CAUCHY – Jurnal Matematika Murni dan Aplikasi
Volume 11 (1) (2026), Pages 724-735
p-ISSN: 2086-0382; e-ISSN: 2477-3344

Numerical Pricing of Discrete-Dividend European Options: An Empirical Case Study

Muhammad Gibran Elgiffary and Rudianto Artiono*

Department of Mathematics, Universitas Negeri Surabaya, Indonesia

Abstract

This applied quantitative study implements Simpson's 1/3 numerical integration to systematically evaluate the pricing of discrete-dividend-adjusted European options. Using an anonymized technology stock (XYZ Tech Corp.) as an empirical case study, this research pursues two primary objectives: validating the computational convergence of the numerical algorithm against the exact Black-Scholes analytical benchmark using Relative Error, and analyzing the model's empirical pricing deviation against real market observations across various moneyness zones. Computational tests demonstrate that the Simpson's 1/3 method, inherently bounded by a fourth-order truncation error, achieves optimal and rapid convergence. By establishing the grid partition at $N = 200$, the algorithm successfully suppresses the relative error strictly below a 0.001% threshold compared to the analytical solution, executing efficiently under 0.005 seconds. Empirically, while the theoretical model exhibits high accuracy for In-The-Money (ITM) options with minimal deviation, it consistently reveals a significant overvaluation bias for Out-Of-The-Money (OTM) Call contracts, whereas OTM Put valuations exhibit a structurally different deviation pattern. This valuation asymmetry suggests a potential limitation of the constant historical volatility assumption, which appears inadequate to fully capture the implied volatility skew and shifting risk perceptions prevalent in actual market microstructures.

Keywords: Black-Scholes; Discrete Dividend; Moneyness; Numerical Integration; Simpson's 1/3 Method.

1. Introduction

Investment is an asset management strategy that aims to increase wealth in the future [1]. This activity generally takes place in capital markets that bring together sellers and buyers of various financial instruments such as stocks, bonds, and currencies [2, 3]. However, direct investment in stocks often exposes investors to the risk of extreme and unpredictable market volatility [4]. As a hedging solution, derivative instruments are available with values that depend on the performance of their underlying assets. One of the most common derivative products is the European-style stock option, which is a contract that gives the right to buy (call) or sell (put) an asset at a specific price and maturity date [5, 6].

In the context of option valuation, the Black-Scholes model introduced in 1973 has become the gold standard because it provides a closed-form solution [7, 8]. The model assumes a Brownian

***Corresponding author**. E-mail: rudiantoartiono@unesa.ac.id

Submitted: February 07, 2026 Reviewed: February 22, 2026 Accepted: March 19, 2026
DOI: https://doi.org/10.18860/cauchy.v11i1.40995

motion in stock prices and a normal distribution in returns [9]. However, this standard model has a fundamental flaw, namely the assumption that there are no dividend payments during the life of the option. In reality, discrete dividend payments are common corporate events that theoretically lower stock prices on the ex-date, thereby significantly affecting option premiums [10]. Ignoring this factor can certainly cause valuation biases that are detrimental to investors.

A number of approaches have been developed to accommodate the dividend factor, both analytically and numerically. Research by [11] models option prices with discrete dividends in a constant market, but the study has not classified the accuracy of the model based on the price zone of execution (moneyness). On the other hand, numerical approaches such as the finite difference method [12] and Simpson's numerical integration method [13] have begun to be widely applied to option valuation. The Simpson's 1/3 method was chosen because it is an efficient deterministic approach inherently bounded by a truncation error of $O(h^4)$ [14]. For single-asset European options, this method theoretically avoids the random sampling noise and heavy computational iteration loads typically required by stochastic simulations (Monte Carlo) to achieve stable decimal precision [15]. However, previous studies have generally been limited to simulating assumption parameters and have not tested the reliability of this numerical method on technology assets with high volatility in today's real market.

Although the numerical integration of dividend-adjusted Black-Scholes models has been widely discussed, there remains a practical need to systematically evaluate how this deterministic framework performs amidst actual market dynamics. Many previous studies have tended to focus primarily on proving pure mathematical convergence, without critically examining the empirical implications of the model's fundamental inputs such as the constant historical volatility assumption when confronted with real-world option quotes. Addressing this literature gap, this paper presents a structured numerical case study designed to bridge theoretical calculation with market reality. The study approximates the risk-neutral expectation integral representation of the option price using Simpson's 1/3 rule, serving as a straightforward alternative to repetitive stochastic simulations.

Therefore, the primary objective of this study is to implement and evaluate the Simpson's 1/3 numerical integration framework for pricing discrete-dividend European options. As an empirical case study, the computation is applied to a prominent US blue-chip technology stock. To preserve scientific reproducibility while preventing potential market misinterpretations regarding theoretical valuation claims, the instrument's ticker is anonymized as XYZ Tech Corp. The research focuses on two distinct objectives: (1) validating the convergence stability of the computational algorithm against its exact analytical benchmark, and (2) analyzing the model's valuation deviation against real market observations (last traded prices) across various moneyness spectrums. Through this evaluation framework, the research aims to provide a critical review of the limitations of constant volatility-based models in capturing anomalies in volatility expectations within turbulent stock markets.

2. Methods

This section describes the mathematical and numerical methodologies used in this study to determine the price of European-style options with discrete dividend payments. The discussion covers the principles of data acquisition, parameter estimation, dividend adjustment formulation, and the Simpson's 1/3 numerical integration procedure.

2.1. Data and Parameter Estimation

This research is an applied quantitative study aimed at evaluating the pricing of European-style stock options with discrete dividend adjustments. The theoretical framework utilizes the Black-Scholes model, with its expectation integral solved numerically via Simpson's 1/3 method. Model performance is assessed based on computational precision and pricing accuracy against actual market prices across various moneyness zones.

Secondary data includes the adjusted closing prices of XYZ Tech Corp. stock (Yahoo Finance) during the 2025 period to estimate historical volatility, as well as risk-free interest rate data from the Federal Reserve Bank of St. Louis (FRED) [16, 17]. To avoid timestamp mismatch bias, all market variables (S_0, K, option last prices, and r) were recorded simultaneously at market close on the same date. Option samples were selected via purposive sampling to proportionally represent the In-The-Money (ITM), At-The-Money (ATM), and Out-Of-The-Money (OTM) zones for both call and put options. The fundamental parameters used for the computation are summarized in Table 1.

Table 1: Parameter Description

Parameter	**Description**
S_0	Initial Stock Price
K	Strike Price
r	Risk-free Interest Rate
σ	Annualized Historical Volatility
D	Discrete Dividend Amount
T	Time to Maturity
t_d	Time to Ex-Dividend date

Parameter estimation begins with the calculation of daily logarithmic returns (r_t) from stock closing price data (S_t). The logarithmic return equation is defined as follows:

$$r_t = \ln\left(\frac{S_t}{S_{t-1}}\right), \tag{1}$$

where S_t is the stock price on day t and S_{t-1} is the stock price on the previous day.

Next, the annual historical volatility (σ) is estimated based on the sample standard deviation (s) of the log-return data, adjusted for the number of active trading days in a year ($T = 252$ days). The volatility formulation is:

$$s = \sqrt{\frac{1}{n-1}\sum_{t=1}^{n}(r_t - \bar{r})^2}, \tag{2}$$

and

$$\sigma = s\sqrt{252}, \tag{3}$$

where n is the number of data observations and $\bar{r}$ is the mean of the log-return.

2.2. Model Formulation

Stock price movements in the capital market are assumed to move randomly and follow Brownian Motion or the Wiener process [2]. Mathematically, changes in stock price S are formulated in the Stochastic Differential Equation as follows:

$$dS = \mu S dt + \sigma S dW, \tag{4}$$

where μ represents the expected rate of return, σ is the volatility of stock prices, and dW is a random component that follows the Wiener process [3].

2.2.1. Dividend

Dividends are a percentage of a company's profits delivered to its shareholders. In the capital market, this distribution occurs on a regular and discrete basis. In theory, on the ex-dividend date, the share price will fall by the amount of the dividend paid, if no other factors influence it [18]. This relation can be stated mathematically as:

$$S_{t+} = S_{t-} - D, \tag{5}$$

where S_{t+} represents the stock price immediately after time t, S_{t-} represents the stock price immediately before maturity time T, and D represents the dividend per share [10].

2.2.2. Black-Scholes Model

The Black-Scholes model is derived based on the formation of a risk-free hedge portfolio and the principle of no arbitrage [2]. The main assumptions underlying it include: Stock price movements follow Geometric Brownian Motion, there are no dividends during the life of the option, no transaction costs, continuous trading, constant risk-free interest rates, and options can only be exercised at maturity (European).

The price dynamics of the underlying stock S are assumed to move stochastically while exhibiting a specific drift, modeled via Geometric Brownian Motion (GBM). The fundamental premise of the GBM model is that the continuous compounding returns (log-returns) of the stock are normally distributed. This strict mathematical property ensures that the simulated stock prices strictly remain positive and inherently follow a log-normal distribution [7]. Changes in the underlying asset's price over an infinitesimally small time interval dt, technically referred to as an increment, follow the stochastic process previously defined in Eq. (4).

The hedge portfolio π is formed by holding one option (valued at V) and selling short Δ shares, written as:

$$\pi = V - \Delta S, \tag{6}$$

So the change in portfolio value over the time interval dt is:

$$d\pi = dV - \Delta dS, \tag{7}$$

Using Itô's lemma to find dV from $V(S,t)$ and performing substitution, we obtain:

$$d\pi = \left(\frac{\partial V}{\partial t} + \mu S\frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2 S^2\frac{\partial^2 V}{\partial S^2} - \Delta\mu S\right)dt + \sigma S\left(\frac{\partial V}{\partial S} - \Delta\right)dW, \tag{8}$$

Stochastic risk can be eliminated by choosing $\Delta = \frac{\partial V}{\partial S}$, known as delta hedging. Such a risk-free portfolio must generate a return equal to the risk-free interest rate r:

$$d\pi = r\pi dt, \tag{9}$$

By substituting $\pi = V - \frac{\partial V}{\partial S}S$, we obtain the Black-Scholes partial differential equation:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2\frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0, \tag{10}$$

Solving this equation requires terminal conditions and boundary conditions. For European call options, the terminal condition at maturity T is $C(S,T) = \max(S-K,0)$. The analytical solution, namely the Black-Scholes formula for call options, is expressed as:

$$C(S,t) = SN(d_1) - Ke^{-r(T-t)}N(d_2), \tag{11}$$

$$d_1 = \frac{\ln(S/K) + (r + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}}, \tag{12}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}, \tag{13}$$

where $N(\cdot)$ is the standard normal cumulative distribution function. The price of a European put option can be derived through the put-call parity relationship:

$$P(S,t) = Ke^{-r(T-t)}N(-d_2) - SN(-d_1) \tag{14}$$

2.2.3. Modification of the Black-Scholes Model with Discrete Dividend

The standard Black-Scholes model assumes no dividends [2]. In order for the model to be used to value options on stocks that pay dividends, modifications are necessary. A common approach is to adjust the stock price by discounting the present value of dividends to be paid.

Suppose there are n discrete dividend payments, each equal to D_i, which will be paid at time $t_{d,i}$ before the option expires ($t < t_d < T$). The present value of all dividends is calculated as:

$$PV(D) = \sum_{i=1}^{n} D_i e^{-rt_{d,i}} \tag{15}$$

The adjusted stock price, denoted by S^*, is then defined:

$$S^* = S_0 - PV(D) \tag{16}$$

The S^* value represents the component of the stock price that is not affected by claims on future dividends. Based on Eq. (11) we obtain the modified Black-Scholes formula for call options:

$$C(S,t) = S^* N(d_1) - Ke^{-r(T-t)} N(d_2), \tag{17}$$

$$d_1 = \frac{\ln(S^*/K) + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}, \tag{18}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}, \tag{19}$$

For put options, the modified formula becomes:

$$P(S,t) = Ke^{-r(T-t)} N(-d_2) - S^* N(-d_1) \tag{20}$$

2.3. Numerical Method

2.3.1. Simpson's 1/3 Method

Simpson's 1/3 method is a deterministic numerical integration technique that approximates the value of a definite integral using second-degree polynomials (parabolas) over each sub-interval partition. By interpolating the integrand with quadratic polynomials, Simpson's 1/3 method bounds the global truncation error at $O(h^4)$. This characteristic ensures a significantly faster mathematical convergence rate compared to lower-order algorithms, achieving high decimal precision without requiring an excessively large number of partitions (N).

Within the European option valuation framework, the current option price is represented as the discounted expected value of the future payoff under the risk-neutral probability measure Q [9]. In the context of European option valuation, the option price can be expressed as the discounted expected value of the future option payoff [9]. For a Call Option, the option price (C) is formulated as:

$$C = e^{-rT}\mathbb{E}^Q[\max(S_T - K, 0)] = e^{-rT} \int_0^{\infty} \max(S_T - K, 0) f(S_T) dS_T \tag{21}$$

As for Put Options, the option price (P) is expressed as:

$$P = e^{-rT}\mathbb{E}^Q[\max(K - S_T, 0)] = e^{-rT} \int_0^{\infty} \max(K - S_T, 0) f(S_T) dS_T \tag{22}$$

The function $f(S_T)$ represents the Probability Density Function (PDF) of the underlying stock price at maturity (T). Following the Geometric Brownian Motion assumption in the Black-Scholes model, S_T is assumed to be lognormally distributed [1]. By incorporating the initial stock price parameter corrected by the discrete dividend adjustment (S^*), the density function is formulated as follows:

$$f(S_T) = \frac{1}{S_T \sigma \sqrt{2\pi T}} \exp\left(-\frac{(\ln(S_T/S^*) - (r - \frac{1}{2}\sigma^2)T)^2}{2\sigma^2 T}\right) \tag{23}$$

To solve the integral numerically using Simpson's 1/3 rule, the integration interval $[a, b]$ is divided into N partitions (sub-intervals) with a step width $h = \frac{b-a}{N}$. The main condition for applying this method is that the number of partitions N must be an even number. The Simpson's 1/3 integral approximation algorithm is formulated as [5]:

$$\int_a^b g(S_T)dS_T \approx \frac{h}{3}\left[g(S_{T,0}) + 4\sum_{i=1,3,5,\ldots}^{N-1} g(S_{T,i}) + 2\sum_{i=2,4,6,\ldots}^{N-2} g(S_{T,i}) + g(S_{T,N})\right] \tag{24}$$

The function $g(S_T)$ in the algorithm above represents the product of the option payoff function and the lognormal PDF $f(S_T)$. To ensure that the numerical computation can be executed with precision and to avoid mathematical errors, the numerical integration bounds $[a, b]$ are strictly defined with the following justifications:

1. **For Call Options:** The lower bound of integration is set to the strike price $(a = K)$, as the call option payoff is zero for all $S_T < K$. Given that the theoretical upper bound is infinity (∞), the numerical upper bound (b) is truncated at $S_{max} = 4S^*$. This specific truncation bound is mathematically justified by a tail-probability argument: under the lognormal distribution properties, the cumulative probability of the stock price exceeding $4S^*$ within the short maturity timeframe of this study is statistically negligible (approaching zero). Therefore, cutting off the integration domain at this point ensures that the truncation error at the extreme right tail does not materially distort the calculated option value, safeguarding the precision, especially for OTM Call pricing [10].
2. **For Put Options:** The theoretical lower bound of integration is zero. However, since the evaluation of the lognormal PDF involves the term $\ln(S_T)$, the numerical lower bound is set to a very small epsilon approximation $(a = 0.0001)$. This approach is a standard technique in the numerical evaluation of improper integrals, aimed exclusively at circumventing the mathematical singularity caused by $\ln(0)$ without compromising the precision level of the Simpson's method itself [14]. The upper bound of integration is set to the strike price $(b = K)$, considering that the put option payoff is absolutely zero for $S_T > K$.

2.4. Evaluation Metrics

Mean Absolute Percentage Error (MAPE) is a statistical measurement method used to test the accuracy of a model compared to actual data. The main function of this method is to calculate the average error (deviation) in percentage form to evaluate the reliability of the model being tested. However, considering that the use of percentages can be biased when the actual option price is very small, this study also incorporates the Mean Absolute Error (MAE). MAE serves to compute the average of the absolute differences between the predicted values and the actual observed values nominally.

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{V_i - \hat{V}_i}{V_i}\right| \times 100\% \tag{25}$$

$$MAE = \frac{1}{n}\sum_{i=1}^{n}\left|V_i - \hat{V}_i\right| \tag{26}$$

In these equations, n indicates the total number of observations or option contract samples, V_i represents the actual value of the option price in the market, and $\hat{V}_i$ is the calculated value or model prediction.

3. Results and Discussion

This section presents the computational results and analysis of discrete-dividend European option valuation using the Simpson's 1/3 method. The discussion encompasses four essential stages:

fundamental parameter estimation, mathematical convergence testing, accuracy evaluation against real market prices, and empirical analysis of valuation deviations.

3.1. Research Data and Parameters

The data used for the valuation computation in this study were recorded simultaneously based on the market closing conditions on January 7, 2026. Based on the observation on that date, the underlying asset price of XYZ Tech Corp. (Yahoo Finance) was recorded at $S_0 = 189.11$ USD, and the risk-free interest rate from the Federal Reserve Bank of St. Louis (FRED) instrument was at the 3.52% level [16, 17]. This discount basis value was subsequently converted mathematically into a continuous compounding rate with $r = \ln(1 + 0.0352) \approx 3.46\%$ to satisfy the Black-Scholes model parameters. Furthermore, to accommodate the discrete dividend friction, the payment parameter was calibrated based on the historical quarterly dividend distribution policy of XYZ Tech Corp. Consequently, an anticipated discrete dividend of $D = 0.01$ USD is projected to occur 64 days prior to the option expiration ($t_d = 64/365$). Based on Eq. (16), the underlying asset price was subsequently adjusted to $S^* = 189.11 - 0.01 \cdot e^{-0.0346 \cdot (64/365)} = 189.10$ USD.

Subsequently, the historical volatility (σ) was estimated using the daily closing price movements of XYZ Tech Corp. over the 2025 trading period. Based on the standard deviation calculation of the daily log-returns, the annualized historical volatility was obtained at 49.91%.

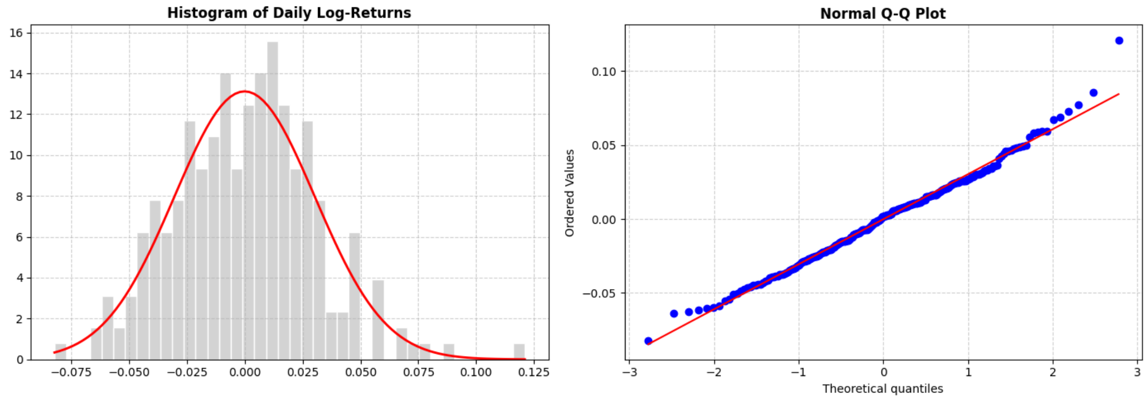

Fig. 1: Histogram and Normal Q-Q Plot of Daily Log-Returns

Fig. 1 visualizes the log-return distribution of the historical data. Visually, the empirical histogram forms a symmetrical bell curve, and the Normal Q-Q Plot shows the observed quantiles aligning closely with the theoretical line around the mean. However, consistent with widely recognized stylized facts of financial time series, minor deviations at the extreme tails are observable. While these visual diagnostics sufficiently justify the use of the normal assumption for baseline Black-Scholes computational modeling, they also indicate the presence of empirical market frictions (such as fat-tails) that constant-volatility models fundamentally fail to capture.

All processed and validated fundamental parameters are summarized in Table 2. These parameters are designated as constant inputs to be integrated into the Simpson's 1/3 algorithm in the subsequent convergence test phase.

In addition to the fundamental parameters of the underlying asset, this study utilized 10 samples of European option contracts (5 call options and 5 put options) from XYZ Tech Corp. expiring on March 20, 2026, as the primary computational objects. The five strike prices were purposively selected to proportionally represent the spectrum of In-The-Money (ITM), At-The-Money (ATM), and Out-Of-The-Money (OTM) zones based on their ratio to the actual underlying asset price. The real market prices (last prices) of these ten instruments were also recorded on the same valuation date (January 7, 2026), which will be maintained as the real-world market benchmark in the final evaluation stage. The details of the sample option contracts along with their moneyness zone classifications are presented in Table 3.

Table 2: Parameter Value

Parameter	Value	Information
S_0	189.11 USD	Initial underlying asset price at market close
S^*	189.10 USD	Adjusted underlying asset price
r	3.46%	Continuous compounding risk-free interest rate
K	See Table 3	Strike prices representing various moneyness zones
σ	49.91%	Annualized historical volatility
T	0.1972 years	Time to maturity (72/365 years)
t_d	0.1753 years	Time to ex-dividend date (64/365 years)
N	10, 20, 50, 100, 200, 500	Number of grid partitions for numerical integration

Table 3: Sample Option Contracts and Moneyness Classifications

Strike Price (USD)	Call		Put	
	Zone	Last Price (USD)	Zone	Last Price (USD)
140	ITM	51.40	OTM	1.40
155	ITM	38.85	OTM	3.11
190	ATM	14.67	ATM	14.00
220	OTM	4.60	ITM	34.10
240	OTM	1.89	ITM	51.40

3.2. Numerical Model Convergence Test

The Black-Scholes equation possesses an exact analytical closed-form solution that is mathematically absolute. Therefore, this analytical solution serves as the ground truth to validate the performance of the Simpson's 1/3 numerical integration method and to measure the degree of discretization error. The convergence testing is evaluated using the Relative Error (E_R) metric and is conducted comprehensively across five strike price levels to ensure the algorithm's reliability over the entire spectrum of moneyness zones.

Table 4: Convergence Test Results for Call Options

N	E_R 140 ITM (%)	E_R 155 ITM (%)	E_R 190 ATM (%)	E_R 220 OTM (%)	E_R 240 OTM (%)	Time (Sec)
10	8.239291	5.911636	1.099434	5.679140	7.594442	0.008420
20	0.152209	0.369449	0.255827	0.158528	0.410270	0.001877
50	0.000588	0.005361	0.007139	0.001883	0.008911	0.001960
100	0.000034	0.000313	0.000443	0.000101	0.000543	0.003994
200	0.000002	0.000019	0.000027	0.000007	0.000035	0.004483
500	0.000000	0.000000	0.000000	0.000001	0.000003	0.011576

Table 5: Convergence Test Results for Put Options

N	E_R 140 OTM (%)	E_R 155 OTM (%)	E_R 190 ATM (%)	E_R 220 ITM (%)	E_R 240 ITM (%)	Time (Sec)
10	0.033799	0.167914	0.074225	0.079619	0.106160	0.001890
20	0.002163	0.009434	0.003606	0.000399	0.001090	0.001763
50	0.000054	0.000234	0.000092	0.000009	0.000027	0.001751
100	0.000003	0.000015	0.000006	0.000001	0.000002	0.001699
200	0.000000	0.000001	0.000000	0.000000	0.000000	0.001852
500	0.000000	0.000000	0.000000	0.000000	0.000000	0.001856

Based on Table 4 and Table 5, the Simpson's 1/3 method consistently demonstrates a high level of stability and converges toward the exact analytical Black-Scholes solution. Although this high precision is achieved generally, there is a fundamental mathematical distinction between Call and Put options underlying this computational process. For Put options, the risk-neutral expectation formulation is integrated over a finite range (from 0 to the strike price K), allowing the numerical method to directly achieve optimal precision as the number of partitions increases.

Conversely, Call options possess a theoretical upper bound that approaches infinity, thereby necessitating the establishment of a numerical integration truncation bound. Consequently, the convergence rate for Call options is highly sensitive to the calibration of this upper bound and does not solely depend on the partition resolution.

Despite the differences in integral boundary treatments, this convergence explicitly establishes $N = 200$ as a globally robust computational parameter, specifically optimized for the underlying asset characteristics in this study. By scaling the partition setting to $N = 200$, the relative errors for all option samples are simultaneously suppressed to a strict tolerance below the 0.001% threshold. Furthermore, this mathematical precision is achieved with high computational efficiency, recording execution times of less than 0.005 seconds for simultaneous multizone valuations. Such empirical performance firmly highlights the practical feasibility of the one-dimensional deterministic integration approach. Achieving absolute mathematical precision in under 0.005 seconds demonstrates that the Simpson's 1/3 algorithm is highly efficient and computationally lightweight for standalone European option valuations.

3.3. Accuracy Evaluation Against Real Market Prices

The valuation results of the discrete-dividend Black-Scholes model, executed using the Simpson's 1/3 numerical integration at the optimal parameter $N = 200$, were evaluated against real market observations (last price).

Table 6: Pricing Accuracy of the Simpson's 1/3 Method for Call Options

Zone	Strike Price (USD)	Last Price (USD)	Simpson Price (USD)	Absolute Error (USD)	Abs. Percentage Error (%)
ITM	140	51.40	51.4025	0.0025	0.0049
ITM	155	38.85	38.7490	0.1010	0.2600
ATM	190	14.67	16.8682	2.1982	14.9841
OTM	220	4.60	6.9542	2.3542	51.1782
OTM	240	1.89	3.5856	1.6956	89.7122
Average				**1.2703**	**31.23**

Table 7: Pricing Accuracy of the Simpson's 1/3 Method for Put Options

Zone	Strike Price (USD)	Last Price (USD)	Simpson Price (USD)	Absolute Error (USD)	Abs. Percentage Error (%)
OTM	140	1.40	1.3503	0.0497	3.5474
OTM	155	3.11	3.5948	0.4848	15.5879
ATM	190	14.00	16.4759	2.4759	17.6852
ITM	220	34.10	36.3579	2.2579	6.6215
ITM	240	51.40	52.8533	1.4533	2.8274
Average				**1.3443**	**9.2539**

Based on Table 6 and Table 7, the overall valuation projection yields a Mean Absolute Error (MAE) of 1.2703 USD for call instruments and 1.3443 USD for put instruments. Proportionally, the Mean Absolute Percentage Error (MAPE) is recorded at 31.23% (call) and 9.25% (put).

Analyzed across the moneyness spectrum, the distribution of deviations exhibits a highly asymmetrical pattern. For call instruments, the highest accuracy is consistently clustered in the In-The-Money (ITM) zone, with an APE below 0.26%. However, the deviation escalates sharply as the price moves toward the Out-Of-The-Money (OTM) zone, peaking at 89.71%. Conversely, put instruments display a different anomaly; the highest percentage deviation is not located in the extreme zones but is concentrated around the At-The-Money (ATM) area at 17.69%, with deviations in the deep-ITM and deep-OTM zones remaining more contained below 7%.

3.4. Valuation Deviation and Market Dynamics Analysis

Considering that the convergence stability of the Simpson's 1/3 method has been mathematically validated and proven free from discretization anomalies, the asymmetrical disparity between the theoretical numerical prices and real market prices strongly suggests a potential friction between the fundamental assumptions of the Black-Scholes model and the actual market microstructure.

The primary factor interpreting the high percentage of deviation (particularly in the OTM zone) is the assumption of a constant historical volatility ($\sigma = 49.91\%$). In the reality of the options market, market participants generally anticipate the risk of extreme price movements (tail risk) by assigning additional premiums to OTM options. This psychological dynamic creates a curved implied volatility structure, widely recognized as the volatility smile or skew. While this study strictly employs constant historical volatility and does not empirically estimate the implied volatility surface, the observed overvaluation of OTM instruments is highly consistent with this well-documented skew phenomenon. Models utilizing a flat historical volatility theoretically struggle to capture such dynamic risk perceptions, which offers a robust interpretation for the pricing disparities found in these OTM contracts.

Furthermore, this disparity is amplified by liquidity frictions and data recording metrics (market microstructure). The use of the last price on option contracts with low trading volumes (such as deep-OTM) potentially introduces a time-synchronization bias, where the recorded price no longer reflects the true closing price of the underlying asset. Additionally, the absence of bid-ask spread data causes very small absolute nominal differences (AE) (e.g., 1.69 USD on an OTM call option) to escalate into mathematically massive percentage errors. Despite these market assumption limitations, the Simpson's 1/3 numerical integration combined with a discrete dividend correction is proven to provide an exact, efficient, and mathematically precise valuation framework for evaluating risk-neutral expectations.

4. Conclusion

This study implements an applied quantitative approach using Simpson's 1/3 numerical integration to evaluate the pricing of European options within a discrete-dividend-adjusted framework. The numerical scheme was systematically applied to evaluate both the mathematical consistency of the integral approximation and its empirical pricing performance using a real-world technology stock dataset.

From a computational perspective, the numerical results demonstrate that the Simpson's 1/3 method is highly stable and mechanically reliable. The integration method precisely approximates the risk-neutral expectation integral, converging seamlessly toward the exact analytical Black-Scholes benchmark. Specifically, by establishing the partition grid at $N = 200$, the relative error is effectively suppressed strictly below a 0.001% threshold across all option samples. This structural consistency confirms that the deterministic numerical integration provides a highly accurate operational framework without contradicting the standard mathematical theory.

However, the empirical evaluation against observed market prices reveals systematic valuation deviations that are likely influenced by the model's fundamental inputs. While the theoretical model aligns closely with the market's last traded prices for In-The-Money (ITM) options, it consistently exhibits an overvaluation bias for Out-Of-The-Money (OTM) Call options. Conversely, OTM Put options demonstrate a mixed deviation pattern due to the asymmetrical nature of market risk perception. This structural discrepancy suggests that the reliance on a constant historical volatility assumption appears inadequate to fully capture the complex risk perceptions inherent in highly volatile market environments. These OTM deviations align with the implied volatility skew phenomenon widely recognized in financial literature, though explicitly mapping this skew remains beyond the scope of this historical-volatility-based study.

The main contribution of this research lies in presenting a structured empirical evaluation framework for deterministic numerical integration. Nevertheless, this study acknowledges several limitations. The empirical scope is constrained to a single underlying asset, a single maturity date, a limited sample of option contracts, and the reliance on last traded prices rather than bid-ask midpoints. Therefore, future research is highly recommended to extend this computational framework by incorporating Implied Volatility surfaces or Stochastic Volatility models, and utilizing broader multi-maturity datasets to provide a more comprehensive pricing evaluation in the presence of market frictions.

CRediT Authorship Contribution Statement

Muhammad Gibran Elgiffary: Conceptualization, Methodology, Software, Formal Analysis, Writing-Original Draft, Writing-Review & Editing. **Rudianto Artiono:** Supervision, Validation, and Project Administration.

Declaration of Generative AI and AI-assisted technologies

During the preparation of this manuscript, the author used Gemini 3 Pro to validate the syntax and verify the code calculations, especially for numerical calculations using the Simpson's 1/3 method. DeepL was also used to help refine the language and explain technical points. All content has been reviewed and approved by the authors to ensure its accuracy and integrity.

Declaration of Competing Interest

The authors declare no conflicts of interest.

Funding and Acknowledgments

This research was funded by personal funds and did not receive any specific grants from public, commercial, or non-profit funding agencies.

Data and Code Availability

Stock price data for XYZ Tech Corp. was obtained from Yahoo Finance at https://finance.yahoo.com/, and daily 3-Month Treasury Bill interest rates were sourced from Federal Reserve Economic Data (FRED), Federal Reserve Bank of St. Louis at https://fred.stlouisfed.org/series/DTB3. The data and code supporting the findings of this study are available from the corresponding author upon reasonable request.

References

[1] F. H. Hanifa et al. "Pengujian model Black-Scholes dan model Binomial pada kontrak opsi saham NFLX". In: *Jurnal Manajemen dan Keuangan* 14.1 (July 2025). DOI: 10.33059/jmk.v14i1.10705.

[2] M. N. Mooy, A. Rusgiyono, and R. Rahmawati. "Penentuan Harga Opsi Put Dan Call Tipe Eropa Terhadap Saham Menggunakan Model Black-Scholes". In: *Jurnal Gaussian* 6 (2017), pp. 407–417. DOI: 10.14710/j.gauss.6.3.407-417.

[3] N. A. Purnamasari. "Model Black-Scholes put-call parity harga opsi tipe Eropa dengan Pembagian Dividen pada Penutupan Harga Saham Media Nusantara Citra Tbk". In: *PARAMETER: Jurnal Matematika, Statistika dan Terapannya* (Aug. 2023), pp. 67–78. DOI: 10.30598/parameterv2i01pp67-78.

[4] A. W. Firdaus and R. Artiono. "Pemodelan Harga Saham dengan Pembagian Dividen menggunakan Geometric Brownian Motion dan Simulasi Monte Carlo". In: *MATHunesa: Jurnal Ilmiah Matematika* 13.2 (Aug. 2025). DOI: 10.26740/mathunesa.v13n2.

[5] R. L. McDonald. *Derivatives Markets*. 3rd. Pearson, 2012.

[6] J. Hull. *Options, Futures, and Other Derivatives Global Edition*. Pearson Deutschland, 2021. https://elibrary.pearson.de/book/99.150005/9781292410623.

[7] F. Black and M. Scholes. "The Pricing of Options and Corporate Liabilities". In: *Journal of Political Economy* 81 (1973), pp. 637–654. DOI: 10.1086/260062.

[8] R. C. Merton. "Theory of Rational Option Pricing". In: *The Bell Journal of Economics and Management Science* 4 (1973), pp. 141–183. DOI: 10.2307/3003143.

[9] S. E. Shreve. *Stochastic Calculus for Finance II Continuous-Time Models*. Vol. 11. New York: Springer, 2004.

[10] P. Wilmott. *Quantitative Finance*. 2nd. John Wiley & Sons, 2006.

[11] E. Wahyuni, R. Lestari, and M. Syafwan. "Model Black-Scholes Opsi Call dan Opsi Put Tipe Eropa dengan Dividen pada Keadaan Constant Market". In: *Jurnal Matematika UNAND* 6.2 (2017), pp. 43–49. DOI: 10.25077/jmu.6.2.43-49.2017.

[12] C. E. Murwaningtyas et al. "Finite difference method for pricing of Indonesian option under a mixed fractional Brownian motion". In: *Mathematics and Statistics* 8.5 (2020), pp. 610–619. DOI: 10.13189/ms.2020.080516.

[13] D. Purwandari. "Penentuan Nilai Opsi Call Eropa Dengan Pembayaran Dividen". In: *Jurnal Sainsmat* 5 (2017). DOI: 10.35580/sainsmat5232442016.

[14] R. L. Burden and J. D. Faires. *Numerical Analysis*. 9th. Richard Stratton, 2010.

[15] P. Glasserman. *Monte Carlo Methods in Financial Engineering*. New York, NY: Springer, 2003. DOI: 10.1007/978-0-387-21617-1.

[16] Yahoo! Finance. *Yahoo Finance*. https://finance.yahoo.com. Accessed: Jan. 07, 2026. 2026.

[17] Federal Reserve Bank of St. Louis. *3-Month Treasury Bill Secondary Market Rate, Discount Basis [DTB3]*. FRED, Federal Reserve Bank of St. Louis. https://fred.stlouisfed.org/series/DTB3. Accessed: Jan. 07, 2026. 2026.

[18] D. Purwandari. "Pengaruh Pembagian Dividen melalui Model Black-Scholes". In: *Lebesgue: Jurnal Ilmiah Pendidikan Matematika, Matematika dan Statistika* 2 (2021). DOI: 10.46306/lb.v2i3.111.