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Remodeling and Application of Stock Option Price Based on Skewed Laplace Distribution Approach

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Abstract

This paper proposed a new model to price a stock option based on the Skewed Laplace distribution approach (SLOP). The approach was considered to provide a better option price than Black Scholes Option Price (BSOP) because Skewed Laplace distribution (SL) has a shape parameter that can capture excess skewness and kurtosis frequently found in stock return underlying the option price. In this study, SL's shape parameter was estimated using a mixture of the Moment Method and Fourth-Order Taylor Series approach. The estimator was different from the majority of prior SL's shape parameter that was obtained by Maximum Likelihood Estimation (MLE). The proposed shape parameter was easier to obtain relative to the prior parameter because it did not require a Likelihood Function (LF) and the maximization of LF where involved a complicated numerical method. The performance of SLOP was applied to eleven different enterprises that trade stock options at several strike prices. For the option contracts examined in this study, the SLOP produces a lower aggregate MSE (65.6667) than the Black–Scholes model (87.1059). The APE values of 0.0425 for SLOP and -0.1929 for Black–Scholes further indicate a tendency toward underpricing and overpricing, respectively. These findings indicate the potential of the Skewed Laplace distribution as an alternative approach for European call option pricing, although its performance may vary across individual option contracts.

Keywords: asymmetric; black-scholes; exponential-power.

1. Introduction

Black Scholes Option Price (BSOP) was developed based on several assumptions. One of the assumptions is that stock returns follow a normal distribution, while stock prices follow a lognormal distribution. In the financial industry, return data are frequently found in a condition that is not normally distributed. In other words, they are often found in excess kurtosis conditions documented by [1–8]. Thus, the assumption of BSOP is less able to explain the existence of this excess kurtosis.

The condition has motivated many researchers to build some models to price the option in terms of the asset-return distribution approach. This approach enables the researchers to model the characteristics of return data used for analysis properly. Skewed Laplace Distribution (SL) is one of the distributions that can be utilized to achieve the purpose.

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Therefore, this paper attempted not only to reanalyze the model of the European call option conducted by [9] and written based on [10] but also to analyze the performance of the SL Approach in pricing the European call option. In [9], we derived a skewness parameter estimator based on a third-order Taylor approximation. The present study builds upon that analytical framework by extending the approximation to the fourth order, providing a more comprehensive discussion of parameter estimation methods reported in the literature, and demonstrating the application of the resulting estimator in European stock option pricing.

The paper is organized as follows. Section two presents some theoretical aspects utilized as a basic concept that will be used for the next part of the paper. The third section summarizes the derivation of SL's parameter. Then, section four derives the model of the European call option price based on the SL Approach (SLOP). Meanwhile, section five gives a practical example to demonstrate the performance of SLOP in real financial data, and finally, in the last section, a conclusion and recommendation for further research will be provided.

2. Methods

This section presents the methodological framework for the proposed SLOP. The BSOP is first introduced as the benchmark, followed by the formulation and parameter estimation of the SL.

2.1. Black Scholes Option Price (BSOP)

BSOP is a seminal option price that has been modified and developed by numerous researchers. BSOP with corresponding payoff $f_T = (S_T - K, 0)^+$, symbolized as C_{BS}, can be expressed as follows [11]

$$C_{BS} = S_0 N(d_1) - K e^{-r\tau} N(d_2) \tag{1}$$

which S_0 is the price of the stock at time zero, K is the strike price, r is the continuously compounded risk-free rate, σ is the volatility of the stock, τ is the time to maturity of the option,

$$d_1 = \frac{ln(S_0/K) + \tau(r + \sigma^2/2)}{\sigma\sqrt{\tau}}, \tag{2}$$

$$d_2 = \frac{ln(S_0/K) + \tau(r - \sigma^2/2)}{\sigma\sqrt{\tau}} = d_1 - \sigma\sqrt{\tau}, \tag{3}$$

and

$$N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp \frac{-y^2}{2} \, dy$$

is the cumulative standard normal distribution. BSOP expressed by Equation (1) is derived based on the solution of a partial differential equation emerging from a stochastic differential equation.

BSOP has been frequently used to be compared with other models in terms of European call option pricing. BSOP provides a closed-form option price, making the model applicable practically. Even though it offers easiness, the assumption of normally distributed asset return is regarded as impractical. In addition, the latter assumption cannot represent the real condition of the option market.

The pricing option requires a probability distribution that can accommodate the skewness of log-returns. For achieving this objective, such distribution must have the ability to define all of the moments (moment generating function) [8]. SL, as a special case of Skewed Generalized Error Distribution (SGE) (see [8]) is one of the distributions which fulfills the requirement needed in pricing option.

2.2. Skewed Laplace Distribution

SL is a skewed version of Laplace Distribution (L). The other name of L is Double Exponential Distribution (DE) because it can be considered as two Exponential Distributions joined together (with one additional location parameter). Many research exploring the extension and the application of L have been published in numerous journals.

There are several classes of SL. The first type of SL was obtained by relaxing the assumption of finite variance of the random variables. The SL for this version is called Asymmetric Laplace distribution (AL). This type of SL was analyzed by [12]. Then, the second type of SL is grouped based on Azzalini's method. The SL for this type was generated using a genuine probability density presented in [13]. Next, type III of SL was developed by [14]. Jagannathan (2005) [14] defined a Skewed DE (another name for SL) from the two i.i.d standard Laplace variables. Then, Type IV of SL was developed based on the connection between AL and quantile regression. Arslan and Genc (2009) [15] analyzed a family of skewed generalized t distribution (SGT) firstly developed by [10], namely Skewed Exponential Power distribution (SEP). SEP is a special case in the family of SGT. Meanwhile, SEP (in other articles called Generalized error distribution (GE)) is included SL as a special case. Arslan and Genc (2009) [15] characteristics of the SGT and utilized Maximum Likelihood Estimation (MLE) to estimate the parameters.

Marin and Sucarrat (2012) [16] investigated the properties of two methods for financial density selection of SEP. Marin and Sucarrat (2012) [16] estimated parameters of SEP by MLE via fGarch package in R. Then, [17], where the parameters are different from the parameters of AL constructed by [18], developed a new risk measure, namely Skewed-EWMA-based VaR. The estimation parameter of AL in this research paper was conducted by MLE. Next, [19] introduced a novel multivariate SEP (MSEP). MSEP is also called p-dimensional SEP. They estimated the parameters using an iterative procedure, namely the Generalized Expectation Minimization (GEM) algorithm.

In this paper, a new generalization of L called SL, which is a particular case of SGE [20], SGT [21] and multivariate SL [22] will be applied to model European call option price. SL is a particular case of SGE when a scale parameter that specifies both the (fat) tails and shape (degree of leptokurtosis) of the distribution is one [20]. [9] modifies a probability density function of SL proposed by [8] so that a continuous random variable Y follows SL, which has μ, σ, and ϖ as its parameters and has a probability density function as follows

$$f(y \mid \mu, \sigma, \varpi) = \frac{\zeta}{\sigma} \exp\left(-\frac{1}{[1 + sign(y)\varpi]\vartheta\sigma}|y|\right) \tag{4}$$

where μ, σ, and ϖ consecutively are expected value, standard deviation, and skewness parameter (that obeys a value between -1 and 1) of the random variable. Meanwhile, *sign* represents of Signum Function, $\zeta = \dfrac{\sqrt{2}(\sqrt{1+\varpi^2})}{2}$, and $\vartheta = \dfrac{1}{\sqrt{2}(\sqrt{1+\varpi^2})}$ ([8]).

Parameter estimators of the SL family in the previous research using MLE were conducted by [8, 15–18], and [23–25]. Meanwhile, the Moment Method (MM) used to estimate the parameters were utilized by [14], and [26].

The shape parameter of SL in this study was estimated using a mixture of the Moment Method and Fourth-Order Taylor Series approach. The estimator was different from the majority of prior SL's shape parameter that was obtained by MLE. The proposed parameter estimator in this article is also different from the previous SL's shape parameter resulted from the Moment Method. The proposed shape parameter was easier to obtain relative to the prior parameter because it did not require a Likelihood Function (LF) and the maximization of LF where involved a complicated numerical method. Derivation of parameter estimations of SL will be shown in the next paragraphs.

It will be easy to derive that the first and the second non central moment of SL are

$E(Y)=\dfrac{\sqrt{2}\sigma}{\sqrt{1+\varpi^2}}$ and $Var(Y)=\sigma^2$ while the first of three central moments of SL successively are $\Psi_1=0,\Psi_2=\sigma^2$, and $\Psi_3=\dfrac{1}{(\sqrt{1+\varpi^2})}\sigma^3[3\sqrt{2}\varpi-\dfrac{2\sqrt{2}}{1+\varpi^2}\varpi^3]$. Then, the central moments and the non-central moments of SL will be utilized to obtain estimators of SL's parameters using moment method ([9]). This procedure will gain $\hat{\mu}=\dfrac{\sum_{t=1}^{n}Y_t}{n}=\bar{Y}$ and $\hat{\sigma^2}=\dfrac{\sum_{t=1}^{n}(Y_t-\bar{Y})^2}{n}$. On the other hand, ϖ can be derived by finding the roots of Equation (5) as follows

$$\begin{aligned}&6\sqrt{2}\varpi D_1^2(D_2)^{-3/2}-3\sqrt{2}\varpi(D_2)^{-1/2}\\&-2\sqrt{2}\varpi^3(D_2)^{-3/2}D_3-D_4=0.\end{aligned}\tag{5}$$

where $D_1=(1+\varpi),D_2=1+\varpi^2,D_3=\left[\frac{1}{n}\sum_{t=1}^{n}(Y_t-\bar{Y})^2\right]^{3/2}$, and $D_4=\frac{1}{n}\sum_{t=1}^{n}(Y_t-\bar{Y})^3$. Using a similar procedure in obtaining skewness parameter (ϖ) based on Taylor Series approximation (see ([9])), it can be obtained estimator of ϖ, namely

$$\begin{aligned}\hat{\varpi}_1&=-\frac{1}{2}\sqrt{\beta_*+1},\\\hat{\varpi}_2&=\frac{1}{2}\sqrt{\beta_*+1},\\\hat{\varpi}_3&=-\frac{1}{2}\sqrt{1-\beta_*},\\\hat{\varpi}_4&=\frac{1}{2}\sqrt{1-\beta_*}.\end{aligned}$$

where $\beta_*=\sqrt{5-4b}$ and $b=\dfrac{1}{6n\sigma^4}\sum_{t=1}^{n}(Y_t-\bar{Y})^4$. The estimator $\hat{\varpi}$ is selected as the real-valued root satisfying the admissible parameter space $-1<\hat{\varpi}<1$. The selected root is then substituted into the corresponding skewness equation to verify its mathematical validity.

3. Results and Discussion

This section will explain the derivation of option price based on the SL Approach stated by [20] and [7]. Call option price, assumed that the underlying stock pays no dividends during the life of the option, can be expressed as Equation (6)

$$C_{SL}=e^{-r\tau}\int_K^\infty (S_T-K)\left[A_1+A_2\right]dS_t\tag{6}$$

where $A_1=P(S_T>K)f(S_T\mid S_t>K)$ and $A_2=P(S_T\le K)f(S_T\mid S_T\le K)$ (Topaloglou et al., 2008). For simplification purpose, let $\rho=P(S_T>K)$, so that Equation (6) can be written as

$$C_{SL}=e^{-r\tau}E(S_T\mid S_T>K)\rho-e^{-r\tau}K\rho\tag{7}$$

Using Ito's Lemma, Brownian Motion of stock price, and also risk-neutral assumption in the market, $S_T>K$, can be written as

$$z>-\frac{ln\frac{S_0}{K}+(r-0.5\sigma^2)\tau}{\sigma\sqrt{\tau}}=-d_2\tag{8}$$

in which d_2 is stated at Equation (3). Next, ρ can be expressed by

$$\rho=P(z>-d_2)=1-P(z<-d_2)=1-M(-d_2),\tag{9}$$

which $M_{(-d_2)} = \int_{-\infty}^{-d_2} f(z, \varpi)$ is a cumulative distributive function of SL. The call option price of SL Approach written by Equation (10) can be modelled by substitution Equation (9) into Equation (7)

$$C_{SL} = e^{-0.5\sigma^2\tau} S_0 \int_{-d_2}^{\infty} e^{\sigma\sqrt{\tau}z} f(z;\varpi)dz - e^{-r\tau} K\rho. \tag{10}$$

Meanwhile, Equation (4) was re-parameterized by simple algebra and can be written as Equation (11)

$$f(z;\varpi) = \begin{cases} \Upsilon \exp(\sqrt{2}\eta_1(z+\varrho))^2, & \text{for } z \leq -\varrho \\ \Upsilon \exp(-\sqrt{2}\eta_2(z+\varrho))^2, & \text{for } z \geq -\varrho \end{cases} \tag{11}$$

with its cumulative standardised LD is expressed by

$$L(z^*) = \begin{cases} \frac{1}{2} exp(\sqrt{2}z^*), & \text{for } z^* \leq 0 \\ 1 - \frac{1}{2} exp(-\sqrt{2}z^*) & \text{for} z^* > 0 \end{cases} \tag{12}$$

where $\Xi(\varpi) = \sqrt{1 + (3 - 4(\mathfrak{d})^2)\varpi^2}$, $\Upsilon = \frac{\Xi(\varpi)}{\sqrt{2\pi}}$, $\eta_1 = \frac{\Xi(\varpi)}{1-\varpi}$, $\eta_2 = \frac{\Xi(\varpi)}{1+\varpi}$, $\mathfrak{d} = \sqrt{\frac{2}{\pi}}$, and $\varrho = 2\varpi\mathfrak{d}\Xi(\varpi)^{-1}$ ([20]).

The re-parameterization process was conducted to simplify the formulation of European call option price based on SL Approach. Then, for getting a close form SLOP, it is considered two cases for European call option price based on SL Approach as follows ([20]).

$$\rho = \begin{cases} \frac{1}{2}(1+\varpi)\exp(-\sqrt{2}\eta_2|d_2-\varrho|), & \text{for } d_2 - \varrho < 0 \\ \varpi + (1-\varpi)L[\eta_1(d_2-\varrho)] & \text{for} d_2 - \varrho > 0 \end{cases} \tag{13}$$

Next, let $\psi = \exp^{-0.5\sigma^2\tau} S_0 \int_{-d_2}^{\infty} \exp^{\sigma\sqrt{\tau}z} f(z;\varpi)dz$ and consider the two cases

$$\psi = \begin{cases} S_0 \exp(-\varrho\sigma\sqrt{\tau} - 0.5\sigma^2\tau) \dfrac{2\Upsilon}{\sqrt{2}\eta_2 - \sigma\sqrt{\tau}} L\left[q_1\right] \\ S_0 \Upsilon exp(-\varrho\sigma\sqrt{\tau} - 0.5\sigma^2\tau) \left[q_2 + \dfrac{1}{\sqrt{2}\eta_2 - \sigma\sqrt{\tau}}\right] \end{cases} \tag{14}$$

where

$$q_1 = (\eta_2 - \frac{1}{\sqrt{2}}\sigma\sqrt{\tau})(d_2 - \varrho)$$

and

$$q_2 = \frac{1 - \exp(-(\sqrt{2}\eta_1 + \sigma\sqrt{\tau}|d_2 - m|))}{\sqrt{2}\eta_1 + \sigma\sqrt{\tau}}.$$

The first cases given in Equation (14) is for $d_2 - \varrho < 0$ while the second cases is for $d_2 - \varrho > 0$.

Finally, a close form of SLOP for $d_2 - \varrho < 0$ is given in Equation (15) while a close form of SLOP for $d_2 - \varrho > 0$ is given in Equation (16). Both equation can be obtained by substitution of Equation (13) and Equation (14) into Equation (10)

$$C_0 = S_0\beta\Delta_1 - \exp^{-r\tau} K \frac{1}{2}(1+\varpi)exp(-(\sqrt{2}\eta_2) \mid d_2 - \varrho \mid). \tag{15}$$

$$C_0 = S_0\beta\Delta_2 - \exp^{-r\tau} K\Delta_3. \tag{16}$$

In Equation (15) and (16),

$$\beta = \Upsilon \exp(-\varrho\sigma\sqrt{\tau} - 0.5\sigma^2\tau),$$

$$\Delta_1 = \left[\frac{\exp(-\left(\sqrt{2}\eta_2 - \sigma\sqrt{\tau}\right) \mid d_2 - \varrho \mid)}{\sqrt{2}\eta_2 - \sigma\sqrt{\tau}} \right],$$

$$\Delta_2 = \frac{1 - \exp(-\left(\sqrt{2}\eta_1 + \sigma\sqrt{\tau}\right) \mid d_2 - \varrho \mid)}{\sqrt{2}\eta_1 + \sigma\sqrt{\tau}} + \frac{1}{\sqrt{2}\eta_2 - \sigma\sqrt{\tau}},$$

and

$$\Delta_3 = \left[1 - \frac{1}{2}(1 - \varpi) \exp\left(-\sqrt{2}\eta_1 \mid d_2 - \varrho \mid\right)\right].$$

4. Performance of SL Approach in Pricing Option

In this section, the application of European call option pricing based on the SL Approach was utilized to price the stock options of eleven enterprises listed in Table 1. The historical stock prices underlying the options which were utilized to calculate SLOP were accessed from finance.yahoo.com. The period of the stock prices analyzed in this article was from May 14, 2018 to May 14, 2019. The period was taken before covid 19 pandemy. Calculating the SLOP requires some information such as the price of the stock at time zero (S_0) and the Strike Price (K) of each stock that can be obtained directly in the market (finance.yahoo.com). Meanwhile, volatility for each corresponding stock price was calculated as the standard deviation of the corresponding log return. Summary statistics of daily-log return of stocks underlying the options are given in Table 2.

Table 1: Eleven enterprises utilized in the analysis and its ticker code

Name of Enterprise	Ticker Code of Enterprise's Stock (TC)
The Estee Lauder Companies Inc	EL
Kellog Company	K
Visa Inc	V
Xerox Corporation	XRX
Abbott Laboratories	ABT
Charter Coomunications, Inc	CHTR
The Coca Cola Company	KO
The Procter and Gamble Company	PG
Walmart Inc	WMT
Apple Inc	AAPL
Amazon.com, Inc	AMZN

Table 2: Summary statistics of daily-log return of stocks underlying the options

TC	Minimum	Median	Mean	Maximum
EL	-0.0772	0.0014	0.0006	0.1101
K	-0.0929	0.0012	-0.0003	0.0429
V	-0.0489	0.0015	0.0008	0.0675
XRX	-0.1384	0.0009	0.0003	0.1079
ABT	-0.0483	0.0061	0.0008	0.0601
CHTR	-0.0649	-0.0004	0.0012	0.1327
KO	-0.0881	0.0007	0.0005	0.0295
PG	-0.0409	0.0014	0.0015	0.0084
WMT	-0.0368	0.0002	0.0007	0.0892
AAPL	-0.1049	0.0012	-0.0001	0.0681
AMZN	-0.0814	0.0014	0.0005	0.0903

Besides the price of the stock at time zero (S_0), the Strike Price of each stock (K), and counted volatility of each stock returns underlying the options, we also need data of maturity time (τ) and also risk-free interest rate (r). In this case, we assumed that the maturity of time is ten days while the rate follows The Federal Reserve, namely 2.5 percent. Those components were utilized to derive both SLOP and BSOP, which consecutively are calculated by Equation (15) and Equation (1).

For SLOP, besides the previous components, we also require an estimator of skewness parameter corresponding to each log return of the stock. The estimators were obtained using Equation (6). The values of the estimators are summarized in the fourth column of Table 3. The nearly close value skewness parameter estimates occur because the fourth-order approximation produces very close values of β_*, which consequently yield similar estimates of $\hat{\varpi}$.

After having all the mentioned components, SLOP and BSOP are calculated by Equation (16) and Equation (1), respectively. The price of SLOP will be compared to BSOP in order to analyze SLOP performance in real data. Option prices of several stocks which are counted by both models are shown in Table 3 and Table 4.

Fig. 1 illustrates the comparison between the estimated option prices obtained from the SLOP and BSOP and the corresponding MOP across different strike prices. In general, all three price series exhibit a similar downward trend, indicating that option prices decrease as the strike price increases. Visually, the SLOP appears to produce estimates that are generally closer to the market option prices than the BSOP for most analyzed stocks. However, this observation will be validated using Average of Pricing Error (APE) and Mean Squared Error (MSE) to objectively assess the pricing performance of the two models.

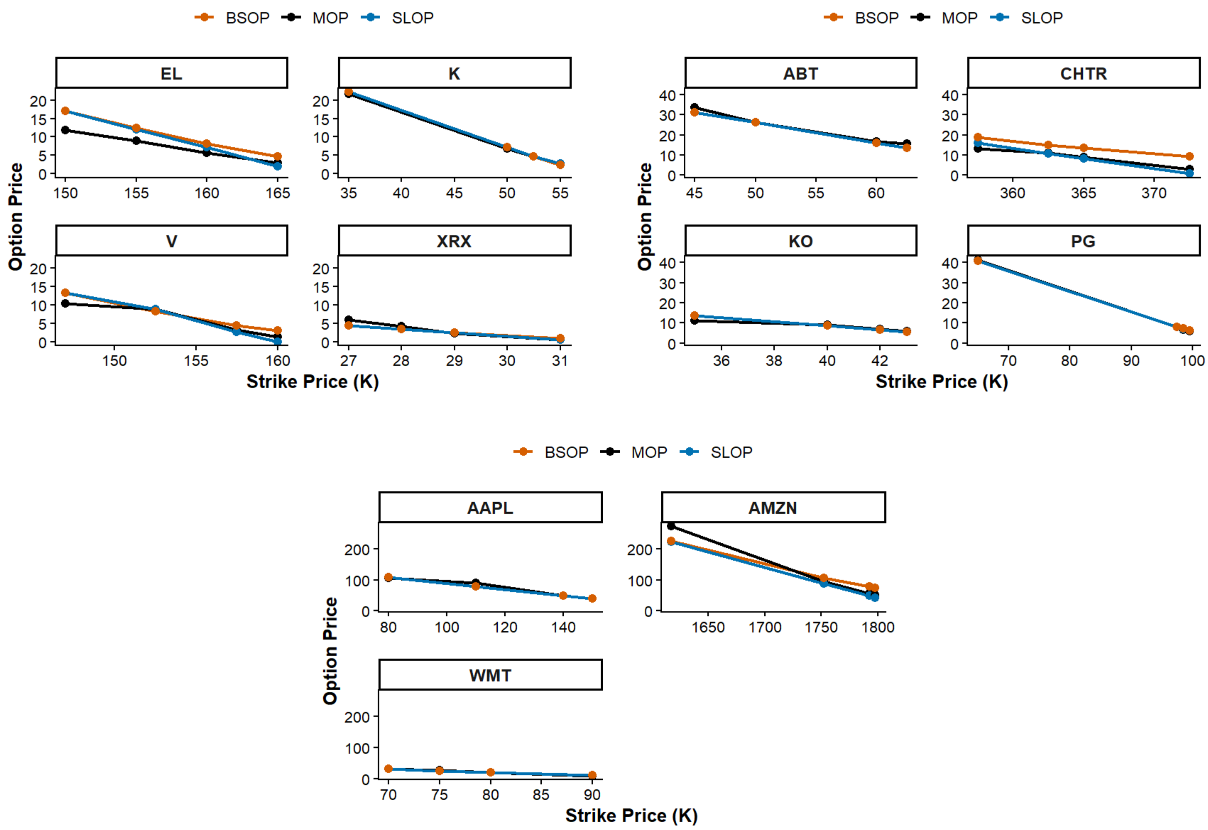

Fig. 1: Comparison of estimated SLOP and BSOP with MOP

An option price model is considered to be well-specified by investigating its Mean Squared Error (MSE) and Average of Pricing Error (APE). The MSE is obtained by counting the average squared difference between the option price in the market (MOP) and the option price calculated by the specified model. Meanwhile, APE is calculated as the average relative difference between the observed market option prices (MOP) and the option prices computed by SLOP, following

Table 3: Comparison of several option price of SLOP and BSOP

TC	S_0	K	$\hat{\varpi}$	MOP	SLOP	BSOP
EL	167	150	0.899453684	11.83	17.0243	17.0957
EL	167	155	0.899453684	9	12.0292	12.3638
EL	167	160	0.899453684	5.7	7.2595	8.1202
EL	167	165	0.899453684	2.95	2.0258	4.7194
K	57.17	35	0.899453719	21.6	22.1751	22.1735
K	57.17	50	0.899453719	6.9	7.1766	7.1757
K	57.17	52.5	0.899453719	4.7	4.6769	4.6993
K	57.17	55	0.899453719	2.8	2.7352	2.4319
V	160.21	147	0.899453515	10.45	13.2199	13.2897
V	160.21	152.5	0.899453515	8.9	8.7497	8.2358
V	160.21	157.5	0.899453515	3.3	2.7211	4.4723
V	160.21	160	0.899453515	1.3	0.0212	3.0477
XRX	31.44	27	0.899453714	6	4.4453	4.4466
XRX	31.44	28	0.899453714	4.25	3.4454	3.4640
XRX	31.44	29	0.899453714	2.25	2.4479	2.5253
XRX	31.44	31	0.899453714	0.5	0.4456	1.0091
ABT	76.03	45	0.899453442	33.59	31.0368	31.0345
ABT	76.03	50	0.899453442	26.22	26.0373	26.0350
ABT	76.03	60	0.899453442	16.6	16.0383	16.0360
ABT	76.03	62.5	0.899453442	15.6	13.5385	13.5363
CHTR	373.15	357.5	0.899453202	13.3	15.9433	18.6556
CHTR	373.15	362.5	0.899453202	10.9	10.7649	15.0578
CHTR	373.15	365	0.899453202	8.8	8.2633	13.4130
CHTR	373.15	372.5	0.899453202	2.95	0.7638	9.1422
KO	48.69	35	0.899453562	11.1	13.6941	13.6925
KO	48.69	40	0.899453562	9.17	8.6946	8.6940
KO	48.69	42	0.899453562	6.82	6.6948	6.6942
KO	48.69	43	0.899453562	6	5.6949	5.6944
PG	105.6	65	0.899448249	41.19	40.6084	40.6065
PG	105.6	97.5	0.899448249	8	8.1116	8.1307
PG	105.6	98.5	0.899448249	6.56	7.1117	7.1539
PG	105.6	99.5	0.899448249	5.85	6.1118	6.1968
WMT	100.29	70	0.899453408	30.8	30.2990	30.2970
WMT	100.29	75	0.899453408	25.9	25.2995	25.2975
WMT	100.29	80	0.899453408	20.95	20.3000	20.2980
WMT	100.29	90	0.899453408	9.2	10.3010	10.3019
AAPL	188.66	80	0.899453720	105.87	108.6720	108.6680
AAPL	188.66	110	0.899453720	90.35	78.6784	78.6710
AAPL	188.66	140	0.899453720	49.55	48.6778	48.6741
AAPL	188.66	150	0.899453720	39.25	38.6788	38.6751
AMZN	1840.12	1617.5	0.899453702	272.8	222.9988	224.2808
AMZN	1840.12	1752.5	0.899453702	95.38	88.4780	105.5980
AMZN	1840.12	1792.5	0.899453702	55.5	48.0170	77.9100
AMZN	1840.12	1797.5	0.899453702	51.65	43.0170	74.7770

the APE formulation employed by [27]. In this study, the specified models are BSOP and SLOP. In this case, the MSE of the proposed model (SLOP) will be compared to the MSE of the older model, namely BSOP. Based on the result of the analysis, it can be calculated that the MSE of BSOP and MSE of SLOP covering all stocks with various K are 87.1059 and 65.6667, successively.

The MSE of SLOP for all cases listed in Table 3 is smaller than the MSE of BSOP. Within the empirical setting considered in this study, SLOP yielded smaller pricing deviations from the observed market prices than the Black–Scholes model according to the evaluation criteria employed. This result suggests that incorporating return asymmetry through the SL may improve the representation of option prices for the contracts analyzed. Moreover, based on the results

Table 4: Comparison of option pricing performance between the SLOP and BSOP models

TC	K	MOP	SLOP	BSOP	SE (SLOP)	SE (BSOP)	PE (SLOP)	PE (BSOP)
EL	150	11.83	17.0243	17.0957	26.9807	27.7276	-0.4390	-0.4451
EL	155	9.00	12.0292	12.3638	9.1760	11.3151	-0.3365	-0.3737
EL	160	5.70	7.2595	8.1202	2.4320	5.8573	-0.2736	-0.4246
EL	165	2.95	2.0258	4.7194	0.8541	3.1307	0.3132	-0.5998
K	35	21.60	22.1751	22.1735	0.3307	0.3289	-0.0266	-0.0265
K	50	6.90	7.1766	7.1757	0.0765	0.0760	-0.0400	-0.0399
K	52.5	4.70	4.6769	4.6993	0.0005	0.0000	0.0049	0.0001
K	55	2.80	2.7352	2.4319	0.0042	0.1354	0.0231	0.1314
V	147	10.45	13.2199	13.2897	7.6723	8.0638	-0.2650	-0.2717
V	152.5	8.90	8.7497	8.2358	0.0225	0.4411	0.0168	0.0746
V	157.5	3.30	2.7211	4.4723	0.3351	1.3742	0.1754	-0.3552
V	160	1.30	0.0212	3.0477	1.6353	3.0544	0.9836	-1.3443
XRX	27	6.00	4.4453	4.4466	2.4170	2.4130	0.2591	0.2589
XRX	28	4.25	3.4454	3.4640	0.6473	0.6177	0.1893	0.1849
XRX	29	2.25	2.4479	2.5253	0.0391	0.0757	-0.0879	-0.1223
XRX	31	0.50	0.4456	1.0091	0.0029	0.2591	0.1086	-1.0182
ABT	45	33.59	31.0368	31.0345	6.5188	6.5305	0.0760	0.0760
ABT	50	26.22	26.0373	26.0350	0.0333	0.0342	0.0069	0.0070
ABT	60	16.60	16.0383	16.0360	0.3155	0.3180	0.0338	0.0339
ABT	62.5	15.60	13.5385	13.5363	4.2497	4.2588	0.1321	0.1322
CHTR	357.5	13.30	15.9433	18.6556	6.9870	28.6824	-0.1987	-0.4026
CHTR	362.5	10.90	10.7649	15.0578	0.0182	17.2873	0.0123	-0.3814
CHTR	365	8.80	8.2633	13.4130	0.2880	21.2797	0.0609	-0.5242
CHTR	372.5	2.95	0.7638	9.1422	4.7794	38.3433	0.7410	-2.0990
KO	35	11.10	13.6941	13.6925	6.7293	6.7210	-0.2337	-0.2335
KO	40	9.17	8.6946	8.6940	0.2260	0.2265	0.0518	0.0519
KO	42	6.82	6.6948	6.6942	0.0156	0.0158	0.0183	0.0184
KO	43	6.00	5.6949	5.6944	0.0930	0.0933	0.0508	0.0509
PG	65	41.19	40.6084	40.6065	0.3382	0.3404	0.0141	0.0141
PG	97.5	8.00	8.1116	8.1307	0.0124	0.0170	-0.0139	-0.0163
PG	98.5	6.56	7.1117	7.1539	0.3043	0.3527	-0.0841	-0.0905
PG	99.5	5.85	6.1118	6.1968	0.0685	0.1202	-0.0447	-0.0592
WMT	70	30.80	30.2990	30.2970	0.2510	0.2530	0.0162	0.0163
WMT	75	25.90	25.2995	25.2975	0.3606	0.3630	0.0231	0.0232
WMT	80	20.95	20.3000	20.2980	0.4225	0.4251	0.0310	0.0311
WMT	90	9.20	10.3010	10.3019	1.2122	1.2141	-0.1196	-0.1197
AAPL	80	105.87	108.6720	108.6680	7.8512	7.8288	-0.0264	-0.0264
AAPL	110	90.35	78.6784	78.6710	136.2262	136.3990	0.1291	0.1292
AAPL	140	49.55	48.6778	48.6741	0.7607	0.7672	0.0176	0.0176
AAPL	150	39.25	38.67883	38.6751	0.3262	0.3305	0.0145	0.0146
AMZN	1617.5	272.80	222.9988	224.2808	2480.1600	2354.1130	0.1825	0.1778
AMZN	1752.5	95.38	88.4780	105.5980	47.6376	104.4075	0.0723	-0.1071
AMZN	1792.5	55.50	48.0170	77.9100	55.9952	502.2081	0.1348	-0.4037
AMZN	1797.5	51.65	43.0170	74.7770	74.5286	534.8581	0.1671	-0.4477

in Table 3, the Average Price Error (APE) values for the SLOP and BSOP models are 0.0425 and -0.1929, respectively. According to the definition of APE, a positive value indicates that the computed option prices tend to be lower than the observed market prices (underpricing), whereas a negative value indicates that the computed prices tend to exceed the observed market prices (overpricing). Therefore, the positive APE of SLOP indicates a tendency to underprice the option contracts, while the negative APE of BSOP indicates a tendency to overprice them.

5. Conclusion

The research summarized that a mixture of the moment method and fourth-order Taylor series approach can be applied to derive parameters of SL. Furthermore, the European call option price based on SL can be obtained in a closed-form model. Even though the model is much more complicated, the calculation of the option price can be conducted by developing an R Program. For the analyzed dataset, SLOP yields a lower aggregate MSE (65.6667) than Black–Scholes (87.1059), indicating a smaller overall pricing error. The APE results also show different pricing tendencies, with SLOP tending to underprice and Black–Scholes tending to overprice the observed market option prices. Overall, these findings support the potential of SLOP as an alternative approach for European call option pricing, although its lower aggregate MSE does not imply better performance for every individual option contract.

CRediT Authorship Contribution Statement

Evy Sulistianingsih: Conceptualization, Methodology, Writing–Original Draft, Formal Analysis. **Ferdi Afrizal:** Writing–Review, Software, Investigation, Visualization & Editing. **Muhammad Fikri:** Writing–Review & Editing. **Pitriani:** Writing–Review & Editing.

Declaration of Generative AI and AI-assisted technologies

AI-assisted technologies were employed throughout the development of this manuscript for converting the script from Word to LaTeX and to increase the clarity and readability of specific sentences of the document. After utilising this tool, we examined and modified the content and take full responsibility for the content of the published article.

Declaration of Competing Interest

The authors declare no competing interests.

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Data and Code Availability

The dataset and code analyzed during the current study are publicly available in the yahoo finance[1]. Meanwhile, the code utilized in analyzing the data is available form the corresponding authors.

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