On The Metric Dimension of Some Operation Graphs

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ABSTRACT

Let be a simple, finite, and connected graph. An ordered set of vertices of a nontrivial connected graph is and the -vector represent vertex that respect to , where and is the distance between vertex and for . The set called a resolving set for if different vertex of have different representations that respect to . The minimum of cardinality of resolving set of G is the metric dimension of , denoted by . In this paper, we give the local metric dimension of some operation graphs such as joint graph, amalgamation of parachute, amalgamation of fan, and .

**Keywords**: metric dimension, resolving set, operation graphs.

# INTRODUCTION

All graphs in this paper are simple, finite and connected, for basic definition of graph we can see in [1]. [2] define the length of a shortest path between two vertices and is the distance between two vertices in a connected graph G. An ordered set of vertices of a nontrivial connected graph is and the -vector represent vertex that respect to . The set called a resolving set for if different vertex of have different representations that respect to . The minimum of cardinality of resolving set of G is the metric dimension of , denoted by [3].

There are many article that explained about metric dimension such as [2], [4], [5], [6], and [7]. [8] defined a shackle graphs constructed by nontrivial connected graphs such that and have no a common vertex for every with , and for every , and share exactly one common vertex (called linkage vertex) and the linking vertices are all different. [9] defined an amalgamation of graphs constructed from isomorphic connected graphs and the choice of the vertex as a terminal is irrelevant. For any positive integer, we denote such an amalgamation by , where denotes the number of copies of .

**Proposition 1**. *[2] Let*  *be a connected graph or order , then the following hold:*

1. *Then if and only if graph is a path graph*
2. *Then if and only if graph is a complete graph*
3. *For more than or equal 3,*
4. *For more than or equal to 4, if and only if .*

# Results and Discussion

**Theorem 2.1**. *For dan* , the metric dimension and non-isolated resolving set of joint graph *is .*

**Proof.** The joint of path and cycle graph, denoted by is a connected graph with vertex set and edge set . The cardinality of vertex set and edge set, respectively are and .

 For and , the lower bound which can be shown with take resolving set such that we obtained the representation of vertices and respect to as follows.

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If we show that which is the lower bound of dan , then we will show the best lower bound namely , but . This can be shown with take resolving set so that it obtained the representation of the vertices and respect to .

 It can be seen that there is at least two vertices in which have the same representation respect to , one of them is and such that we have the cardinality of resolving set of .

 Hence, we have the best lower bound . Furthermore, we will prove that with determine the resolving set . So, we have the cardinality of resolving set of namely . The representation of the vertices and respect to as follows.

, where

 , where

It can be seen that every vertex in have distinct representation respect to , such that the cardinality of resolvng set in is or . Thus, we conclude that for and .

**Theorem 2.2**. *For , the metric dimension of amalgamation of parachute is .*

**Proof.** The amalgamation of parasut graph, denoted by is a connected graph with vertex set and edge set . The cardinality of vertex set and edge set, respectively are and .

 For , the lower bound which can be shown with take resolving set such that we obtained the representation of vertices and respect to as follows.

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If we show that which is the lower bound of , then we will show the best lower bound namely . This can be shown with take resolving set so that it obtained the representation of the vertices and respect to . The representation of the vertices and respect to as follows.

It can be seen that there is at least two vertices in which have the same representation respect to , one of them is and such that we have the cardinality of resolving set of . Hence, we have the best lower bound . Furthermore, we will prove that with determine the resolving set . So, we have the cardinality of resolving set of namely . The representation of the vertices and respect to as follows.

, where

 , where

It can be seen that every vertex in have distinct representation respect to , such that the cardinality of resolvng set in is or . Thus, we conclude that .

**Theorem 2.3**. *For , the metric dimension and non-isolated resolving set of amalgamation of fan graph is:*

**Proof.** The amalgamation of fan graph, denoted by is a connected graph with vertex set and edge set . The cardinality of vertex set and edge set, respectively are and .

 For and is even, the lower bound which can be shown with take resolving set such that we obtained the representation of vertices and respect to as follows.

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If we show that which is the lower bound of and is even, then we will show the best lower bound namely . This can be shown with take resolving set so that it obtained the representation of the vertices and respect to . It can be seen that there is at least two vertices in which have the same representation respect to , one of them is and such that we have the cardinality of resolving set of . Hence, we have the best lower bound . Furthermore, we will prove that with determine the resolving set . So, we have the cardinality of resolving set of namely . The representation of the vertices and respect to as follows.

, where

 , where

It can be seen that every vertex in have distinct representation respect to , such that the cardinality of resolvng set in is or . Thus, we conclude that .

**Theorem 2.4**. *For , the metric dimension of is .*

**Proof.** The shackle of fan graph, denoted by is a connected graph with vertex set and edge set . The cardinality of vertex set and edge set, respectively are and .

 The proof that the lower bound of is . Based on Proposition 1, that if only if . The graph does not isomorphic to path such that . Furthermore, we proof that the upper bound of is , we choose the resolving set .

The representation of the vertices respect to as follows.

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Vertex are distict. So, we have the cardinality of resolving set is . Thus, the upper bound of is . It conclude that .

# Conclusions

In this paper, the result show that the local metric dimension of some graph operation such as joint graph, amalgamation of parachute, amalgamation of fan, and .

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