On The Metric Dimension of Some Operation Graphs

Marsidi1, Ika Hesti Agustin2, Dafik3, Ridho Alfarisi4, Hendrik Siswono5

1Mathematics Edu. Depart. IKIP PGRI Jember Indonesia

2Mathematics Depart. University of Jember Indonesia

3Mathematics Edu. Depart. University of Jember Indonesia

4Elementary School Teacher Edu. Depart. University of Jember Indonesia

5Majoring in Early Childhood Edu. Depart. IKIP PGRI Jember Indonesia

Email: [marsidiarin@gmail.com, ikahesti@fmipa.unej.ac.id](mailto:marsidiarin@gmail.com,%20ikahesti@fmipa.unej.ac.id), [d.dafik@gmail.com](mailto:d.dafik@gmail.com).

ABSTRACT

Let be a simple, finite, and connected graph. An ordered set of vertices of a nontrivial connected graph is and the -vector represent vertex that respect to , where and is the distance between vertex and for . The set called a resolving set for if different vertex of have different representations that respect to . The minimum of cardinality of resolving set of G is the metric dimension of , denoted by . In this paper, we give the local metric dimension of some operation graphs such as joint graph, amalgamation of parachute, amalgamation of fan, and .

**Keywords**: metric dimension, resolving set, operation graphs.

# INTRODUCTION

All graphs in this paper are simple, finite and connected, for basic definition of graph we can see in [1]. [2] define the length of a shortest path between two vertices and is the distance between two vertices in a connected graph G. An ordered set of vertices of a nontrivial connected graph is and the -vector represent vertex that respect to . The set called a resolving set for if different vertex of have different representations that respect to . The minimum of cardinality of resolving set of G is the metric dimension of , denoted by [3].

There are many article that explained about metric dimension such as [2], [4], [5], [6], and [7]. [8] defined a shackle graphs constructed by nontrivial connected graphs such that and have no a common vertex for every with , and for every , and share exactly one common vertex (called linkage vertex) and the linking vertices are all different. [9] defined an amalgamation of graphs constructed from isomorphic connected graphs and the choice of the vertex as a terminal is irrelevant. For any positive integer, we denote such an amalgamation by , where denotes the number of copies of .

**Proposition 1**. *[2] Let*  *be a connected graph or order , then the following hold:*

1. *Then if and only if graph is a path graph*
2. *Then if and only if graph is a complete graph*
3. *For more than or equal 3,*
4. *For more than or equal to 4, if and only if .*

# Results and Discussion

**Theorem 2.1**. *For dan* , the metric dimension and non-isolated resolving set of joint graph *is .*

**Proof.** The joint of path and cycle graph, denoted by is a connected graph with vertex set and edge set . The cardinality of vertex set and edge set, respectively are and .

For and , the lower bound which can be shown with take resolving set such that we obtained the representation of vertices and respect to as follows.

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If we show that which is the lower bound of dan , then we will show the best lower bound namely , but . This can be shown with take resolving set so that it obtained the representation of the vertices and respect to .

It can be seen that there is at least two vertices in which have the same representation respect to , one of them is and such that we have the cardinality of resolving set of .

Hence, we have the best lower bound . Furthermore, we will prove that with determine the resolving set . So, we have the cardinality of resolving set of namely . The representation of the vertices and respect to as follows.

, where

, where

It can be seen that every vertex in have distinct representation respect to , such that the cardinality of resolvng set in is or . Thus, we conclude that for and .

**Theorem 2.2**. *For , the metric dimension of amalgamation of parachute is .*

**Proof.** The amalgamation of parasut graph, denoted by is a connected graph with vertex set and edge set . The cardinality of vertex set and edge set, respectively are and .

For , the lower bound which can be shown with take resolving set such that we obtained the representation of vertices and respect to as follows.

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If we show that which is the lower bound of , then we will show the best lower bound namely . This can be shown with take resolving set so that it obtained the representation of the vertices and respect to . The representation of the vertices and respect to as follows.

It can be seen that there is at least two vertices in which have the same representation respect to , one of them is and such that we have the cardinality of resolving set of . Hence, we have the best lower bound . Furthermore, we will prove that with determine the resolving set . So, we have the cardinality of resolving set of namely . The representation of the vertices and respect to as follows.

, where

, where

It can be seen that every vertex in have distinct representation respect to , such that the cardinality of resolvng set in is or . Thus, we conclude that .

**Theorem 2.3**. *For , the metric dimension and non-isolated resolving set of amalgamation of fan graph is:*

**Proof.** The amalgamation of fan graph, denoted by is a connected graph with vertex set and edge set . The cardinality of vertex set and edge set, respectively are and .

For and is even, the lower bound which can be shown with take resolving set such that we obtained the representation of vertices and respect to as follows.

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If we show that which is the lower bound of and is even, then we will show the best lower bound namely . This can be shown with take resolving set so that it obtained the representation of the vertices and respect to . It can be seen that there is at least two vertices in which have the same representation respect to , one of them is and such that we have the cardinality of resolving set of . Hence, we have the best lower bound . Furthermore, we will prove that with determine the resolving set . So, we have the cardinality of resolving set of namely . The representation of the vertices and respect to as follows.

, where

, where

It can be seen that every vertex in have distinct representation respect to , such that the cardinality of resolvng set in is or . Thus, we conclude that .

**Theorem 2.4**. *For , the metric dimension of is .*

**Proof.** The shackle of fan graph, denoted by is a connected graph with vertex set and edge set . The cardinality of vertex set and edge set, respectively are and .

The proof that the lower bound of is . Based on Proposition 1, that if only if . The graph does not isomorphic to path such that . Furthermore, we proof that the upper bound of is , we choose the resolving set .

The representation of the vertices respect to as follows.

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Vertex are distict. So, we have the cardinality of resolving set is . Thus, the upper bound of is . It conclude that .

# Conclusions

In this paper, the result show that the local metric dimension of some graph operation such as joint graph, amalgamation of parachute, amalgamation of fan, and .

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