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Minor-Eigenvalue Spectral Analysis in Intuitionistic Fuzzy Soft Sets for Multicriteria Decision Making

Silfiatis Sabila Azra Shofa, Siti Amiroch*, and Awawin Mustana Rohmah

Mathematics Department, Faculty of Science and Technology, Universitas Islam Darul 'Ulum, Lamongan, Indonesia

Abstract

Multi-criteria decision making (MCDM) under uncertain data requires a framework capable of capturing ambiguity and non-linear interactions among criteria. This study develops an Intuitionistic Fuzzy Soft Set (IFSS)-based MCDM model using spectral analysis of aggregation matrices constructed with the Einstein operator. Unlike approaches that rely mainly on global eigenvalues, the proposed method utilizes dominant eigenvalues of principal minors to capture local structural variations among alternatives. The method is validated using subdistrict-level economic facility data from Lamongan Regency. The results produce spectral scores ranging from 0.0683 to 2.0000, with Bluluk obtaining the lowest score and Lamongan obtaining the highest score. Several alternatives with comparable global structural characteristics also exhibit distinct minor-eigenvalue responses, indicating that the proposed approach can reveal local structural variations that may not be reflected in global spectral analysis. These findings suggest that minor-eigenvalue-based spectral analysis provides an alternative local perspective for distinguishing alternatives within the IFSS framework. The proposed framework contributes theoretically to IFSS-based spectral modeling and practically supports decision-makers in prioritizing subdistrict development based on local structural characteristics.

Keywords: Einstein Operator; Intuitionistic Fuzzy Soft Set; Minor Eigenvalue; Multi-Criteria Decision Making; Spectral Analysis.

1. Introduction

Multi-Criteria Decision Making (MCDM) is a fundamental topic in applied mathematics that evaluates and ranks alternatives based on multiple interacting criteria with different levels of importance [1]. Classical MCDM approaches generally use crisp data, so each criterion value is represented deterministically [2]. Although simple and easy to implement, this approach has limitations in representing uncertainty, ambiguity, and data imprecision in complex decision systems [3]. These limitations have encouraged the development of more flexible mathematical models for uncertain multi-criteria data [4].

Fuzzy set theory, introduced by Zadeh, provides a mathematical basis for representing uncertainty through membership degrees on the interval $[0, 1]$ [5]. Its development has produced several generalizations, including Type-2 Fuzzy Sets, Hesitant Fuzzy Sets, and Intuitionistic Fuzzy

*Corresponding author. E-mail: siti.amiroch@unisda.ac.id

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Sets (IFS) [6]. IFS, introduced by Atanassov, extends classical fuzzy sets by describing each element through membership, non-membership, and hesitation degrees [7]. Various similarity measures have also been developed in the IFS environment to represent the closeness between objects under ambiguous and incomplete data [8].

IFS-based approaches are widely used because they can handle incomplete and imprecise data [9]. They have also been shown to produce more stable ranking structures than crisp approaches [10]. In addition, entropy-based approaches in intuitionistic fuzzy environments are used to quantify information uncertainty [11]. These developments show that intuitionistic fuzzy modeling provides a relevant mathematical foundation for decision-making problems involving uncertainty and hesitation.

Soft set theory, introduced by Molodtsov, provides a flexible parametric framework without requiring rigid membership functions [12]. The integration of soft set theory and intuitionistic fuzzy sets leads to the concept of Intuitionistic Fuzzy Soft Set (IFSS) [13]. IFSS enables the evaluation of alternatives based on parameters while simultaneously considering membership, non-membership, and hesitation degrees [14]. This characteristic makes IFSS suitable for high-dimensional MCDM problems with complex relationships between criteria and alternatives [15].

In the IFSS framework, the relationship between alternatives is commonly represented through a matrix that encodes interaction patterns among alternatives [16]. Various measures have been developed for this purpose, including distance-based, correlation-based, and energy-based approaches [17]. These measures provide different perspectives on comparing alternatives under uncertain and parameterized information.

The selection of aggregation operators also plays an important role in maintaining the consistency and stability of IFSS-based models. Conventional algebraic operators, such as algebraic sums and products, tend to produce linear changes in values and may be less sensitive to non-linear interactions among criteria. Einstein operators based on T-norm and S-norm offer a smoother aggregation mechanism and may reduce information distortion in membership and non-membership data with high ambiguity [18]. Several studies have reported that Einstein operators yield consistent ranking structures compared with conventional aggregation operations [19].

Further developments in IFSS analysis include the use of linear algebra concepts, particularly eigenvalues, to represent the structural characteristics of relational matrices [20]. Dominant eigenvalues reflect connectivity patterns through the spectral characteristics of a matrix [21]. The distribution of eigenvalues can also reveal structural variation and heterogeneity in complex systems [22]. This approach is consistent with spectral analysis in network systems, where eigenvalues are used to study internal structure and system dynamics [23].

From a spectral graph theory perspective, an aggregation matrix in MCDM can be interpreted as a weighted adjacency structure. Global eigenvalues summarize the overall connectivity of the system, but they may overlook differences caused by the removal of a particular alternative. By constructing a principal minor through the deletion of the i-th row and column, the resulting eigenvalue describes how the remaining system reorganizes after alternative A_i is removed.

This mechanism can be interpreted as a structural influence indicator. If excluding an alternative produces a noticeable spectral response, then the alternative contributes meaningfully to the relational structure of the system. Principal minor eigenvalues therefore provide an additional perspective beyond global eigenvalues, especially when alternatives have similar relational patterns. This interpretation is exploratory and is intended as a local structural indicator rather than a universally established priority measure.

Graph theory perspectives emphasize the importance of representing relationships among elements in network-based decision problems [24]. However, most IFSS studies using spectral analysis still focus on the global structure of the aggregation matrix. The use of eigenvalues derived from principal minors to represent alternative-specific structural responses remains limited in the literature [25]. This limitation creates a research gap in IFSS-based spectral analysis, especially for distinguishing alternatives with similar interaction patterns.

Accordingly, this study aims to develop a mathematical framework for MCDM based on spectral analysis in an IFSS environment using principal minor eigenvalues. The main contribution lies in the use of eigenvalues of principal minors derived from Einstein aggregation matrices as representations of alternative-specific structural responses. This approach extends IFSS-based spectral analysis from global matrix structures toward local structural evaluation.

The contributions of this study are: (1) constructing membership-based and non-membership-based aggregation matrices using the Einstein operator in the IFSS framework, (2) extracting principal minors to capture alternative-level structural effects, and (3) utilizing dominant minor eigenvalues as local spectral response indicators for ranking alternatives in MCDM problems. This combination has not been widely explored in IFSS-based studies. Compared with conventional spectral MCDM approaches that rely on global eigenvalues, the proposed method provides an alternative perspective for distinguishing alternatives with similar relational structures.

A case study of economic development priorities in Lamongan Regency is used to validate the proposed framework. The ranking results illustrate how the model captures structural characteristics in the data, while comparative analysis with conventional methods provides an empirical benchmark. The remainder of this paper is organized as follows: Section 2 presents the methodology, Section 3 discusses the results, and Section 4 concludes the study.

2. Methods

This study adopts a computational and analytical research design to develop and evaluate a spectral-based MCDM framework within the IFSS environment. The proposed approach combines mathematical modelling and numerical analysis to investigate the structural characteristics of aggregation matrices. Empirical data are used for numerical validation to assess the stability, consistency, and sensitivity of the proposed model in distinguishing structurally similar alternatives.

2.1. Research Flow

The overall procedure of the proposed method is summarized in Fig. 1. The workflow shows how the data are transformed from economic facility indicators into IFSS components, processed through two parallel aggregation paths, and finally converted into priority rankings.

Fig. 1 illustrates the main computational stages of the proposed IFSS-based spectral framework. The process begins with data collection for 27 subdistricts and ten economic facility criteria, followed by cost-type min–max normalization. The normalized values are then transformed into intuitionistic fuzzy soft set components consisting of membership, non-membership, and hesitation degrees.

The IFSS components are processed through two parallel paths: the membership-based aggregation path and the non-membership-based aggregation path. In each path, Einstein aggregation is used to construct a relational matrix, from which principal minors are extracted by deleting the row and column associated with each alternative. The dominant eigenvalues of these minors are computed using the Power Iteration method and then transformed into perturbation-based spectral sensitivity measures. Finally, the normalized perturbation components are combined to obtain the priority score P_i, where smaller values indicate higher development priority.

2.2. Data Sources and Analysis Units

The data used in this study are secondary data obtained from the 2025 Village Potential Statistics (PODES) published by the Central Statistics Agency (BPS) of Lamongan Regency. The analysis units consist of 27 subdistricts in Lamongan Regency, which are treated as decision

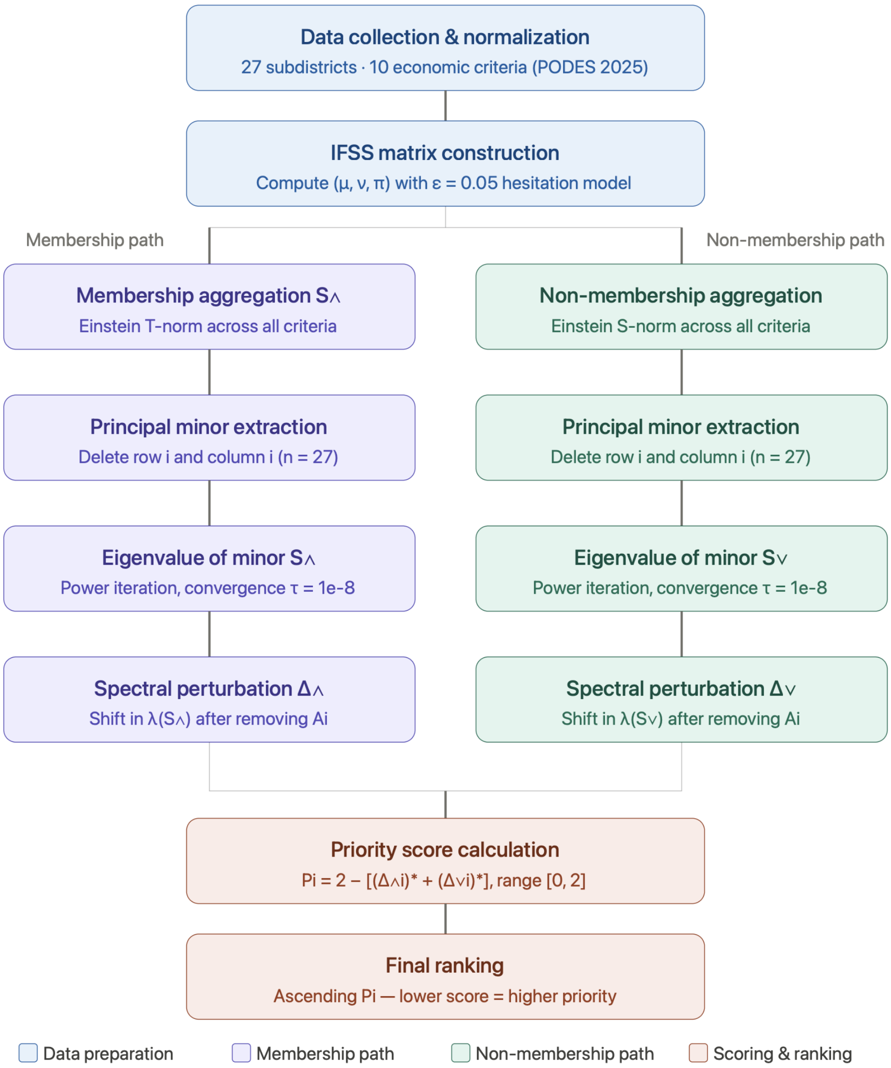

Fig. 1: Workflow of the proposed IFSS-based minor-eigenvalue spectral framework for economic development priority ranking

alternatives. Each alternative is evaluated based on ten economic facility criteria, namely minimarkets/supermarkets (C1), restaurants/eateries (C2), hotels and lodgings (C3), shops (C4), permanent building markets (C5), banks (C6), savings and loan cooperatives (C7), People's Business Credit/KUR (C8), micro and small industries (C9), and leading products (C10).

The value of each criterion represents the number of villages within each subdistrict that possess the corresponding economic facility. All criteria are treated as cost-type attributes, where smaller values indicate lower availability of economic facilities and higher development priority. This interpretation supports the objective of identifying subdistricts that require stronger economic infrastructure development.

2.3. Criteria Data Normalization

All criteria are normalized using the min–max method to obtain values within the interval $[0, 1]$. Since all criteria are cost-type attributes, smaller original values produce larger normalized values. To ensure numerical stability, the normalization is defined as

$$x'_{ij} = \frac{\max(x_j) - x_{ij}}{\max(x_j) - \min(x_j) + \delta}, \tag{1}$$

where x_{ij} denotes the original value of alternative A_i under criterion C_j, x'_{ij} denotes the normalized value, and δ is a small positive constant.

The smoothing parameter δ in Eq. (1) avoids numerical instability when $\max(x_j) = \min(x_j)$. It prevents division by zero while keeping the normalization well-defined for all criteria. In this study, $\delta = 0.0001$ is used because it is sufficiently small to preserve the original data scale while maintaining numerical stability.

2.4. Intuitionistic Fuzzy Soft Set (IFSS) Matrix Formation

Each alternative A_i under criterion C_j is represented as an intuitionistic fuzzy triple $(\mu_{ij}, \nu_{ij}, \pi_{ij})$ satisfying

$$0 \leq \mu_{ij} + \nu_{ij} + \pi_{ij} \leq 1. \tag{2}$$

The membership degree is obtained directly from the normalized data:

$$\mu_{ij} = x'_{ij}. \tag{3}$$

The hesitation degree is modeled as

$$\pi_{ij} = \varepsilon(1 - \mu_{ij}), \tag{4}$$

where ε is a fixed calibration parameter controlling the magnitude of hesitation. The hesitation degree is not assigned as a constant, but adapts to the membership value for each alternative–criterion pair.

The non-membership degree is calculated as

$$\nu_{ij} = 1 - \mu_{ij} - \pi_{ij}. \tag{5}$$

Eqs. (3)–(5) follow the classical intuitionistic fuzzy set structure, where the three components satisfy

$$\mu_{ij} + \nu_{ij} + \pi_{ij} = 1. \tag{6}$$

Thus, all intuitionistic fuzzy triples remain within a valid feasible region. The parameter ε regulates the level of hesitation without violating the IFSS structure. In this study, $\varepsilon = 0.05$ is adopted to represent a low level of residual uncertainty in secondary statistical data. When $\mu_{ij} = 1$, then $\pi_{ij} = 0$ and $\nu_{ij} = 0$, preserving the consistency of the IFSS representation.

2.5. Einstein Aggregation Matrix Construction

The relational structure between alternatives is constructed using Einstein aggregation operators. Instead of defining a strict similarity measure, this study integrates membership and non-membership information across multiple criteria into aggregation-based relational matrices. These matrices do not necessarily satisfy classical similarity properties such as reflexivity, i.e., $S_{ii} = 1$, and are therefore interpreted as structural interaction matrices.

(a) **Membership-Based Aggregation Matrix** $(S^{\wedge})$
The membership-based aggregation matrix is constructed using the Einstein product, namely the T-norm, defined as

$$T_E(\mu_1, \mu_2) = \frac{\mu_1 \mu_2}{1 + (1 - \mu_1)(1 - \mu_2)}. \tag{7}$$

For each pair of alternatives A_i and A_k, the aggregated membership-based relational value is computed as

$$S^{\wedge}_{ik} = \frac{1}{m} \sum_{j=1}^{m} T_E(\mu_{ij}, \mu_{kj}), \tag{8}$$

where m denotes the number of criteria. Equation (8) captures the average membership-based interaction between alternatives over all criteria.

(b) **Non-Membership-Based Aggregation Matrix** ($S^{\vee}$)
The non-membership-based aggregation matrix is constructed using the Einstein sum, namely the S-norm, defined as

$$S_E(\nu_1, \nu_2) = \frac{\nu_1 + \nu_2}{1 + \nu_1 \nu_2}. \tag{9}$$

The aggregated non-membership relational value is defined as

$$S_{ik}^{\vee} = \frac{1}{m} \sum_{j=1}^{m} S_E(\nu_{ij}, \nu_{kj}). \tag{10}$$

For a decision problem with n alternatives, both $S^{\wedge}$ and $S^{\vee}$ are square matrices of size $n \times n$. The two matrices provide complementary information because one is constructed from membership degrees and the other from non-membership degrees.

Unlike classical similarity matrices, the diagonal elements of these matrices are not constrained to be one. The aggregation process reflects cumulative interactions across criteria rather than direct self-similarity. Consequently, the resulting matrices are suitable as weighted relational structures for spectral analysis. The use of Einstein operators also provides smooth aggregation behavior in intuitionistic fuzzy environments [18].

2.6. Aggregation Matrix Minor Formation

After constructing $S^{\wedge}$ and $S^{\vee}$, matrix minors are generated for each alternative by removing the i-th row and i-th column from each aggregation matrix. This operation observes how the relational structure changes when one alternative is excluded. For each alternative A_i, the membership-based and non-membership-based aggregation minors are denoted as

$$S_i^{\wedge} = S_{-i,-i}^{\wedge}, \qquad S_i^{\vee} = S_{-i,-i}^{\vee}. \tag{11}$$

From spectral matrix theory, Eq. (11) corresponds to extracting a principal submatrix. The eigenvalues of principal submatrices satisfy the eigenvalue interlacing property, so the spectrum of each minor remains bounded by the spectrum of the original matrix. Therefore, the dominant eigenvalues of the minors may reflect localized structural responses induced by the removal of alternative A_i.

To quantify this effect, a perturbation-based sensitivity measure is introduced. For each aggregation matrix $S \in \{S^{\wedge}, S^{\vee}\}$, define

$$\Delta_i = \left| \lambda(S) - \lambda(S_{-i,-i}) \right|, \tag{12}$$

where $\lambda(S)$ denotes the dominant eigenvalue of the full aggregation matrix and $\lambda(S_{-i,-i})$ denotes the dominant eigenvalue of the corresponding principal minor.

The quantity Δ_i measures the magnitude of spectral deviation caused by excluding alternative A_i. Larger values indicate stronger structural perturbation and greater influence of the excluded alternative within the relational system. Thus, the ranking mechanism is not based only on the interlacing theorem. The theorem guarantees bounded spectral stability, while Δ_i measures the actual shift of the dominant structural mode. In this framework, Δ_i is used as the main alternative-specific spectral indicator, whereas the raw minor eigenvalue is treated as an intermediate computational component.

2.7. Minor Eigenvalue Calculation

The dominant eigenvalues of each aggregation matrix minor are computed using the Power Iteration method because it is efficient for dense positive matrices. All numerical calculations are implemented in Python using NumPy and SciPy. The same procedure is applied to both membership-based and non-membership-based aggregation minors.

Let $A \in \mathbb{R}^{n\times n}$ denote an aggregation matrix minor. The iterative procedure begins with the normalized vector

$$x^{(0)} = \frac{1}{\sqrt{n}}[1, 1, \ldots, 1]^T. \tag{13}$$

At iteration k, the vector is updated as

$$y^{(k)} = Ax^{(k-1)}, \tag{14}$$

followed by normalization:

$$x^{(k)} = \frac{y^{(k)}}{\|y^{(k)}\|_2}. \tag{15}$$

The dominant eigenvalue is estimated using the Rayleigh quotient:

$$\lambda^{(k)} = \frac{(x^{(k)})^T Ax^{(k)}}{(x^{(k)})^T x^{(k)}}. \tag{16}$$

The iteration continues until

$$\left|\lambda^{(k)} - \lambda^{(k-1)}\right| < \tau, \tag{17}$$

where $\tau = 10^{-8}$ is the convergence tolerance, with a maximum iteration limit of 1000.

The resulting dominant eigenvalues of the membership-based and non-membership-based minors are denoted by $\lambda_i^{\wedge}$ and $\lambda_i^{\vee}$, respectively. These values are then transformed into perturbation quantities:

$$\Delta_i^{\wedge} = \left|\lambda(S^{\wedge}) - \lambda_i^{\wedge}\right|, \qquad \Delta_i^{\vee} = \left|\lambda(S^{\vee}) - \lambda_i^{\vee}\right|. \tag{18}$$

In Eq. (18), $\lambda(S^{\wedge})$ and $\lambda(S^{\vee})$ denote the dominant eigenvalues of the full membership-based and non-membership-based aggregation matrices. Thus, minor eigenvalues serve as intermediate spectral quantities, while perturbation values are used as alternative-specific structural sensitivity indicators in the priority score calculation.

2.8. Priority Score Calculation

The final priority score combines perturbation indicators from the membership-based and non-membership-based aggregation structures. Since the two perturbation components may have different numerical ranges, both are normalized into the interval $[0, 1]$. The normalized membership-based component is defined as

$$(\Delta_i^{\wedge})^* = \frac{\Delta_i^{\wedge} - \min(\Delta^{\wedge})}{\max(\Delta^{\wedge}) - \min(\Delta^{\wedge})}. \tag{19}$$

Larger values of $\Delta_i^{\wedge}$ indicate stronger perturbation in the membership-based relational structure. Since the membership component is derived from cost-type normalized criteria, stronger perturbation reflects greater structural deficiency and higher development priority.

For the non-membership-based component, inverse normalization is applied:

$$(\Delta_i^{\vee})^* = \frac{\max(\Delta^{\vee}) - \Delta_i^{\vee}}{\max(\Delta^{\vee}) - \min(\Delta^{\vee})}. \tag{20}$$

Larger values of $\Delta_i^{\vee}$ indicate stronger resistance within the non-membership relational structure, which corresponds to alternatives with relatively better facility conditions. Inverse normalization aligns this component with the priority interpretation used in the model.

The final spectral priority score is defined as

$$P_i = 2 - \left[(\Delta_i^{\wedge})^* + (\Delta_i^{\vee})^*\right], \tag{21}$$

where $0 \leq P_i \leq 2$. Smaller values of P_i indicate weaker structural support, lower facility connectivity, and higher regional development priority. Therefore, alternatives are ranked in ascending order of P_i, so those requiring greater development attention appear at the top of the priority list.

2.9. Sensitivity Analysis

A parameter sensitivity analysis is conducted for ε and δ to examine the robustness of the proposed model. The parameter ε influences the hesitation degree, while δ affects the normalization process in degenerate or near-degenerate cases. Several values of both parameters are tested, and the resulting rankings are compared using rank correlation measures. This analysis evaluates whether the final rankings remain stable under reasonable parameter variations.

2.10. Reproducibility

To facilitate independent verification, the supplementary materials include the complete raw dataset, normalized matrix, IFSS matrix, aggregation matrices, minor eigenvalues, ranking results, and Python implementation code. These materials allow all computational stages to be reproduced from normalization to final priority ranking. Reproducibility is important because the proposed method involves sequential transformations that must remain transparent and verifiable.

3. Results and Discussion

This section presents the numerical results of the proposed IFSS-based spectral framework and discusses their interpretation in the context of regional development priority. The discussion begins with an overview of the research data, followed by normalization results, IFSS construction, aggregation matrices, minor-eigenvalue computation, priority ranking, sensitivity analysis, and comparison with conventional and global spectral approaches.

3.1. Overview of Research Data

The dataset consists of 27 alternatives evaluated across ten criteria. Each alternative is represented in the IFSS framework and analyzed using Einstein operator-based aggregation matrices. The data are sourced from the 2025 Lamongan Regency Village Potential Statistics (PODES), where each criterion represents the number of villages within a subdistrict that possess a specific type of economic facility. The initial data are presented in Table 1.

Due to space limitations, only selected elements of the aggregation matrices are presented in this section. The complete normalized dataset, IFSS matrix, aggregation matrices, and minor matrices are provided in the supplementary materials. This allows the main numerical patterns to be discussed while preserving full reproducibility.

Table 1: Initial Data on the Distribution of Economic Facilities

Alternative	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
Sukorame	3	0	0	1	3	2	1	9	9	0
Bluluk	0	0	0	2	2	2	3	9	9	0
Ngimbang	3	2	1	3	2	2	4	19	19	1
Sambeng	4	0	0	0	2	2	3	22	22	3
Mantup	5	0	0	3	4	4	6	15	15	1

Table 1 is visualized in a bar chart to show variation in economic facility distribution among subdistricts. This visual summary provides an initial overview of facility availability before IFSS-based transformation.

Fig. 2 shows the heterogeneity of economic facility coverage in Lamongan Regency. This heterogeneity motivates the use of a multi-criteria framework because each subdistrict may

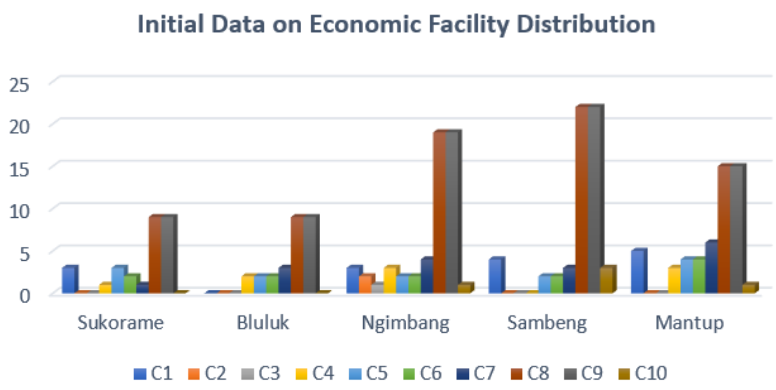

Fig. 2: Initial data on economic facility distribution

show different patterns across the observed facilities. The visualization therefore serves as the descriptive context before the IFSS and spectral relational analysis.

3.2. Criteria Data Normalization

All quantitative data were normalized using Eq. (1) to obtain values in the range $[0, 1]$. Since all criteria are cost-type attributes, smaller original facility counts produce larger normalized values. This transformation makes the criteria comparable and aligns the numerical scale with the interpretation of development priority.

For illustration, the normalization of an alternative under criterion C1 is computed as follows:

$$x'_{ij} = \frac{13-3}{13-0+0.0001} = \frac{10}{13.0001} = 0.7692.$$

Some of the normalization results are presented in Table 2. The normalized values form the basis for determining membership degrees in the IFSS construction.

Table 2: Criterion Normalization Results

Alternative	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
Sukorame	0.7692	1.0000	1.0000	0.8889	0.7273	0.9167	0.9444	1.0000	1.0000	1.0000
Bluluk	1.0000	1.0000	1.0000	0.7778	0.8182	0.9167	0.8333	1.0000	1.0000	1.0000
Ngimbang	0.7692	0.8182	0.9000	0.6667	0.8182	0.9167	0.7778	0.4737	0.5000	0.9412
Sambeng	0.6923	1.0000	1.0000	1.0000	0.8182	0.9167	0.8333	0.3158	0.3500	0.8235
Mantup	0.6154	1.0000	1.0000	0.6667	0.6364	0.7500	0.6667	0.6842	0.7000	0.9412

The normalized values in Table 2 show that subdistricts with limited facilities obtain higher normalized values. Consequently, these subdistricts receive higher membership degrees in the IFSS framework. This supports the interpretation that larger membership values represent higher development priority.

3.3. Intuitionistic Fuzzy Soft Set Matrix Formation

Based on the normalization results, the IFSS matrix is constructed by defining membership degree (μ_{ij}), non-membership degree (ν_{ij}), and hesitation degree (π_{ij}) for each alternative–criterion pair. The membership, hesitation, and non-membership degrees are computed using Eqs. (3)–(5). These three components form a complete intuitionistic fuzzy representation of each criterion value.

For example, for the alternative–criterion pair A1–C1 with $x'_{ij} = 0.7692$, we obtain

$$\mu_{ij} = 0.7692,$$
$$\pi_{ij} = 0.05(1 - 0.7692) = 0.0115,$$
$$\nu_{ij} = 1 - 0.7692 - 0.0115 = 0.2192.$$

Thus, the resulting intuitionistic fuzzy triple is approximately $(0.7692, 0.2192, 0.0115)$. Using the same procedure, the IFSS matrix is constructed for all alternative–criterion pairs. This formulation satisfies Eq. (2) and remains consistent with intuitionistic fuzzy set theory.

Table 3: IFSS Matrix

Alternative	C1	C2	C3
Sukorame	(0.77; 0.22; 0.01)	(1; 0; 0)	(1; 0; 0)
Bluluk	(1; 0; 0)	(1; 0; 0)	(1; 0; 0)
Ngimbang	(0.77; 0.22; 0.01)	(0.82; 0.17; 0.01)	(0.90; 0.09; 0.01)
Sambeng	(0.69; 0.29; 0.02)	(1; 0; 0)	(1; 0; 0)
Mantup	(0.61; 0.37; 0.02)	(1; 0; 0)	(1; 0; 0)

The resulting IFSS matrix is partially presented in Table 3. Each entry contains the membership, non-membership, and hesitation degrees for an alternative–criterion pair. This matrix becomes the basis for constructing membership-based and non-membership-based aggregation matrices.

3.4. Einstein Aggregation Matrix Construction

The relational matrix between alternatives is constructed using membership and non-membership degrees aggregated by the Einstein operator. This matrix represents aggregated structural interactions among alternatives within the IFSS space, rather than a strict similarity measure. Separate aggregation matrices are constructed so that membership and non-membership dimensions can be analyzed complementarily.

(a) **Membership-Based Aggregation Matrix** ($S^{\wedge}$)

For the membership degree (μ), the Einstein product operator in Eq. (7) is used. For example, the aggregation of two membership values is computed as

$$T_E(\mu_1, \mu_2) = \frac{(0.7692)(1.0000)}{1+(1-0.7692)(1-1.0000)} = 0.7692.$$

For each pair of alternatives, this operation is applied across all criteria and averaged using Eq. (8). Some elements of the $S^{\wedge}$ matrix are shown below:

$$S^{\wedge} = \begin{bmatrix} 0.3076 & 0.3127 & \cdots & 0.0562 & 0.2190 \\ 0.3127 & 0.3180 & \cdots & 0.0574 & 0.2229 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0.0562 & 0.0574 & \cdots & 0.0090 & 0.0380 \\ 0.2190 & 0.2229 & \cdots & 0.0380 & 0.1532 \end{bmatrix}.$$

The values in $S^{\wedge}$ reflect aggregated structural interactions between alternatives based on the degree of criterion fulfillment. They are interpreted as relational strengths rather than normalized similarity scores, since diagonal values are not forced to be one.

(b) **Non-Membership-Based Aggregation Matrix** ($S^{\vee}$)

For the non-membership degree (ν), the Einstein sum operator in Eq. (9) is used. For example, the aggregation of two non-membership values is computed as

$$S_E(\nu_1, \nu_2) = \frac{0.2192+0.0000}{1+(0.2192)(0.0000)} = 0.2192.$$

Similarly, this operation is applied across all criteria and averaged using Eq. (10). Some elements of the $S^{\vee}$ matrix are shown below:

$$S^{\vee} = \begin{bmatrix} 0.1505 & 0.1484 & \cdots & 0.1732 & 0.1672 \\ 0.1484 & 0.1463 & \cdots & 0.1711 & 0.1651 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0.1732 & 0.1711 & \cdots & 0.1956 & 0.1897 \\ 0.1672 & 0.1651 & \cdots & 0.1897 & 0.1838 \end{bmatrix}.$$

The numerical differences between $S^\wedge$ and $S^\vee$ indicate structural variation between membership and non-membership dimensions. These matrices are then used to extract principal minors and compute spectral responses.

3.5. Aggregation Matrix Minor Formation

For each alternative, aggregation matrix minors are constructed by removing the corresponding row and column from the main aggregation matrix, as defined in Eq. (11). Since the full matrix size is 27×27, only representative minor matrices are shown. This presentation demonstrates the structure of the computed minors without displaying all matrices in full.

An example of minor matrices for alternative A_1 is given below:

$$S_1^\wedge = \begin{bmatrix} 0.3180 & \cdots & 0.0574 & 0.2229 \\ \vdots & \ddots & \vdots & \vdots \\ 0.0574 & \cdots & 0.0090 & 0.0380 \\ 0.2229 & \cdots & 0.0380 & 0.1532 \end{bmatrix},$$

and

$$S_1^\vee = \begin{bmatrix} 0.1463 & \cdots & 0.1711 & 0.1651 \\ \vdots & \ddots & \vdots & \vdots \\ 0.1711 & \cdots & 0.1956 & 0.1897 \\ 0.1651 & \cdots & 0.1897 & 0.1838 \end{bmatrix}.$$

Each minor matrix represents the relational structure among the remaining alternatives after excluding A_i. From a spectral perspective, this removal introduces a structured perturbation in the aggregation matrix. The dominant eigenvalue of the resulting minor may therefore reflect how the relational structure reorganizes in the absence of the excluded alternative.

3.6. Minor Eigenvalue Calculation

The dominant eigenvalues of each minor matrix were computed using the Power Iteration procedure described in the methodology section. The calculation was applied to membership-based and non-membership-based aggregation matrix minors for all alternatives. The resulting dominant eigenvalues are denoted as $\lambda_i^\wedge$ and $\lambda_i^\vee$, respectively.

To obtain alternative-specific structural sensitivity measures, these eigenvalues were transformed into perturbation quantities using Eq. (18). The dominant eigenvalues of the full aggregation matrices were obtained as

$$\lambda(S^\wedge) = 3.9120, \qquad \lambda(S^\vee) = 4.8840.$$

Some computational results are summarized in Table 4. The table reports the minor eigenvalues and corresponding perturbation values for selected alternatives.

Table 4: Recapitulation of Minor Eigenvalues and Perturbation Values

Alternative	$\lambda_i^\wedge$	$\lambda_i^\vee$	$\Delta_i^\wedge$	$\Delta_i^\vee$
Sukorame	3.1656	4.6271	0.7464	0.2569
Bluluk	3.1541	4.6309	0.7579	0.2531
Ngimbang	3.3519	4.6095	0.5601	0.2745
Sambeng	3.2950	4.5998	0.6170	0.2842
Mantup	3.3358	4.6190	0.5762	0.2650

The values of λ_i represent residual spectral responses of the minor matrices, whereas Δ_i quantifies the structural perturbation caused by removing each alternative. Larger Δ_i values indicate stronger structural influence of the excluded alternative. This interpretation supports the use of perturbation values in the priority score calculation.

3.7. Priority Score Calculation

The perturbation indicators from both aggregation structures were combined using Eq. (21). For illustration, the score for Sukorame was computed as

$$P_1 = 2 - \left(\frac{\Delta_1^{\wedge} - \min(\Delta^{\wedge})}{\max(\Delta^{\wedge}) - \min(\Delta^{\wedge})} + \frac{\max(\Delta^{\vee}) - \Delta_1^{\vee}}{\max(\Delta^{\vee}) - \min(\Delta^{\vee})}\right) = 0.1803.$$

The complete ranking results are summarized in Table 5. The score P_i is interpreted in ascending order, where smaller values indicate higher development priority.

Table 5: Priority Ranking of Alternatives

Alternative	$\Delta_i^{\wedge}$	$\Delta_i^{\vee}$	Score (P_i)
Bluluk	0.7579	0.2531	0.0683
Sukorame	0.7464	0.2569	0.1803
Sarirejo	0.7815	0.2747	0.4484
...	...	...	...
Babat	0.4508	0.2975	1.8783
Paciran	0.4439	0.3014	1.9790
Lamongan	0.4366	0.3014	2.0000

The ranking results show that Bluluk obtained the lowest priority score of 0.0683, followed by Sukorame with 0.1803 and Sarirejo with 0.4484. These values indicate relatively limited economic facility distribution and higher development priority. Conversely, Lamongan obtained the highest score of 2.0000, followed by Paciran with 1.9790 and Babat with 1.8783, reflecting stronger structural connectivity and relatively better facility availability.

The dispersion of scores suggests that the proposed perturbation-based spectral framework can differentiate alternatives meaningfully. This differentiation is relevant for alternatives with similar facility distributions but different local structural responses within the aggregation matrices.

3.8. Sensitivity Analysis

To evaluate robustness, sensitivity analysis was conducted on ε and δ, which influence the hesitation degree and normalization process, respectively. The parameter ε controls the magnitude of hesitation through Eq. (4), while δ prevents numerical instability in Eq. (1). The tested values are

$$\varepsilon \in \{0.01, 0.03, 0.05, 0.07, 0.10\}, \qquad \delta \in \{10^{-5}, 10^{-4}, 10^{-3}\}.$$

For each parameter combination, the IFSS matrix, aggregation matrices, minor eigenvalues, and final scores were recomputed. The resulting rankings were compared using rank correlation analysis. The results show that variations in ε and δ produced only minor numerical changes and did not significantly alter the overall ranking structure.

The top-ranked and bottom-ranked alternatives remained stable across all tested configurations. No rank reversal was observed for the three highest-priority and three lowest-priority alternatives. The Spearman rank correlation with the baseline configuration ($\varepsilon = 0.05, \delta = 0.0001$) ranged from 0.963 to 1.000, indicating very high ranking stability. These findings support the robustness of the proposed framework under reasonable parameter variations.

3.9. Interpretation and Discussion of IFSS Results

The variation in spectral scores is governed by a sequential transformation from cost-type criteria to the final spectral representation. Since all criteria are cost-type attributes, lower original values x_{ij} indicate lower availability of economic facilities. Through min–max normalization, smaller x_{ij} values produce larger normalized values x'_{ij}. Consequently, alternatives with lower facility availability obtain higher membership degrees because $\mu_{ij} = x'_{ij}$.

The hesitation degree $\pi_{ij} = \varepsilon(1 - \mu_{ij})$ and the non-membership degree $\nu_{ij} = 1 - \mu_{ij} - \pi_{ij}$ are then derived to preserve the intuitionistic fuzzy structure. These components are aggregated using the Einstein operators in Eqs. (7) and (9). The resulting matrices reflect relational patterns shaped by the original cost-based characteristics of the data.

Spectral analysis is performed on principal minors derived from these aggregation matrices. The dominant eigenvalues $\lambda_i^{\wedge}$ and $\lambda_i^{\vee}$ represent structural responses of the membership-based and non-membership-based minors after each alternative is removed. Smaller perturbations indicate that an alternative is embedded in a relatively homogeneous connectivity pattern, while larger perturbations indicate stronger structural influence within the network.

The final spectral score P_i is constructed using Eq. (21) by combining the normalized membership-based and inverse-normalized non-membership-based perturbation components. This construction aligns both components with the priority interpretation used in the study. As a result, alternatives associated with stronger structural deficiency obtain smaller final scores, so lower spectral scores indicate higher development priority.

From an applied perspective, weaker structural connectivity corresponds to limited economic facility distribution. Bluluk, which has the lowest spectral score, reflects a relatively sparse distribution of economic facilities. In contrast, Lamongan exhibits stronger structural connectivity and more balanced facility distribution. Thus, the spectral score can be interpreted as a quantitative indicator of regional development disparity, where lower values correspond to areas requiring targeted infrastructure expansion.

3.10. Comparison with Conventional and Global Spectral Approaches

The proposed method was compared with the Simple Additive Weighting (SAW) method using the same normalized dataset. SAW was selected as a representative conventional MCDM approach because of its simplicity and interpretability. Equal criterion weights were used for all ten criteria to ensure a fair comparison.

The comparative results indicate that the proposed IFSS-based spectral method largely preserves the priority structure identified by SAW. Bluluk, Sukorame, and Sarirejo consistently appear among the highest-priority alternatives, while Lamongan, Paciran, and Babat remain among the lowest-priority alternatives under both approaches. This consistency suggests that the proposed method is aligned with conventional additive decision models in identifying extreme priority groups.

Moderate differences were observed in several middle-ranked alternatives. For example, Mantup, Ngimbang, and Sambeng changed positions between the two methods. This suggests that the proposed framework may provide additional discrimination by incorporating local relational responses from principal minor eigenvalues, whereas SAW relies solely on additive aggregation of criterion scores.

Table 6 presents a partial comparison of the ranking outcomes.

Table 6: Illustrative Comparison of Rankings between the Proposed Method and SAW

Alternative	Proposed Rank	SAW Rank
Bluluk	1	1
Sukorame	2	2
Sarirejo	3	4
Mantup	4	5
Ngimbang	6	6
Sambeng	7	8
Babat	25	25
Paciran	26	26
Lamongan	27	27

A comparison was also conducted with a global spectral approach based on the dominant eigenvalue of the full aggregation matrix without minor extraction. The global spectral approach produces a single structural indicator for the complete relational matrix and cannot directly distinguish individual alternatives. In contrast, the proposed minor-eigenvalue framework generates alternative-specific local spectral responses by evaluating the system after the exclusion of each alternative.

These comparative results do not establish universal superiority of the proposed method. Rather, they show that the minor-eigenvalue framework can complement conventional MCDM and global spectral approaches. While SAW captures additive criterion performance and global eigenvalues summarize overall connectivity, the proposed framework captures localized perturbation effects that may distinguish alternatives with similar aggregated values. Future studies may compare this framework with TOPSIS, AHP, PROMETHEE, VIKOR, or other ranking methods to clarify its relative behavior in different decision environments.

4. Conclusion

This study proposes a minor-eigenvalue-based spectral framework within the IFSS environment using principal minors derived from membership-based and non-membership-based Einstein aggregation matrices. The proposed framework provides a local spectral perspective by capturing structural responses associated with the removal of each alternative, thereby complementing global spectral approaches that mainly summarize overall matrix connectivity.

The numerical results show stable and internally consistent spectral responses. In the Lamongan Regency case study, lower spectral scores indicate lower economic facility availability and higher development priority. Bluluk obtained the lowest score of 0.0683 and was identified as the highest-priority subdistrict, while Lamongan obtained the highest score of 2.0000, reflecting stronger economic infrastructure. Sensitivity analysis on ε and δ also showed high ranking stability, with Spearman rank correlation values ranging from 0.963 to 1.000 and no rank reversal among the three highest-priority and three lowest-priority alternatives.

The comparison with SAW showed that the extreme priority groups remained broadly consistent, indicating that the proposed method preserves the general priority pattern obtained from a conventional MCDM approach. Differences in several middle-ranked alternatives suggest that the local spectral mechanism may provide additional discrimination among alternatives with similar aggregated characteristics. Nevertheless, this study is limited by the use of a constant uncertainty parameter and static cross-sectional data. Future research may compare the proposed method with TOPSIS, AHP, PROMETHEE, and other MCDM methods, investigate alternative perturbation indicators, and extend the framework to larger or dynamic datasets.

CRediT Authorship Contribution Statement

Silfiatis Sabila Azra Shofa: Conceptualization, Methodology, Data Collection, Data Analysis, Writing – Initial Draft. **Siti Amiroch:** Supervision, Validation, Writing – Review and Editing. **Awawin Mustana Rohmah:** Supervision, Validation, Writing – Review and Editing.

Declaration of Generative AI and AI-Assisted Technologies

During the preparation of this manuscript, the authors used AI-assisted tools for language refinement, grammar checking, and improving the clarity of presentation. The authors reviewed and edited all AI-assisted outputs and take full responsibility for the content of the published article.

Declaration of Competing Interest

The authors declare that there are no conflicts of interest, either financial or non-financial, that could influence the results of this study.

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Data and Code Availability

The data used in this study are derived from the 2025 Lamongan Regency Village Potential Statistics (PODES) published by the Central Statistics Agency (BPS). The processed data and source code are available at Google Drive Dataset.[1]

The repository contains the normalized dataset, IFSS matrix, aggregation matrices, principal minor matrices, eigenvalue results, ranking outputs, and Python scripts used for the computations.

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