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A Modified RESET and Hybrid Truncated Spline-Fourier Nonparametric Path Model for Waste-Behavior Data

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Abstract

Nonparametric path analysis is a statistical approach that does not require the functional form of relationships between variables to be known a priori. Classical path analysis assumes linearity, which can be tested using the Ramsey Regression Specification Error Test (RESET). If the linearity test indicates that the relationships between variables are nonlinear, a nonparametric model can be applied. The purpose of this study is to develop a modified Ramsey RESET to identify nonparametric relationships modeled using truncated spline and Fourier series. The modified Ramsey RESET algorithm was successfully implemented to detect the optimal functional form of the nonparametric truncated spline and Fourier series and was subsequently applied to behavioral data on waste management practices. Furthermore, this study proposes an estimator for a hybrid nonparametric path model combining truncated spline and Fourier series approaches. The analysis results reveal that the best model integrates truncated spline with one and two knot points and a Fourier series with one oscillation. The model achieved an adjusted coefficient of determination of 0.942, indicating that it explains 94.2% of the variation in the Behavior of Transforming Waste into Economic Value, while the remaining 5.8% is explained by other unobserved factors outside the model.

Keywords: nonparametric path analysis; truncated spline; Fourier series; Ramsey RESET; waste management behavior

1 Introduction

Path analysis examines how variables influence one another through direct and indirect effects, with some serving as causal factors and others as outcomes. According to [1], classical path analysis requires several assumptions, including that the relationships among variables must be linear and additive. This linearity assumption is essential to verify before applying the method; when it is satisfied, parametric path analysis is appropriate. A commonly used procedure to assess linearity is the Ramsey RESET test.

Linearity testing with the Ramsey Regression Specification Error Test (RESET) is performed by comparing two related regression models. The first model captures the direct relationship between exogenous and endogenous variables, while the second model incorporates additional

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predictors generated from the first model's fitted values, specifically the squared ($\hat{Y}_i^2$) and cubic ($\hat{Y}_i^3$) terms. This procedure follows the idea of an auxiliary regression, where these added terms help identify nonlinear patterns that the initial model may miss. The outcome of the Ramsey RESET test indicates whether the relationships among variables are linear or non-linear, where "non-linear" can also include certain nonparametric forms [2].

Nonparametric path analysis allows the data to determine the shape of relationships among variables without specifying a functional form beforehand. A commonly used method is the spline, especially the truncated spline, which is effective for capturing sharp changes or turning points through its knot structure. Another frequently applied estimator is the Fourier series, which represents periodic behavior using sums of sine and cosine terms. Its main advantage is the ability to model repeating or seasonal patterns in both theoretical and applied settings.

Nonparametric path models that use only one type of nonparametric function are limited by two key assumptions: all exogenous variables are expected to follow the same relationship pattern, and a single estimator is applied uniformly across them. In reality, each exogenous variable may display a different data pattern, making a single estimator unable to capture these variations. As a result, the estimated curves may not represent the true structure of the data, leading to less accurate model estimates and higher error rates [3].

This study builds upon several previous works relevant to the development of nonparametric analysis methods. Research by [4] employed nonparametric path analysis with a truncated spline approach to obtain the best path function estimation for longitudinal data using PLS and PWLS methods. Meanwhile, [5] applied nonparametric regression using Fourier series to analyze factors influencing poverty levels in South Sulawesi, finding that the best model occurred at an oscillation level of three, as it effectively captured recurring and seasonal data patterns.

Previous research has relied on only one nonparametric method, either truncated spline or Fourier series without combining the two in a single framework, and without a testing procedure to identify the specific functional form of the nonparametric relationship. Most studies continue to use the traditional Ramsey RESET test, which detects deviations from linearity but cannot distinguish particular nonparametric patterns. This creates a clear gap in both detecting and modeling specific nonparametric functional forms within path analysis. To fill this gap, the present study introduces two key novelties: a modified Ramsey RESET that can distinguish whether a nonparametric relationship arises from truncated spline or Fourier series, and a hybrid nonparametric path model that combines both estimators within a single analytical framework.

The modification of the Ramsey RESET test plays a crucial role in path analysis because the accuracy of this model heavily depends on correctly identifying the functional form of relationships among variables. Traditional RESET can only signal the presence of nonlinearity without specifying its nature, which often leads to model misspecification and biased estimates of direct or indirect effects. By enhancing the test to detect and differentiate specific nonparametric forms such as truncated spline and Fourier series, the modified RESET ensures that each causal pathway is modeled according to its true structure.

Path analysis models are widely used to understand the interrelationships among social, economic, and environmental factors influencing waste management behavior[6]. However, these relationships are often nonlinear and complex, making traditional linear models insufficient for accurate representation. This condition necessitates a more flexible nonparametric approach, yet most previous studies have employed only a single estimator, such as the truncated spline or Fourier series. In reality, waste management behavior data often exhibit both piecewise patterns (characteristic of splines) and periodic tendencies (characteristic of Fourier functions) [7]. Therefore, this study proposes a hybrid truncated spline–Fourier nonparametric path model equipped with a Modified Ramsey RESET capable of identifying specific forms of nonlinearity. This approach is crucial as it enables a more realistic and accurate modeling of waste management behavior, capturing the underlying social and environmental dynamics that shape it.

To provide a clear flow of discussion, this paper is organized as follows: Section 2 presents the methodology, including the modification of the Ramsey RESET and the formulation of the hybrid model; Section 3 discusses the empirical results and their implications; and Section 4 concludes the paper.

2 Methods

Following the introduction, this part provides a detailed explanation of the methodological framework used in the study. The research methods are systematically structured to ensure that the developed model can be clearly explained and empirically tested. This section begins with a description of the data sources, followed by the development of the research model, which serves as the foundation for the subsequent testing and analysis methods

2.1 Data Sources

The data used in this study consist of primary data and simulation data. The primary data were obtained through a survey of 210 respondent in Bumiaji District using multiple Likert-scale items to measure the latent variables. The Likert responses were first rescaled using a summated rating scale approach to approximate interval-level data, after which the item scores for each construct were averaged to form composite measures. All instruments passed validity and reliability testing, supporting the use of these composite scores in subsequent analysis. Because the Fourier terms in the regression require inputs within the 0–1 domain, all composite variables were normalized using min–max scaling. Throughout data collection, no missing data were found, so no additional handling procedures were required.

2.2 Research Model

This study involves two exogenous variables, namely Environmental Quality (X_1) and Use of Waste Banks (X_2), one intervening variable, namely the Use of the 3R Principle (Y_1), and one endogenous variable, namely the Economic Benefits of Waste (Y_2). The model used in this study is presented in Fig. 1.

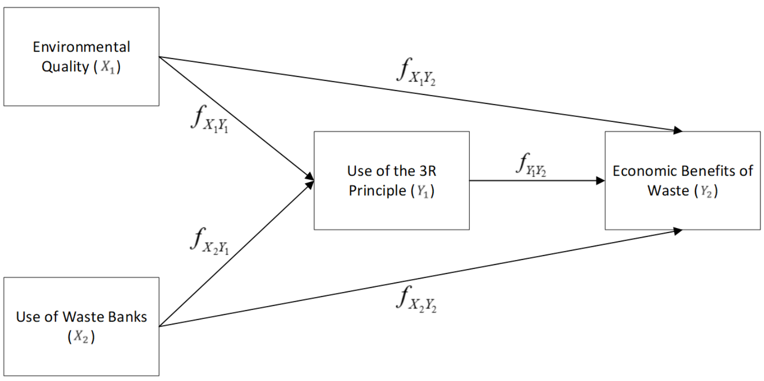

Figure 1: Research Framework of the Study

The directed arrows indicate causal pathways modeled using nonparametric functions $f_{X_1Y_1}$, $f_{X_2Y_1}$, $f_{X_1Y_2}$, and $f_{Y_1Y_2}$, each representing distinct functional relationships. The framework is developed based on primary waste-management data from Batu City collected.

In path analysis, verifying the correct functional form of variable relationships is essential. A linearity test is needed before estimating path coefficients to determine whether the links among exogenous, intervening, and endogenous variables are linear or nonlinear. The Ramsey RESET test is commonly used for this purpose, providing a baseline method for detecting specification errors and potential nonlinearities in the model.

2.3 Ramsey RESET

This study requires a linearity test to determine whether parametric path analysis can be used (the linearity assumption is met) or not (the linearity assumption is not met), so a linearity test is needed. Ramsey RESET is used to test the form of the linear relationship between exogenous and endogenous variables. The hypothesis testing used is as follows [8].

H_0: $\beta_2 = \beta_3 = 0$
H_1: There is at least one $\beta_j \neq 0, \quad j = 2, 3$

Eq. 1 is an equation used to perform a linearity test.

$$\begin{gathered} \hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_{1i} \\ \hat{Y}_i^* = \hat{\beta}_0^* + \hat{\beta}_1^* X_{1i} + \hat{\beta}_2^* \hat{Y}_i^2 + \hat{\beta}_3^* \hat{Y}_i^3 \end{gathered} \tag{1}$$

From Eq. 1, the coefficient of determination of each model is then calculated to obtain R_1^2 and R_2^2. Subsequently, a linearity test is performed using the F-test as shown in Eq. 2.

$$F_{\text{calculated}} = \frac{(R_2^2 - R_1^2)/m}{(1 - R_2^2)/(n - p - 1 - m)} \sim F_{m, n-p-1-m} \tag{2}$$

Where:

R_1^2	:	Coefficient of determination from the original regression equation
R_2^2	:	Coefficient of determination from the auxiliary regression equation
p	:	Number of initial predictor variables
m	:	Number of additional predictors
n	:	Number of observations

If the resulting p-value > 0.05, then the form of the relationship obtained is linear. Meanwhile, if the p-value < 0.05, it means that the form of the relationship is non-linear.Although the classical Ramsey RESET test can reveal misspecification or global nonlinearity, it does not indicate the correct alternative model. Rejecting linearity does not guarantee that the true form is quadratic, cubic, or any specific polynomial. To avoid arbitrary polynomial choices, this study introduces a modified RESET that uses a flexible nonparametric basis capable of universally approximating smooth functions. This approach allows the detection of a broad range of nonlinear patterns while extending the RESET framework in a more principled way.

2.4 Truncated Spline Nonparametric Path Analysis

A truncated spline is a polynomial function divided into segments of a certain degree or order m, within which there are specific points known as knots. According to [9], in general, a truncated spline function of order m with knots $u = 1, 2, \ldots, r$ is any function that can be expressed in the form of Eq. 3.

$$f(X_{1i}) = \sum_{k=0}^{m} \beta_k X_{1i}^k + \sum_{u=1}^{r} \beta_{m+u} (X_{1i} - K_u)_+^m \tag{3}$$

The + sign in $(X_{1i} - K_u)_+^m$ indicates that there are two possibilities, namely:

$$(X_{1i} - K_u)_+^m = \begin{cases} (X_{1i} - K_u)^m, & X_{1i} \geq K_u \\ 0, & X_{1i} < K_u \end{cases} \tag{4}$$

Where:
k = 1, 2, . . . , m ; m: spline order
u = 1, 2, . . . , r ; r: number of knot points

Next, to obtain the estimator of β, the Weighted Least Squares (WLS) method will be used, which minimizes the sum of squared errors.

$$\underset{\sim}{\hat{\beta}} = \left(X[\underset{\sim}{K}]'\,\hat{\boldsymbol{\Sigma}}^{-1}\,X[\underset{\sim}{K}]\right)^{-1}\,X[\underset{\sim}{K}]'\,\hat{\boldsymbol{\Sigma}}^{-1}\,\underset{\sim}{y} \tag{5}$$

In the nonparametric truncated spline path, determining the optimal knot point is crucial because it strongly influences the shape of the resulting regression curve [10]. One commonly used method for selecting the optimal knot location is Generalized Cross-Validation (GCV), which can be expressed in Eq. 6.

$$\text{GCV}(\underset{\sim}{K}) = \frac{\text{MSE}(\underset{\sim}{K})}{\left[\frac{1}{n}\text{trace}(\mathbf{I} - \mathbf{A}(\underset{\sim}{K}))\right]^2} \tag{6}$$

Where $MSE(\underset{\sim}{K}) = n^{-1}\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2$ and $\underset{\sim}{K}$ are knot points and the matrix $\mathbf{A}(\underset{\sim}{K})$ is obtained from:

$$\hat{f}(X_i) = \mathbf{A}[\underset{\sim}{K}]\,\underset{\sim}{y} \tag{7}$$

$$\mathbf{A}[\underset{\sim}{K}] = X[\underset{\sim}{K}]\left(X[\underset{\sim}{K}]'\hat{\boldsymbol{\Sigma}}^{-1}X[\underset{\sim}{K}]\right)^{-1}X[\underset{\sim}{K}]'\hat{\boldsymbol{\Sigma}}^{-1} \tag{8}$$

In this study, the candidate knot points were chosen based on the observed data range, starting from the lowest to the highest values.

2.5 Fourier Series Nonparametric Path Analysis

The Fourier series is a flexible trigonometric polynomial that adapts well to local data patterns [11] and is effective for modeling sine–cosine characteristics [12]. In nonparametric regression, it may use additive combinations of sine, cosine, or linear–trigonometric functions [13]. For many smoothing tasks, a cosine-only basis is often sufficient for capturing smooth, low-frequency variation[14]. The additive linear–cosine form used to model the curve f is given in Eq. 9 [15].

$$\hat{f}(X_i) = \frac{1}{2}\hat{a}_0 + \hat{b}_1 X_i + \sum_{k=1}^{L}\gamma_k\cos(kX_i) \tag{9}$$

Where:

$\hat{f}(X_i)$	:	Function estimator at the i-th observation point
γ_k	:	Fourier coefficient for the k-th cosine component
$\hat{b}_1$	:	Regression coefficient for variable xx
$\hat{a}_0$	:	Constant coefficient for the zero-order cosine component
$k = 1, 2, \ldots, L;\ L$	:	Number of oscillations

In this study, the data were normalized to the 0–1 range to match the domain required for the Fourier estimator

2.6 Combined Nonparametric Path of Truncated Spline and Fourier Series

Nonparametric path analysis is often applied using a single estimator, assuming all functional relationships share the same data pattern. In practice, however, different paths may exhibit different patterns, making a single approach insufficient. For this reason, this study employs a combined nonparametric path analysis using both Truncated Spline and Fourier Series.

Let the data be given as $(x_{1i}, x_{2i}, y_{1i}, y_{2i})$, and the relationships among the exogenous, intervening, and endogenous variables are assumed to follow a nonparametric path model:

$$\begin{aligned} Y_{1i} &= f_1(X_{1i}, X_{2i}) + \varepsilon_i \\ Y_{2i} &= f_2(X_{1i}, X_{2i}, Y_{1i}) + \varepsilon_i, \quad i = 1, 2, \ldots, n \end{aligned} \tag{10}$$

The forms of the curves $f_1(X_{1i}, X_{2i})$ and $f_2(X_{1i}, X_{2i}, Y_{1i})$ in Eq. 10 are assumed to be unknown and are only assumed to be smooth, meaning continuous and differentiable. Furthermore, the regression curves $f_1(X_{1i}, X_{2i})$ and $f_2(X_{1i}, X_{2i}, Y_{1i})$ are assumed to be additive. Additive nonparametric models are widely adopted because they provide flexible smooth effects while intentionally excluding higher-order interactions unless explicitly specified, thereby avoiding the curse of dimensionality and improving interpretability [16]. Consistent with this framework, the present analysis assumes additive smooth components and does not include interaction terms, as the theoretical structure of the studied variables does not suggest strong interactive effects. So they can be written in the following form:

$$\begin{aligned} f_1(X_{1i}, X_{2i}) &= f_{11}(X_{1i}) + f_{12}(X_{2i}) \\ f_2(X_{1i}, X_{2i}, Y_{1i}) &= f_{21}(X_{1i}) + f_{22}(X_{2i}) + f_{23}(Y_{1i}) \end{aligned} \tag{11}$$

In Eq. 11, it is assumed that the regression curves $f_{11}(X_{1i})$ and $f_{22}(X_{2i})$ follow truncated spline regression, while $f_{12}(X_{2i})$, $f_{21}(X_{1i})$, and $f_{23}(Y_{1i})$ follow Fourier series regression. The estimator of the combined nonparametric path model is as follows:

$$\begin{aligned} \hat{f}_1(X_{1i}, X_{2i}) &= \hat{f}_{11}(X_{1i})_{\text{trunc}} + \hat{f}_{12}(X_{2i})_{\text{fourier}} \\ \hat{f}_2(X_{1i}, X_{2i}, Y_{1i}) &= \hat{f}_{21}(X_{1i})_{\text{fourier}} + \hat{f}_{22}(X_{2i})_{\text{trunc}} + \hat{f}_{23}(Y_{1i})_{\text{fourier}} \end{aligned} \tag{12}$$

2.7 Model Validity

A model is considered valid when its underlying assumptions are met, and one way to assess this validity is through the total coefficient of determination [1]. This measure reflects the proportion of data variability explained by the model, indicating how well the model captures the relationships among variables. The computation of total explained variability is given in Eq. 13[4].

$$R^2_{T,\text{adj}} = 1 - (1 - R^2_{1,\text{adj}})(1 - R^2_{2,\text{adj}}) \tag{13}$$

Where:

$$R^2_{k,\text{adj}} = 1 - \frac{\sum_{i=1}^{n}(y_{ki} - \hat{f}_{ki})^2/(n-m-1)}{\sum_{i=1}^{n}(y_{ki} - \bar{y}_k)^2/(n-1)} \tag{14}$$

Description:

$R^2_{T,\text{adj}}$: Adjusted total coefficient of determination
$R^2_{k,\text{adj}}$: Adjusted coefficient of determination for the k-th equation ; k=1,2
n : Number of observations
m : Number of parameters in the model

Next, K-Fold cross-validation is performed to evaluate the model. K-fold cross-validation is a statistical method used to evaluate the performance of a model to be developed[17]. In addition, k-fold cross-validation is an effective approach for combining attributes and parameter settings in machine learning to train a better prediction model[18]. In this study, the dataset is divided into 5 folds of equal size, resulting in five data subsets. Another advantage of using k-fold cross-validation is its ability to minimize the risk of overfitting by training and testing the model across multiple data combinations[19].

2.8 Hypothesis Testing with Resampling Jackknife

According to [20], resampling is the process of repeatedly drawing samples from an existing dataset to form new samples, either with or without replacement. Before the bootstrap was introduced, simple resampling methods such as the jackknife had already been widely used. The jackknife reduces estimation bias by systematically removing observations and examining the resulting changes in parameter estimates. It is also used to estimate standard deviation. In its simplest form, the delete-1 jackknife removes one observation at a time and repeats the process for all n observations [21].

$$t = \frac{\bar{\hat{\beta}}^{*}_{jp}}{SE(\hat{\beta}^{*}_{jp})} \sim t_{(n-1)} \tag{15}$$

3 Results and Discussion

This section presents the findings derived from the statistical analysis using the integrated approach of Modified Ramsey RESET in Combined Truncated Spline–Fourier Nonparametric Path Analysis. The discussion is structured to first address the linearity test conducted with Ramsey's RESET, followed by the analysis of the relationships between key variables. Each subsection emphasizes the empirical evidence supporting the theoretical framework of waste management behavior, providing insights into whether linear or nonlinear approaches best capture these relationships.

3.1 Linearity Test Using Ramsey RESET

Relationships that exhibit a linear form are addressed using a parametric approach, while nonlinear relationships can be handled using nonparametric methods. The linearity test in this study was conducted using Ramsey's RESET. The results of the linearity test are presented in Table 1.

Table 1: Results of the Ramsey RESET Linearity Test

Relationship between Variables	P-value	Results
Environmental Quality (X_1) → Use of the 3R Principle (Y_1)	0.001	Not linear
Use of Waste Banks (X_2) → Use of the 3R Principle (Y_1)	0.003	Not linear
Environmental Quality (X_1) → Economic Benefits of Waste (Y_2)	0.019	Not linear
Use of Waste Banks (X_2) → Economic Benefits of Waste (Y_2)	<0.001	Not linear
Use of the 3R Principle (Y_1) → Economic Benefits of Waste (Y_2)	0.008	Not linear

Based on Table 1, all p-values are below $\alpha = 0.05$, leading to the rejection of H_0. This indicates that all variable relationships are nonlinear, although the specific nonlinear form is unknown. Therefore, a nonparametric path analysis is applied, beginning with a modified Ramsey RESET test to identify the potential nonparametric functional form before estimating the functions.

3.2 Development of the Modified Ramsey RESET

The development of the Ramsey RESET was carried out by modifying the second regression equation using nonparametric path equations based on truncated spline and Fourier series approaches. The first modification of the Ramsey RESET aims to detect nonparametric relationships modeled using a linear-order truncated spline with one knot point. The hypotheses used are as follows.

H_0: $\beta_2 = 0$
H_1: $\beta_2 \neq 0$

The equation used is presented in the following formula.

$$\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_{1i}$$
$$\hat{Y}_i^* = \hat{\beta}_0^* + \hat{\beta}_1^* X_{1i} + \hat{\beta}_2 (X_{1i} - K_{11})_+ \tag{16}$$

If the p-value obtained from the Ramsey RESET test for the linear truncated spline with one knot is less than 0.05, it indicates that the identified pattern follows a linear-order truncated spline with one knot. The same interpretation applies to the case with two knot points.

The second modification of the Ramsey RESET aims to detect nonparametric relationships modeled using the Fourier series with one oscillation. The hypotheses used are as follows.

$H_0 : \gamma_1 = 0$
$H_1 : \gamma_1 \neq 0$

The equation used is presented in the following formula.

$$\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_{1i}$$
$$\hat{Y}_i^* = \hat{\beta}_0^* + \hat{\beta}_1^* X_{1i} + \gamma_1 \cos X_{1i} \tag{17}$$

If the p-value obtained from the Ramsey RESET test for the Fourier series at oscillation level 1 is less than 0.05, it indicates that the identified pattern follows a Fourier series with one oscillation. The same interpretation applies to oscillation levels 2 and 3. The calculation of the p-value follows the same procedure as in the classical Ramsey RESET, by computing the coefficient of determination for each equation and then performing P-value from F-test.

Based on the linearity test, the relationships among variables were found to be nonlinear, although the specific form of nonlinearity remains unknown. Therefore, the analysis proceeded using the modified Ramsey RESET. In this study, the modified RESET was applied across five conditions to identify the appropriate nonparametric functional forms. The results for these five conditions are presented below.

Table 2: Results of the Modified Ramsey RESET

Relationship Variables	P-value ($R^2_{adj,auxiliary}$)					Results
	RRT1K	RRT2K	RRF1O	RRF2O	RRF3O	
$X_1 \to Y_1$	0.049 (0.543)	**0.031** (**0.748**)	0.044 (0.601)	0.037 (0.653)	0.041 (0.633)	**RRT2K**
$X_2 \to Y_1$	0.019 (0.671)	0.047 (0.601)	**0.002** (**0.796**)	0.022 (0.648)	0.031 (0.611)	**RRF1O**
$X_1 \to Y_2$	0.045 (0.555)	**0.011** (**0.740**)	0.024 (0.696)	0.038 (0.691)	0.045 (0.556)	**RRT2K**
$X_2 \to Y_2$	0.045 (0.630)	0.036 (0.657)	**<0.001** (**0.812**)	0.003 (0.779)	0.014 (0.766)	**RRF1O**
$Y_1 \to Y_2$	**0.013** (**0.716**)	0.029 (0.699)	0.041 (0.654)	0.049 (0.643)	0.017 (0.706)	**RRT1K**

Description:
RRT1K : Ramsey RESET Truncated Spline 1 Knot ($df = 1,\ n - 3$)
RRT2K : Ramsey RESET Truncated Spline 2 Knot ($df = 2,\ n - 4$)
RRF1O : Ramsey RESET Fourier Series 1 Oscillation ($df = 1,\ n - 3$)
RRF2O : Ramsey RESET Fourier Series 2 Oscillation ($df = 2,\ n - 4$)
RRF3O : Ramsey RESET Fourier Series 3 Oscillation ($df = 3,\ n - 5$)

Based on Table 2, the selection of the functional form was carried out by considering both the smallest p-value and the highest auxiliary adjusted R^2. For the relationships $X_1 \to Y_1$ and $X_1 \to Y_2$, the most appropriate specification is the truncated spline linear with two knots

(RRT2K), as it provides the lowest p-value and the largest improvement in adjusted R^2. For the relationships $X_2 \to Y_1$ and $X_2 \to Y_2$, the best specification is the first-order Fourier series (RRF1O), which consistently yields the smallest p-value and the highest adjusted R^2 among the candidate models. Meanwhile, the relationship $Y_1 \to Y_2$ is best represented by the truncated spline linear with one knot (RRT1K), which also demonstrates the strongest performance on both criteria.

3.3 Evaluation of the Effectiveness Modified Ramsey RESET in Simulation Studies

For each simulation design with 1000 replications, we record whether the procedure correctly identifies the true nonlinear specification. Let p_{mod} be the proportion of correct identifications produced by the modified RESET, and let p_{base} be the corresponding proportion obtained from the baseline (standard) RESET test. The effectiveness ratio is then defined as:

$$ER = \frac{p_{\text{mod}}}{p_{\text{base}}} \tag{18}$$

Thus, an $ER > 1$ indicates that the modified RESET is more effective than the baseline RESET in correctly recovering the true functional form.

Table 3: Effectiveness of the Modified Ramsey RESET on Simulation Data

Pattern of Simulation	P_{mod}	Monte Carlo SE	P_{base}	Monte Carlo SE	ER
Truncated 1 Knot	0.812	0.012	0.503	0.015	**1.614**
Truncated 2 Knot	0.845	0.011	0.618	0.015	**1.367**
Fourier 1 Osilasi	0.840	0.011	0.700	0.014	**1.200**
Fourier 2 Osilasi	0.880	0.010	0.680	0.014	**1.294**
Fourier 3 Osilasi	0.780	0.013	0.840	0.014	**0.928**

Based on Table 3, the relative efficiency (ER) shows that the Modified Ramsey RESET provides a significant performance improvement over the baseline in most nonparametric patterns. For truncated splines with one and two knots, the modified method is 1.61 times and 1.37 times more effective, respectively, in identifying the correct functional form. For Fourier series with one and two oscillations, the improvements are also consistent, with the method being 1.20 times and 1.29 times more effective than the baseline. Only in the case of three Fourier oscillations does the ER fall below one (0.93), indicating that the baseline performs slightly better at that level of complexity. Furthermore, the small Monte Carlo standard error (SE) values across all scenarios indicate that the estimated success proportions are stable and reliable.

3.4 Selection of Optimal Knot Points

The criterion used to determine the optimal knot points is the Generalized Cross Validation (GCV) value, which is a statistical measure used to assess the goodness of fit of a model while penalizing for model complexity. The optimal knot points are identified as those corresponding to the model with the smallest GCV value, indicating the best balance between model accuracy and complexity. The process of selecting optimal knot points for each model based on GCV values is presented in Table 4.

Table 4: Optimal Knot Points

Relationship	Optimal Knot	GCV	$R^2_{T,adj}$
Environmental Quality (X_1) → Use of the 3R Principle (Y_1)	$K_{11} = 0.25$ $K_{12} = 0.83$		
Use of Waste Banks (X_2) → Use of the 3R Principle (Y_1)	-		
Environmental Quality (X_1) → Economic Benefits of Waste (Y_2)	$K_{21} = 0.31$ $K_{22} = 0.77$	0.006	0.942
Use of Waste Banks (X_2) → Economic Benefits of Waste (Y_2)	-		
Use of the 3R Principle (Y_1) → Economic Benefits of Waste (Y_2)	$K_{31} = 0.45$		

Based on Table 4, the results of the selection of optimal knot points for each relationship between variables in the model are presented. The optimal knot points were determined using the Generalized Cross Validation (GCV) criterion, where a smaller GCV value indicates a model with better fit and appropriate complexity. After obtaining the optimal knot points, it was found that the GCV value is 0.006 and the adjusted R^2 is 0.942. This means that the constructed model can explain 94.2% of the variability in the Economic Benefits of Waste variable, while the remaining 5.8% is explained by factors outside the model. After the optimal knot points were determined, the next step is to estimate the hybrid nonparametric path functions combining truncated spline and Fourier series.

3.5 Model Validation Results

Model validation ensures that the proposed hybrid nonparametric path model performs reliably and generalizes beyond the estimation sample. The process used five-fold cross validation with a 70% training and 30% testing split in each iteration, allowing every subset to serve as the test set. This approach provides a comprehensive assessment of the model's consistency and its ability to generalize to unseen data.

Table 5: K-Fold Cross-Validation Results

Fold	Adjusted R^2_{total}	GCV Model
1	0.931	0.008
2	0.949	0.007
3	0.890	0.011
4	0.911	0.005
5	0.880	0.016
Mean	**0.912**	**0.009**

Base on Table 5, the k-fold cross-validation results indicate that the hybrid nonparametric path model combining truncated spline and Fourier series demonstrates stable performance and strong generalization ability. The Adjusted R^2_{total} values across the folds fall within a narrow range (0.880 to 0.949), with an average of 0.9122. This consistency shows that the model is able to explain a substantial proportion of data variation in each split without signs of overfitting. In addition, the GCV values are low and stable (ranging from 0.005 to 0.016, with an average of 0.0094), reflecting optimal curve smoothness and well-controlled model complexity. Overall, the combination of high Adjusted R^2 values and low GCV values confirms that the model achieves strong predictive accuracy and generalizes well beyond the training sample.

3.6 Results of Hybrid Nonparametric Path Modeling Using Truncated Spline–Fourier Series

The estimation of the hybrid nonparametric path function combining truncated spline and Fourier series was carried out after obtaining the optimal knot points. The results of the estimated hybrid nonparametric path function combining truncated spline and Fourier series are as follows.

$$\begin{aligned}\hat{f}_{1i} &= 0.431 - 0.114X_{1i} + 0.465(X_{1i} - 0.25)_+ - 0.153(X_{1i} - 0.83)_+ + 0.264X_{2i} \\ &\quad - 0.126\cos X_{2i} \\ \hat{f}_{2i} &= 0.336 + 0.187X_{1i} - 0.169(X_{1i} - 0.31)_+ + 0.213(X_{1i} - 0.77)_+ + 0.177X_{2i} \\ &\quad - 0.311\cos X_{2i} + 0.436Y_{1i} + 0.296(Y_{1i} - 0.45)_+\end{aligned} \tag{19}$$

Where the truncated function is:

$$(X_{1i} - 0.25)_+ = \begin{cases} X_{1i} - 0.25, & X_{1i} \geq 0.25 \\ 0, & X_{1i} < 0.25 \end{cases}$$

$$(X_{1i} - 0.83)_+ = \begin{cases} X_{1i} - 0.83, & X_{1i} \geq 0.83 \\ 0, & X_{1i} < 0.83 \end{cases}$$

$$(X_{1i} - 0.31)_+ = \begin{cases} X_{1i} - 0.31, & X_{1i} \geq 0.31 \\ 0, & X_{1i} < 0.31 \end{cases}$$

$$(X_{1i} - 0.77)_+ = \begin{cases} X_{1i} - 0.77, & X_{1i} \geq 0.77 \\ 0, & X_{1i} < 0.77 \end{cases}$$

$$(Y_{1i} - 0.45)_+ = \begin{cases} Y_{1i} - 0.45, & Y_{1i} \geq 0.45 \\ 0, & Y_{1i} < 0.45 \end{cases}$$

After obtaining the path function coefficients from the combined truncated spline–Fourier nonparametric model, the next step is to conduct a hypothesis test to examine the significance of each variable's effect within the model. This test aims to confirm whether the identified relationships are statistically significant and meaningfully contribute to the dependent variable in the constructed path structure.

3.7 Hypothesis Testing Using Resampling Jackknife

Significance testing using the t-test was carried out on the best model, beginning with jackknife resampling. In this study, jackknife resampling was performed by randomly removing five observations at each resampling stage. The resampling procedure was repeated 1000 times. The hypotheses used in the hypothesis testing with jackknife resampling are as follows:

Hypothesis of the direct effect of X_1 on Y_1:
$H_0 : \beta_{1k} = 0;\ k = 1, 2, 3$ Vs $H_1 : \beta_{1k} \neq 0;\ k = 1, 2, 3$
Hypothesis of the direct effect of X_2 on Y_1:
$H_0 : \beta_{14} = 0$ Vs $H_1 : \beta_{14} \neq 0$
$H_0 : \gamma_{11} = 0$ Vs $H_1 : \gamma_{11} \neq 0$
Hypothesis of the direct effect of X_1 on Y_2:
$H_0 : \beta_{2k} = 0;\ k = 1, 2, 3$ Vs $H_1 : \beta_{2k} \neq 0;\ k = 1, 2, 3$
Hypothesis of the direct effect of X_2 on Y_2:
$H_0 : \beta_{24} = 0$ Vs $H_1 : \beta_{24} \neq 0$
$H_0 : \gamma_{21} = 0$ Vs $H_1 : \gamma_{21} \neq 0$
Hypothesis of the direct effect of Y_1 on Y_2:
$H_0 : \beta_{2k} = 0;\ k = 5, 6$ Vs $H_1 : \beta_{2k} \neq 0;\ k = 5, 6$

The results of the nonparametric path modeling combining truncated spline and Fourier series show that the relationships $X_1 \rightarrow Y_1$, $X_1 \rightarrow Y_2$, and $Y_1 \rightarrow Y_2$ follow a truncated spline pattern, while the relationships $X_2 \rightarrow Y_1$ and $X_2 \rightarrow Y_2$ follow a Fourier series pattern. The data patterns modeled using the nonparametric truncated spline approach can still be interpreted, whereas those modeled with the nonparametric Fourier series approach cannot be directly interpreted. However, the Fourier components can still be tested in terms of the significance of their functional elements.

The direct effect of variable X_1 on Y_1, assuming that the relationships among other variables remain constant, can be seen in the following equation.

$$\hat{f}_{1i} = -0.114X_{1i} + 0.465(X_{1i} - 0.25)_+ - 0.153(X_{1i} - 0.83)_+ \tag{20}$$

With the truncated spline conditions as follows.

$$\hat{f}_{1i} = \begin{cases} -0.114X_{1i}, & \text{when } X_{1i} < 0.25, \\ -0.116 + 0.351X_{1i}, & \text{when } 0.25 \leq X_{1i} < 0.83, \\ -0.011 + 0.198X_{1i}, & \text{when } X_{1i} \geq 0.83. \end{cases} \tag{21}$$

The effect of variable X_2 on Y_1, assuming that the relationships among other variables remain constant, can be seen in the following equation.

$$\hat{f}_{1i} = 0.264X_{2i} - 0.126\cos X_{2i} \tag{22}$$

The direct effect of variable X_1 on Y_2, assuming that the relationships among other variables remain constant, can be seen in the following equation.

$$\hat{f}_{2i} = 0.187X_{1i} - 0.169(X_{1i} - 0.31)_+ + 0.213(X_{1i} - 0.77)_+ \tag{23}$$

With the truncated spline conditions as follows.

$$\hat{f}_{2i} = \begin{cases} 0.187X_{1i}, & \text{when } X_{1i} < 0.31, \\ 0.052 + 0.018X_{1i}, & \text{when } 0.31 \leq X_{1i} < 0.77, \\ -0.112 + 0.231X_{1i}, & \text{when } X_{1i} \geq 0.77. \end{cases} \tag{24}$$

The effect of variable X_2 on Y_2, assuming that the relationships among other variables remain constant, can be seen in the following equation.

$$\hat{f}_{2i} = 0.177X_{2i} - 0.311\cos X_{2i} \tag{25}$$

The direct effect of variable Y_1 on Y_2, assuming that the relationships among other variables remain constant, can be seen in the following equation.

$$\hat{f}_{2i} = 0.436\,Y_{1i} + 0.296(Y_{1i} - 0.45)_+ \tag{26}$$

With the truncated spline conditions as follows.

$$\hat{f}_{2i} = \begin{cases} 0.436\,Y_{1i}, & \text{when } Y_{1i} < 0.45, \\ -0.133 + 0.732\,Y_{1i}, & \text{when } Y_{1i} \geq 0.45. \end{cases} \tag{27}$$

The results of the hypothesis testing for direct effects in the nonparametric path model combining truncated spline and Fourier series, as presented in the previous equation, can be seen in Table 6.

Table 6: Results of Hypothesis Testing for the Best Model

Relationship	Coefficient	P-value	CI
$X_1 \to Y_1$	$\beta_{11}X_{1i}$	0.007*	(−0.208, −0.020)
	$\beta_{11}X_{1i} + \beta_{12}(X_{1i} - K_{11})_+$	< 0.001*	(0.142, 0.560)
	$\beta_{11}X_{1i} + \beta_{12}(X_{1i} - K_{11})_+ + \beta_{13}(X_{1i} - K_{12})_+$	0.008*	(0.040, 0.356)
$X_2 \to Y_1$	$\beta_{18}X_{2i}$	< 0.001*	(0.107, 0.421)
	$\gamma_{11} \cos X_{2i}$	0.021*	(−0.235, −0.017)
$X_1 \to Y_2$	$\beta_{21}X_{1i}$	0.008*	(0.038, 0.336)
	$\beta_{21}X_{1i} + \beta_{22}(X_{1i} - K_{21})_+$	0.024*	(0.003, 0.033)
	$\beta_{21}X_{1i} + \beta_{22}(X_{1i} - K_{21})_+ + \beta_{23}(X_{1i} - K_{22})_+$	< 0.001*	(0.093, 0.369)
$X_2 \to Y_2$	$\beta_{24}X_{2i}$	0.003*	(0.056, 0.298)
	$\gamma_{21} \cos X_{2i}$	0.001*	(−0.508, −0.114)
$Y_1 \to Y_2$	$\beta_{25}Y_{1i}$	< 0.001*	(0.176, 0.696)
	$\beta_{25}Y_{1i} + \beta_{26}(Y_{1i} - K_{31})_+$	0.003*	(0.126, 1.338)

*Notes: *Significant with* $\alpha = 0.05$

Based on Table 6, it is known that there are 12 function components with p-values less than 0.05, indicating a decision to reject H_0. Therefore, these function components are significant to the model. In addition, the decision is also supported by the confidence intervals (CI), where none of the intervals include the value 0, confirming the significance of all components. Furthermore, the direct effects can be interpreted as follows.

1) **Environmental Quality (X_1) on the Use of the 3R Principle (Y_1)**
The direct effect between Environmental Quality (X_1) and the Use of the 3R Principle (Y_1) is divided into three regimes. When the Environmental Quality (X_1) value is below 0.25, its effect on the Use of the 3R Principle (Y_1) is negative. This means that at low levels of environmental quality, efforts to improve it may not immediately encourage 3R behaviors (Reduce, Reuse, Recycle); instead, individuals or communities may still lack the awareness or infrastructure to support sustainable practices. However, when X_1 is in the range of $0.25 \leq X_1 < 0.83$, the effect becomes positive, indicating that improvements in environmental quality begin to foster stronger adoption of 3R practices. This stage reflects growing environmental awareness and access to supporting facilities. Furthermore, when $X_1 \geq 0.83$, the positive effect continues but with decreasing intensity, suggesting that at already high levels of environmental quality, the marginal benefit of further improvement becomes stable. This aligns with the behavioral saturation theory, where motivation to change behavior tends to level off once optimal conditions are achieved.
2) **Environmental Quality (X_1) on the Economic Benefits of Waste (Y_2)**
The direct effect between Environmental Quality (X_1) and the Economic Benefits of Waste (Y_2) is also divided into three regimes. When the level of environmental quality is still low ($X_1 < 0.31$), the effect is positive but weak, meaning that a small improvement in environmental quality does not yet significantly impact the economic value derived from waste management. In the moderate range ($0.31 \leq X_1 < 0.77$), the positive effect persists but with lower intensity, suggesting that environmental efforts have not yet fully contributed to increasing economic benefits. However, when environmental quality is high ($X_1 \geq 0.77$), the direct effect becomes strongly positive, indicating that a clean, well-organized, and sustainable environment can generate greater economic opportunities, such as through recycling activities, organic waste utilization, or circular economy industries.
3) **Use of the 3R Principle (Y_1) on the Economic Benefits of Waste (Y_2)**
The direct effect between the Use of the 3R Principle (Y_1) and the Economic Benefits

of Waste (Y_2) is divided into two regimes. At a low level of 3R application (below 0.45), the effect is moderately positive, showing that 3R practices begin to generate economic value, although not yet optimally. However, when the 3R principle is applied more intensively (≥ 0.45), the effect increases sharply, indicating that the more consistently people implement the 3R principles, the greater the economic benefits obtained. This reinforces environmental economic theory, which states that comprehensive sustainable behavior can transform waste management from a burden into a tangible source of economic value.

4 Conclusion

This study aimed to address the limitation of previous research that relied on a single nonparametric approach and lacked a systematic method for identifying the appropriate model form. To overcome this, a modified Ramsey RESET procedure was proposed and evaluated as a diagnostic tool for detecting potential nonparametric relationships within path analysis models, particularly those combining truncated spline and Fourier series estimators. The findings suggest that the modified RESET shows promising performance in identifying nonparametric patterns across different functional forms.

The best-fitting hybrid nonparametric path model in this study was obtained when the relationships between Environmental Quality (X_1) and both the Use of the 3R Principle (Y_1) and Economic Benefits of Waste (Y_2) were modeled using truncated splines with two knot points; between the Use of Waste Banks (X_2) and both Y_1 and Y_2 using Fourier series with one oscillation; and between Y_1 and Y_2 using a truncated spline with one knot point. This model yielded an adjusted total coefficient of determination ($R^2_{T,\text{adj}}$) of 0.942 and a GCV value of 0.006, reflecting a strong in-sample fit for the data used. These empirical results illustrate the potential of integrating two nonparametric estimators within a unified path analysis framework.

Theoretically, this study contributes by introducing a diagnostic approach that may assist in identifying diverse nonparametric structures, and practically, it offers initial insights into how environmental quality, community behavior, and waste management systems may interact to generate economic value. Future research is recommended to examine higher-order spline forms, such as quadratic and cubic orders, and to incorporate interaction terms, to further refine the model's ability to capture more complex nonlinear relationship patterns.

CRediT Authorship Contribution Statement

Moh Zhafran Hidayatulloh: Conceptualization, Methodology, Formal Analysis, Software, visualization, Writing-Original Draft. **Solimun:** Conceptualization, Data Curation, Resources, Validation, Writing-Review & Editing. **Adji Achmad Rinaldo Fernandes:** Methodology, Formal Analysis, Supervision, Writing—Review & Editing. **Anggun Fadhila Rizqia:** Data curation, Validation, Visualization, Writing—Review & Editing. **Fachira Haneinanda Junianto:** Project Administration, Resources, Supervision, Writing—Review & Editing.

Declaration of Generative AI and AI-assisted technologies

The use of AI-assisted technology in preparing this manuscript was limited to ChatGPT version 5. The AI was employed to assist with grammar checking, translation from Indonesian to English, and conversion and formatting of the manuscript into Latex to meet academic writing standards. No AI tools were used for data analysis, graph generation, or model development.

Declaration of Competing Interest

The authors declare no competing interest.

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Data and Code Availability

The data and code supporting the findings of this study are available from the corresponding author upon reasonable request and subject to confidentiality agreements. The analysis code and reproducible workflow (excluding confidential data) are openly accessible via the project's GitHub repository [1].

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