



Numerical Pricing of European Stock Options Using Black-Scholes Interval Model

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Abstract

This study presents the numerical implementation of the Black-Scholes interval model for determining the price of European call options under parameter uncertainty. Unlike the classic Black-Scholes model, which assumes fixed volatility and risk-free interest rates, this study represents both parameters as limited intervals constructed based on historical data. Daily stock closing prices obtained from Yahoo Finance and risk-free interest rate data from the Federal Reserve Bank of St. Louis were used to define the parameter ranges. The interval Black-Scholes model was discretized in the asset price and time domains and solved numerically by formulating an optimization problem that minimizes the residual error of the interval partial differential equation with terminal payoff constraints. The numerical solution produces lower and upper bounds for option prices in the specified domain. Simulation results show that classic Black-Scholes option prices lie within the calculated interval price bounds, demonstrating numerical consistency between the deterministic and interval formulations. This study provides a structured numerical case study showing how the Black-Scholes interval model can be implemented to generate option price bounds under bounded parameter uncertainty.

Keywords: Stock Options; Black-Scholes Model; Interval Parameters; Discretization; Numerical Solution.

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1. Introduction

Economic and financial developments in the present day have created an increasingly complex and dynamic investment environment. Investment is a strategy used by individuals and institutions to manage funds or other valuable assets in certain instruments with the aim of obtaining profits or increasing wealth in the future [1, 2]. In making investment decisions, there are two fundamental factors that need to be considered, namely return and risk. Basic principles in finance show a positive relationship between these two factors, whereby high potential returns also carry high risks [3, 4]. However, investors strive to minimize high risks while maintaining potential profits. To mitigate risk, derivative products can be used as financial instruments to manage and protect investment portfolios from market volatility [5, 6]. Derivatives are not only a hedging tool, but can also be used by investors to speculate on asset price movements.

One of the derivative instruments used is stock options. Stock options are derivative contracts whereby the holder is given the right (not the obligation) to buy (call option) or sell (put option)

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a stock at a strike price and time to maturity [7, 8]. There are two types of stock options: European-style and American-style. European-style options can only be exercised at maturity, while American-style options can be exercised at any time until maturity [1]. In determining the accurate price of an option, the fundamental step is to assess its suitability as an investment instrument or hedge. Various models have been developed to determine the price of an option, one of which is the Black-Scholes model.

The Black-Scholes model was first published by Fisher Black and Myron Scholes in 1973. The Black-Scholes model is a partial differential equation that estimates the price of a European option over time [9, 10]. This mathematical model can be used to determine the price of European-style stock options by considering parameters such as stock price, strike price, time to maturity, risk-free interest rate, and volatility [11]. The Black-Scholes model has the ability to provide an objective and standardized fair value for stock options, which previously depended on intuition and bargaining.

However, the classic Black-Scholes formulation relies on strict assumptions, particularly constant volatility and risk-free interest rates, which rarely hold in real financial markets [12]. In practice, both parameters experience significant fluctuations triggered by market dynamics and macroeconomic uncertainty, which can lead to model specification errors when single-point parameter estimates are used [13]. Therefore, this approach is considered unsuitable for stock market dynamics.

Based on these issues, various model developments have been carried out. Research conducted by Ding and Meng shows that tree-based methods, namely binomial trees, can outperform other models in terms of accuracy [14, 15]. Further development was carried out by integrating a more realistic volatility model, such as the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model, which combines past residuals and past variances to better capture volatility dynamics using fewer parameters [16]. In addition, the trinomial tree method is an improvement on the binomial model by allowing for three possible price movements: up, down, and unchanged at each time step [3]. Although these methods are effective for obtaining point estimates under stochastic or discrete dynamics, they still rely on fixed parameter values and therefore provide limited insight into the pricing uncertainty arising from parameter ambiguity.

To overcome the limitations of the classic Black-Scholes model in representing market uncertainty. Interval analysis was developed as an alternative modeling framework, in which uncertain parameters such as volatility and risk-free interest rates are expressed in the form of intervals. This approach has been formalized through the interval Black-Scholes model proposed by [17], which shows that parameter uncertainty can be systematically integrated into the option pricing framework.

Despite these developments, existing studies have largely focused on the theoretical formulation of interval-based Black-Scholes models, while practical numerical implementation and empirical validation using real market data remain limited. In particular, the computational realization of interval models as optimization problems and their comparison with classic Black-Scholes prices have not been adequately explored. Therefore, this study focuses on the implementation of numerical optimization-based discretization of the Black-Scholes interval model, as well as proving its validity using real market data.

The proposed approach differs from standard finite difference schemes in that the discretized equations are solved via global optimization of residuals rather than via time-stepping or direct linear solvers. Instead, the solution is obtained by minimizing the residual of the Black-Scholes equation over the entire computational domain, resulting in upper and lower bounds for the option price that explicitly capture parameter uncertainty. This provides empirical validation of the interval model as a pricing framework that approximates reality and supports more robust risk assessment and hedging strategies under conditions of parameter uncertainty.

2. Methods

This section describes the research methodology adopted in this study, including the data sources, parameter construction, and analytical procedures used to address the research objectives.

2.1. Data and Parameter Construction

This research is a literature study that examines concepts and theories through various literature sources and simulations using mathematical computing software. Literature sources were obtained from various sources related to the topic being studied. This research is quantitative research with the aim of determining the price of European-style stock options by applying volatility and risk-free interest rate parameters as intervals using the Black-Scholes model.

The type of data used is secondary data on daily stock prices obtained from Yahoo Finance and risk-free interest rate data from the Federal Reserve Bank of St. Louis for a specific observation period. The data used in this study is historical data in the period from June 23, 2025, to December 23, 2025, and the data used is closing price data. The variables and parameters used in the study include:

Table 1: Parameter Description

Parameter	Description
S	Stock Price
K	Strike Price
T	Time Maturity
r	Risk-Free Interest Rate
\underline{r}	Lower Limit of Risk-Free Interest Rate
\bar{r}	Upper Limit of Risk-Free Interest Rate
σ	Stock Price Volatility
$\underline{\sigma}$	Minimum Stock Price Volatility
$\bar{\sigma}$	Maximum Stock Price Volatility

This study uses two models, namely the classic Black-Scholes model and the interval Black-Scholes model. The data processing stage begins with calculating the return and volatility of the shares to be used in the Black-Scholes model. Daily share returns are calculated using daily share price data with the following formula:

$$R_t = \ln \left(\frac{S_t}{S_{t-1}} \right), \quad (1)$$

where R_t is the stock return at time t , S_t is the stock price at time t , and S_{t-1} is the stock price at time $t - 1$. Volatility is calculated based on the standard deviation of stock returns:

$$\sigma_{daily} = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (R_i - \bar{R})^2}, \quad (2)$$

Thus, volatility can be calculated by describing the level of stock price fluctuations during the observation period [18]. The estimate is based on the standard deviation of stock price returns with k factor adjustment, formulated as follows:

$$\sigma = \sqrt{k\sigma_{daily}^2},$$

where $k = 252$ denotes the assumed number of trading days per year. To account for parameter uncertainty, volatility is modeled as an interval using a tolerance parameter α :

$$[\underline{\sigma}, \bar{\sigma}] = [(1 - \alpha)\sigma, (1 + \alpha)\sigma],$$

Next, we find the risk-free interest rate. Risk-free interest rates obtained from Federal Reserve Bank of St. Louis are reported in annual percentage form and are converted into decimals prior to use in the model. Let \underline{r} and \bar{r} denote the minimum and maximum annualized rates observed during the study period. The risk-free rate is therefore modeled as

$$r \in [\underline{r}, \bar{r}].$$

2.2. Classical Black-Scholes Model

The Black-Scholes model is derived from the assumption that stock prices follow Geometric Brownian Motion, namely:

$$dS = \mu S dt + \sigma dB,$$

for example, $V(S, t)$ represents the option price at time t relative to the stock price $S(t)$. Then V in the Taylor series is as follows:

$$dV = \frac{\partial V}{\partial t} dt + \frac{\partial V}{\partial S} dS + \frac{1}{2} \frac{\partial^2 V}{\partial S^2} (dS)^2 + \frac{\partial^2 V}{\partial S \partial t} dS dt + \frac{1}{2} \frac{\partial^2 V}{\partial t^2} (dt)^2 + (hot),$$

with hot or higher order terms representing terms with higher exponents. Then form a risk-free portfolio using delta hedging, namely:

$$\Delta = \frac{\partial V}{\partial S},$$

this yields the following partial differential equation:

$$\frac{\partial V}{\partial t} = -rS(t) \frac{\partial V}{\partial S} - \frac{1}{2} \sigma^2 S^2(t) \frac{\partial^2 V}{\partial S^2} + rV(S, t) \quad (3)$$

For European call options, the terminal condition at maturity $t = T$ is defined by the payoff function $V(S, T) = (S - K)^+$. Which indicates that the option value equals the difference between the stock price and the strike price when $S > K$ and equals zero when $S < K$. The boundary condition is specified as $V = (0, t) = 0$. Since the option becomes worthless when the stock price is zero. Furthermore, as $S \rightarrow \infty$ the option value converges to $V(S, t) = S - Ke^{(-r(T-t))}$ reflecting the fact that the option is almost certain to be exercised in this case.

Through the transformation $x = \ln S, \tau = T - t$ and $u(x, \tau) = e^{r\tau} V(S, t)$ Eq. (3) is reduced to the heat equation, namely:

$$\frac{\partial u}{\partial \tau} = \frac{1}{2} \sigma^2 \frac{\partial^2 u}{\partial x^2},$$

so that the analytical solution for European call options is:

$$V(S, T) = SN(d_1) - N(d_2)Ke^{-r(T-t)},$$

with

$$d_1 = \frac{\ln(\frac{S}{K}) + (r + \frac{\sigma^2}{2})(T - t)}{\sigma\sqrt{T - t}},$$

$$d_2 = d_1 - \sigma\sqrt{T - t},$$

The above expressions represent the closed-form solution of the Black-Scholes model for European call options, where S denotes the underlying asset price, K is the strike price, r is the risk-free interest rate, σ is the volatility of the underlying asset, and $T - t$ is the time to maturity. The function $N(\cdot)$ denotes the cumulative distribution function of the standard normal distribution. This analytical solution will be used as a baseline for determining the price of classical options and for benchmarking the performance of the proposed modeling framework.

2.3. Black-Scholes Interval Model

The classic Black-Scholes model is modified by replacing the constant parameters r and σ with interval parameters, namely $r = [\underline{r}, \bar{r}]$ and $\sigma = [\underline{\sigma}, \bar{\sigma}]$. Thus, the Black-Scholes equation is no longer a single partial differential equation, but rather a family of equations that depend on the realization of the parameters in that interval. As a result, a unique classical solution is generally not available.

Suppose $V(S, t)$ is the candidate option price function. To determine the extent to which this function satisfies the Black-Scholes equation at the extreme limits of the parameters, two error functions (interval error functions) are defined as follows.

lower limit error:

$$\underline{A}(S, t) = \frac{\partial V}{\partial t} + \underline{r}S(t) \frac{\partial V}{\partial S} + \frac{1}{2} \underline{\sigma}^2 S^2(t) \frac{\partial^2 V}{\partial S^2} - \underline{r}V,$$

upper limit error:

$$\bar{A}(S, t) = \frac{\partial \bar{V}}{\partial t} + \bar{r}S(t) \frac{\partial \bar{V}}{\partial S} + \frac{1}{2} \bar{\sigma}^2 S^2(t) \frac{\partial^2 \bar{V}}{\partial S^2} - \bar{r}\bar{V},$$

these two error functions measure the residuals of the Black-Scholes equation when the parameters are at the extreme values of the interval. If $\underline{A}(S, t)$ and $\bar{A}(S, t)$ throughout the domain, then $V(S, t)$ is a valid solution for all parameter values within the interval. However, since exact solutions are rarely available, a numerical approach is used by minimizing the error globally.

Based on Theorem 4.3 in [17], a valid interval solution can be obtained by finding the function $V(S, t)$ that minimizes the total residue of both interval errors across the entire space-time domain. Intuitively, this approach is based on the principle that the solution to a classical partial differential equation is characterized by zero residuals, whereas in the interval case, a feasible solution is a function that minimizes the maximum violation of all possible equations. Thus, the total error objective function is defined:

$$\text{Minimize } TA = \sum_{i=0}^{m-1} \sum_{j=0}^{n-1} \int_{S_i}^{S_{i+1}} \int_{t_j}^{t_{j+1}} (\underline{A}(S, t) + \bar{A}(S, t)) dt dS,$$

with boundary conditions for call options:

- (1) $V(0, j) = 0, \quad \forall j = 0, 1, \dots, n,$
- (2) $V(i, n) = \max(i\Delta s - K, 0), \quad \forall i = 0, 1, \dots, m,$
- (3) $V(m, j) = m\Delta s - K \exp(-r\Delta t(n - j)), \quad \text{as } m \rightarrow \infty,$

the total interval difference minimization approach does not constitute a rigorous mathematical proof of closure in the sense of verified interval analysis. Instead, the obtained lower and upper bounds represent a numerically consistent price range induced by bounded parameter uncertainty. This approach ensures that the discretized Black-Scholes equation is satisfied as closely as possible at extreme parameter values, following the numerical framework established in [17].

Then the domain $[0, S_{max}] \times [0, T]$ is discretized into m price subinterval and n time subinterval, with $S_i = i\Delta S$ and $t_j = j\Delta t$, where $\Delta S = \frac{S_{max}}{m}$ and $\Delta t = \frac{T}{n}$.

To solve the minimization problem computationally, an approximation is made using the trapezoidal rule on the grid formed as follows:

$$TA = \frac{\Delta S \Delta t}{4} \sum_{i=0}^{m-1} \sum_{j=0}^{n-1} \underline{A}(S_{i+1}, t_{j+1}) + \underline{A}(S_{i+1}, t_j) + \underline{A}(S_i, t_{j+1}) + \underline{A}(S_i, t_j) + \bar{A}(S_{i+1}, t_{j+1}) + \bar{A}(S_{i+1}, t_j) + \bar{A}(S_i, t_{j+1}) + \bar{A}(S_i, t_j) dt dS$$

this formulation avoids duplication of terms and ensures that error evaluation is performed consistently at each grid point. The resulting nonlinear optimization problem is then solved numerically using an interior-point algorithm in open source mathematics application.

2.4. Validation Model

The interval model is considered consistent if the classic Black-Scholes price within the obtained interval $[\underline{V}, \overline{V}]$. These criteria follow the principle of restriction in interval analysis, where deterministic solutions are expected to lie within the interval solutions when deterministic parameters lie within the assumed limits.

3. Results and Discussion

This section presents the empirical results obtained from the proposed model and provides a discussion of the findings in relation to the research questions and existing literature.

3.1. Stock Option Pricing

Stock option pricing is determined using numerical simulation, with the optimal solution to the optimization problem obtained using open source mathematics application. The data used is historical data with an observation period from June 23, 2025 to December 23, 2025 and using daily closing prices obtained from Yahoo Finance. The closing stock price was obtained at \$664.94, with the strike price assumed to be \$655 and the maturity period set at 0.3917808 years.

The first stage begins with calculating the daily return of shares based on the logarithmic change in closing prices between days using Eq. (1). Based on the data processing results, positive and negative return values were obtained, indicating stock price fluctuations during the observation period. The results of the stock return calculations are shown in Table 2.

Table 2: Daily Stock Return

t	Date	S_t	S_{t-1}	R_t
2	June 23, 2025	712.2	712.2	0.0193806
3	June 24, 2025	698.53	712.2	0.0049546
\vdots	\vdots	\vdots	\vdots	\vdots
128	December 21, 2025	658.77	661.5	0.0051868
129	December 22, 2025	661.5	664.94	0.0041355

The average daily return is -0.0003850 or -0.04% per day. This result shows that the stock price decreased by 0.04% every day. Meanwhile, based on Eq. (2), the standard deviation value obtained was 0.0204419, which illustrates that the daily stock price fluctuations were quite high. Furthermore, based on the volatility equation, the annual volatility is 0.3245055. By applying an uncertainty tolerance factor of $\alpha = 5\%$, the volatility interval is obtained:

$$\sigma \in [0.3082802, 0.3407307]$$

Furthermore, the risk-free interest rate used was based on the Federal Reserve Bank of St. Louis for the observation period throughout 2025, namely:

$$r \in [0.0378, 0.0422]$$

The following are all the parameter values summarized in Table 3.

Table 3: Parameter Values

Parameter	Value
S	\$664.94
K	\$655
T	0.3917808
r	0.04

Parameter	Value
r	0.0378
\bar{r}	0.0422
σ	0.3245055
$\underline{\sigma}$	0.3082802
$\bar{\sigma}$	0.3407307

The parameter values in Table 3 are used as inputs to determine the stock option price. In the Black-Scholes interval model, discretization and interior point optimization were performed in Open source mathematics application with a grid of $m = 100$ and $n = 1000$ yielding the following results for European call options price:

Table 4: European Call Option Price

Model	European Call Option Price
Black-Scholes Interval	$\underline{V} = \$61.84, \bar{V} = \67.75
Classical Black-Scholes	\$63.64

Table 4 shows the results of determining the price of European call options using the classic Black-Scholes model and the Black-Scholes interval model. The interval model produces a range of option prices bounded by lower and upper values, reflecting the uncertainty of the volatility and risk-free interest rate parameters.

The classic option price lies between the lower and upper limits of the interval price, indicating that the deterministic solution is a special case of the interval solution when the parameters are single values.

3.2. Validation Model

The validity of the Black-Scholes interval model in this study is numerically verified through visualization of the relationship between the option prices resulting from the interval model and the classic Black-Scholes option prices. Fig. 1 shows a comparison between the lower and upper bounds of the option prices generated by the interval model and the classic Black-Scholes option prices in the same stock price domain.

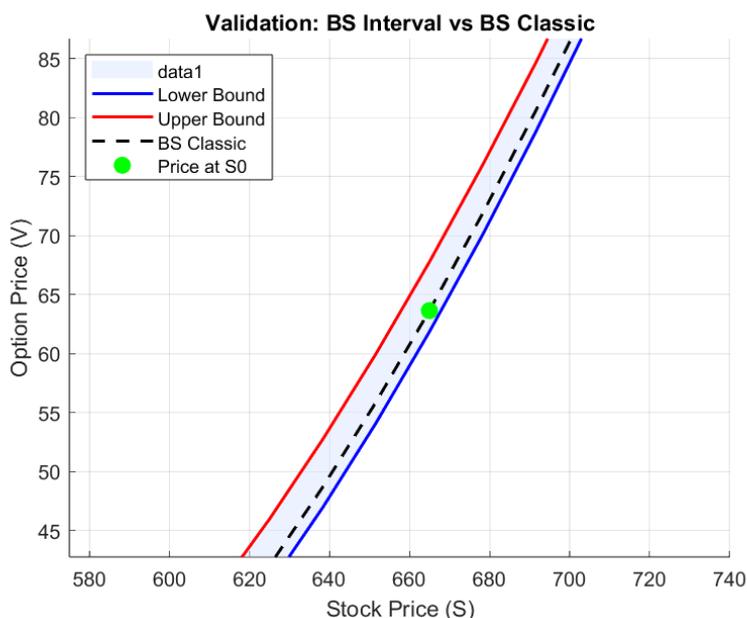


Fig. 1: Results For European-style Call Option Prices

In the figure, the two interval boundary curves represent the minimum and maximum option price solutions obtained from solving the Black-Scholes interval optimization problem, while the classic Black-Scholes curve shows the deterministic solution with a single parameter. It can be seen that the classic option price curve lies within the envelope formed by the lower and upper limits of the interval option prices across the entire analyzed stock price domain.

This condition shows that the classic Black-Scholes solution is numerically consistent with the interval formulation when the volatility and risk-free interest rate parameters are assumed to be within a certain range. Therefore, the existence of classic option prices within the interval is not interpreted as an increase in market accuracy, but rather as evidence that the interval formulation and optimization approach used produce numerically consistent and stable solutions.

4. Conclusion

This study presents a numerical implementation of the Black-Scholes interval model for determining the price of European call options, taking into account the uncertainty of volatility and risk-free interest rate parameters in the form of intervals. The approach used focuses on the numerical solution of the Black-Scholes interval equation through residual error formulation and optimization, thereby producing lower and upper bounds for the option price in the specified domain.

The numerical results show that the option prices obtained from the classical Black-Scholes model are within the price range generated by the interval model. This finding indicates structural consistency between deterministic and interval solutions, and suggests that the interval formulation used does not contradict the standard Black-Scholes theoretical framework.

The main contribution of this research lies in the presentation of a structured numerical case study on the application of the Black-Scholes interval model, including parameter interval construction, optimization problem formulation, and illustration of numerical option price results. Thus, this research serves as an applied numerical study that shows how interval models can be implemented and analyzed computationally.

This study has several limitations. First, the level of parameter uncertainty is represented through empirically determined static intervals, without considering time dynamics. Second, the numerical study was conducted on a single stock and a single grid configuration, so the results obtained are illustrative and are not intended as a convergence analysis or market generalization. Therefore, further development can be directed towards the use of dynamic intervals, more comprehensive numerical sensitivity studies, application to American-style options, or integration of stochastic volatility models into the Black-Scholes interval framework.

CRediT Authorship Contribution Statement

Muhammad Iqbal Aditama: Conceptualization, Methodology, Data Collection, Data Curation, Formal Analysis, Software Implementation, Validation, Visualization, Writing—Original Draft, and Writing—Review and Editing. **Rudianto Artiono:** Supervision and Validation.

Declaration of Generative AI and AI-assisted technologies

No generative AI or AI-assisted technologies were used during the preparation of this manuscript.

Declaration of Competing Interest

The authors declare no competing interests

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Data and Code Availability

The data used in this study consist of historical daily closing stock prices obtained from publicly available sources on Yahoo Finance, and risk-free interest rate data retrieved from the Federal Reserve Bank of St. Louis. These datasets are openly accessible and can be obtained directly from the respective official websites using the same sampling period described in this study.

The numerical implementation and optimization routines used in this study were developed by the authors for research purposes. Due to institutional and supervisory constraints, the source code cannot be publicly shared at this time. However, the methodological framework, model formulation, and numerical procedures have been described in sufficient detail to allow independent replication using standard numerical and optimization software.

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