On the Local Adjacency Metric Dimension of Generalized Petersen Graphs

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ABSTRACT

The local adjacency metric dimension is one of graph topic. Suppose there are three neighboring vertex $a$, $b$, $c$ in path $a-c$. Path $a-c$ is called local if $a, b, c$ where each has representation: a is not equals $b$ and $a$ may equals to $c$. Let’s say, $x, y \in V(G)$. For an order set of vertices $H=\{h\_{1}, h\_{2}, . . . , h\_{k}\}$, the adjacency representation of $v$ with respect to $H$ is the ordered $k$-tuple $r\_{A}(x\_{j}|H)=(d\_{A}(x, h\_{1}), d\_{A}(x, h\_{2}), . . . , d\_{A}(x, h\_{k}))$, where $d\_{A}(x, h)$ represents the adjacency distance $x-h$. The distance $d\_{A}(x, h)$ defined by 0 if $x=h\_{i}$, 1 if $x$ adjacent with $h$, and 2 if $x$ does not adjacent with $h$. The set $H$ is a local adjacency resolving set of $G$ if for every two distinct vertices $x$, $y$ and $x$ adjacent with y then $r\_{A}(x\_{j}|H)=r\_{A}(y\_{j}|H)$. A minimum local adjacency resolving set in $G$ is called local adjacency metric basis. The cardinality of vertices in the basis is a local adjacency metric dimension of $G$, denoted by $(dim\_{A,l}(G))$. Next, we investigate the local adjacency metric dimension of generalized petersen graph.

**Keywords**: Local Resolving Set; Local (Adjacency) Metric Dimension; Adjacency Metric Dimension; Generalized Petersen.

# INTRODUCTION

This section presents about some definitions which are used. A graph $G$ is defined by set of $V(G)$ and $E(G)$, the set of vertices and the set of edges of $G$, for more details of the definition in [1,2]. The metric dimension is one of an interesting studied graph topic. Local means that every neighbouring vertex or edge has distinct representation. Let’s say, there are three neighboring vertex in a path, $a$, $b$, $c$ where each has representation: $a=b$ and a may equals $c$. Then, the path $a-c$ is called local. The local adjacency metric dimension is combination of local metric dimension and adjacency metric dimension [3]. Let $G=(V, E)$ be a connected simple finite graph and $u$, $v$ in $G$. For an order set of vertices $X=\{x\_{1}, x\_{2}, . . . , x\_{k}\}$, the adjacency representation of $v$ with respect to $X$ is the ordered $k$-tuple $r\_{A}(v|X)=(d\_{A}(v, x\_{1}), d\_{A}(v, x\_{2})$, . . . , $d\\_A (v, x\\_k))$, where $d\_{A}(u,v)$ represents the adjacency distance $u-v$. $d\_{A}(u,v)$ defines by 0 if $u=v\_{i}$, 1 if $u∼v$, and 2 if $u≉v$. We called $X$ is a local adjacency resolving set of G if for every two distinct vertices $u$, $v$ and $u∼v$ then $r\_{A}(u|X) =r\_{A}(v|X)$. A local adjacency metric basis of G is a minimum local adjacency resolving set in G. The cardinality of vertices in the basis is a local adjacency metric dimension of $G(dim\_{A,l}(G))$.

The research is begun with Rodriguez, et al. [3] about local adjacency metric dimension of corona graphs. Marsidi, et. al. Determined the local metric dimension of line graph of special graphs. Then in 2017 Rinurwati, et al. [5] researched about local adjacency metric dimension of some wheel related graphs with pendant points. Recently, Darmaji, et al. [4] studied about Local adjacency metric dimension of sun graph and stacked book graph this year of course they really motivate us to present this research.

# Results and Discussion

In this section, we investigate the local adjacency metric dimension of generalized petersen graph $GP(n, k)$ for $k = 2$ as follows

**Theorem 2.1**. The local adjacency metric dimension of $GP(n, 2)$ is $dim\_{A,l}(GP(n, 2))=\frac{3n+2}{7}$, for $n≡4 mod 7$.

**Proof.** Vertex set of $GP(n, 2)$ is $V(GP(n, 2))=\{x\_{i}, y\_{i} :1 \leq i \leq n\}$. We choose the local adjacensy resolving set $W=\left\{y\_{i}; i ≡ 1 mod 7\right\}\bigcup\_{}^{}\{y\_{n}-2\}$ such that we have the vertex representation respect to $H$ as follows.

$r\left(H\right)=\{2, 2, …, 2\}$, for $i≡3 mod 7$

$\frac{3n+2}{7}$

$r\left(H\right)=\{2, 2, …, 2\}$, for $i≡6 mod 7$

$\frac{3n+2}{7}$

$$r\left(H\right)=\{2, 2,…, 2\}$$

$\frac{3n+2}{7}$

$$r\left(H\right)=\{2, 2, …, 2\}$$

$\frac{3n+2}{7}$

$r\left(H\right)=\{2, 2,…, 2, 1, 2, 2,…, 2\}$, for $i≡2 mod 7$

$\frac{i-2}{7}$ $\frac{3n-i-3}{7}$

$r\left(H\right)=\{2, 2, 2, …, 2, 2, 2, 2, …, 2, 1, 2,…, 2, 2, 2, 2, …, 2, 2, 2\}$, for $i≡4, 5 mod 7$

$\frac{i-3}{7}$ $\frac{i-2}{7}$ $\frac{3n-i-3}{7}$ $\frac{3n-i-4}{7}$

$r\left(H\right)=\{2, 2, 2, …, 2, 2, 2, 2, …, 2, 1, 2,…, 2, 2, 2, 2, …, 2, 2, 2\}$, for $i≡0 mod 7$

$\frac{i-3}{7}$ $\frac{i-2}{7}$ $\frac{3n-i-3}{7}$ $\frac{3n-i-4}{7}$

For representation in vertex $x\_{i}\in V(GP(n,2))$ follow the representation in vertex $y\_{i}$ such that we have the cardinality of local adjacency resolving set is $\left|H\right|=\left|\left\{v\_{i};i≡1 mod 7\right\}∪\left\{v\_{n-2}\right\}\right|=\frac{3n+2}{7}$. Thus, we get the upper bound of local adjacency metric dimension of $GP(n,2)$ is $dim\_{A,l}(GP(n,2))\leq \frac{3n+2}{7}$. Furthermore, we prove the lower bound of local adjacency metric dimension of $GP\left(n,2\right)$ is $dim\_{A,l}(GP(n,2))\geq \frac{3n+2}{7}$. Assume that $dim\_{A,l}(GP(n,2))<\frac{3n+2}{7}$, we choose $\left|H\right|=\frac{3n+2}{7}-1$ such that we have the same representation for two adjacent vertices in $GP(n,2)$ namely $r\left(H\right)\ne r\left(H\right)=\left\{2, …, 2\right\}$. Thus, the local adjacency metric dimension of $GP(n,2)$ is $dim\_{A,l}\left(GP\left(n,2\right)\right)$ =$\frac{3n+2}{7}$,

$$\frac{3n-5}{7}$$

for $n≡4 mod 7$. $∎$

**Theorem 2.2**. The local adjacency metric dimension of $GP(n,2)$ is $dim\_{A,l}\left(GP\left(n,2\right)\right)=\frac{3n+6}{7}$, for $n≡5 mod 7$.

**Proof.** Vertex set of $GP(n, 2)$ is $V(GP(n, 2))=\{x\_{i}, y\_{i} :1 \leq i \leq n\}$. We choose the local adjacensy resolving set $H=\left\{y\_{i}; i ≡ 1 mod 7\right\}\bigcup\_{}^{}\{y\_{n}-2\}$ such that we have the vertex representation respect to $H$ as follows.

$r\left(H\right)=\{2, 2, …, 2\}$, for $i≡3 mod 7$

$\frac{3n+2}{7}$

$r\left(H\right)=\{2, 2, …, 2\}$, for $i≡6 mod 7$

$\frac{3n+2}{7}$

$$r\left(H\right)=\{2, 2,…, 2\}$$

$\frac{3n+2}{7}$

$$r\left(H\right)=\{2, 2,…, 2\}$$

$\frac{3n+2}{7}$

$r\left(H\right)=\{2, 2, …, 2, 1, 2, 2,…, 2\}$, for $i≡2 mod 7$

$\frac{i-2}{7}$ $\frac{3n-i-3}{7}$

$r\left(H\right)=\{2, 2, 2, …, 2, 2, 2, 2, …, 2, 1, 2,…, 2, 2, 2, 2, …, 2, 2, 2\}$, for $i≡4, 5 mod 7$

$\frac{i-3}{7}$ $\frac{i-2}{7}$ $\frac{3n-i-3}{7}$ $\frac{3n-i-4}{7}$

$r\left(H\right)=\{2, 2, 2, …, 2, 2, 2, 2, …, 2, 1, 2,…, 2, 2, 2, 2, …, 2, 2, 2\}$, for $i≡0 mod 7$

$\frac{i-3}{7}$ $\frac{i-2}{7}$ $\frac{3n-i-3}{7}$ $\frac{3n-i-4}{7}$

For representation in vertex $x\_{i}\in V(GP(n,2))$ follow the representation in vertex $y\_{i}$ such that we have the cardinality of local adjacency resolving set is $\left|H\right|=\left|\left\{y\_{i};i≡1 mod 7\right\}∪\left\{y\_{n-2}\right\}\right|=\frac{3n+6}{7}$. Thus, we get the upper bound of local adjacency metric dimension of $GP(n,2)$ is $dim\_{A,l}(GP(n,2))\leq \frac{3n+6}{7}$. Furthermore, we prove the lower bound of local adjacency metric dimension of $GP\left(n,2\right)$ is $dim\_{A,l}(GP(n,2))\geq \frac{3n+6}{7}$. Assume that $dim\_{A,l}(GP(n,2))<\frac{3n+6}{7}$, we choose $\left|H\right|=\frac{3n+6}{7}-1$ such that we have the same representation for two adjacent vertices in $GP(n,2)$ namely $r\left(H\right)\ne r\left(H\right)=\left\{2, …, 2\right\}$. Thus, the local adjacency metric dimension of $GP(n,2)$ is $dim\_{A,l}\left(GP\left(n,2\right)\right)$=$\frac{3n+6}{7}$,

$$\frac{3n-5}{7}$$

for $n≡5 mod 7$. $∎$

**Theorem 2.3**. The local adjacency metric dimension of $GP(n,2)$ is $dim\_{A,l}\left(GP\left(n,2\right)\right)=\frac{3n+3}{7}$, for $n≡6 mod 7$.

**Proof.** Vertex set of $GP(n, 2)$ is $V(GP(n, 2))=\{x\_{i}, y\_{i} :1 \leq i \leq n\}$. We choose the local adjacensy resolving set $H=\left\{y\_{i}; i ≡ 1 mod 7\right\}\bigcup\_{}^{}\{y\_{n}-2\}$ such that we have the vertex representation respect to $H$ as follows.

$r\left(H\right)=\{2, 2, …, 2\}$, for $i≡3 mod 7$

$\frac{3n+2}{7}$

$r\left(H\right)=\{2, 2,…, 2\}$, for $i≡6 mod 7$

$\frac{3n+2}{7}$

$$r\left(H\right)=\{2, 2,…, 2\}$$

$\frac{3n+2}{7}$

$$r\left(H\right)=\{2, 2, …, 2\}$$

$\frac{3n+2}{7}$

$r\left(H\right)=\{2, 2, …, 2, 1, 2, 2,…, 2\}$, for $i≡2 mod 7$

$ \frac{i-2}{7}$ $\frac{3n-i-3}{7}$

$r\left(H\right)=\{2, 2, 2, …, 2, 2, 2, 2, …, 2, 1, 2,…, 2, 2, 2, 2, …, 2, 2, 2\}$, for $i≡4, 5 mod 7$

$\frac{i-3}{7}$ $\frac{i-2}{7}$ $\frac{3n-i-3}{7}$ $\frac{3n-i-4}{7}$

$r\left(H\right)=\{2, 2, 2, …, 2, 2, 2, 2, …, 2, 1, 2,…, 2, 2, 2, 2, …, 2, 2, 2\}$, for $i≡0 mod 7$

$\frac{i-3}{7}$ $\frac{i-2}{7}$ $\frac{3n-i-3}{7}$ $\frac{3n-i-4}{7}$

For representation in vertex $x\_{i}\in V(GP(n,2))$ follow the representation in vertex $y\_{i}$ such that we have the cardinality of local adjacency resolving set is $\left|H\right|=\left|\left\{y\_{i};i≡1 mod 7\right\}∪\left\{y\_{n-2}\right\}\right|=\frac{3n+3}{7}$. Thus, we get the upper bound of local adjacency metric dimension of $GP(n,2)$ is $dim\_{A,l}(GP(n,2))\leq \frac{3n+3}{7}$. Furthermore, we prove the lower bound of local adjacency metric dimension of $GP\left(n,2\right)$ is $dim\_{A,l}(GP(n,2))\geq \frac{3n+3}{7}$. Assume that $dim\_{A,l}(GP(n,2))<\frac{3n+3}{7}$, we choose $\left|H\right|=\frac{3n+3}{7}-1$ such that we have the same representation for two adjacent vertices in $GP(n,2)$ namely $r\left(H\right)\ne r\left(H\right)=\left\{2, …, 2\right\}$. Thus, the local adjacency metric dimension of $GP(n,2)$ is $dim\_{A,l}\left(GP\left(n,2\right)\right)$ =$\frac{3n+3}{7}$,

$$\frac{3n-5}{7}$$

for $n≡6 mod 7$. $∎$

**Theorem 2.4**. The local adjacency metric dimension of $GP(n,2)$ is $dim\_{A,l}\left(GP\left(n,2\right)\right)=\frac{3n}{7}$, for $n≡0 mod 7$.

**Proof.** Vertex set of $GP(n, 2)$ is $V(GP(n, 2))=\{x\_{i}, y\_{i} :1 \leq i \leq n\}$. We choose the local adjacensy resolving set $H=\left\{y\_{i}; i ≡ 1 mod 7\right\}\bigcup\_{}^{}\{y\_{n}-2\}$ such that we have the vertex representation respect to $H$ as follows.

$r\left(H\right)=\{2, 2, …, 2\}$, for $i≡3 mod 7$

$\frac{3n+2}{7}$

$r\left(H\right)=\{2, 2, …, 2\}$, for $i≡6 mod 7$

$\frac{3n+2}{7}$

$$r\left(H\right)=\{2, 2, …, 2\}$$

$\frac{3n+2}{7}$

$$r\left(H\right)=\{2, 2, …, 2\}$$

$\frac{3n+2}{7}$

$r\left(H\right)=\{2, 2, …, 2, 1, 2, 2,…, 2\}$, for $i≡2 mod 7$

$ \frac{i-2}{7}$ $\frac{3n-i-3}{7}$

$r\left(H\right)=\{2, 2, 2, …, 2, 2, 2, 2, …, 2, 1, 2,…, 2, 2, 2, 2, …, 2, 2, 2\}$, for $i≡4, 5 mod 7$

$ \frac{i-3}{7}$ $\frac{i-2}{7}$ $\frac{3n-i-3}{7}$ $\frac{3n-i-4}{7}$

$r\left(H\right)=\{2, 2, 2, …, 2, 2, 2, 2, …, 2, 1, 2,…, 2, 2, 2, 2, …, 2, 2, 2\}$, for $i≡0 mod 7$

$ \frac{i-3}{7}$ $\frac{i-2}{7}$ $\frac{3n-i-3}{7}$ $\frac{3n-i-4}{7}$

For representation in vertex $x\_{i}\in V(GP(n,2))$ follow the representation in vertex $y\_{i}$ such that we have the cardinality of local adjacency resolving set is $\left|H\right|=\left|\left\{y\_{i};i≡1 mod 7\right\}∪\left\{y\_{n-2}\right\}\right|=\frac{3n+2}{7}$. Thus, we get the upper bound of local adjacency metric dimension of $GP(n,2)$ is $dim\_{A,l}(GP(n,2))\leq \frac{3n}{7}$. Furthermore, we prove the lower bound of local adjacency metric dimension of $GP\left(n,2\right)$ is $dim\_{A,l}(GP(n,2))\geq \frac{3n}{7}$. Assume that $dim\_{A,l}(GP(n,2))<\frac{3n}{7}$, we choose $\left|H\right|=\frac{3n}{7}-1$ such that we have the same representation for two adjacent vertices in $GP(n,2)$ namely $r\left(H\right)\ne r\left(H\right)=\left\{2, 2, …, 2\right\}$. Thus, the

 $\frac{3n-5}{7}$

 local adjacency metric dimension of $GP(n,2)$ is $dim\_{A,l}\left(GP\left(n,2\right)\right)$=$\frac{3n}{7}$, for $n≡0 mod 7$.

$$∎$$

**Theorem 2.5**. The local adjacency metric dimension of $GP(n,2)$ is $dim\_{A,l}\left(GP\left(n,2\right)\right)=\frac{3n+4}{7}$, for $n≡1 mod 7$.

**Proof.** Vertex set of $GP(n, 2)$ is $V(GP(n, 2))=\{x\_{i}, y\_{i} :1 \leq i \leq n\}$. We choose the local adjacensy resolving set $H=\left\{y\_{i}; i ≡ 1 mod 7\right\}\bigcup\_{}^{}\{y\_{n}-2\}$ such that we have the vertex representation respect to $H$ as follows.

$r\left(H\right)=\{2, 2, …, 2\}$, for $i≡3 mod 7$

$\frac{3n+2}{7}$

$r\left(H\right)=\{2, 2, …, 2\}$, for $i≡6 mod 7$

$\frac{3n+2}{7}$

$$r\left(H\right)=\{2, 2, …, 2\}$$

$\frac{3n+2}{7}$

$$r\left(H\right)=\{2, 2, …, 2\}$$

$ \frac{3n+2}{7}$

$r\left(H\right)=\{2, 2, …, 2, 1, 2, 2,…, 2\}$, for $i≡2 mod 7$

$ \frac{i-2}{7}$ $\frac{3n-i-3}{7}$

$r\left(H\right)=\{2, 2, 2, …, 2, 2, 2, 2, …, 2, 1, 2,…, 2, 2, 2, 2, …, 2, 2, 2\}$, for $i≡4, 5 mod 7$

$\frac{i-3}{7}$ $\frac{i-2}{7}$ $\frac{3n-i-3}{7}$ $\frac{3n-i-4}{7}$

$r\left(H \right)=\{2, 2, 2, …, 2, 2, 2, 2, …, 2, 1, 2,…, 2, 2, 2, 2, …, 2, 2, 2\}$, for $i≡0 mod 7$

$\frac{i-3}{7}$ $\frac{i-2}{7}$ $\frac{3n-i-3}{7}$ $\frac{3n-i-4}{7}$

For representation in vertex $x\_{i}\in V(GP(n,2))$ follow the representation in vertex $y\_{i}$ such that we have the cardinality of local adjacency resolving set is $\left|H\right|=\left|\left\{y\_{i};i≡1 mod 7\right\}∪\left\{y\_{n-2}\right\}\right|=\frac{3n+4}{7}$. Thus, we get the upper bound of local adjacency metric dimension of $GP(n,2)$ is $dim\_{A,l}(GP(n,2))\leq \frac{3n+4}{7}$. Furthermore, we prove the lower bound of local adjacency metric dimension of $GP\left(n,2\right)$ is $dim\_{A,l}(GP(n,2))\geq \frac{3n+4}{7}$. Assume that $dim\_{A,l}(GP(n,2))<\frac{3n+4}{7}$, we choose $\left|H\right|=\frac{3n}{7}-1$ such that we have the same representation for two adjacent vertices in $GP(n,2)$ namely $r\left(H\right)\ne r\left(H\right)=\left\{2, …, 2\right\}$. Thus, the local adjacency metric dimension of $GP(n,2)$ is $dim\_{A,l}\left(GP\left(n,2\right)\right)$= $\frac{3n+4}{7}$,

$$\frac{3n-5}{7}$$

for $n≡1 mod 7$. $∎$

**Theorem 2.6**. The local adjacency metric dimension of $GP(n,2)$ is $dim\_{A,l}\left(GP\left(n,2\right)\right)=\frac{3n+1}{7}$, for $n≡2 mod 7$.

**Proof.** Vertex set of $GP(n, 2)$ is $V(GP(n, 2))=\{x\_{i}, y\_{i} :1 \leq i \leq n\}$. We choose the local adjacensy resolving set $H=\left\{y\_{i}; i ≡ 1 mod 7\right\}\bigcup\_{}^{}\{y\_{n}-2\}$ such that we have the vertex representation respect to $H$ as follows.

$r\left(H\right)=\{2, 2, …, 2\}$, for $i≡3 mod 7$

$\frac{3n+2}{7}$

$r\left(H\right)=\{2, 2, …, 2\}$, for $i≡6 mod 7$

$\frac{3n+2}{7}$

$$r\left(H\right)=\{2, 2, …, 2\}$$

$\frac{3n+2}{7}$

$$r\left(H\right)=\{2, 2, …, 2\}$$

$\frac{3n+2}{7}$

$r\left(H\right)=\{2, 2, …, 2, 1, 2, 2,…, 2\}$, for $i≡2 mod 7$

$ \frac{i-2}{7}$ $\frac{3n-i-3}{7}$

$r\left(H\right)=\{2, 2, 2, …, 2, 2, 2, 2, …, 2, 1, 2,…, 2, 2, 2, 2, …, 2, 2, 2\}$, for $i≡4, 5 mod 7$

$\frac{i-3}{7}$ $\frac{i-2}{7}$ $\frac{3n-i-3}{7}$ $\frac{3n-i-4}{7}$

$r\left(H\right)=\{2, 2, 2, …, 2, 2, 2, 2, …, 2, 1, 2,…, 2, 2, 2, 2, …, 2, 2, 2\}$, for $i≡0 mod 7$

$\frac{i-3}{7}$ $\frac{i-2}{7}$ $\frac{3n-i-3}{7}$ $\frac{3n-i-4}{7}$

For representation in vertex $x\_{i}\in V(GP(n,2))$ follow the representation in vertex $y\_{i}$ such that we have the cardinality of local adjacency resolving set is $\left|H\right|=\left|\left\{y\_{i};i≡1 mod 7\right\}∪\left\{y\_{n-2}\right\}\right|=\frac{3n+1}{7}$. Thus, we get the upper bound of local adjacency metric dimension of $GP(n,2)$ is $dim\_{A,l}(GP(n,2))\leq \frac{3n+1}{7}$. Furthermore, we prove the lower bound of local adjacency metric dimension of $GP\left(n,2\right)$ is $dim\_{A,l}(GP(n,2))\geq \frac{3n+1}{7}$. Assume that $dim\_{A,l}(GP(n,2))<\frac{3n+1}{7}$, we choose $\left|H\right|=\frac{3n+1}{7}-1$ such that we have the same representation for two adjacent vertices in $GP(n,2)$ namely $r\left(H\right)\ne r\left(H\right)=\left\{2, …, 2\right\}$. Thus, the local adjacency metric dimension of $GP(n,2)$ is $dim\_{A,l}\left(GP\left(n,2\right)\right)$= $\frac{3n+1}{7}$,

$\frac{3n-5}{7}$

for $n≡2 mod 7$. $∎$

**Theorem 2.7**. The local adjacency metric dimension of $GP(n,2)$ is $dim\_{A,l}\left(GP\left(n,2\right)\right)=\frac{3n-2}{7}$, for $n≡3 mod 7$.

**Proof.** Vertex set of $GP(n, 2)$ is $V(GP(n, 2))=\{x\_{i}, y\_{i} :1 \leq i \leq n\}$. We choose the local adjacensy resolving set $H=\left\{y\_{i}; i ≡ 1 mod 7\right\}\bigcup\_{}^{}\{y\_{n}-2\}$ such that we have the vertex representation respect to $H$ as follows.

$r\left(H\right)=\{2, 2, …, 2\}$, for $i≡3 mod 7$

$\frac{3n+2}{7}$

$r\left(H\right)=\{2, 2, …, 2\}$, for $i≡6 mod 7$

$\frac{3n+2}{7}$

$$r\left(H\right)=\{2, 2,…, 2\}$$

$\frac{3n+2}{7}$

$$r\left(H\right)=\{2, 2, …, 2\}$$

$\frac{3n+2}{7}$

$r\left(H\right)=\{2, 2,…, 2, 1, 2, 2,…, 2\}$, for $i≡2 mod 7$

$ \frac{i-2}{7}$ $\frac{3n-i-3}{7}$

$r\left(H\right)=\{2, 2, 2, …, 2, 2, 2, 2, …, 2, 1, 2,…, 2, 2, 2, 2, …, 2, 2, 2\}$, for $i≡4, 5 mod 7$

$\frac{i-3}{7}$ $\frac{i-2}{7}$ $\frac{3n-i-3}{7}$ $\frac{3n-i-4}{7}$

$r\left(H\right)=\{2, 2, 2, …, 2, 2, 2, 2, …, 2, 1, 2,…, 2, 2, 2, 2, …, 2, 2, 2\}$, for $i≡0 mod 7$

$\frac{i-3}{7}$ $\frac{i-2}{7}$ $\frac{3n-i-3}{7}$ $\frac{3n-i-4}{7}$

For representation in vertex $x\_{i}\in V(GP(n,2))$ follow the representation in vertex $y\_{i}$ such that we have the cardinality of local adjacency resolving set is $\left|H\right|=\left|\left\{y\_{i};i≡1 mod 7\right\}∪\left\{y\_{n-2}\right\}\right|=\frac{3n-2}{7}$. Thus, we get the upper bound of local adjacency metric dimension of $GP(n,2)$ is $dim\_{A,l}(GP(n,2))\leq \frac{3n-2}{7}$. Furthermore, we prove the lower bound of local adjacency metric dimension of $GP\left(n,2\right)$ is $dim\_{A,l}(GP(n,2))\geq \frac{3n-2}{7}$. Assume that $dim\_{A,l}(GP(n,2))<\frac{3n-2}{7}$, we choose $\left|H\right|=\frac{3n-2}{7}-1$ such that we have the same representation for two adjacent vertices in $GP(n,2)$ namely $r\left(H\right)\ne r\left(H\right)=\left\{2, …, 2\right\}$. Thus, the local adjacency metric dimension of $GP(n,2)$ is $dim\_{A,l}\left(GP\left(n,2\right)\right)$= $\frac{3n-2}{7}$,

$\frac{3n-5}{7}$

for $n≡3 mod 7$. $∎$

# Conclusions

We have discussed about the local adjacency metric dimension of generalized Petersen graph $GP\left(n,k\right)$ for $k=2$. Such that, we have some problem for $k\geq 3$ as follows.

**Open Problem 1.** Find the local adjacency metric dimension of generalized Petersen graph $GP\left(n,k\right)$ for $k\geq 3 ?$.

# Acknowledgments

We gratefully acknowledge the support from IKIP PGRI Jember 2019 and CGANT-University of Jember of year 2019.

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