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Optimizing Profit-to-Cost Ratios in Bakery Production Using the Hasan–Acharjee Fractional Programming Method

Fahliza Adisty* and Riri Syafitri Lubis

Department of Mathematics, Universitas Islam Negeri Sumatera Utara, Medan, Indonesia

Abstract

Production optimization under resource constraints can be effectively modeled using Linear Fractional Programming (LFP), where the objective function is defined as a profit-to-cost ratio. This study applies the Hasan–Acharjee method to optimize production planning in a household-scale bakery enterprise in Indonesia, considering four product types and three resource constraints (materials, labor, and equipment). The model was reformulated as a single linear program and solved using LINGO 21.0. Validation against the classical Charnes–Cooper transformation confirmed identical optimal solutions, demonstrating the robustness of the Hasan–Acharjee approach. Sensitivity and trade-off analyses further revealed how variations in costs and production capacity influence profitability. The results highlight both the theoretical relevance of the Hasan–Acharjee method in fractional programming and its practical applicability to small and medium-sized enterprises seeking efficient resource utilization under limited conditions.

Keywords: Charnes-Cooper; Hasan-Acharjee; Linear Fractional Programming; Optimization.

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1 Introduction

Micro, small, and medium enterprises (MSMEs) play a pivotal role in economic growth and employment worldwide. In Indonesia, MSMEs contribute more than 60% to the national gross domestic product (GDP) and employ over 97% of the workforce [1]. Among these, the food industry—particularly bakery and pastry businesses—is a rapidly expanding subsector, driven by population growth, urbanization, and evolving consumption patterns. Despite such promising prospects, household-scale bakeries continue to face critical resource limitations, including constrained access to raw materials, labor, production equipment, and capital. These challenges underscore the importance of developing efficient and quantitative production strategies that can maximize profit under limited resources.

Within the field of operations research, problems of this nature are naturally modeled as optimization problems. Linear programming and its extensions have been widely applied to support decision-making in MSMEs [2]. A particularly relevant extension is *linear fractional programming* (LFP), where the objective function is expressed as a ratio of profit to cost. This formulation explicitly captures efficiency, making it especially suitable for small-scale enterprises where the balance between profitability and resource utilization is critical [3]–[5].

*__Corresponding author__. E-mail: fahliza0703212057@uinsu.ac.id

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Prior studies have demonstrated the effectiveness of mathematical approaches for solving LFP models. For example, the Hasan–Acharjee method has been applied to optimize selling prices and initial capital allocation in small industries, showing that transforming an LFP into a solvable linear program via the simplex method can significantly improve the profit-to-cost ratio [6]. Similarly, the Charnes–Cooper transformation has been employed in production planning studies, yielding multiple optimal solutions that inform decision-making under different market conditions [7], [8]. Comparative analyses have further suggested that the Hasan–Acharjee method offers simplicity and flexibility relative to Charnes–Cooper when applied across diverse problem structures [9]. More broadly, LFP-based approaches have been used for resource allocation and scheduling in SMEs and service operations [10]–[12].

Despite these advances, research on the empirical application of fractional programming in household-scale bakeries remains scarce. Previous studies have largely focused on theoretical validation or non-food-related case studies, leaving a gap in understanding how such methods can be implemented in small-scale food enterprises. This gap is noteworthy, as household bakeries represent one of the most common and relevant forms of MSMEs in emerging economies. Furthermore, while the Charnes–Cooper transformation is well established, limited work has explicitly tested the Hasan–Acharjee method in real-world MSME contexts, particularly to assess its robustness and practical implications under realistic resource constraints [13], [14].

To address this gap, the present study applies the Hasan–Acharjee method of LFP to a household-scale bakery enterprise in Indonesia, considering four product types and three resource constraints (raw materials, labor, and equipment). The contributions of this study are twofold. First, it demonstrates the empirical applicability and robustness of the Hasan–Acharjee method in solving LFP problems for real production planning scenarios, validated against the classical Charnes–Cooper transformation. Second, it provides practical managerial insights through sensitivity and trade-off analyses, illustrating how variations in costs and capacity affect optimal profitability. By combining methodological rigor with real-world application, this study contributes both to the theoretical development of fractional programming and to the practical design of production strategies for MSMEs operating under resource limitations.

The remainder of this paper is structured as follows. Section 2 presents the research methodology, including the formulation of the Linear Fractional Programming (LFP) model and the transformation procedures using the Hasan–Acharjee and Charnes–Cooper methods. Section 3 discusses the results and analysis, covering the baseline solution, comparison of methods, sensitivity analysis, and capacity–profit trade-off. Section 4 concludes the study by summarizing the main findings, theoretical contributions, practical implications, and directions for future research.

2 Methods

This study adopts a quantitative approach with a case study design on a representative household-scale bakery enterprise in Indonesia. The research focuses on optimizing business profits through a mathematical formulation based on Linear Fractional Programming (LFP). The model incorporates four decision variables representing production quantities of caramel sponge cake, roll cake, canoe cake, and round cake. Three key resource constraints are considered: raw material costs, labor costs, and equipment costs [15].

The dependent variable in this study is the business profit level, defined as the ratio of sales revenue to total production costs. Primary data were obtained through direct observation and structured interviews with the business owner regarding production volumes, labor capacity, material costs, and equipment limitations. Secondary data were gathered from financial records and internal business documents. The general structure of the LFP model follows the framework of Charnes & Cooper and Schaible, ensuring mathematical validity and equivalence under transformation.

2.1 Linear Fractional Programming Formulation

The general form of the LFP model is expressed as

$$\max\ Z(x) = \frac{c^T x + \alpha}{d^T x + \beta}, \tag{1}$$

subject to

$$Ax \leq b, \quad x \geq 0, \tag{2}$$

where:

- $x \in \mathbb{R}^n$: vector of decision variables (production quantities),
- $c \in \mathbb{R}^n$: revenue coefficient vector,
- $d \in \mathbb{R}^n$: cost coefficient vector,
- $A \in \mathbb{R}^{m \times n}$: resource consumption matrix,
- $b \in \mathbb{R}^m$: vector of available resource capacities,
- $\alpha, \beta \in \mathbb{R}$: constants.

The objective function in (1) maximizes the ratio of total revenue to total cost, subject to linear resource constraints (2). This fractional structure captures efficiency explicitly, making the formulation suitable for small-scale enterprises where both profitability and resource utilization are critical.

2.2 Hasan–Acharjee Transformation Method

The original LFP problem is nonlinear because of its fractional objective. The Hasan–Acharjee method reformulates it into an equivalent linear program by introducing a normalization condition and appropriately scaling the constraints.

First, impose the normalization

$$d^T x + \beta = 1, \tag{3}$$

which effectively removes the denominator in the objective function. Under this condition, the objective simplifies to

$$\max\ c^T x + \alpha. \tag{4}$$

Next, the constraints are rescaled as

$$Ax \leq b\,(d^T x + \beta). \tag{5}$$

Substituting the normalization condition (3), these constraints reduce to

$$Ax \leq b, \tag{6}$$

together with the nonnegativity condition $x \geq 0$.

The transformed problem is therefore a standard linear program, which can be solved efficiently using the simplex method [16]. Once the normalized solution y^* is obtained, the optimal solution to the original LFP can be recovered by the back-transformation:

$$x^* = \frac{y^*}{d^T y^* + \beta}. \tag{7}$$

This procedure ensures equivalence between the original fractional model and the transformed linear program, guaranteeing that feasibility and optimality are preserved [17], [18].

2.3 Charnes–Cooper Transformation Method

In contrast, the classical Charnes–Cooper transformation linearizes the fractional model through explicit variable substitution:

$$y = \frac{x}{d^T x + \beta}, \qquad t = \frac{1}{d^T x + \beta}.$$

The equivalent linear program is then expressed as

$$\max \ c^T y + \alpha t \tag{8}$$

subject to

$$Ay \leq bt, \quad d^T y + \beta t = 1, \quad y \geq 0, \ t > 0. \tag{9}$$

Unlike the Hasan–Acharjee method, this transformation explicitly introduces the auxiliary pair (y, t). Its advantage is generality: it provides a systematic and widely recognized procedure applicable to many forms of LFP problems. The equivalence between the fractional and linear models is mathematically guaranteed, and the resulting LP can be efficiently solved by the simplex method, originally developed by Dantzig [7].

2.4 Research Workflow

The methodological workflow proceeds as follows. First, production, cost, and resource data were collected and processed to construct the LFP model (covering four product categories and three resource constraints, measured over a weekly planning horizon). Second, the problem was solved using two alternative transformation approaches: Hasan–Acharjee and Charnes–Cooper. Third, the resulting linear or equivalent models were optimized using the simplex method in LINGO. Fourth, the solutions obtained from both methods were compared to assess equivalence and robustness. Finally, sensitivity analysis (on material, labor, equipment, and price parameters) and trade-off analysis (between production capacity and profitability) were conducted to evaluate model stability and managerial implications.

This dual-method approach provides a novelty contribution: whereas most previous studies adopt either Charnes–Cooper or Hasan–Acharjee independently, this study empirically validates their equivalence in a real-world SME context, demonstrating both robustness and practical applicability.

2.5 Software

The optimization models were implemented in LINGO 21.0 for solving both linear and nonlinear programs. Microsoft Excel was used for data preprocessing and tabulation, and Matplotlib (Python) for generating sensitivity and trade-off visualizations. This combination of tools ensures computational reliability, reproducibility, and clarity of presentation. The detailed dataset used to calibrate the model is presented in Section 3, Table 1.

3 Results and Discussion

This section reports the empirical results of the proposed Linear Fractional Programming (LFP) model and discusses their implications for production planning under resource constraints. We first summarize the data used to calibrate the model, then present the baseline optimal solution and interpret its economic meaning. Subsequent subsections compare the Hasan–Acharjee and Charnes–Cooper transformations, examine parameter sensitivity, analyze the capacity–profit trade-off, and draw managerial insights.

3.1 Data

Table 1 reports the actual dataset used to implement the optimization model, complementing the methodological description in Section 2. The dataset includes product-level information (unit selling price and resource requirements) and weekly capacity limits for materials, labor, and equipment. These values were obtained through direct observation and owner interviews, and cross-checked against internal records.

Table 1: Data on Fajar Bakery: product resource requirements, market demand, and selling prices

Types of Product	Material Costs (IDR Million)	Employee Salaries (IDR Million)	Equipment Costs (IDR Million)	Market Demand (packs/week)	Selling Price (IDR Million)
Caramel Sponge Cake	15.0	2.0	4.0	7,210	27.0
Roll Cakes	5.0	0.7	1.0	2,650	10.0
Canoe Cakes	8.0	1.0	2.0	3,845	14.0
Round Cakes	8.0	1.0	2.0	3,905	14.0

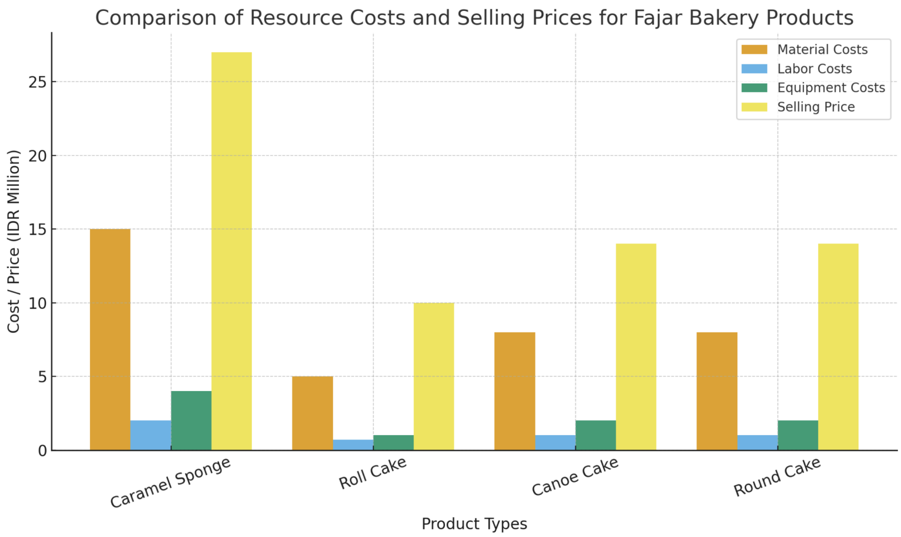

Figure 1: Comparison of resource costs and selling prices for Fajar Bakery products. Caramel sponge cakes have the highest selling price but also the largest resource demands, while roll cakes are the most cost-efficient. Canoe and round cakes show nearly identical intermediate profiles.

From Table 1 and Figure 1, several patterns are immediately evident. Caramel sponge cakes command the highest selling price (IDR 27 million per weekly unit), but they also require disproportionately large amounts of materials and equipment, giving them a high-risk, high-return profile. In contrast, roll cakes are the least resource-intensive across all categories while still generating a reasonable selling price of IDR 10 million, suggesting strong cost efficiency. Canoe cakes and round cakes occupy a middle ground: their resource requirements and prices are nearly identical, both positioned between the extremes of caramel sponge and roll cakes.

These contrasts underscore the trade-offs that producers must navigate when allocating scarce resources. Products with higher revenue potential (e.g., caramel sponge) may not be optimal if their resource draw is too heavy relative to the constraints, whereas more modestly priced products (e.g., roll cakes) may dominate in terms of efficiency. The Linear Fractional Programming (LFP) model is designed precisely to formalize these trade-offs and identify the production plan that maximizes the profit-to-cost ratio under binding resource limitations.

3.2 Baseline Optimal Solution and Interpretation

Solving the model under the baseline parameters yields a unique production plan that maximizes the profit-to-cost ratio. The optimal solution is:

$$X_1 = 0, \quad X_2 = 10, \quad X_3 = 0, \quad X_4 = 0, \qquad Z^* = 1.388889.$$

In other words, the model recommends allocating all production resources exclusively to *roll cakes* (X_2), while other product types are excluded from the optimal mix. This baseline plan is summarized in Table 2.

Table 2: Baseline optimal production plan and objective value

Metric	X_1	X_2	X_3	X_4	Z^*
Optimal value	0	10	0	0	1.388889

At this optimum, the *material* and *equipment* constraints are binding, meaning these resources are fully utilized. By contrast, the *labor* constraint remains slack, indicating unused worker capacity. This implies that material inputs and equipment availability are the primary bottlenecks in production, while labor does not currently limit profitability.

The dominance of roll cakes in the optimal solution arises from their favorable balance between revenue and resource use. Although caramel sponge cakes promise higher revenue, their high consumption of materials and equipment reduces their efficiency relative to roll cakes. Canoe and round cakes, while moderate in both price and costs, are not competitive once resource trade-offs are formalized. The resulting optimal ratio $Z^* = 1.388889$ therefore reflects the best achievable balance between profit and cost, given the current technological and financial constraints.

3.3 Comparison of Charnes–Cooper and Hasan–Acharjee Methods

Both transformation techniques—the classical Charnes–Cooper and the more recent Hasan–Acharjee—were applied to the same dataset to test consistency of results. Despite relying on different reformulations, the two methods produced identical production decisions and objective values. The comparison is summarized in Table 3.

Table 3: Comparison of optimal solutions under Charnes–Cooper and Hasan–Acharjee methods

Variable / Metric	**Charnes–Cooper (LP)**	**Hasan–Acharjee (NLP)**
X_1 / Y_1	0	0
X_2 / Y_2	10 (from $Y_2 = 0.138889$, $t = 0.138889$)	10
X_3 / Y_3	0	0
X_4 / Y_4	0	0
Scaling variable t	0.138889	–
Objective value Z^*	1.388889	1.388889
Binding constraints	Material, Equipment	Material, Equipment
Inactive constraint	Labor	Labor
Solution status	Global optimum	Local optimum[1]

As shown in Table 3, both approaches yield the same production plan: producing only roll cakes ($X_2 = 10$) and omitting other products. The optimal objective value $Z^* = 1.388889$ is likewise identical. The fact that two mathematically distinct transformations lead to equivalent solutions strengthens the validity of the results and demonstrates the robustness of the Hasan–Acharjee approach, even though solvers classify it as a "local" optimum.

The comparison highlights an important methodological insight. The Charnes–Cooper method, as a linear programming (LP) reformulation, guarantees a global optimum via simplex-based solution procedures. In contrast, the Hasan–Acharjee method retains the problem in its nonlinear programming (NLP) form, which solvers typically report as a local optimum. However, the pseudoconcave nature of the fractional programming problem ensures equivalence, making the Hasan–Acharjee method a simpler yet equally valid alternative for applied contexts. For practitioners, this means that even without adopting variable substitution and scaling, reliable results can still be achieved.

[1] For LFP problems with positive denominators, the objective function is pseudoconcave, ensuring that any local optimum found under the Hasan–Acharjee method is also globally optimal.

3.4 Sensitivity Analysis

Sensitivity analysis was conducted to evaluate how changes in key parameters—raw material costs, labor costs, equipment costs, and selling price—affect the optimal solution. This procedure tests the robustness of the model under plausible shifts in cost structures and market conditions.

Table 4: Sensitivity analysis results under parameter variations

Scenario	Z^*	X_1	X_2	X_3	X_4
Baseline (100%)	1.38889	0	10	0	0
Material +10%	1.29536	0	10	0	0
Material –10%	1.50318	0	10	0	0
Labor +10%	1.36078	0	10	0	0
Labor –10%	1.41940	0	10	0	0
Equipment +10%	1.29126	0	10	0	0
Equipment –10%	1.51759	0	10	0	0
Price +10%	1.52878	0	10	0	0
Price –10%	1.24910	0	10	0	0

Table 4 shows that the optimal production decision remains unchanged across all scenarios: the model consistently selects roll cakes ($X_2 = 10$) as the sole product. However, the objective value Z^* varies in predictable ways with parameter shifts.

- `Raw materials and equipment.` A 10% increase in either raw material or equipment costs substantially reduces the profit-to-cost ratio (to 1.29536 and 1.29126, respectively). Conversely, a 10% decrease in these costs increases Z^* to above 1.50. This confirms that materials and equipment are the critical bottleneck resources identified earlier.
- `Labor.` Changes in labor costs have a smaller effect: raising labor costs by 10% lowers Z^* only to 1.36078, while reducing them by 10% increases it slightly to 1.41940. This aligns with the earlier finding that the labor constraint is slack at the optimum.
- `Selling price.` A 10% increase in the selling price yields the highest improvement in profitability ($Z^* = 1.52878$), whereas a 10% decrease causes the sharpest decline ($Z^* = 1.24910$). This indicates that pricing policy is a highly sensitive lever for profit optimization.

The analysis demonstrates that the optimal solution is structurally stable (decision variables remain unchanged) but economically sensitive to cost and price fluctuations. For practitioners, this means that while the production mix recommendation is robust, profitability is highly contingent on maintaining favorable input costs and market prices. Strategic negotiations with suppliers and dynamic pricing adjustments thus represent critical tools for sustaining efficiency.

3.5 Capacity–Profit Trade-Off Analysis

To further understand the scalability of production, a trade-off analysis was performed by varying total production capacity from 80% to 130% of the baseline. The resulting optimal profit-to-cost ratios are reported in Table 5.

Table 5: Trade-off between production capacity and profitability

Capacity Level (%)	Optimal Z^*
80	1.05
90	1.22
100	1.38
110	1.50
120	1.57
130	1.63

Table 5 shows a clear upward trend in Z^* as production capacity increases. At the baseline

(100%), the optimal ratio is 1.38889. Reducing capacity by 10% or 20% lowers profitability significantly to 1.22 and 1.05, respectively, because fixed costs are spread over a smaller production scale. Conversely, expanding capacity to 110%–130% improves profitability to the range of 1.50–1.63.

Although profitability rises with capacity, the rate of improvement slows as capacity expands. For instance, moving from 100% to 110% yields an increase of 0.11 in Z^*, but the gain shrinks to only 0.06 when moving from 120% to 130%. This diminishing return reflects the fact that other constraints—especially raw materials and equipment—become tighter, limiting further gains from scaling.

The trade-off analysis suggests that moderate capacity expansion (around 110–120%) is beneficial, but beyond that, the marginal improvement in profitability is relatively small. For managers, this implies that investment in expanding production should be balanced with parallel efforts to relax binding resource constraints. Without addressing raw material and equipment limitations, simply adding production capacity may not be cost-effective.

3.6 Managerial Insights and Limitations

The findings of this study provide several practical takeaways for small-scale enterprises such as Fajar Bakery:

- `Focus on high-efficiency products.` The model consistently recommends roll cakes as the sole product under the given resource structure. This indicates that, when resources are limited, concentrating on products with the highest profit-to-cost ratio maximizes overall efficiency.
- `Monitor input costs closely.` Sensitivity analysis shows that profitability is highly responsive to changes in raw material and equipment costs. Business owners should prioritize negotiations with suppliers, bulk purchasing, or alternative sourcing strategies to stabilize these critical inputs.
- `Adopt flexible pricing strategies.` Among all parameters, selling price fluctuations exert the greatest influence on profitability. A modest increase in unit price has a disproportionately positive effect on the profit-to-cost ratio, suggesting that pricing decisions are as important as production planning.
- `Scale with caution.` The trade-off analysis reveals that expanding capacity can increase profits, but diminishing returns appear beyond 120% capacity. Managers should carefully evaluate whether further investments in capacity yield sufficient returns unless accompanied by broader resource expansion.

Despite these valuable insights, the study has several limitations that must be acknowledged:

- `Single case study.` The analysis is restricted to one household-scale bakery in Medan City. While the results are illustrative, they may not generalize to other SMEs in different sectors or regions without adjustment.
- `Static model.` The optimization assumes fixed prices, costs, and demand over a single planning horizon. In reality, these parameters may vary dynamically due to market conditions or seasonal effects.
- `Simplified constraints.` Only three categories of resources (materials, labor, equipment) were considered. Other important factors such as energy costs, distribution capacity, or quality requirements were not modeled.
- `Solver dependency.` Although results were consistent across Charnes–Cooper and Hasan–Acharjee methods, both rely on mathematical transformations and solver precision. In larger, more complex cases, numerical stability and computational efficiency may become relevant concerns.

Acknowledging these limitations not only clarifies the scope of this study but also points toward future research directions. Extending the model to multi-period planning, incorporating stochastic demand, or applying the Hasan–Acharjee method to larger datasets across different industries would further validate and expand the practical applicability of fractional programming approaches.

4 Conclusion

This study applied the Hasan–Acharjee method of Linear Fractional Programming (LFP) to optimize production planning at Fajar Bakery, a household-scale enterprise in Medan City. By modeling four decision variables (different cake products) and three resource constraints (raw materials, labor, and equipment), the analysis demonstrated that the Hasan–Acharjee method can be reliably used to reformulate the fractional model into a linear programming structure and solve it efficiently with the simplex method. The optimal solution obtained was a profit-to-cost ratio of $Z^* = 1.388889$, with roll cakes emerging as the most efficient product to produce under the given constraints.

Validation against the classical Charnes–Cooper transformation confirmed identical results, reinforcing the robustness of the Hasan–Acharjee method for fractional optimization problems with positive denominators. Sensitivity analysis revealed that profitability is highly sensitive to raw material and equipment costs, as well as selling price fluctuations, while labor costs exert minimal influence. Trade-off analysis further showed that increasing production capacity can improve profitability, but with diminishing returns beyond 120% capacity.

From a managerial perspective, the study highlights four practical lessons: (1) prioritize high-efficiency products when resources are limited, (2) monitor and negotiate input costs, especially raw materials and equipment, (3) adopt dynamic pricing strategies to safeguard profitability, and (4) expand production capacity cautiously, recognizing the onset of diminishing returns.

Nevertheless, this research has limitations. It is based on a single case study, employs a static one-period model, and simplifies constraints by focusing on only three resource categories. Future research should extend the Hasan–Acharjee method to multi-period, stochastic, or multi-objective formulations, and apply it to a broader range of industries to test generalizability.

In conclusion, the findings confirm the effectiveness of the Hasan–Acharjee method as a practical and theoretically sound approach to fractional programming. Beyond contributing to the optimization literature, the study provides actionable guidance for small and medium-sized enterprises seeking to maximize efficiency under resource limitations.

CRediT Authorship Contribution Statement

Fahliza Adisty: Conceptualization, Methodology, Formal analysis, Writing – Original Draft, Visualization. **Riri Syafitri Lubis**: Supervision, Validation, Writing – Review & Editing.

Declaration of Generative AI and AI-assisted Technologies

During the preparation of this work, the authors used OpenAI's ChatGPT to assist in drafting, editing, and refining sections of the manuscript, particularly in improving the clarity of explanations and restructuring some methodological descriptions. After using this tool, the authors carefully reviewed, verified, and edited the content to ensure accuracy, originality, and alignment with the research findings. All mathematical derivations, results, and analyses were independently conducted by the authors using standard optimization software (e.g., LINGO).

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data and Code Availability

The datasets and codes used and/or analyzed during the current study are not publicly available, but can be obtained from the corresponding author upon reasonable request.

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