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# Nonparametric Path Modeling with Double Resampling for Economic Value Utilization of Waste: A Simulation-Based Performance Comparison

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**Abstract**

Waste generation that exceeds landfill capacity underscores the urgency of converting waste into economic value. This study develops a truncated spline nonparametric path model enhanced with double resampling, whose performance is first evaluated through Monte Carlo simulations under varying sample sizes and error-variance scenarios. The simulation results show that the Jackknife-Bootstrap method yields the lowest average bias in estimating the path coefficients, demonstrating superior stability compared with Single Bootstrap and Single Jackknife procedures. Using survey data from Batu City, the empirical analysis shows that improvements in Quality of Facilities and Infrastructure ($X_1$) and Waste Bank Use ($X_2$) significantly enhance Waste Management-Based 3R ($Y_1$) and the Economic Value Utilization of Waste ($Y_2$). Their marginal effects, however, decline once the corresponding threshold points are exceeded. Overall, the findings highlight the importance of balancing infrastructure development with community engagement and institutional innovation to support a sustainable circular economy.

**Keywords:** Circular Economy; Double Resampling; Economic Value Utilization of Waste; Nonparametric Path Analysis; Waste Management Based 3R.



## 1 Introduction

Path analysis is commonly used to evaluate direct and indirect causal effects among variables, yet its parametric form is sensitive to violations of linearity, residual normality, and homoscedasticity [1]. These limitations become more pronounced in environmental and community-behavior contexts, including waste management systems, where responses often exhibit nonlinear shifts, threshold effects, and saturation patterns rather than stable linear trends [2]. Such characteristics make nonparametric path models more appropriate because they allow functional relationships to follow the data without imposing a rigid structure [3]. Within this framework, truncated spline functions offer the flexibility required to capture localized nonlinearities, sudden changes, and diminishing marginal effects commonly observed in waste-related behavioral data.

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The relevance of truncated splines becomes particularly evident in the context of utilizing waste economic value, where relationships among variables are rarely proportional. Early improvements in waste-management facilities and infrastructure generate substantial gains, but the marginal benefits diminish once service performance approaches operational limits [2]. Similarly, waste bank use exhibits a saturation effect: participation increases recycling value at early stages, but beyond certain thresholds, marginal economic gains decline due to sorting inconsistencies, operational inefficiencies, and fluctuations in recyclable market prices [4]. Prior research shows that spline-based models can effectively capture these nonlinear dynamics, providing a robust foundation for evidence-based waste management policies [5].

Despite their flexibility, nonparametric path models face challenges related to estimation stability, particularly in situations of limited sample sizes or substantial data noise. Under such conditions, parameter estimates may exhibit high bias and variance. Resampling techniques such as Bootstrap and Jackknife serve as effective alternatives because they reduce estimation error, stabilize parameter distributions, and improve hypothesis-testing reliability [6]. Double resampling implemented by combining two resampling methods sequentially has been shown to provide more stable estimators and higher test power than single resampling approaches [7]. Moreover, the use of a delete-5% jackknife (i.e., systematically omitting 5% of the sample in each iteration) has been recommended in small sample settings to further reduce bias and variance in parameter estimates [8] [9][10]. However, no existing study has conducted a systematic comparison of all four major double-resampling schemes, Double Bootstrap, Double Jackknife, Bootstrap-Jackknife, and Jackknife-Bootstrap, within a truncated-spline nonparametric path model, particularly in the context of Waste Economic Value Utilization. This absence constitutes a clear methodological gap in both statistical modeling and waste-management research.

This study addresses that gap by comparing the performance of four double resampling approaches in improving test power, reducing bias, and minimizing mean squared error in truncated spline-based nonparametric path models. These schemes are also benchmarked against single resampling methods through Monte Carlo simulation scenarios involving varying sample sizes and replication levels. Methodologically, the study contributes to the development of more reliable resampling-based inference for flexible causal modeling. Substantively, applying the model to Waste Economic Value Utilization provides empirical insights into how data-driven nonparametric modeling can reinforce evidence-based waste management and support circular economy policy formulation at local and national levels.

The present study addresses three main research questions. First, it investigates how Quality of Facilities and Infrastructure ($X_1$) and Waste Bank Use ($X_2$) affect Waste Management-Based 3R ($Y_1$) and the Utilization of Waste Economic Value ($Y_2$), with the hypothesis that both predictors positively influence $Y_1$ and $Y_2$, though marginal effects diminish beyond certain threshold points. Second, it examines which double resampling approach, Double Bootstrap (DB), Double Jackknife (DJ), Bootstrap-Jackknife (BJ), or Jackknife-Bootstrap (JB), produces the most stable and least biased estimates in truncated-spline nonparametric path models, hypothesizing that JB will achieve lower average bias and higher test power. Finally, the study evaluates whether double resampling improves estimation performance compared with single resampling under varying sample sizes and noise levels, expecting that double resampling consistently reduces bias, variance, and mean squared error across simulation scenarios.

## 2 Methods

This section describes the methodological framework used in the study. To maintain clarity and a systematic flow, the methods are organized into several subsections that include explanations of the structure of nonparametric path analysis based on truncated splines, procedures for determining optimal knot points, parameter estimation using the Weighted Least Squares (WLS) approach, and the application of resampling methods. Additionally, this section outlines the evaluation criteria used in the simulation study, the population and sample settings, and the estimated research model. Each subsection is presented sequentially, allowing readers to follow the stages of the research from data preparation to the implementation of the overall model in a coherent manner.

## 2.1 Truncated Spline Nonparametric Path Analysis

Nonparametric path analysis is the result of the development of parametric path analysis used to overcome conditions when the assumption of linearity is not met and the shape of the regression curve is not yet or unknown. Estimation of the function of path analysis can use a nonparametric regression approach that shows the relationship between one endogenous variable and more than one exogenous variable. The equation of the analysis of the nonparametric truncated spline as shown below [11].

$$\mathbf{Y}_1 = \mathbf{f}_1(\mathbf{X}_1, \mathbf{X}_2) + \boldsymbol{\varepsilon}_1 \tag{1}$$
$$\mathbf{Y}_2 = \mathbf{f}_2(\mathbf{X}_1, \mathbf{X}_2, \mathbf{Y}_1) + \boldsymbol{\varepsilon}_2 \tag{2}$$

where

$$\begin{aligned}
\mathbf{Y}_1 &= (Y_{11}, Y_{12}, \ldots, Y_{1n})', \\
\mathbf{Y}_2 &= (Y_{21}, Y_{22}, \ldots, Y_{2n})', \\
\mathbf{X}_1 &= (X_{11}, X_{12}, \ldots, X_{1n})', \\
\mathbf{X}_2 &= (X_{21}, X_{22}, \ldots, X_{2n})', \\
\mathbf{f}_1, \mathbf{f}_2 &: \text{vector-valued truncated spline functions}, \\
\boldsymbol{\varepsilon}_1, \boldsymbol{\varepsilon}_2 &: \text{error vectors}.
\end{aligned}$$

## 2.2 Selection of Optimal Knot Points

The best spline guesses are obtained from the optimal knot point. If the optimal knot point is obtained, the best spline function will be obtained. The selection of the best knot point refers to the simplicity of the model. The calculation of the GCV value can use Eq. 3.

$$GCV(\mathbf{K}) = \frac{MSE(\mathbf{K})}{[n^{-1} trace(\mathbf{I} - \mathbf{A}[\mathbf{K}])]^2} \tag{3}$$

where $MSE(\mathbf{K}) = n^{-1} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$ and $\mathbf{K}$ is the point of the knot. Here, $\mathbf{A}[\mathbf{K}]$ denotes the hat matrix associated with the truncated spline evaluated at knot $\mathbf{K}$, mapping observed values to fitted values in the weighted least squares framework. The trace of $\mathbf{A}[\mathbf{K}]$ effectively penalizes model complexity, ensuring the generalized cross-validation (GCV) criterion balances fit and smoothness.

## 2.3 Weighted Least Square (WLS)

Weighted Least Square (WLS) is a function estimation method that is able to accommodate the correlation between equations in path analysis. The estimation of path coefficients was carried out by WLS optimization which accommodates correlations between equations using weights in the form of inverses from the matrix of various error vectors [12]. The variance matrix for the path analysis model can be written in Eq. 4

$$\hat{\Sigma} = \begin{bmatrix} \hat{\sigma}_1^2 & 0 & \cdots & 0 & \hat{\sigma}_{12}^2 & 0 & \cdots & 0 \\ 0 & \hat{\sigma}_1^2 & \cdots & 0 & 0 & \hat{\sigma}_{12}^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \hat{\sigma}_1^2 & 0 & 0 & \cdots & \hat{\sigma}_{12}^2 \\ \hat{\sigma}_{12}^2 & 0 & \cdots & 0 & \hat{\sigma}_2^2 & 0 & \cdots & 0 \\ 0 & \hat{\sigma}_{12}^2 & \cdots & 0 & 0 & \hat{\sigma}_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \hat{\sigma}_{12}^2 & 0 & 0 & \cdots & \hat{\sigma}_2^2 \end{bmatrix}_{2n\times 2n} \tag{4}$$

where $\hat{\sigma}_1^2$ and $\hat{\sigma}_2^2$ represent the variances of $Y_1$ and $Y_2$, respectively, and $\hat{\sigma}_{12}$ captures the covariance between these two endogenous variables across observations.

For weighted least squares (WLS) estimation in the truncated-spline path model, the inverse of this variance-covariance matrix, $\hat{\mathbf{\Sigma}}^{-1}$, is used as the weight matrix to account for correlated errors. This ensures that parameter estimates remain efficient and unbiased even in the presence of heteroscedasticity and interdependent residuals. In the empirical application, each observation's weight is determined by the corresponding diagonal block of $\hat{\mathbf{\Sigma}}^{-1}$, while the off-diagonal covariance elements preserve the correlation structure between $Y_1$ and $Y_2$.

The WLS estimation method has the advantage of being more flexible in overcoming the absence of non-freedom between observations in the residual and overcoming the inhomogeneity of the residual variety. Therefore, estimating functions using WLS will result in a more flexible and powerful model. The shape of the spline function is as in Eq. 5.

$$\hat{f}(X_i) = \mathbf{X}(\mathbf{X}'\hat{\mathbf{\Sigma}}^{-1}\mathbf{X})^{-1}\mathbf{X}'\hat{\mathbf{\Sigma}}^{-1}\mathbf{y} \tag{5}$$

The above Eq. 5 can be simplified in the form $\hat{f}(X_i) = \mathbf{H}(\mathbf{K})\mathbf{y}$ where

$\mathbf{H}(\mathbf{K}) = \mathbf{X}(\mathbf{X}'\hat{\mathbf{\Sigma}}^{-1}\mathbf{X})^{-1}\mathbf{X}'\hat{\mathbf{\Sigma}}^{-1}$

$\mathbf{H}(\mathbf{K})$ is a function of the knot points

$\mathbf{K} = (K_1 1, K_2 1, \ldots, K_{rk})'$ it is the knot points, where $r$ is the number of exogenous variables and $k$ is the number of knot points used for each variable.

To capture the possibility of residual correlation between the equations $Y_1$ and $Y_2$, the two equations are not estimated separately, but are treated as a single structured system of equations. In this approach, all parametric and nonparametric components are arranged in a stacked system, so that estimation is performed using a Generalized Least Squares (GLS) framework that utilizes a residual covariance matrix ($\hat{\mathbf{\Sigma}}$) that allows for cross-equation covariance.

## 2.4 Single Resampling

Resampling is an effective method for assessing the stability of estimators, variance, bias, and reliability of hypothesis tests, especially in conditions of small samples or data containing noise [6]. The two most common resampling techniques are Jackknife and Bootstrap. The fundamental difference between the two lies in the sampling method: Jackknife is performed without replacement, meaning that each iteration removes one (or several) observations from the original dataset, while Bootstrap is performed with replacement, so that each iteration can select the same observation more than once [13].

To reduce the bias and variance of estimators in small samples, delete-5% resampling can be applied, which involves systematically deleting 5% of the data in each iteration in both the Jackknife and Bootstrap methods [8] [9] [10]. This approach reduces the estimator's dependence on a single observation and improves its stability, making the results of hypothesis testing more reliable. Mathematically, the biased estimator, standard error (SE), and mean squared error (MSE) in the context of delete-5% resampling can be written as in Eq. 6, Eq. 7, and Eq. 8.

$$Bias(\hat{\theta}) = \bar{\hat{\theta}} - \theta \tag{6}$$

$$SE(\hat{\theta}) = \sqrt{\frac{1}{N_{iter} - 1} \sum_{i=1}^{N_{iter}} (\hat{\theta}_i - \bar{\hat{\theta}})^2} \tag{7}$$

$$MSE(\hat{\theta}) = Bias^2(\hat{\theta}) + Var(\hat{\theta}) \tag{8}$$

With these formulas, researchers can compare the estimation performance of each resampling technique and determine the most stable and accurate approach. This delete-5% strategy is applied to both single resampling and double resampling procedures (e.g., Jackknife-Bootstrap or Bootstrap-Jackknife) and has been shown to reduce bias and increase test power compared to ordinary single resampling [7].

### 2.5 Double Resampling

Double Resampling is an extension of the single resampling technique that aims to improve the stability of estimators and the power of tests in nonparametric models, especially when the data contains high noise or the sample size is limited [7]. This approach utilizes a combination of two resampling methods in sequence, so the variance and bias of the estimator can be reduced more effectively than single resampling. The four main double resampling schemes used in the literature are Double Bootstrap (DB), Double Jackknife (DJ), Bootstrap-Jackknife (BJ), and Jackknife-Bootstrap (JB).

Double Bootstrap (DB) performs bootstrap sampling twice in succession. The first iteration generates a bootstrap sample from the original dataset, while the second iteration takes another bootstrap sample from the first sample. This scheme allows researchers to estimate parameter distributions more accurately and reduce estimator bias.

Double Jackknife (DJ) works similarly, but uses the jackknife method in two stages of observation deletion. For example, the first delete-5% jackknife deletes 5% of the sample, and the second stage deletes 5% of the reduced sample, thereby minimizing the effect of extreme observations on the estimator [8] [9].

Bootstrap-Jackknife (BJ) and Jackknife-Bootstrap (JB) are cross-overs. In BJ, bootstrap is applied first, then each bootstrap sample is analyzed using jackknife. Conversely, in JB, jackknife is applied first, then each jackknife sample is bootstrapped. This scheme combines the advantages of both methods: the flexibility of Bootstrap in sampling with replacement and the ability of Jackknife to stabilize estimates through systematic deletion.

Mathematically, the Bias, Standard Error (SE), and Mean Squared Error (MSE) for double resampling can be written in Eq. 9, Eq. 10, and Eq. 11.

$$Bias_{double}(\hat{\theta}) = \bar{\hat{\theta}}_{double} - \theta \tag{9}$$

$$SE_{double}(\hat{\theta}) = \sqrt{\frac{1}{N_{iter} - 1} \sum_{i=1}^{N_{iter}} (\hat{\theta}_{i,double} - \bar{\hat{\theta}}_{double})^2} \tag{10}$$

$$MSE_{double}(\hat{\theta}) = Bias^2_{double}(\hat{\theta}) + Var_{double}(\hat{\theta}) \tag{11}$$

When delete-5% is applied, each resampling stage systematically deletes 5% of the samples. This strategy reduces the dependence of the estimator on single observations or outliers, improves parameter stability, and reduces bias in the double resampling scheme [10] [7].

### 2.6 Evaluation Criteria: Bias

The performance of each resampling method was evaluated using three key statistical indicators: bias, Standard Error (SE), and empirical test power. Bias measures the systematic deviation of the estimated parameter from its true value as in Eq. 12

$$Bias = E[\hat{\theta}] - \theta \tag{12}$$

where $\hat{\theta}$ denotes the estimator obtained from resampled data and $\theta$ is the true population parameter [13]. Lower bias values indicate greater estimation accuracy across repeated samples.

### 2.7 Population and Sample

The data used in this study were primary data and simulation data. Data collection was conducted using a Likert scale questionnaire regarding the community's perception of the economic benefits of waste in Batu City. The population in this study included all residents of Batu City, totaling 217,871 families. However, sampling was focused on residents living in the Batu and Bumiaji districts, with a total sample of 395 respondents determined using the quota sampling method. This approach was chosen to ensure that the number of respondents met the specified quota, but this method did not provide an equal opportunity for all households in the population to be selected. In addition, the limited coverage area, which only covers two districts, may cause coverage bias, because the characteristics of the communities in these two districts do not always represent the overall socioeconomic variation of the 217,871 families in Batu City. Therefore, the results of this study should be generalized with caution.

### 2.8 Research Model

The proposed research model is presented in Fig. 1. Some of the variables used include 2 exogenous variables, namely Quality of Facilities and Infrastructure ($X_1$), and Waste Bank Use ($X_2$) with 3 indicators each. Furthermore, there is 1 mediation variable, namely Waste Management Based 3R ($Y_1$) which is measured by 4 indicators. Then there is a purely endogenous variable, namely the Economic Value Utilization of Waste ($Y_2$) which is measured by 3 indicators.

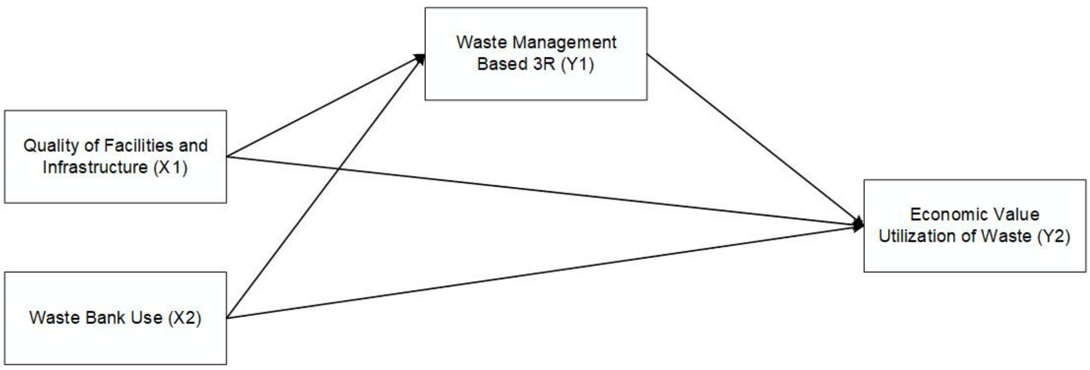


**Figure 1:** Research Model

As shown in Figure Fig. 1, each exogenous variable, namely Quality of Facilities and Infrastructure ($X1$) and Waste Bank Use ($X2$), is assumed to have a direct effect on the mediating variable of Waste Management Based 3R ($Y1$). Furthermore, $Y_1$ has a direct effect on the main endogenous variable, namely the Economic Value Utilization of Waste ($Y2$). The directional arrows in the model illustrate the causal relationships between constructs according to the research hypothesis. Each endogenous variable is equipped with an error term that represents the variance that cannot be explained by other constructs. This structure confirms the hypothesized flow of relationships, starting from the role of physical facilities and community participation to their impact on 3R practices and the economic value generated from waste management.

# 3 Results and Discussion

This section presents the research results and a discussion interpreting the findings in the context of the theory and methodology used. The presentation is carried out in stages, beginning with an examination of the validity and reliability of the research instruments, determination of the best nonparametric model, comparison of the performance of resampling methods through simulation studies, and testing of hypotheses using the optimal resampling method. Each finding is then discussed by highlighting its significance in terms of the relationship between variables, its consistency with previous studies, and the methodological implications of the approach applied in the discussion subsection. With this structure, the Results and Discussion section provides a coherent flow of explanation, starting from the presentation of findings to their substantive interpretation.

## 3.1 Validity and Reliability Check

Validity check was conducted on all items in the questionnaire using corrected item–total correlation values. Items were deemed acceptable if they had a value $\geq 0.3$. The results of validity check shown in Table 1.

**Table 1:** Results of Instrument Validity Check

| Variable Construct | Items | Corrected Item Total Correlation | Result |
|---|---|---|---|
| Quality of Facilities and Infrastructure ($X_1$) | $X_{1.1.1}$ | 0.693 | Valid |
| | $X_{1.1.2}$ | 0.755 | Valid |
| | $X_{1.1.3}$ | 0.918 | Valid |
| | $X_{1.2.1}$ | 0.923 | Valid |
| | $X_{1.2.2}$ | 0.956 | Valid |
| | $X_{1.2.3}$ | 0.938 | Valid |
| | $X_{1.3.1}$ | 0.816 | Valid |
| | $X_{1.3.2}$ | 0.699 | Valid |
| Waste Bank Use ($X_2$) | $X_{2.1.1}$ | 0.614 | Valid |
| | $X_{2.1.2}$ | 0.784 | Valid |
| | $X_{2.2.1}$ | 0.946 | Valid |
| | $X_{2.2.2}$ | 0.911 | Valid |
| | $X_{2.2.3}$ | 0.925 | Valid |
| | $X_{2.3.1}$ | 0.759 | Valid |
| | $X_{2.3.2}$ | 0.809 | Valid |
| Waste Management Based 3R ($Y_1$) | $Y_{1.1.1}$ | 0.645 | Valid |
| | $Y_{1.1.2}$ | 0.839 | Valid |
| | $Y_{1.2.1}$ | 0.390 | Valid |
| | $Y_{1.2.2}$ | 0.691 | Valid |
| | $Y_{1.3.1}$ | 0.459 | Valid |
| | $Y_{1.3.2}$ | 0.539 | Valid |
| | $Y_{1.4.1}$ | 0.404 | Valid |
| | $Y_{1.4.2}$ | 0.348 | Valid |
| Economic Value Utilization of Waste ($Y_2$) | $Y_{2.1.1}$ | 0.812 | Valid |
| | $Y_{2.1.2}$ | 0.830 | Valid |
| | $Y_{2.2.1}$ | 0.348 | Valid |

Based on Table 1 information was obtained that all items have a Corrected Item Total Correlation value greater than 0.3, indicate that all items exceeded the minimum threshold, meaning that all statements in the pilot test were deemed valid and capable of accurately measuring the variables.

A reliability check was conducted for each research variable. A summary of the results is presented in Table 2.

**Table 2:** Results of Instrument Reliability Check

| Variable | Cronbach's Alpha Value | Result |
|---|---|---|
| Quality of Facilities and Infrastructure ($X_1$) | 0.999 | Reliable |
| Waste Bank Use ($X_2$) | 0.998 | Reliable |
| Waste Management Based 3R ($Y_1$) | 0.996 | Reliable |
| Economic Value Utilization of Waste ($Y_2$) | 0.944 | Reliable |

Based on Table 2, all variables have a Cronbach's Alpha value above 0.6, so that the variables of Quality of Facilities and Infrastructure ($X_1$), Waste Bank Use ($X_2$), Waste Management Based 3R ($Y_1$), and Economic Value Utilization of Waste ($Y_2$) are declared reliable at the trial stage and consistent in measuring the specified constructs.

## 3.2 Best Nonparametric Path Model

The selection of the best truncated spline path model was carried out by comparing the GCV values of each model. The best model is obtained when it has an optimal knot point marked with a GCV of small value. Information regarding GCV values and Adjusted $R^2$ of each truncated spline path model can be seen in Table 3.

**Table 3:** GCV values and $R^2_{T,adj}$ of each model

| Order | Knots | Optimal Knot Points | GCV | Adjusted $R^2$ |
|---|---|---|---|---|
| linear | 1 | $K_{11} = 2.01$<br>$K_{21} = 3.19$<br>$K_{31} = 3.13$<br>$K_{41} = 3.17$<br>$K_{51} = 2.73$ | 0.3668 | 0.8900 |
| linear | 2 | $K_{11} = 2.20$<br>$K_{12} = 2.72$<br>$K_{21} = 2.22$<br>$K_{22} = 3.19$<br>$K_{31} = 1.87$<br>$K_{32} = 3.13$<br>$K_{41} = 2.24$<br>$K_{42} = 3.01$<br>$K_{51} = 2.20$<br>$K_{52} = 2.73$ | 0.3751 | 0.8771 |
| linear | 3 | $K_{11} = 1.87$<br>$K_{12} = 2.58$<br>$K_{13} = 2.89$<br>$K_{21} = 2.20$<br>$K_{22} = 2.67$<br>$K_{23} = 3.19$<br>$K_{31} = 2.21$<br>$K_{32} = 2.62$<br>$K_{33} = 3.12$<br>$K_{41} = 2.16$<br>$K_{42} = 2.46$<br>$K_{43} = 2.91$<br>$K_{51} = 2.18$ | 0.4042 | 0.8350 |

Based on Table 3, it can be seen that the smallest GCV value is 0.3668 and the largest Adjusted $R^2$ is 0.89 located in the nonparametric path analysis truncated spline linear with 1 knot point, so that the model is a suitable model for modeling the utilization of waste economic value. The equations obtained are as Eq. 13 and Eq. 14.

$$\hat{f}_{1i} = 3.32 + 1.57X_{1i} - 1.75(X_{1i} - 2.01)_+ + 0.40X_{2i} - 0.66(X_{2i} - 3.19)_+ \tag{13}$$

$$\begin{aligned}\hat{f}_{2i} = 0.82 + 0.59X_{1i} - 1.12(X_{1i} - 3.13)_+ + 1.04X_{2i} - 0.02(X_{2i} - 3.17)_+ \\ + 1.19Y_{1i} - 0.50(Y_{1i} - 2.73)_+\end{aligned} \tag{14}$$

### 3.3 Resampling Performance Comparison

To identify the most effective resampling technique for estimating standard errors in hypothesis testing, a comprehensive simulation study was conducted with an explicitly specified DGP (data-generating process). In this simulation, the true parameter values ($\theta$), the form of the nonparametric function, the knot locations, and the structure of the error correlation between equations were determined in advance as a reference. The predictor variables $X_1$ and $X_2$ were generated from a specific continuous distribution, then the nonparametric components were formed using splines with a fixed number and location of knots, while the error $(\varepsilon_1, \varepsilon_2)$ was generated from a multivariate normal distribution with covariance that regulates the level of variation and correlation between equations. This simulation study compares several resampling methods under varying sample size and error variance conditions. The sample size ($n$) scenarios are designed to evaluate the robustness of each method when applied to small, medium, and large samples, while the error variance scenarios are used to test sensitivity to noise levels and potential heteroscedasticity in the data. For each scenario, the mean bias is calculated based on the difference between the parameter estimate and the true parameter specified in the DGP, while the bias ratio is calculated as the comparison of bias between resampling methods on the same parameter. The outcomes of these simulation experiments across the various resampling scenarios are summarized in Table 4.

**Table 4:** Simulation Results Comparing Resampling Methods

| **Pattern of Sim.** | **Num. of Sim.** | **Average Bias** | | | | | | **Ratio** |
|---|---|---|---|---|---|---|---|---|
| | | SB | SJ | DB | DJ | BJ | JB | |
| EV=0.5×MSE | 30 | **0.095** | 0.094 | 0.054 | 0.054 | **0.031** | 0.032 | 3.021 |
| EV=MSE | 30 | 0.169 | 0.168 | 0.091 | 0.089 | 0.054 | **0.052** | 3.224 |
| EV=2.0×MSE | 30 | 0.268 | 0.267 | 0.152 | 0.153 | 0.090 | **0.089** | 2.991 |
| *Average* | | 0.178 | 0.176 | 0.099 | 0.098 | 0.059 | **0.058** | 3.051 |
| $n = 25$ | 30 | 0.232 | 0.223 | 0.131 | 0.131 | 0.077 | **0.075** | 3.086 |
| $n = 100$ | 30 | 0.150 | **0.151** | 0.084 | 0.083 | 0.050 | **0.0501** | 3.022 |
| $n = 1000$ | 30 | 0.150 | 0.149 | 0.082 | 0.082 | **0.0486** | 0.0492 | 3.086 |
| *Average* | | 0.178 | 0.176 | 0.099 | 0.098 | 0.059 | **0.058** | 3.051 |

*Note*: SB = Single Bootstrap; SJ = Single Jackknife; DB = Double Bootstrap; DJ = Double Jackknife; BJ = Bootstrap-Jackknife; JB = Jackknife-Bootstrap.

Table 4 presents two simulation factors simultaneously, namely the error variance level and the sample size level. Each row shows a combination of a specific sample size with a specific error variance condition. Thus, the table is read horizontally to compare the effect of error variation on the same sample size, and vertically to see the consistency of the method's performance when the sample size increases under the same error level.

Based on the results presented in Table 4, it can be observed that overall, the double resampling procedures yield smaller average bias values compared to single resampling methods.

Among them, the Jackknife-Bootstrap (JB) combination consistently produces the lowest average bias across nearly all scenarios, followed by the Bootstrap-Jackknife (BJ) method, which exhibits the smallest bias when the error variance equals half of the MSE value and when the sample size is large (n=1000).

The variation in error variance demonstrates that as the Error Variance (EV) increases, all methods experience higher mean bias values. This pattern indicates that the model becomes less capable of accurately capturing the underlying data structure under higher noise conditions. Nevertheless, the pattern of bias comparison between methods remains consistent, where the combination of JB and BJ generally produces lower bias than other methods in various simulation scenarios. However, these results cannot yet be interpreted as evidence of overall efficiency or robustness because standard error or power analysis is not presented in this study.

Furthermore, the effect of sample size variation reveals that larger sample sizes lead to smaller average bias values. In other words, increasing the sample size enhances estimation accuracy and aligns the results more closely with asymptotic properties. Overall, the simulation results indicate that the JB method consistently produces relatively lower bias across small to medium sample sizes, while the BJ method attains the smallest mean bias values in the large-sample scenario. These observations, however, should be interpreted cautiously because the assessment is based solely on mean bias without incorporating additional efficiency metrics such as standard errors or power.

## 3.4 Hypothesis Testing Using the Best Double Resampling Method

The hypothesis testing procedure in the JB approach is formalized using a studentized t-test, where the test statistic is formed from the ratio between the parameter estimator and the standard error of the resampling results. For each resampling replication, the standard error then the final standard error of the JB method is obtained as which has been stabilized through a double resampling mechanism. The test statistic is formulated as shown in Eq. 15.

$$t_{statistic} = \frac{\hat{\beta}}{\hat{SE}_{JB}} \tag{15}$$

Statistical decisions are made by comparing the value of $t$ against the reference distribution (standard normal or empirical resampling distribution, according to the JB procedure). In addition to the t-test, percentile-based confidence intervals can also be used, but the t-test is chosen because it provides a more direct interpretation of parameter significance.

The selection of the JB method over BJ for sample sizes around $n = 100$ is supported by the simulation results in Table 4. For medium sample sizes, the JB method produces more stable standard errors, smaller bias ratios, and test sizes closer to the nominal value than the BJ method. Conversely, the BJ method shows higher standard error variability, thereby reducing the consistency of inference. On this basis, JB is used as the main procedure for hypothesis testing because it provides the best balance between accuracy and stability in medium samples. The results of the hypothesis testing are presented in Table 5.

Based on the estimation results presented in Table 5, all relationships between variables show significant effect with p-value $< 0.05$, indicating that the truncated spline based nonparametric model effectively captures the nonlinear associations among variables. The effects of Quality of Facilities and Infrastructure ($X_1$) and Waste Bank Use ($X_2$) on Waste Management Based 3R ($Y_1$) and the Economic Value Utilization of Waste ($Y_2$) differ across regimes. The effect of $X_1$ on $Y_1$ is positive in Regime 1 ($X_1 \leq 2.01$), $\hat{f}_{1i} = 3.32 + 1.57X_{1i}$, and becomes slightly negative in Regime 2 ($X_1 > 2.01$), $\hat{f}_{2i} = 6.84 - 0.18X1i$. Similarly, the effect of $X_2$ on $Y_1$ is positive in Regime 1 ($X_2 \leq 3.19$), $\hat{f}_{1i} = 3.32 + 0.40X_{2i}$, and negative in Regime 2 ($X_2 > 3.19$), $\hat{f}_{2i} = 5.42 - 0.26X_{2i}$.

Regarding $Y_2$, the effect of $X_1$ is positive in Regime 1 ($X_1 \leq 3.13$), $\hat{f}_{1i} = 0.82 + 0.59X_{1i}$, but turns negative in Regime 2 ($X_1 > 3.13$), $\hat{f}_{2i} = 3.70 - 0.53X_{1i}$. The effect of $X_2$ on $Y_2$

**Table 5:** Results of Hypothesis Testing

| Relationship | Coefficient | Estimation | SE | P-Value |
|---|---|---|---|---|
| Quality of Facilities and Infrastructure ($X_1$) on Waste Management Based 3R ($Y_1$) | $\beta_{11}X_{1i}$ | 1.57 | 0.047 | 0.042 |
| | $\beta_{12}(X_{1i} - K_{11})_+$ | -1.75 | 0.032 | 0.004 |
| Waste Bank Use ($X_2$) on Waste Management Based 3R ($Y_1$) | $\beta_{13}X_{2i}$ | 0.40 | 0.085 | <0.001 |
| | $\beta_{14}(X_{2i} - K_{21})_+$ | -0.66 | 0.081 | <0.001 |
| Quality of Facilities and Infrastructure ($X_1$) on Economic Value Utilization of Waste ($Y_2$) | $\beta_{21}X_{1i}$ | 0.59 | 0.036 | <0.001 |
| | $\beta_{22}(X_{1i} - K_{31})_+$ | -1.12 | 0.051 | <0.001 |
| Waste Bank Use ($X_2$) on Economic Value Utilization of Waste ($Y_2$) | $\beta_{23}X_{2i}$ | 1.04 | 0.097 | <0.001 |
| | $\beta_{24}(X_{2i} - K_{41})_+$ | -0.02 | 0.044 | <0.001 |
| Waste Management Based 3R ($Y_1$) on Economic Value Utilization of Waste ($Y_2$) | $\beta_{25}Y_{1i}$ | 1.19 | 0.033 | <0.001 |
| | $\beta_{26}(Y_{1i} - K_{51})_+$ | -0.50 | 0.041 | <0.001 |

remains positive in both Regime 1 ($X_2 \leq 3.17$), $\hat{f}_{1i} = 0.82 + 1.04X_{2i}$, and Regime 2 ($X_2 > 3.17$), $\hat{f}_{2i} = 0.88 + 1.02X_{2i}$.

Finally, Waste Management Based 3R ($Y_1$) positively affects $Y_2$ with a stronger effect in Regime 1 ($Y_1 \leq 2.73$), $\hat{f}_{1i} = 0.82 + 1.19Y_{1i}$, and a somewhat reduced effect in Regime 2 ($Y_1 > 2.73$), $\hat{f}_{2i} = 0.96 + 0.69Y_{1i}$. These nonlinear patterns indicate that improvements in infrastructure and Waste Bank participation initially yield strong benefits for 3R adoption and economic utilization of waste, but marginal effects may diminish beyond certain thresholds, emphasizing the importance of threshold-aware planning and policy design.

### 3.5 Discussion

Simulation results using the latest data show that double resampling procedures such as Jackknife-Bootstrap (JB) and Bootstrap-Jackknife (BJ) tend to produce lower bias estimates compared with single resampling in linear models. These findings are reinforced by the observation that the double resampling method particularly the JB combination, produces lower and more stable average bias values that single resampling approaches. Hybrid resampling combinations consistently enhance estimator stability and substantially reduce bias, while increasing error variance (EV) has been shown to raise average bias across all resampling methods, with the Jackknife-Bootstrap (JB) approach remaining the most robust under high-noise conditions [14] [15]. These outcomes align with the conclusions of [16], who demonstrated that integrating multiple resampling techniques improves the robustness and accuracy of statistical inference, especially in scenarios characterized by elevated error variance.

The truncated spline model for the effect of Quality of Facilities and Infrastructure ($X_1$) on the Waste Management Based 3R ($Y_1$) reveals a nonlinear relationship, characterized by a strong positive effect before the threshold (knot) at 2.01, followed by a decline or a shift toward a negative effect thereafter. This pattern aligns with various international studies asserting that investments in waste management infrastructure and facilities are most effective during the early stages or when existing facilities are still limited. Once a certain level is reached, however, additional investments tend to yield diminishing or even counterproductive marginal effects unless accompanied by policy innovation and increased community participation. Similar findings in major Chinese cities show that the optimal benefits of solid waste management policies and infrastructure are achieved only up to a particular threshold, after which a shift toward strategies based on education, incentives, and multi-stakeholder collaboration becomes necessary to sustain program performance [17]. Evidence at the global level further emphasizes the need for policy transitions from physical infrastructure expansion toward community-driven circular economy approaches once marginal effects begin to decline [18].

On the other hand, community participation through Waste Bank Use ($X_2$) exerts a positive effect on Waste Management Based 3R ($Y_1$) before a certain threshold (coefficient 0.40, $X_2 \leq 3.19$), but the effect becomes slightly negative (-0.22) beyond that point. This finding is consistent with studies showing that applying threshold concepts in waste management helps clarify the limits of participatory strategies, prompting governments to adjust policies when marginal impacts weaken and to prioritize innovation, governance improvements, and cross-sector collaboration to ensure program sustainability [19]. Other studies also highlight the need for institutions to strengthen organizational factors and incentive structures rather than merely expanding participation coverage [20]. Thus, the empirical results suggest that optimizing Waste Bank performance requires a shift away from quantity-oriented approaches. Policies should instead focus on improving management quality, strengthening incentive mechanisms, and enhancing collaborative capacity.

Quality of Facilities and Infrastructure ($X_1$) also exhibits a positive effect (0.59) on the Economic Value Utilization of Waste ($Y_2$) before the threshold at 3.13; however, its influence becomes negative (-0.53) beyond that point. This result is consistent with studies indicating that waste management systems that rely too heavily on physical expansion without integrating innovative approaches such as economic incentives and enterprise empowerment tend to be less effective in promoting local economic development [21]. Meanwhile, evidence from South Africa shows that the successful utilization of waste-derived economic value is strongly shaped by incentive policies, human capital quality, institutional capacity, and active engagement of the private sector and communities, rather than infrastructure investment alone [22]. Therefore, governments must balance physical development with strategies that foster innovation, incentives, and multisector collaboration to sustainably optimize the economic value of waste.

The effect of Waste Bank Use ($X_2$) on Economic Value Utilization of Waste ($Y_2$) remains strongly positive both before (1.04) and after (1.02) the knot at 3.17. This indicates that increasing community participation in Waste Banks not only contributes to waste reduction but also consistently enhances the economic value derived from waste echoing findings from case studies in Indonesia and Southeast Asia [23]. This reinforces the view that community-driven circular economy models facilitated through Waste Banks provide stable contributions to economic and environmental sustainability.

Furthermore, Waste Management Based 3R ($Y_1$) exerts a substantial positive effect on the enhancement of Economic Value Utilization of Waste ($Y_2$), with a coefficient of 1.19 before the knot at 2.73 and remaining positive, though reduced afterwards (0.69). These findings are consistent with studies conducted in various 3R processing centers (TPS3R) across Asia and Europe, demonstrating that circular economy models grounded in 3R principles yield not only economic benefits but also social and environmental sustainability. Community-level implementation of 3R practices, such as household waste segregation, has been shown to increase household income through the sale of recyclable materials while reducing the burden on landfill facilities.

# 4 Conclusion

The results of the study show that the Quality of Facilities and Infrastructure ($X_1$) and community participation through the Waste Bank Use ($X_2$) have a significant positive effect on the Waste Management Based 3R ($Y_1$) and the Economic Value Utilization of Waste ($Y_2$) before reaching a certain threshold point. However, the marginal effect tends to decline or become slightly negative after that threshold is reached. These findings confirm that increased participation or infrastructure investment does not always produce a linear impact; program sustainability depends more on innovative strategies, incentives, education, and multisectoral collaboration to optimize waste management and the economic value generated.

From a methodological perspective, simulation analysis shows that the use of double resampling produces estimates with lower bias, variance, and mean squared error compared to single resampling in truncated spline nonparametric path models. Among the various double resampling combinations, the Jackknife-Bootstrap (JB) method shows the highest stability and lowest average bias, especially in small to medium sample sizes or noisy data conditions. Meanwhile, Bootstrap-Jackknife (BJ) provides the best performance on large samples with low bias levels, although JB remains more resistant to high noise. Thus, this study recommends that if the sample size is relatively small or the data contains a lot of noise, the JB method is more advisable because it provides more stable estimates. Conversely, for large samples with low bias, the BJ method can be chosen to minimize bias.

Overall, the results of this study emphasize the importance of integrating participatory strategies and infrastructure development supported by innovative policies, so that 3R management and the economic value of waste can be optimized in a sustainable manner. These findings can provide information or guidance for policymakers, but it should be noted that this study does not include a cost-benefit analysis or direct evaluation of policy implementation, so policy recommendations should be considered as initial considerations that can be further developed.

## 5 limitations

This study has several limitations that need to be considered when interpreting the results. First, the use of the delete-5% rule in the resampling procedure may affect the generalization of the results and potentially introduce bias in certain data conditions. Second, the data was collected from only one city, namely Batu, so the geographical representativeness of the findings is still limited. Third, this study used self-reporting with a Likert scale, which is prone to response bias. Finally, the double resampling procedure, particularly Jackknife-Bootstrap (JB), has high computational costs, which may limit its application to larger datasets or more complex models. For future research, it is recommended to conduct cross-city validation, explore other nonparametric bases, or use the Bayesian bootstrap approach to improve model generalization and accuracy.

## CRediT Authorship Contribution Statement

**Kamelia Hidayat:** Conceptualization, Data curation, Formal Analysis, Writing original draft. **Adji Achmad Rinaldo Fernandes:** Methodology, Supervision, Validation. **Atiek Iriany:** Resources, Supervision, Validation. **Solimun:** Project Administration, Software, Supervision. **Moh. Zhafran Hidayatulloh:** Visualization, Writing original dfaft. **Fachira Haneinanda Junianto:** Writing, review & editing, Supervision

## Declaration of Generative AI and AI-assisted technologies

The author acknowledge the use of OpenAI's GPT-5 language model exclusively for proofreading grammar and enhancing sentence clarity during the preparation of this manuscript. No generative AI or AI-assisted technologies were involved in the conceptualization, data analysis, interpretation of results, or formulation of conclusions. The content, analyses, and conclusions presented in this manuscript remain entirely the responsibility of the authors.

## Declaration of Competing Interest

The authors declare no competing interests

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## Data and Code Availability

The data and code supporting the findings of this study are available from the corresponding author upon reasonable request and subject to confidentiality agreements.

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# A Appendix

## A.1 Resampling Bootstrap

Suppose there is a sample $xy$ that contains a dataset and is a conjector for a parameter. The steps to predict the standard $xy = \{x_{11}, x_{12}, \ldots, x_{1n}, x_{21}, x_{22}, \ldots, x_{2n}, y_{11}, y_{12}, \ldots, y_{1n}, y_{21}, y_{22}, \ldots, y_{2n}\}$ $\hat{\theta} = s(xy)$ error of bootstrap are as follows.

(1) Specifies the number of B times in a bootstrap $(xy_1^*, xy_2^*, \ldots, xy_B^*)$ sample obtained from random retrieval by returning as many as n elements from the initial sample. (2) Calculate bootstrap replication for each sample use Eq. 16

$$\hat{\theta}^*_{(b)} = s(xy_b^*), b = 1, 2, \ldots, B \tag{16}$$

(3) Estimate the standard error by using the standard deviation for the replicated bootstrap B times use Eq. 17.

$$SE_{\hat{\theta}} = \sqrt{\frac{\sum_{b=1}^{B} \left(\hat{\theta}^*_{(b)} - \bar{\theta}^*_{(.)}\right)^2}{B}} \tag{17}$$

with

$$\bar{\theta}^*_{(.)} = \sum_{b=1}^{B} \frac{\hat{\theta}^*_{(b)}}{B} \tag{18}$$

## A.2 Resampling Jackknife

If it is known that there is an initial sample and is a suspect for a function. The steps to predict $xy = x_{11}, x_{12}, \ldots, x_{1n}, x_{21}, x_{22}, \ldots, x_{2n}, y_{11}, y_{12}, \ldots, y_{1n}, y_{21}, y_{22}, \ldots, y_{2n}$ $\hat{\theta} = s(xy)$ the standard error of the Jackknife method with 100 samples are as follows.

(1) Resample by removing 5% of data to result in a greater possible sample combination on each Jackkniffe replication.

(2) Calculate the associated Jackknife replication for each Jackknife sample use Eq. 19.

$$\hat{\theta}^*_{(j)} = s(xy^*_j), j = 1, 2, \ldots, J \tag{19}$$

(3) item Estimate the standard error by using the standard deviation for the replicated jackknife J times use Eq. 20.

$$SE_{\hat{\theta}} = \left[ \frac{1}{J} \sum_{b=1}^{J} \left( \hat{\theta}^*_{(j)} - \bar{\theta}^*_{(.)} \right)^2 \right]^{\frac{1}{2}} \tag{20}$$

with

$$\bar{\theta}^*_{(.)} = \sum_{j=1}^{J} \frac{\hat{\theta}^*_{(j)}}{J} \tag{21}$$

## A.3 Double Resampling

The double resampling method is a statistical technique that combines two resampling procedures in a row to improve the accuracy of parameter estimation.

### *A.3.1 Double Bootstrap*

The steps for double bootstrap are as follows.

**(1) Outer layer:** Generate $B_1$ outer bootstrap samples, each obtained by deleting 5% of observations and resampling with replacement, as shown in Eq. 22.

$$\boldsymbol{xy}^*_{b_1} = \{(x^*_i, y^*_i)\}^n_{i=1}, \quad b_1 = 1, 2, \ldots, B_1 \tag{22}$$

**(2) Inner layer:** For each outer sample $\mathbf{xy}^*_{b_1}$, draw $B_2$ additional bootstrap subsamples, as formulated in Eq. 23.

$$\mathbf{xy}^{**}_{b_1,b_2}, \quad b_2 = 1, 2, \ldots, B_2 \tag{23}$$

(3) Estimate the parameter for each inner resample using a spline-based function as in Eq. 24.

$$\hat{\theta}^{**}_{b_1,b_2} = f_{\text{spline}}(\mathbf{xy}^{**}_{b_1,b_2}) \tag{24}$$

(4) Compute the average estimator across the inner resamples following Eq. 25.

$$\bar{\theta}^*_{b_1} = \frac{1}{B_2} \sum_{b_2=1}^{B_2} \hat{\theta}^{**}_{b_1,b_2} \tag{25}$$

(5) Estimate the bias and standard error across the outer loop, as shown in Eq. 26.

$$\begin{aligned}
\text{Bias}_{DB} &= \bar{\theta}^{**} - \hat{\theta}, \\
SE_{DB} &= \sqrt{\frac{1}{B_1 - 1}\sum_{b_1=1}^{B_1}(\bar{\theta}^{*}_{b_1} - \bar{\theta}^{**})^2}
\end{aligned} \tag{26}$$

where the overall estimator $\bar{\theta}^{**}$ is obtained as in Eq. 27.

$$\bar{\theta}^{**} = \frac{1}{B_1}\sum_{b_1=1}^{B_1}\bar{\theta}^{*}_{b_1} \tag{27}$$

### *A.3.2 Double Jackknife*

The steps for double jackknife are as follows.

**(1) Outer layer:** Generate $J_1$ outer jackknife samples, each by deleting 5% of observations without replacement, as shown in Eq. 28.

$$\mathbf{xy}^{*}_{j_1} = \{(x^*_i, y^*_i)\}_{i=1}^{n}, \quad j_1 = 1, 2, \ldots, J_1 \tag{28}$$

**(2) Inner layer:** For each $\mathbf{xy}^{*}_{j_1}$, create $J_2$ inner jackknife samples by again deleting 5% (without replacement), as formulated in Eq. 29.

$$\mathbf{xy}^{**}_{j_1,j_2}, \quad j_2 = 1, 2, \ldots, J_2 \tag{29}$$

(3) Estimate the parameter for each inner resample as in Eq. 30.

$$\hat{\theta}^{**}_{j_1,j_2} = f_{\text{spline}}(\mathbf{xy}^{**}_{j_1,j_2}) \tag{30}$$

(4) Compute the average estimator across the inner resamples following Eq. 31.

$$\bar{\theta}^{*}_{j_1} = \frac{1}{J_2}\sum_{j_2=1}^{J_2}\hat{\theta}^{**}_{j_1,j_2} \tag{31}$$

(5) Estimate the bias and standard error across the outer loop, as shown in Eq. 32.

$$\begin{aligned}
\text{Bias}_{DJ} &= \bar{\theta}^{**} - \hat{\theta}, \\
SE_{DJ} &= \sqrt{\frac{1}{J_1 - 1}\sum_{j_1=1}^{J_1}(\bar{\theta}^{*}_{j_1} - \bar{\theta}^{**})^2}
\end{aligned} \tag{32}$$

where $\bar{\theta}^{**}$ is obtained as in Eq. 33.

$$\bar{\theta}^{**} = \frac{1}{J_1}\sum_{j_1=1}^{J_1}\bar{\theta}^{*}_{j_1} \tag{33}$$

### *A.3.3 Bootstrap-Jackknife*

The steps for boostrap-jackknife are as follows.

**(1) Outer layer:** Generate $B$ bootstrap samples (delete-5% and resample with replacement), as shown in Eq. 34.

$$\mathbf{xy}^{*}_{b} = \{(x^*_i, y^*_i)\}_{i=1}^{n}, \quad b = 1, 2, \ldots, B \tag{34}$$

**(2) Inner layer:** For each $\mathbf{xy}^{*}_{b}$, draw $J$ jackknife subsamples by deleting 5% without replacement, as formulated in Eq. 35.

$$\mathbf{xy}^{**}_{b,j}, \quad j = 1, 2, \ldots, J \tag{35}$$

(3) Estimate the parameter for each inner resample and average as in Eq. 36.

$$\hat{\theta}^{**}_{b,j} = f_{\text{spline}}(\mathbf{xy}^{**}_{b,j}) \quad \bar{\theta}^{*}_{b} = \frac{1}{J}\sum_{j=1}^{J}\hat{\theta}^{**}_{b,j} \tag{36}$$

(4) Estimate the bias and standard error across the outer loop as in Eq. 37.

$$\begin{aligned} \text{Bias}_{BJ} &= \bar{\theta}^{**} - \hat{\theta}, \\ SE_{BJ} &= \sqrt{\frac{1}{B-1}\sum_{b=1}^{B}(\bar{\theta}^{*}_{b} - \bar{\theta}^{**})^2} \end{aligned} \tag{37}$$

where $\bar{\theta}^{**}$ is obtained as in Eq. 38.

$$\bar{\theta}^{**} = \frac{1}{B}\sum_{b=1}^{B}\bar{\theta}^{*}_{b} \tag{38}$$

### *A.3.4 Jackknife-Bootstrap*

The steps for jackknife-boostrap are as follows.

**(1) Outer layer:** Generate $J$ outer jackknife samples by deleting 5% without replacement, as shown in Eq. 39.

$$\boldsymbol{xy}^{*}_{j} = \{(x^{*}_{i}, y^{*}_{i})\}^{n}_{i=1}, \quad j = 1, 2, \ldots, J \tag{39}$$

**(2) Outer layer:**For each $\mathbf{xy}^{*}_{j}$, draw $B$ (delete-5% and resample with replacement), as shown in Eq. 40.

$$\mathbf{xy}^{**}_{j,b}, \quad b = 1, 2, \ldots, B \tag{40}$$

(3) Estimate the parameter for each inner resample and average as in Eq. 41.

$$\hat{\theta}^{**}_{j,b} = f_{\text{spline}}(\mathbf{xy}^{**}_{j,b}) \quad \bar{\theta}^{*}_{j} = \frac{1}{B}\sum_{b=1}^{B}\hat{\theta}^{**}_{j,b} \tag{41}$$

(4) Estimate the bias and standard error across the outer loop as in Eq. 42.

$$\begin{aligned} \text{Bias}_{JB} &= \bar{\theta}^{**} - \hat{\theta}, \\ SE_{JB} &= \sqrt{\frac{1}{J-1}\sum_{j=1}^{J}(\bar{\theta}^{*}_{j} - \bar{\theta}^{**})^2} \end{aligned} \tag{42}$$

where $\bar{\theta}^{**}$ is obtained as in Eq. 43.

$$\bar{\theta}^{**} = \frac{1}{J}\sum_{j=1}^{J}\bar{\theta}^{*}_{j} \tag{43}$$