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# Application of the Saint-Venant Model for Simulation of the Kapuas River Flow Using the Finite Difference Method

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**Abstract**

The Kapuas River plays an essential role in transportation, fisheries, tourism, and natural drainage in Pontianak, where river geometry and downstream effects shape its flow conditions. These forces produce variations in discharge and water level over space and time, which can exceed the river's capacity and cause flooding. This study simulates the dynamics of discharge and water level along the Kapuas Kecil River segment. Researchers developed a mathematical flow model using the Saint-Venant framework, based on mass and momentum conservation principles. This study numerically solved the equations using a finite difference approach with a forward-time and central-space scheme. Researchers collected river width and depth data along a 5 km stretch from Pontianak Utara to Jungkat. The study presents simulation results as spatial and temporal profiles of water level and discharge. The results of this study show that water discharge increases along the channel, while water levels generally decrease over time in the direction of flow. These results provide insights into -influenced river flows in urban environments and can support flood-related analysis and management efforts.

**Keywords:** Central Difference; Discharge; Forward Difference; Mass Balance Equation; Momentum Balance Equation.



## 1. Introduction

The Kapuas River is the longest river in Indonesia, stretching for 1,143 km [1]. Located on the island of Borneo, this river starts in Kapuas Hulu Regency and flows to the Karimata Strait, emptying at the river's estuary. The Kapuas Kecil segment forms part of the Kapuas River in Pontianak, West Kalimantan. This segment plays a crucial role in transporting goods and supporting population mobility [2]. Communities use the Kapuas Kecil for water transport, tourism, fisheries, trade, and as part of Pontianak's natural drainage system [3]. The Kapuas Kecil, which flows through the city of Pontianak and Kubu Raya Regency, drains directly into the South China Sea [4]. As the downstream section of the Kapuas, this segment experiences influences from the sea, which significantly affect its flow circulation [5]. fluctuations cause water levels and discharge in the Kapuas Kecil to change across both space and time [6]. Therefore, it is essential to analyze and model the Kapuas River's flow using mathematical methods so that its flow patterns can be understood and simulated, allowing identification of flood risks and supporting effective water resource management planning.

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Previous studies have used Saint-Venant's equation, which is a partial differential equation that represents unsteady flow in regular and irregular channels [7], [8]. There is research that applies the one-dimensional Saint-Venant Equation to describe flow dynamics using a physics-informed neural networks (PINN) approach [9] and numerical flow simulation in a channel using discharge data recorded upstream as boundary conditions [10]. Numerical methods, particularly finite difference methods, can solve Saint-Venant equations for flow models with channel shapes that vary along the flow using a forward scheme for derivatives with respect to time and a central scheme for derivatives with respect to space. This approach replaces the derivatives in the differential equation with a discretized form obtained through Taylor series expansion [11].

Although many studies have applied Saint-Venant's Equation using numerical methods to model flow in open channels, few studies have modeled flow in the Kapuas River. The Kapuas Kecil River has flow characteristics that differ from ideal channels because its width, depth, and cross-sectional shape vary. This study aims to understand the flow patterns of the Kapuas Kecil River through simulations of discharge and water depth from the riverbed. This study applies the Saint-Venant Equation, which is discretized using the finite difference method, that is forward difference scheme for time derivatives and the central difference scheme for spatial derivatives [12]. This study also uses observation data from the Kapuas Kecil River and applies a model that considers variations in the width, depth, and cross-sectional shape of the river. Thus, this study contributes not only to the development of a model of unsteady flow in irregular channels, but also provides a better understanding of the potential flow dynamics that can affect flood risk in the area surrounding the river. Through this simulation, researchers hope to gain a better understanding of the flow patterns in the Kapuas Kecil River.

## 2. Methods

In this section, the researchers explain the research methods and steps. The research flowchart is presented as follows

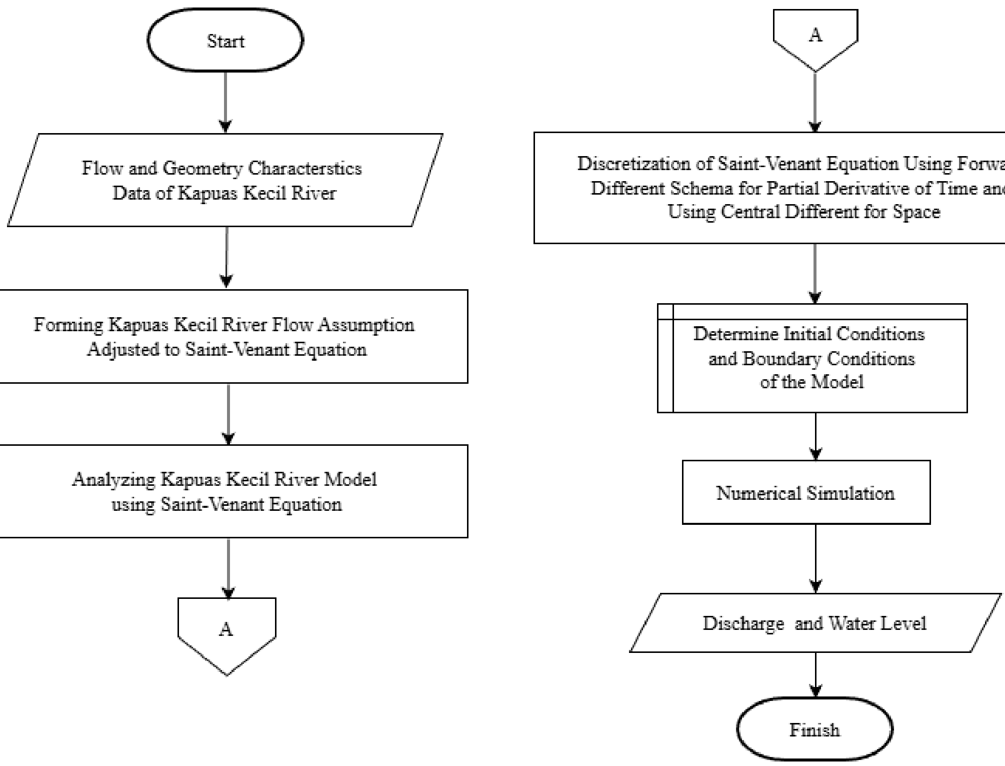


**Fig. 1:** Flowchart of the Application of the Saint-Venant Model for Simulating the Flow of the Kapuas Kecil River using the Finite Difference Method

### 2.1. Kapuas River

The Kapuas River flows from Kapuas Hulu Regency to the city of Pontianak [13]. The Kapuas River in the Pontianak area is the downstream section of the Kapuas River, known as the Kapuas Kecil River, which flows directly into the South China Sea [14]. fluctuations can affect river flow patterns, causing changes in water levels over space and time. River flow, which depends on the characteristics and geometry of the river, also influences these changes in height [15], [16]. The Kapuas Kecil River varies in width, some parts are quite wide, some are narrow, and there are branches and bends in the river [17]. Cross-sectionally, the bottom of the Kapuas Kecil River varies between the middle and the edges of the river. The bottom in the middle of the Kapuas River is deeper than at the edges [18]. The bottom of the Kapuas Kecil River is relatively flat from upstream to downstream, so that the water tends to flow slowly downstream [19].

Based on these geometric characteristics, the researchers selected a section of the Kapuas Kecil River that has no branches, small islands, or sharp bends as the location for the simulation, as shown in Fig. 2.

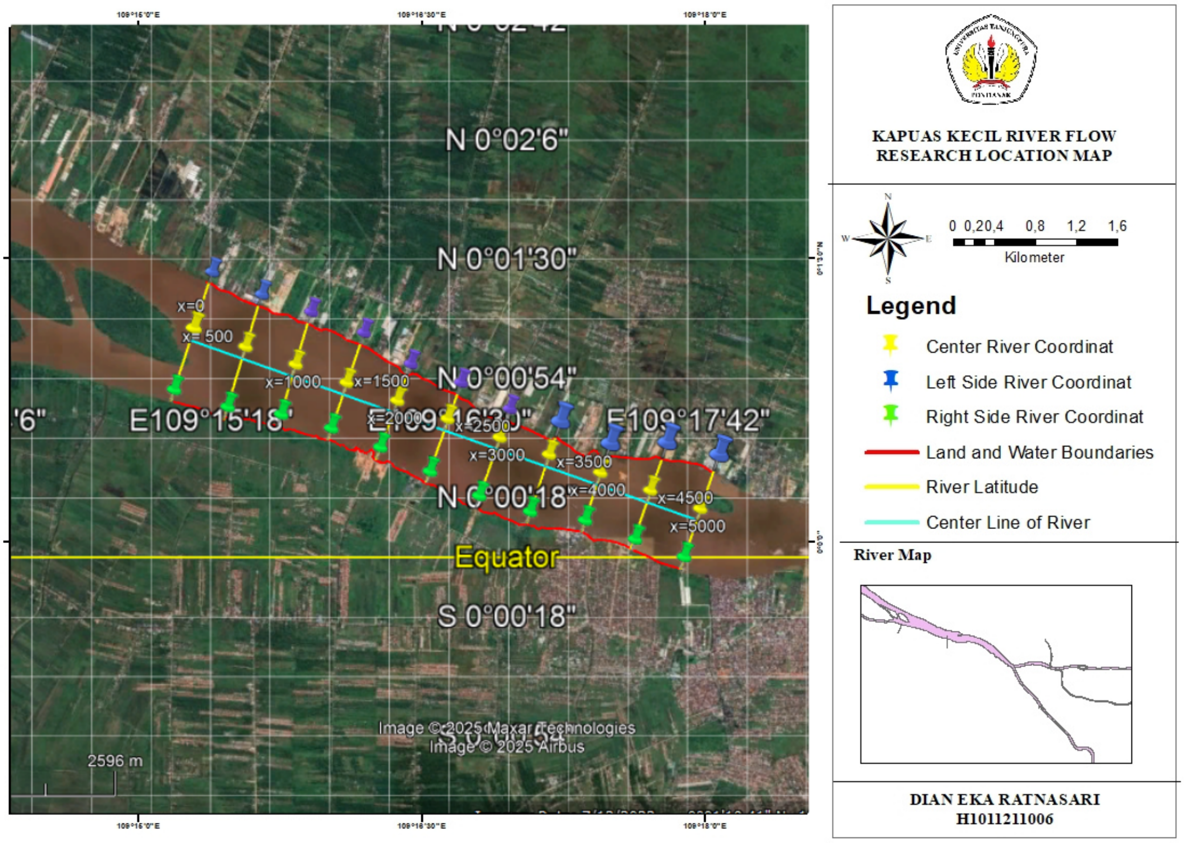


**Fig. 2:** Modeled Flow Location (Source: Google Earth, 2025)

The section of the Kapuas River whose flow was modeled is located in the district of North Pontianak to the district of Jungkat, which is opposite the district of West Pontianak to the district of Sungai Kakap. This river area is about 10 km from the mouth of the Kapuas River. The length of the modeled flow of the Kapuas River is about 5 km. Based on measurements from Google Earth Pro, at coordinates 0°01' 05.60" N 109°15' 11.93" E, marked with a yellow pin $x = 0$ , and 0°00' 11.42" N 109°17' 44.42" E, marked with a yellow pin $x = 5000$, a line was drawn, and the length of the line was found to be 5 km.

From the watershed to be simulated, researchers obtained the following data on river depth and width.

**Table 1:** Depth and Width Data of the Kapuas Kecil River

| Position from the starting point ($x = 0$) | Rivers Depth (m) | Rivers Width (m) |
|---|---|---|
| 0 | 6.91 | 963 |
| 500 | 8.90 | 888 |
| 1000 | 7.17 | 717 |
| 1500 | 6.78 | 842 |
| 2000 | 7.72 | 859 |
| 2500 | 8.17 | 874 |
| 3000 | 9.26 | 838 |
| 3500 | 9.53 | 939 |
| 4000 | 9.73 | 997 |
| 4500 | 9.73 | 1110 |
| 5000 | 9.93 | 1160 |

Water depth data was obtained from direct measurements at the research site at the yellow pin points based on Fig. 2. River depth data was collected on May 25, 2025, from 4:00 p.m. to 6:13 p.m. Water depth data was collected at high tide. Water depth was measured using fishing line wound on a fishing reel, with a weight of approximately 2 kg attached to the end. Depth measurements began at point 0, indicated by the yellow pin ($x = 0$) in Fig. 1. Measurements were taken by dropping the weight attached to the fishing line to the riverbed. Once the weight reached the riverbed, the fishing line at the water surface was marked. Then the iron is lifted up and the length of the line from the iron to the marked line is measured. This length is used as the depth of the river at point 0. Then, with the help of a water motor, move to point $x = 500$ and take the same measurements up dto point $x = 5000$.

In addition to the river height, the width of the river surface at the start of observation is also needed to determine the cross-sectional area at the start of observation. River width measurements also use Google Earth Pro by measuring the river width per 500 m from the starting position, which is from coordinates 0°00'47.13"N 109°15'05.91"E with a green pin symbol to 0°01'22.69"N 109°15'17.34"E with a blue pin symbol, drawn in a straight line like the yellow line in Figure 1. The length of the yellow line is used as the river width. The next river width data was measured using the same method as measuring the width on the initial yellow line ($x = 0$) to the last yellow line ($x = 500$). From these measurements, the river flow width was obtained, ranging from 717 m to 1,160 meters. Complete river width data is presented in Table 1. The river height at the start of observation can be seen in Table 1.

## 2.2. Assumptions of the Kapuas Kecil River Flow Model

Modeling was carried out based on the physical characteristics of the Kapuas Kecil River at that location. Thus, from these various characteristics, Several assumptions in modeling the flow of the Kapuas Kecil River are as follows:

1. One-dimensional flow
2. Unsteady flow
3. The length of the river affected by waves is much greater than the depth of the water.
4. The channel cross-section area is trapezoidal, with the base width being narrower or smaller than the water surface width.
5. The slope of the cross-sectional area or channel side is considered constant ($z = 2$).
6. The flow is continuous.
7. The fluid is incompressible.
8. The pressure distribution in the flow is considered hydrostatic.
9. The slope of the riverbed is relatively small.
10. The geometry of the riverbed is considered fixed (rigid boundary).

11. The effects of external forces such as wind and riverbed infiltration are ignored.
12. The flow velocity is considered uniform at each cross section.
13. Water tends to flow due to gravity
14. There are frictional forces that resist the flow

## 2.3. Saint-Venant Equations

The Saint-Venant equation contains two equations, the mass balance equation using the law of conservation of mass and the momentum balance equation using the law of conservation of momentum, both of which are derived from Newton's Second Law [9]. Fig. 3 shows the forces acting on the Saint-Venant equation.

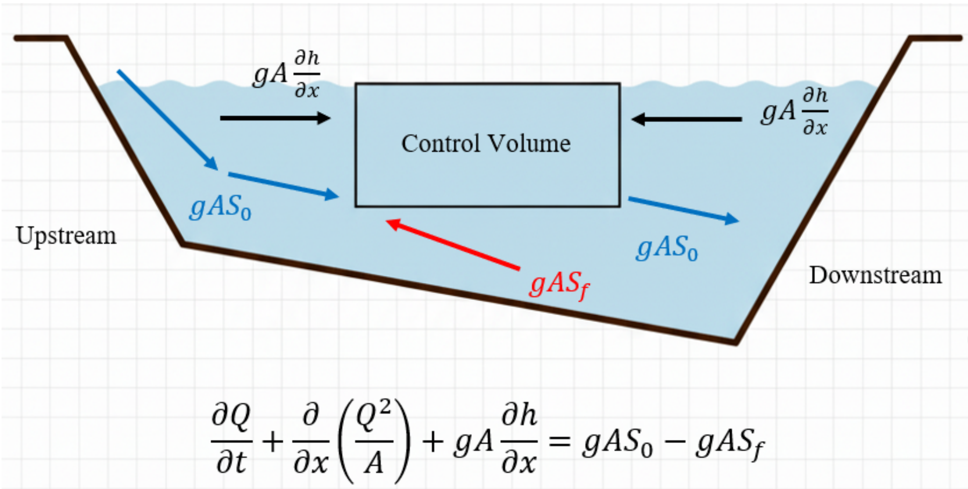


**Fig. 3:** Forces Acting on the Saint-Venant Equation

Modeling of the Kapuas Kecil River flow using the unsteady 1D Saint-Venant Equation. The Saint-Venant Equation can be written as follows

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 \tag{1}$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + gA\frac{\partial h}{\partial x} + gA\left(S_f - S_0\right) = 0 \tag{2}$$

where $A$ is the cross-sectional area of the channel $(m^2)$, $Q$ is the water discharge $(m^3/s)$, $t$ is time $(s)$, $x$ is the position along the flow direction, h is the water depth (m), g is the gravitational force $(m/s^2)$, $S_0$ is the riverbed slope, and $S_f$ is the friction slope [20]. The river cross-section area at the study site is assumed to be trapezoidal, as shown in Fig. 4.

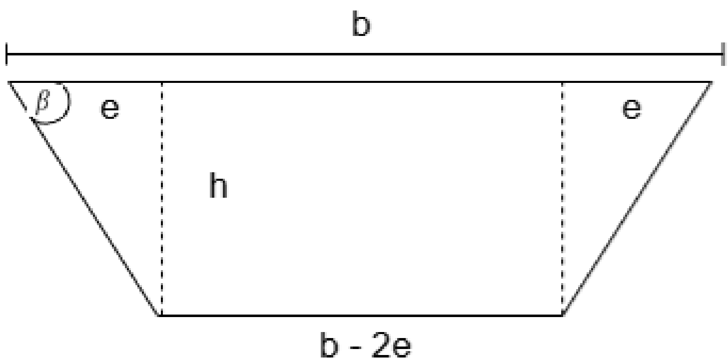


**Fig. 4:** Cross-Section of the Kapuas River Flow Model

Based on Fig. 4, $b$ is the channel width (upper surface) and $z$ is the channel side slope. The $z$ value is 2, which means that the slope of the channel side is approximately 63°. The angle is between 45° and 75°, so the discharge and velocity will increase [20]. In this study, $b$ and $h$ are not constant with respect to $x$, while $z$ is assumed to be constant for simplicity so that the cross-sectional area can be formulated as follows.

$$A(x,t) = (b(x,t) - zh(x,t))h(x,t) \tag{3}$$

with $S_f$ is the channel slope which can be calculated with

$$S_f = \frac{u^2c^2}{R^{4/3}} \tag{4}$$

where $u$ is the water velocity $(m/s)$, $c$ is the Manning coefficient, and $R$ is the hydraulic radius $(m)$. The flow cross-section is trapezoidal in shape, $R$ can be calculated with

$$R = \frac{A}{P} \tag{5}$$

where $P$ is the wet perimeter of the cross-section $(m)$ which is formulated with [8]

$$P = (b - 2zh) + 2h\sqrt{1+z^2} \tag{6}$$

## 2.4. Discretized by Finite Difference Method

This study uses the finite difference method by converting the domain of the independent variable into a limited grid called a mesh, where the dependent variable can be approximated [21]. This approach converts the derivatives in the differential equation into a discretized form based on the Taylor series [22]. Given a function $f$ and its derivatives, $f', f'', f''', \ldots$ continuous in $[a,b]$. Suppose $x_0 \in [a,b]$, then for every $x \in [a,b]$, the value of $f(x)$ can be expanded into a Taylor series around $x_0$ as follows [23]:

$$f(x) = f(x_0) + \frac{x-x_0}{1!}f'(x_0) + \frac{(x-x_0)^2}{2!}f''(x_0) + \cdots + \frac{(x-x_0)^m}{m!}f^{(m)}(x_0) + \cdots \tag{7}$$

There are several differential methods, some of which are as follows [11].

### *2.4.1. Forward Difference*

Based on Eqs. (7), $x - x_0$ is the difference between point $x$ and $x_0$. This difference can be written as $\Delta x$, so that $x = x_0 + \Delta x$. In the forward scheme, the derivative at point $x_i$ is calculated with one point after it, that is $x_i + \Delta x$. By taking the first three terms on the right side of Eqs. (7), we obtain [12]

$$\frac{f(x_i+\Delta x) - f(x_i)}{\Delta x} \approx f'(x_i) + \frac{\Delta x}{2!}f''(x_i) \tag{8}$$

By considering $\frac{\Delta x}{2!}f''(x_i)$ as an error, can obtain by following

$$\frac{df}{dx} \approx \frac{f(x_i+\Delta x) - f(x_i)}{\Delta x} \tag{9}$$

Eqs. (7)–(9) are still in the context of ordinary differential equations used to define Taylor series and finite differences. This study uses partial differential equations, therefore the forward scheme can be formed as follows:

- forward difference space

$$\frac{\partial f}{\partial x} \approx \frac{f_{i+1}^n - f_i^n}{\Delta x} \tag{10}$$

or

$$\frac{\partial f}{\partial x} \approx \frac{f_{i+1}^{n+1} - f_i^{n+1}}{\Delta x} \tag{11}$$

- forward difference time

$$\frac{\partial f}{\partial t} \approx \frac{f_i^{n+1} - f_i^n}{\Delta t} \tag{12}$$

or

$$\frac{\partial f}{\partial t} \approx \frac{f_{i+1}^{n+1} - f_{i+1}^n}{\Delta t} \tag{13}$$

where $i$ is the index for space, $n$ is the index for time, and $\Delta t = t_{n+1} - t_n$.

### *2.4.2. Backward Difference*

In the same way $x = x_0 + \Delta x$. In the backward scheme, the derivative at point $x_i$ is calculated with one point before it, that is $x_i - \Delta x$. By taking the first three terms on the right side of Eq. (7), we obtain [12]

$$\frac{f(x_i - \Delta x) - f(x_i)}{\Delta x} \approx f'(x_i) - \frac{\Delta x}{2!} f''(x_i) \tag{14}$$

By considering $\frac{\Delta x}{2!} f''(x_i)$ as an error, can obtain by following

$$\frac{df}{dx} \approx \frac{f(x_i - \Delta x) - f(x_i)}{\Delta x} \tag{15}$$

Therefore, the backward difference can be shown by

- backward difference space

$$\frac{\partial f}{\partial x} \approx \frac{f_i^n - f_{i-1}^n}{\Delta x} \tag{16}$$

or

$$\frac{\partial f}{\partial x} \approx \frac{f_i^{n+1} - f_{i-1}^{n+1}}{\Delta x} \tag{17}$$

- backward difference time

$$\frac{\partial f}{\partial t} \approx \frac{f_i^n - f_i^{n-1}}{\Delta t} \tag{18}$$

or

$$\frac{\partial f}{\partial t} \approx \frac{f_{i+1}^n - f_{i+1}^{n-1}}{\Delta t} \tag{19}$$

### *2.4.3. Central Difference*

By subtracting the Taylor series in the forward scheme from the Taylor series in the backward scheme [11], we obtain the center difference, which is

$$f'(x_i) \approx \frac{f(x_i + \Delta x) - f(x_i - \Delta x)}{2\Delta x} \tag{20}$$

Therefore, the central difference can be shown by

- central difference space

$$\frac{\partial f}{\partial x} \approx \frac{f_{i+1}^n - f_{i-1}^n}{2\Delta x} \tag{21}$$

or

$$\frac{\partial f}{\partial x} \approx \frac{f_{i+1}^{n+1} - f_{i-1}^{n+1}}{2\Delta x} \tag{22}$$

- central difference time

$$\frac{\partial f}{\partial t} \approx \frac{f_i^{n+1} - f_i^{n-1}}{\Delta t} \tag{23}$$

or

$$\frac{\partial f}{\partial t} \approx \frac{f_{i+1}^{n+1} - f_{i+1}^{n-1}}{\Delta t} \tag{24}$$

By discretizing Eq. (1) and Eq. (2) using forward differences for derivatives with respect to time and central differences for derivatives with respect to space [12], we obtain

$$A_i^{n+1} = A_i^n - \Delta t \left( \frac{Q_{i+1}^n - Q_{i-1}^n}{2\Delta x} \right) \tag{25}$$

$$\begin{aligned} Q_i^{n+1} = Q_i^n - \Delta t \Bigg( & \frac{2Q_i^n Q_{i+1}^n - Q_i^n}{A_i^n \, 2\Delta x} - \frac{(Q_i^n)^2 A_{i+1}^n - A_{i-1}^n}{(A_i^n)^2 \, 2\Delta x} \\ & + gA_i^n \left( \frac{h_{i+1}^n - h_{i-1}^n}{2\Delta x} \right) + gA_i^n \left( S_f^n - S_0 \right) \Bigg) \end{aligned} \tag{26}$$

## 2.5. Initial Conditions and Boundary Conditions

Initial conditions describe the flow conditions at the start of observation along the channel. These initial conditions consist of data on water height, river surface width, cross-sectional area, and water discharge at time $t = 0$ at each observation point along the channel. In this study, direct field measurements for initial condition data in the form of water height and river width were only carried out at 10 points along the channel at 500-meter intervals, at points 0 meters, 500 meters, 1000 meters, and up to 5000 meters. From the water height and river surface width data, the cross-sectional area and water discharge data can be calculated as follows

**Table 2:** Depth, Cross-sectional Area, and Discharge of Kapuas Kecil River

| Channel Station (m) | Cross-sectional Area ($m^2$) | Discharge ($m^3/s$) |
|---|---|---|
| 0 | 7920.10 | 609.85 |
| 500 | 9720.58 | 748.48 |
| 1000 | 7045.67 | 542.52 |
| 1500 | 6274.48 | 483.14 |
| 2000 | 6350.16 | 488.96 |
| 2500 | 7007.08 | 539.55 |
| 3000 | 7782.84 | 599.28 |
| 3500 | 7842.62 | 603.88 |
| 4000 | 6787.06 | 522.60 |
| 4500 | 8450.89 | 650.72 |
| 5000 | 9365.01 | 721.11 |

This study uses boundary conditions to solve Saint-Venant's equation numerically, including water height, river width, cross-sectional area, and water discharge at the starting point of the channel ($x = 0$) and the end of the channel ($x = 5000$), or what can be referred to as the upstream and downstream points, respectively. In this simulation, water height and river width can change in space and time. The start point at 6.91 meters that is, $h(x_0, t_0) = 6.91000, h(x_0, t_1) = 6.91001, h(x_0, t_2) = 6.91002, ..., h(x_0, t_179) = 6.91180$, the end point of the channel is 9.93 meters that is $h(x_{50}, t_n) = 9.93$ . The width at the starting point of the channel is 1160 meters written with, $b(x_0, t_0) = 1160.00, b(x_0, t_1) = 1160.01, b(x_0, t_2) = 1160.02, ..., b(x_0, t_179) = 1160.80$ and the width of the river surface at the end of the channel is 963 meters, $b(x_{50}, t_n) = 963$. In the same way as the initial conditions, the cross-sectional area and water discharge at the starting point of the channel can be obtained, $A(x_0, t) = 7,920.104\ m^2$ , $A(x_{50}, t) = 9,365.38\ m^2$, $Q(x_0, t_n) = 609.848\ m^3/s$, $Q(x_{50}, t_n) = 721.134\ m^3/s$.

## 2.6. Numerical simulation

The numerical simulation process was carried out by solving the Saint-Venant equation system, which had been discretized using the finite difference method. The solution was performed iteratively in space and time. In this study, the 5 km long Kapuas Kecil river basin was partitioned into 100 m sections, which can be written as $\Delta x = 100$, so that the entire area was divided into 50 grid cells.

In this study, direct measurements in the field for initial data in the form of water height and river width were only carried out at 10 points along the channel at 500-meter intervals, namely at points 0 meters, 500 meters, 1000 meters, and 5000 meters. However, for numerical simulation purposes, data with smaller intervals, namely 100-meter intervals, was required. Therefore, data at points that were not measured directly was obtained through linear interpolation using the "FORECAST" function in Microsoft Excel. From this interpolation, water level and river width data with 100-meter intervals along the 5000-meter channel was obtained.

In addition to partitioning the flow length, partitioning is also performed in time at 1-second intervals. To maintain numerical stability, the Courant-Friedrichs-Lewy (CFL) condition is required. The CFL condition can be expressed as follows [24]:

$$\Delta t \leq \frac{\Delta x}{u + \sqrt{gh}} \tag{27}$$

Thus, we obtain

$$\Delta t \leq \frac{100}{0,077 + \sqrt{9,81(9,93)}} \tag{28}$$

$$\Delta t \leq 10,053 \tag{29}$$

In this study using $\Delta t = 1$ which already meets the CFL requirements. The river slope parameter must also be used, because water flows from high to low surfaces. The river slope ($S_0$) can be calculated using the following formula:

$$S_0 = \frac{\text{Water level upstream} - \text{Water level downstream}}{\text{Length of flow}}. \tag{30}$$

Field measurements showed that the upstream elevation in the study area was 9.93 meters and the downstream elevation was 6.91 meters, so that

$$S_0 = \frac{9,93 - 6,91}{5000} = \frac{3,02}{5000} = 0,000604. \tag{31}$$

So the slope of the river is 0.000604 meters. Based on the type of sedimentation and the shape of the riverbed in this study, the Manning coefficient that can be used is 0.03 [25] and the acceleration due to gravity is 9.81 $m/s$ [10]. The parameters used in the simulation are as follows.

**Table 3:** Parameters in the Flow Model of Kapuas Kecil River

| Parameter | Value | Unit | Source / Method |
|---|---|---|---|
| River length ($L$) | 5000 | m | Field measurement |
| Number of partitions ($N$) | 50 | - | Field measurement |
| $\Delta x$ | 100 | m | Field measurement |
| $\Delta t$ | 1 | s | Field measurement |
| Gravitational acceleration | 9.81 | m/s$^2$ | Literature [10] |
| Manning coefficient ($n$) | 0.03 | m/s$^3$ | Literature [25] |
| Bed slope ($S_0$) | 0.000604 | - | Field measurement |
| Flow velocity ($u$) | 0.077 | m/s | Field measurement |

Based on the parameters in Table 3, this study calculates the Saint-Venant Equation, which has been discretized using the finite difference method. These parameters serve as the initial values for the simulation to produce the distribution of water discharge along the channel at a specified time.

## 3. Results and Discussion

The results of the numerical simulation are visualized in a graph showing the relationship between water discharge and river length. Data processing using Ms-Excel produced a simulation of water discharge against channel length, which is shown in Fig. 5.

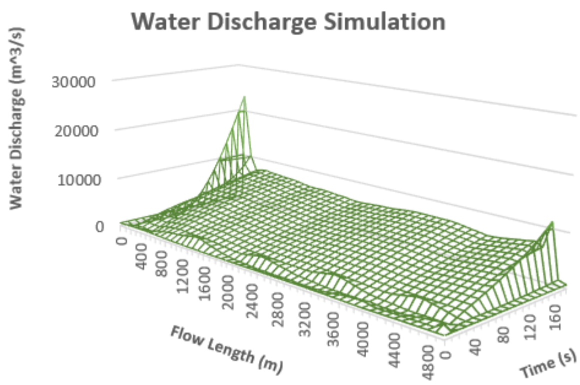


**Fig. 5:** Discharge Water Simulation Graph

The graph simulating water discharge along a 5 km channel shows that at the beginning of the simulation ($t = 60$ seconds), the water discharge was still relatively small throughout the flow. As time increased ($t = 120$ seconds and $t = 180$ seconds), the water discharge increased, especially at the starting point of observation ($x$ approaching 0 m), with a peak discharge of around 23,000 $m^3/s$ at $t = 180$ seconds. The surge occurred due to the response of the discharge to constant initial and boundary conditions. After this peak, the discharge decreases and then tends to be constant at around 7,000 $m^3/s$ along the channel until it approaches the downstream. This pattern shows that the flow wave moves from upstream to downstream and shows the process of adjusting the discharge to the flow boundary conditions.

This simulation also shows a correlation with the actual conditions of the Kapuas Kecil River, that the model is able to describe the waves that occur in the river. The water discharge in the simulation increases over time, in accordance with the increase in discharge that occurs during high tide in the Kapuas Kecil River. In addition to the water discharge simulation, a simulation of the water level from the channel bottom was also obtained and is shown in Fig. 6.

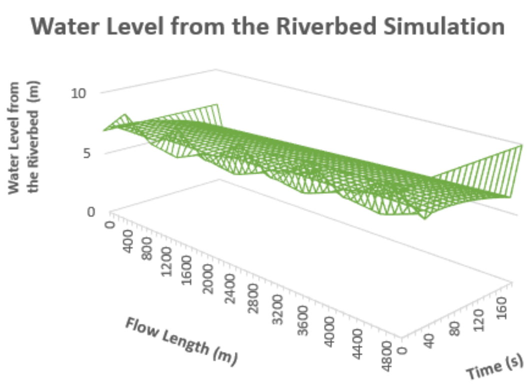


**Fig. 6:** Water Level from Riverbed Simulation Graph

Fig. 6 shows a simulation of the water level from the bottom of the channel with a flow length of 5000 meters at observation times $t = 60$ seconds, $t = 120$ seconds, and $t = 180$ seconds. Based on Fig. 6, the graph shows that the water level tends to decrease from the beginning of the channel to the end of the channel. At the start of observation, i.e., $t = 60$ seconds, the water level was at a relatively high value of around 7 meters. As time increased, the water level decreased at all points along the channel. At observation times $t = 180$ seconds, the decrease was more pronounced, with the water level being 2 to 3 meters lower than the initial condition.

The difference in height between the beginning and end of the flow in the graph shows that the flow wave moves upstream and the water flow spreads along the channel. Based on this pattern, the water level simulation shows that the water level decreases over time. This illustrates the receding conditions on the Kapuas Kecil River, where the water level gradually decreases.

Significant differences at the beginning and end of the flow in the discharge and water level simulations occurred due to the interaction between the boundary conditions applied and the numerical characteristics of the finite difference method.

## 4. Conclusion

The Kapuas Kecil River flow model was developed based on the assumptions of the Saint-Venant 1D model. The finite difference method with a forward scheme for time and a central scheme for space was used to solve the Saint-Venant equations, enabling the model to generate discharge and water level data for approximately 180 seconds along a 5 km channel. The water discharge simulation shows that the water discharge increases over time at all points along the flow. At the beginning of the flow, there is a spike in discharge value due to the response to boundary conditions, but the discharge distribution becomes more even from a distance of 300 meters from the starting point to the end of the flow ($x = 4800$). The water level simulation shows that the water level tends to decrease over time, but its value increases along the flow. This pattern shows the movement of waves towards the end of the channel and the redistribution of water mass. The simulation illustrates the flow pattern of the Kapuas Kecil River. By understanding this pattern, the community can implement flood mitigation measures, regulate sailing schedules, and manage water resources more effectively. Based on the limitations of this study, the suggestion for future work of this research is further research could model flooding in the Kapuas Kecil River by incorporating extreme rainfall, flood discharge, and the direct effects of tides. Researchers could also develop 1D–2D models to map flood-prone areas and design flood mitigation measures, such as the construction of levees, retention ponds, or early warning systems.

## CRediT Authorship Contribution Statement

**Dian Eka Ratnasari:** Conceptualization, Methodology, Data Curation, Writing–Original Draft, Formal Analysis. **Meliana Pasaribu:** Supervision, Validation, Formal Analysis, Writing–Review & Editing, Visualization. **Yudhi:** Software, Validation, Visualization, Supervision, Formal Analysis, Writing-review editing.

## Declaration of Generative AI and AI-assisted technologies

In the process of compiling this work, I used artificial intelligence (AI) tools, specifically ChatGPT and Gemini, solely to assist in writing, such as sentence structure, grammar correction, improving clarity, and creating simple illustrations. No AI tools were used for data analysis, scientific interpretation, modeling, calculations, or the creation of research findings. All academic content, analysis, and research results presented in this work are entirely the result of the author's own thinking and work.

## Declaration of Competing Interest

The authors declare no conflicts of interest.

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## Data Availability

The data in this study is not publicly available. The data was obtained from direct measurements on the Kapuas River and additional measurements using Google Earth Pro.

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