**
   Analytical and simulation-based methodologies will be systematically applied to quantify both finite-time and ultimate ruin probabilities. These evaluations will be conducted under a spectrum of initial surplus conditions, investment return scenarios, and claim intensity variations, thereby facilitating a rigorous and comprehensive assessment of solvency risk within a stochastic framework.
4. **Optimal Stopping Strategy**
   A dynamic programming framework, formally derived from the Hamilton Jacobi Bellman (HJB) equation, will be constructed to determine the optimal stopping rule. This strategy is intended to identify threshold conditions under which fund redistribution or capital intervention should be initiated, with the objective of minimizing the expected loss associated with insolvency events. The formulation will be aligned with Sharia principles, particularly the prohibition of interest-based financial instruments or bailouts, thereby ensuring compliance with Islamic financial ethics.
5. **Investment Optimization**
   Quadratic programming will be utilized to optimize the allocation of assets across permissible Sharia-compliant instruments (such as Sukuk and Islamic mutual funds). The optimization framework aims to maximize the expected return while adhering to predefined levels of risk tolerance and solvency constraints. This formulation will be informed by empirical insights into Takaful investment behavior and by prevailing themes identified in recent bibliometric analyses concerning Islamic portfolio optimization.
6. **Model Validation and Sensitivity Analysis**
   The proposed model will undergo validation through rigorous stress testing and scenario-based analysis, encompassing variations in claim frequency, market volatility (e.g., proxy interest rate fluctuations), and participant withdrawal patterns. The robustness, scalability, and policy relevance of the model will be systematically evaluated to ensure its practical applicability for risk managers and regulatory authorities within the Sharia-compliant insurance sector.

This design directly responds to the identified gaps in the literature—particularly the limited integration of dynamic optimization and solvency analysis in Islamic insurance models—and is aligned with the research objective to propose a sustainable, Sharia-compliant risk management strategy.

To illustrate the structure and sequence of the proposed research process, the following flowchart presents the main steps of the study.

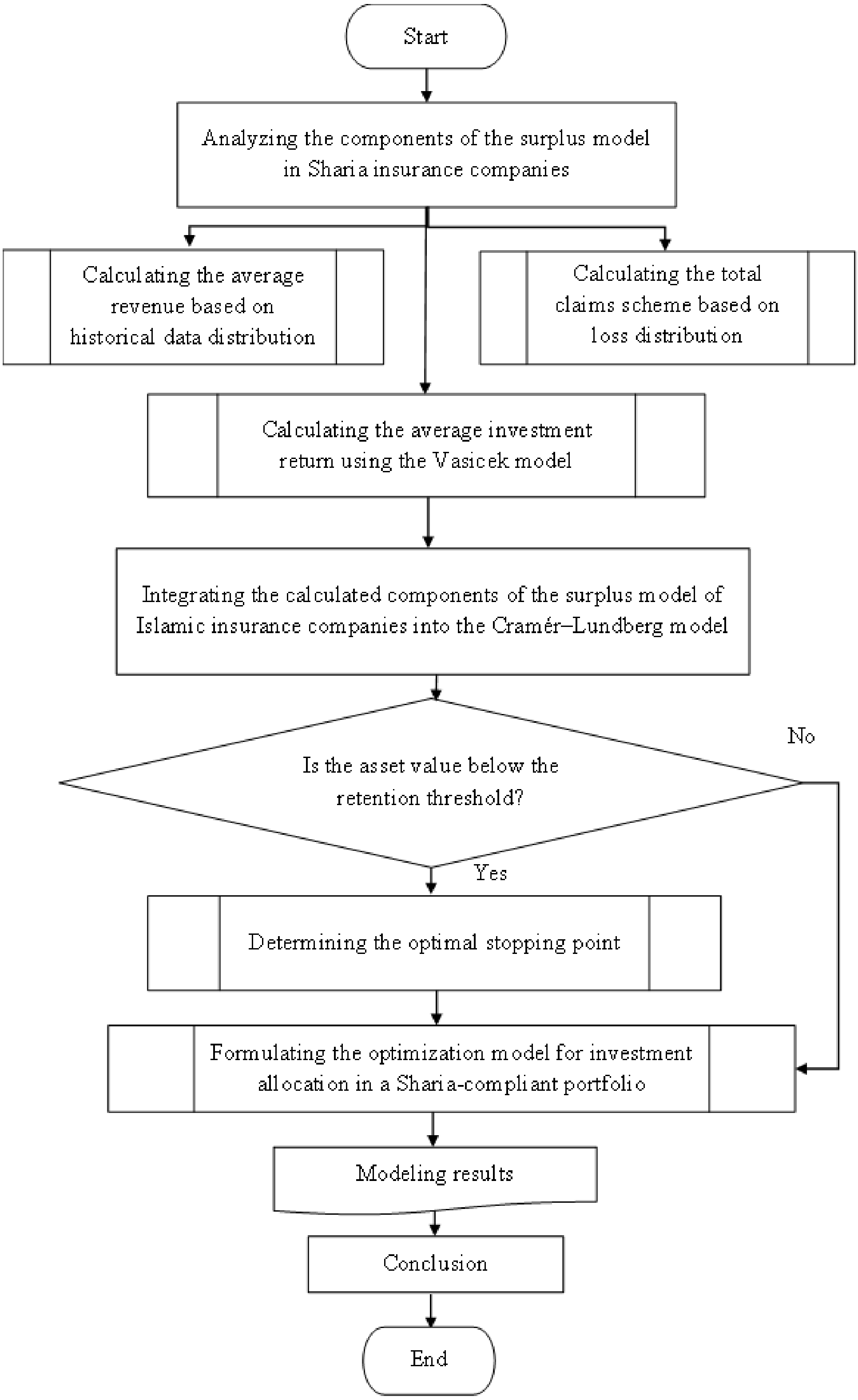


Figure 5: Main Research Flowcart

The following flowchart illustrates the procedure used to calculate the company's revenue based on the integration of contribution and investment income.

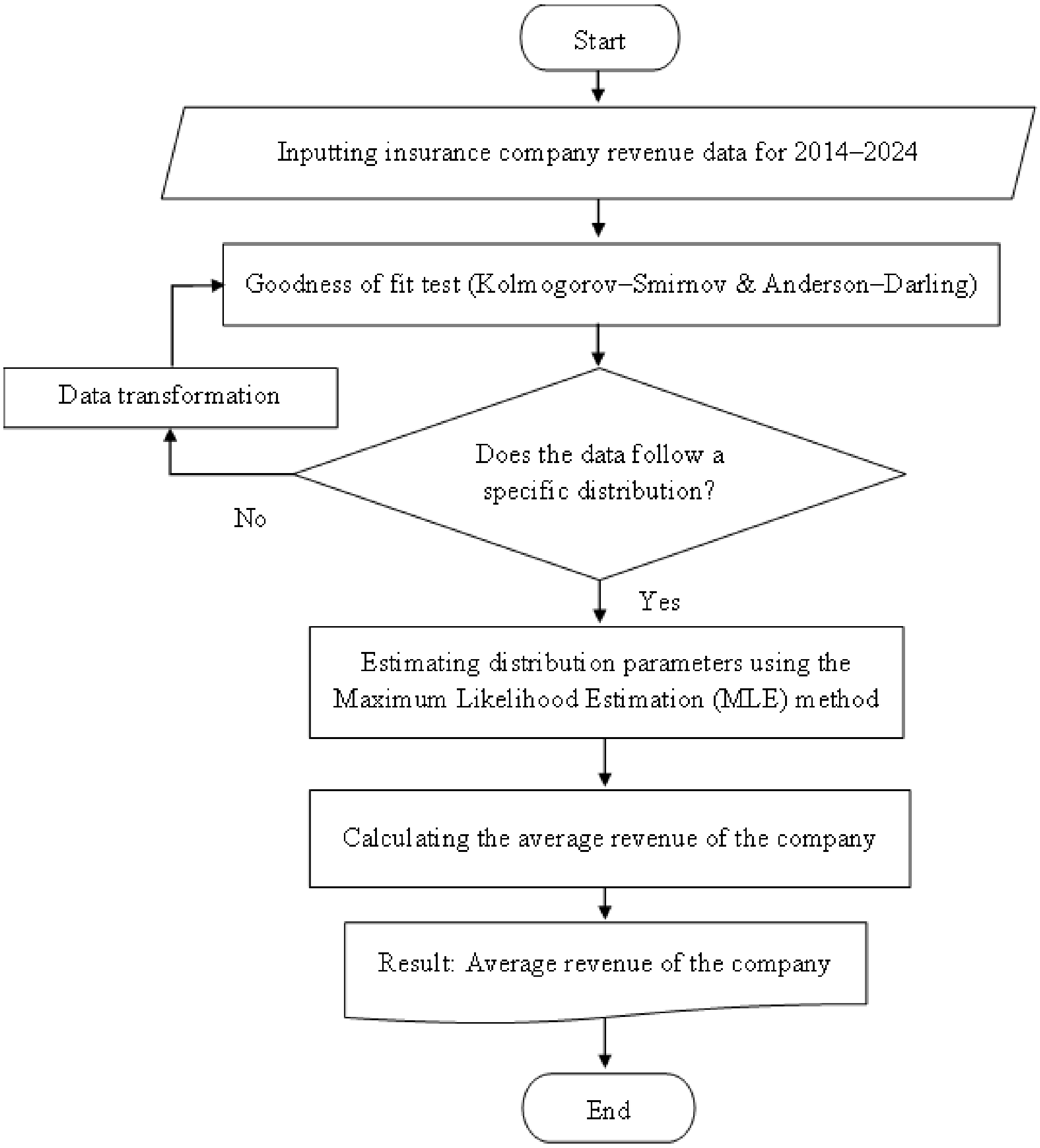


Figure 6: Flowchart of Company Revenue Calculation

The next step involves modeling the investment return using the Vasicek interest rate model, as shown in the flowchart below.

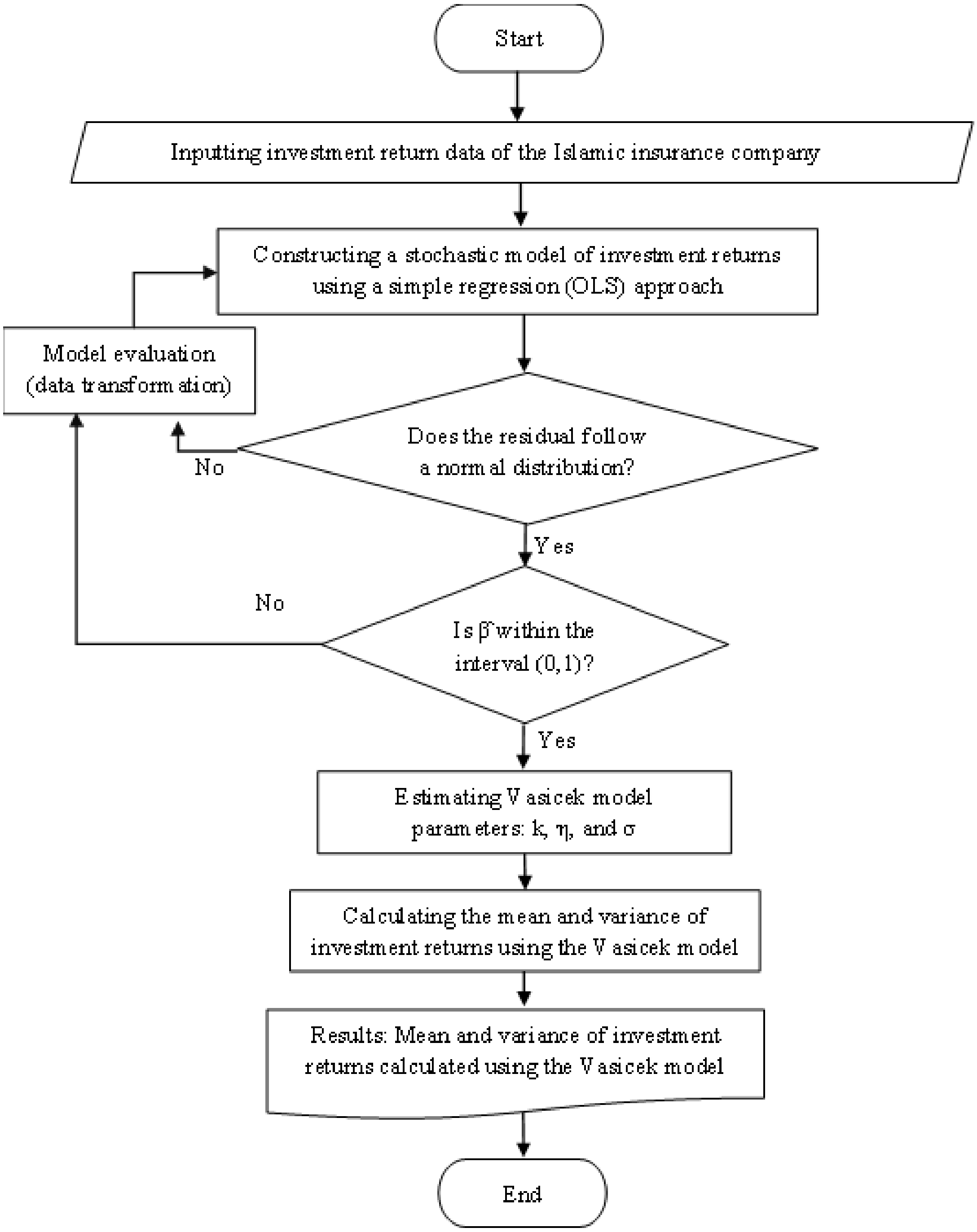


Figure 7: Flowchart of Investment Return Calculation Using the Vasicek Model

To estimate total claims, the following flowchart presents the loss distribution approach used to model claim severity and frequency.

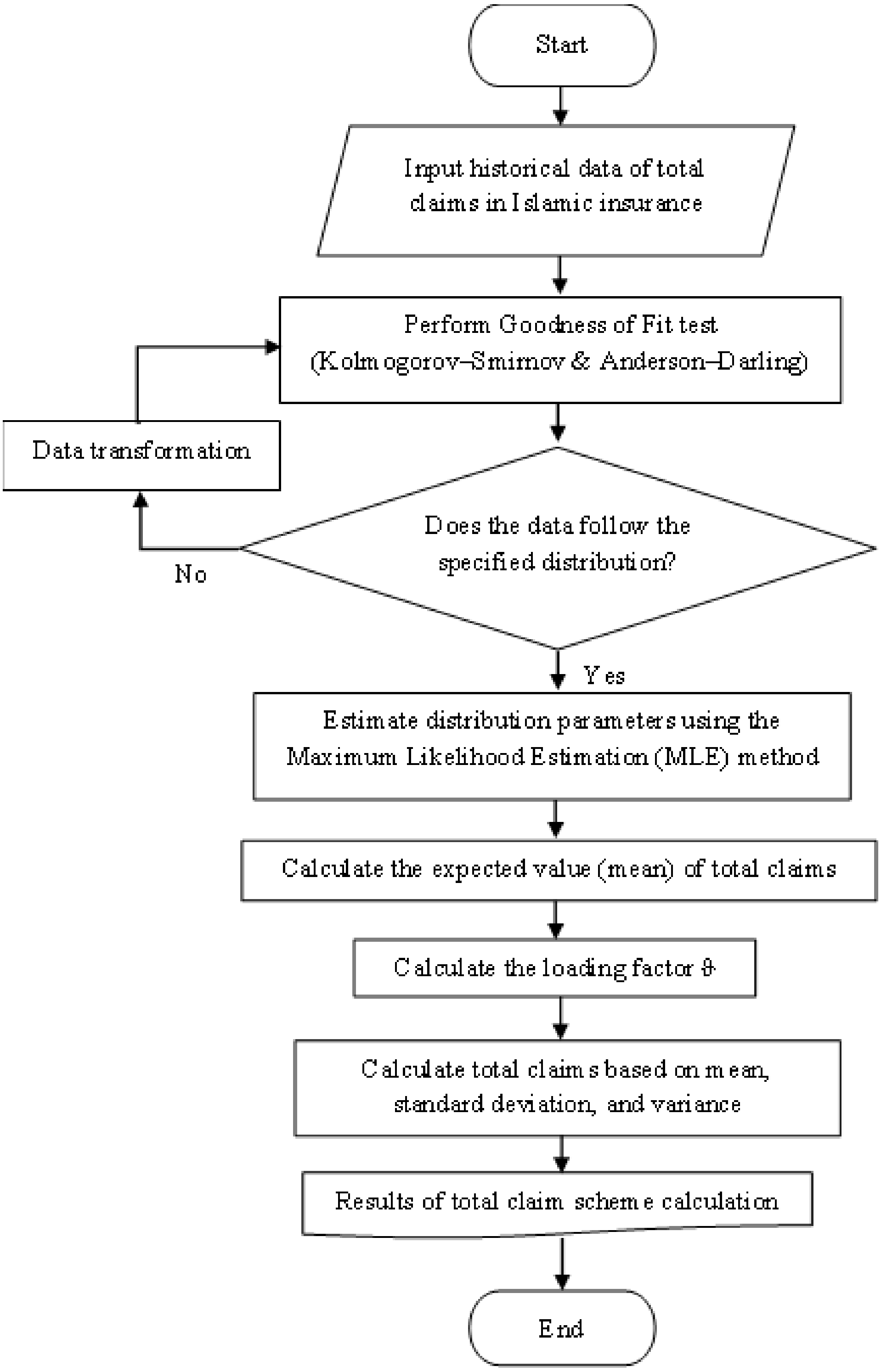


Figure 8: Flowchart of Total Claim Estimation Using the Loss Distribution Approach

The optimal stopping time for minimizing the risk of bankruptcy is determined through the process outlined in the following flowchart.

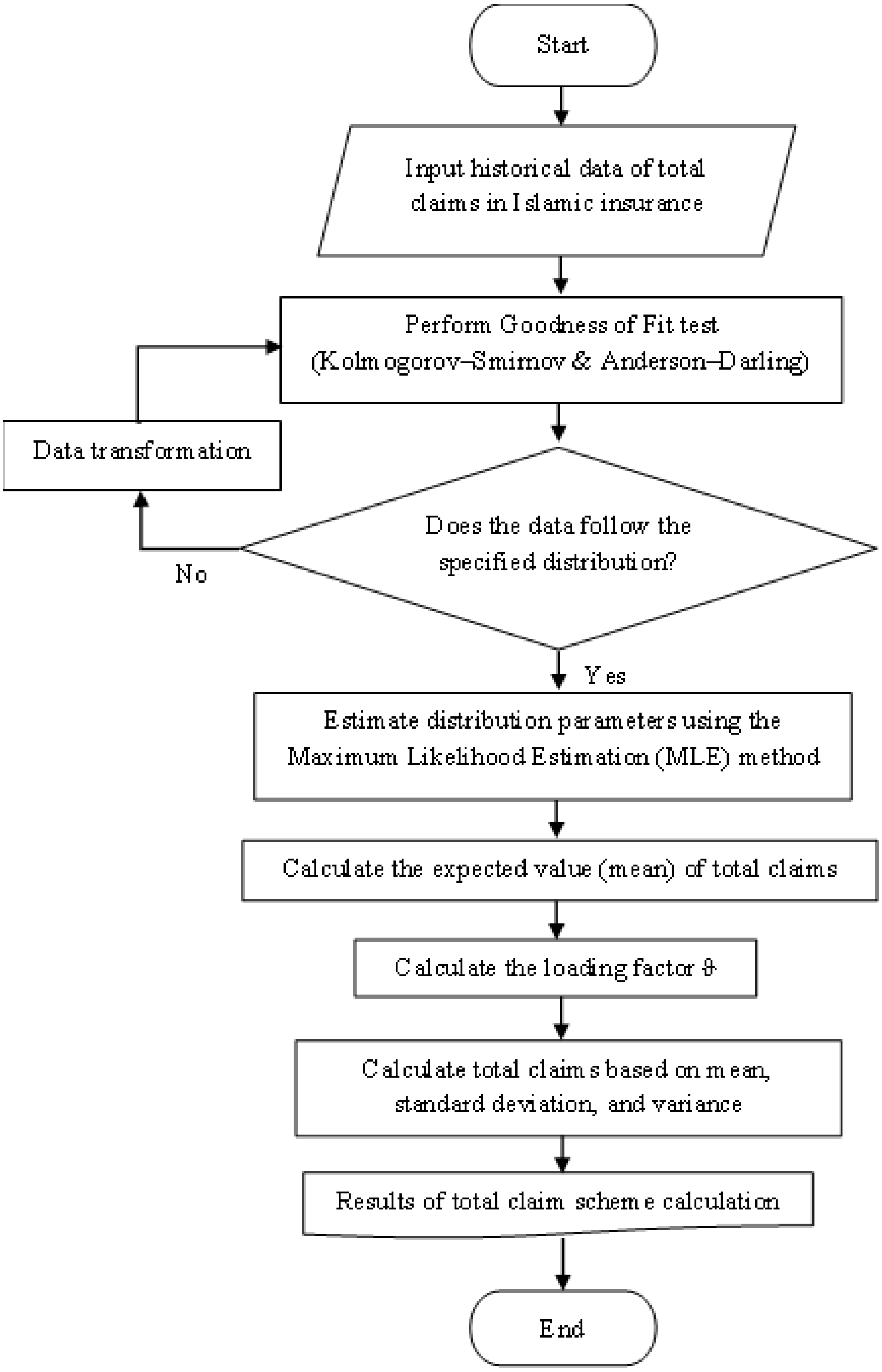


Figure 9: Flowchart of Optimal Stopping Point Determination

Finally, the optimization method used to determine the best financial strategy is depicted in the flowchart below.

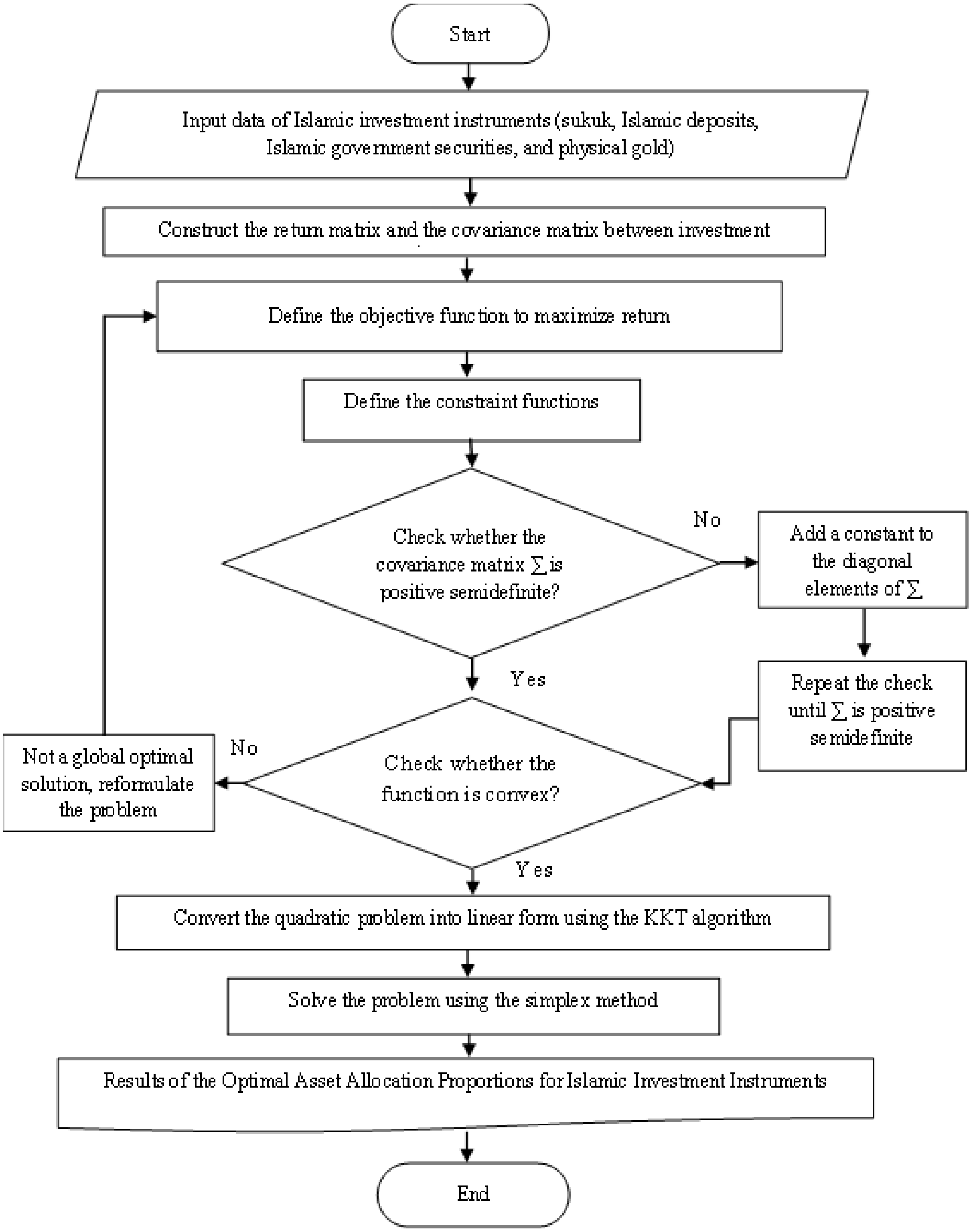


Figure 10: Flowchart of the Optimization Method Applied in the Model

4 Discussion

The proposed research framework presents a holistic model aimed at addressing bankruptcy risk and investment optimization in Sharia-compliant insurance, integrating surplus dynamics, ruin probability, optimal stopping, and Sharia-based investment allocation. Each component of the model is grounded in empirical gaps and thematic trends identified through the systematic literature review and bibliometric analysis. The use of a mean-reverting stochastic return inspired by the Vasicek model—finds theoretical support in financial literature addressing return volatility, yet remains underutilized in Islamic insurance contexts. While many prior models assume constant returns (e.g., deterministic or geometric Brownian motion), our bibliometric co-occurrence analysis indicates a scarcity of works combining stochastic investment dynamics with surplus models in a Sharia context. This justifies a more realistic approach to modeling investment returns that reflect market corrections and central tendencies. The choice of log-logistic distribution for claim severity is not merely technical but supported by literature that acknowledges the heavy-tailed nature of insurance losses (confirmed in keyword clustering around "severity distribution" and "loss modeling"). Despite this, the majority of reviewed studies still rely on exponential or Pareto distributions, which often underestimate tail risk. The log-logistic form allows better tail behavior representation, improving ruin estimates and risk quantification under extreme loss scenarios. The integration of optimal stopping theory as highlighted in the bibliometric cluster linked to "dynamic programming" and "risk control" has been largely theoretical and fragmented in Sharia contexts. Most studies in our corpus do not operationalize stopping rules in conjunction with surplus dynamics or investment behavior. This model introduces a novel synthesis by aligning stopping decisions with surplus thresholds and market signals, thus enhancing proactive risk mitigation. Investment optimization through quadratic programming responds to another identified gap, while the term "portfolio optimization" appears frequently in recent publications, few papers operationalize it within the dual constraints of risk-return trade-off and Sharia compliance. This framework proposes a tractable method for allocating funds to halal instruments while explicitly considering the insurer's surplus trajectory and ruin probabilities.

However, a critical shortcoming in the current literature evidenced by both thematic analysis and publication trend mappi ngis the lack of integrated, data-driven models that simultaneously capture surplus evolution, claim dynamics, and investment policy in Islamic insurance. Furthermore, empirical validation remains limited, with simulation and optimization studies often disconnected from actual financial data. The conceptual model proposed here aims to address these deficiencies by offering an integrated design with concrete implementation steps. Another issue is the fragmentation in methodological approaches, with most studies addressing either ruin probability or investment strategy, but rarely both. This disjointed approach limits the practical application in enterprise-wide risk management. By bringing together multiple techniques—stochastic modeling, maximum likelihood estimation, simulation, optimal stopping, and quadratic optimization—this study seeks to advance an interdisciplinary and actionable methodology. Bibliometric indicators further reinforce the timeliness of this research. For example, the increasing trend in publications (noted from 2015 onwards) related to "Takaful," "risk management," and "stochastic surplus" shows a rising scholarly interest, yet the low co-occurrence with terms like “optimal stopping” or “quadratic programming” underscores the novelty of this study's integration. In summary, while the proposed model is ambitious in scope, it is grounded in both theoretical rigor and bibliometric evidence of gaps in the literature. This study contributes not only a conceptual innovation but also a call for empirically testable, integrated tools to support sustainable Sharia insurance practices.

Based on the proposed model and the future research design, the following recommendations are provided:

- **For Insurance Practitioners:** It is recommended to adopt adaptive surplus management strategies that incorporate real-time financial indicators and risk thresholds to improve the responsiveness to potential ruin scenarios.

- **For Investment Managers:** Sharia insurance companies should continuously optimize their investment portfolios using dynamic and risk-sensitive optimization frameworks that consider market volatility and surplus fluctuations.
- **For Regulators:** Authorities and supervisory institutions are encouraged to support the advancement of sophisticated risk management tools, including optimal stopping models and simulation-based ruin probability estimations, to enhance the financial resilience and stability of Sharia insurance institutions.
- **For Future Researchers:** It is recommended that future research incorporates alternative risk measures, such as Conditional Value-at-Risk (CVaR), and extends the application of the developed models to a broader range of real-world datasets and varying market conditions to strengthen the model's generalizability, robustness, and practical relevance.

5 Conclusion

This study has systematically reviewed and analyzed the development of risk management models in Sharia insurance, with particular emphasis on surplus modeling, probability estimation of ruin, optimal stopping strategies and investment optimization using quadratic programming. The bibliometric findings highlight a notable evolution, from early deterministic models to more sophisticated stochastic frameworks, reflecting the growing interest in dynamic, data-driven decision tools. Building on these developments, the proposed future research aims to construct an integrated stochastic model that realistically captures surplus dynamics, estimates key parameters using robust statistical methods, and establishes optimal stopping rules to manage bankruptcy risk under uncertainty. Furthermore, the research will incorporate a quadratic programming framework to guide Sharia-compliant investment decisions, balancing expected return and ethical constraints. Despite the growing body of literature, the analysis reveals key gaps, including the underrepresentation of unified models that integrate stochastic surplus behavior with optimal risk intervention mechanisms under Sharia principles. Additionally, bibliometric mapping shows limited intersection between themes such as "optimal stopping," "ruin probability," and "Sharia compliance," suggesting a fragmentation of research efforts.

While this study contributes by providing a structured synthesis and identifying pathways for model integration, it is limited to published literature in indexed databases and excludes grey literature or industry reports. As such, the scope may not fully capture practical implementation challenges or the latest regulatory shifts in Takaful operations. The proposed research is expected to enhance decision-making tools in Islamic insurance by offering practical guidance on halting risk exposure and optimizing remaining assets under uncertainty. By accounting for financial volatility and risk complexity, the model aspires to deliver adaptive, real-world solutions for sustainable Sharia insurance management. Future validation and simulation-based experiments will be critical in assessing model performance across various financial scenarios. Moreover, subsequent research is encouraged to explore integration with real-time data analytics and alternative risk transfer instruments (e.g., retakaful and sukuk-linked insurance) to improve resilience and responsiveness in Sharia insurance systems.

CRediT Authorship Contribution Statement

Setyo Luthfi Okta Yohandoko: Conceptualization, Methodology, Software, Data Curation, Formal Analysis and Original draft writing. **Diah Chaerani:** Writing–Review, Editing and Project Administration. **Sukono:** Validation, Supervision.

Declaration of Generative AI and AI-assisted technologies

The author affirms that generative AI technology was utilized to support the preparation of this manuscript. Specifically, OpenAI-developed ChatGPT was used to assist in tasks such as content refinement and language structuring. All outputs generated through this tool were critically evaluated, revised, and integrated by the author to ensure precision, coherence, and adherence to academic standards. The final manuscript reflects the author's independent intellectual contribution and professional judgment.

Declaration of Competing Interest

The authors declare no competing interests

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Data and Code Availability

The data underlying the findings of this study can be obtained from the corresponding author upon reasonable request, subject to the terms of confidentiality and ethical considerations governing the use of such data.

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