



THE SENSE OF UNITY IN THE JAMEH MOSQUE OF VARAMIN: A GEOMETRIC ANALYSIS

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Majid Ahsani

Faculty of Design and Architecture
University Putra Malaysia (UPM)
Serdang, Malaysia

Corresponding Author:

Ahsani.ar@hotmail.com

Sumarni Binti Ismail

Faculty of Design and Architecture
University Putra Malaysia (UPM)
Serdang, Malaysia

Siyamak Nayyeri Fallah

Faculty of Architecture
Islamic Azad University Nour Branch (IAUNOUR)
Nour, Iran

Ali al-Ameen

Faculty of Built Environment
University of Malaya (UM)
Kuala Lumpur, Malaysia

ABSTRACT

The Jameh Mosque of Varamin is one of the few structures manifesting a transformation from Iranian architecture toward Iranian-Islamic architecture. The study presented here tries to deal with some of the geometric principles implemented by Iranian architects to express the concept of unity. The research decodes the sacred architectural elements and illustrates the ideas used to design the Jameh Mosque of Varamin from different aspects. In the first step, geometric principles such as dynamic rectangles, the golden ratio, and the lute of Pythagoras have been discussed. In the second step, the mentioned geometric principles are adapted to the architectural documents of the building. In addition, the general form of the building and the location of the dome are determined, and the hierarchy beginning from the Mosque's entrance to the dome has been analyzed. The results show that the architects tried to induce unity and monotheism through sacred geometry in a predetermined hierarchy. Also, Findings indicate that the Jameh Mosque of Varamin can be considered the intersection of Iranian architectural ideals adapted and integrated with Islamic principles.

KEYWORDS:

Sacred geometry; Multiplicity in unity; Dynamic rectangle; Golden ratio; lute of Pythagoras

INTRODUCTION

Hitherto Iranian-Islamic architecture has implemented geometry to embody the concepts such as multiplicity and unity as the main foundation of Islamic thought. From an Islamic viewpoint, God has set a measure for all things (Qur'an, 65:3). This quasi-geometric idea can be seen in the Arabic term Qadr. Islamic architecture's visual elements and design principles are rooted in religion and a specific meaning. For example, wisdom from an Islamic vision plays a critical role in adapting spirituality, materials, and architectural concepts. The hidden geometry in Islamic architecture relies on the system that defines everything via a metaphor of the spiritual world and the esoteric merit. According to the Qur'an, the origin of nature comes from a predetermined proportion: God ordained a measure for it (25:2). The human beings' eyes were acquainted with the dimensions and proportions by which God shaped nature. Hence, they, the creature, can find everything pleasant and beautiful. The research presented here tries to analyze the role of sacred geometry and the concept of monotheism in different structural dimensions and hidden layers of the Jameh Mosque of Varamin, Tehran, Iran. The Mosque in Islam is a place for aligning

the worship experience and can be considered the most appropriate Islamic structure whose architecture objectifies the concept of multiplicity in unity.

BACKGROUND

Geometry is the basis of many fields, including architecture and engineering, where the plural masses encounter their identity. Everything from a geometric viewpoint finds its boundaries. Geometry can be considered a practical means to regulate building components and identify the ratios leading to spatial integration. Architects have always used geometry and its capacity to transform concepts into space, minimizing erosion in the design process. Such a means results in an architectural design having order and proportions crystallized in the geometric forms. In the legal space of geometry, the components regain their identity, multiplicity, and diversity. That is to say; geometry is a critical factor in integrating the components of the universe. The Iranian-Islamic architects have often focused and been concerned with beauty. Geometry has always been an effective means, making the Iranian muhandis (the Arabic and Farsi term for engineer derived from hindisah, the Arabic term for geometry) able to estimate the

proportions, create equilibrium and harmony, and put beauty in order. In other words, for Iranian-Islamic architecture, geometry, at the same time, is science and art. As mentioned above, the objective of Islamic architecture is placed in the sphere of spirituality and wisdom, so geometry helps architects to build up the bodies that are the manifestation of the sacred [1], [2], [3].

Some studies have focused on describing the sacred geometric features in Iranian-Islamic architecture, including the Mausoleum of Khawaja Ahmed Yasawi (Mankofskaya), the Gawhar Shad (Wilber), hidden geometry in the elevation of Sheikh Lotfollah Mosque (Haji Qasemi), Kabud Mosque (Ansari and Nejad Ebrahimi) [4]. This sacred geometry is rooted in ancient civilizations in which geometry manages the creation of the sacred structures preordained to illustrate the imagination of the universe as the territory of the Absolute, God, the creator. For example, the geometric form of the dome represents the cosmic sphere embracing the Earth. The nature created by Allah, God, is a mathematical and geometric phenomenon (God ordained a measure for it [nature]; Qur'an, 25:2), the highest expression of Divine Wisdom imaged by the mystic and symbolic language of the sacred architecture to represent Divine Order, Harmony, and Beauty [5], [6], [7]. In Islamic architecture, there are three aesthetic ideas, including Ta'lif (composition), I'tidal (harmony), and Nizam (order), directing architects to creativity. Compared with Aristotle, Islamic philosophers like Ibn Rushd (1126-1198 A.D.) and al-Farabi (870-950 A.D.) deal with concepts such as form, order, and proportion according to which aesthetic compositions have been experienced and integrated architecturally through the spatial sequence (proportions), texture (arabesque, muqarnas), material, and light. Iranian-Islamic architecture has always implemented geometry to express the concepts in the design of mosques. Despite the same general patterns, each Mosque has a unique architecture to realize such an objective traditionally, including the dome's design, form, and size, particularly the golden ratio used continuously [8], [9], [4].

THE JAMEH MOSQUE OF VARAMIN

The Jameh Mosque of Varamin, located in Varamin, a city in the south of Tehran, Iran, was built during the reign of Mohammad Khodabande (Öljaitü), the eighth Ilkhanid dynasty ruler from 1304 to 1316 of Iran (Figure 1). Based on the Azeri style (architecture of Azerbaijan, as one of the Iranian architectural styles), the Mosque was designed with considerable geometric precision and numerous Nahaz and Nakhir (Iranian term for protrusion and recess in the structure). There are many courtyards with four porches used for both mosques and schools. The tombs were built in a rectangular manner with a double-skin facade dome [10]. Linked to Islamic architecture, Iranian architects have increasingly tended to design courtyards, patios, garden pits, porches, and pergolas called Shabistan (bedchamber).

There are also various arches whose usage dates back to more than 3400 years ago in Chogha Zanbil temple, Iran.

Furthermore, the structure is another feature of Iranian-Islamic architecture calculated and executed based on geometric principles. The schemas in Jameh Mosque of Varamin are courtyards of four porches. The building is precise, and the additional decorations are stable. The dome was in the form of a double skin facade, which itself collapsed, leaving only the parietal part, which is the Nari dome. The bedchamber has been covered with barrels and four-part arches [10].



Figure 1. Location the Jameh Mosque of Varamin

The Mosque's design is rectangular, with a length of 66 and a width of 43 meters. Among the materials used in the building, one can note bricks with dimensions 24 by 24 cm. The way bricks have been combined in different and simultaneously operated as a decorative element. The building's direction tends to the north and consists of three parts: (1) the entrance of the Mosque, (2) the arched parts, including the eastern and western parts, and (3) the central part of the Mosque and its dome. Vertical frames and crescents that hold the dome directly to the ground have replaced three distinct horizontal sections quadrangular chamber, transition area, and dome. The distinctive feature of the building is the completeness of the four-porch plan, which are compatible with the complex. One can be attracted to the outer entrance to the top of the altar of the prayer hall and the dome dominating the entire building. The formation's order is not symmetrical to any axes in the overall plan (horizontal dimension). The sacred geometry in such a formation begins from the outermost wall to the dome. The vertical dimension of the facade passes through the courtyard and the arch porch at the dome's entrance and the dome itself. As seen in Figure 2, the geometry in the form of hierarchy brings to mind the concept of monotheism.

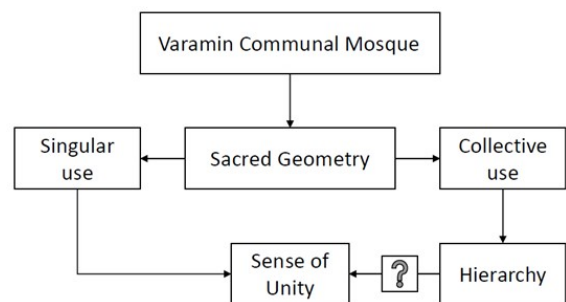


Figure 2. Design elements of the Jameh Mosque of Varamin

METHODOLOGY

The research tries to decode and discover the ideas behind the architectural design of the Jameh Mosque of Varamin, one of the best examples of Iranian-Islamic architecture in which Iranian architecture has been linked to Islamic thoughts. By implementing a semiotics approach, the study analyzes the elements used in architectural design, scenarios, and plan layers in the structure of the Jameh Mosque of Varamin. First, some geometric principles used in the Jameh Mosque of Varamin architecture have been reviewed and re-examined. Second, the study redefines and explains the reasons for implementing such geometric principles and their relation to the concept of monotheism. Third, the geometric principles referred to in the research come from a semantic perspective based on the study done by Ardalan & Bakhtiar [11], [12], [13]. Finally, by comparing the mentioned geometric principles with the architectural documents of the mosque building, including plans, views, and sections, the study tries to decipher the general architectural design scenario of the mosque building in both vertical and horizontal dimensions. In the vertical dimension, one encounters three characteristics when entering the Mosque, including the entrance porch, north porch, and dome, respectively. In the horizontal dimension, the plan has general elements and details which follow a deep order and a hierarchy in semantic layers (Figure 3).

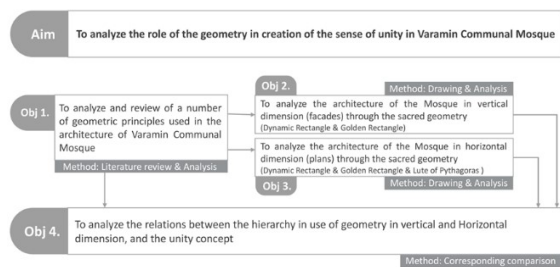


Figure 3. Research objectives and methods (Source: Authors)

DISCUSSION

The Iranian-Islamic architecture emphasizes the monotheistic course of multiplicity in unity and a hierarchy in a larger system outside itself (Figure 4). In this section, some of the geometric principles used in the architectural design of the Jameh Mosque of Varamin are examined, which brings to mind the concept of multiplicity in unity.

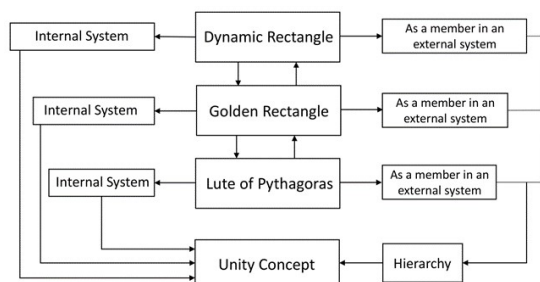


Figure 4. The relation between the Unity concept and Dynamic rectangle, Golden rectangle, and Lute of Pythagoras

THE GEOMETRIC PRINCIPLES

Incommensurable ratios orchestrate diverse spatial elements through endless divisions, achieving unity without compromising the integrity of individual parts. Dynamic symmetry is the term presented by Jay Hambidge to describe this proportioning principle, which he observes in ancient Greek and Egyptian art, the human figure, and other forms of organic life. Dynamic symmetry governs the relationship between individual elements and their relationship to the whole. It is the "perfect modulating process," imparting rhythm and movement to the transition from one level of composition to the next. In contrast, static symmetry has been accomplished by dividing a linear measure into even multiple units or by the radial subdivision of regular geometric figures, crystals, or flowers. Dynamic symmetry, found in shell growth or the arrangement of leaves on a plant, is a more vital and flexible organizing system [13], [14], [15], [16]. For example, a dynamic rectangle is a square whose diameter gives a rectangle $\sqrt{2}$, $\sqrt{3}$, $\sqrt{4}$, $\sqrt{5}$ and so on (Figure 5).

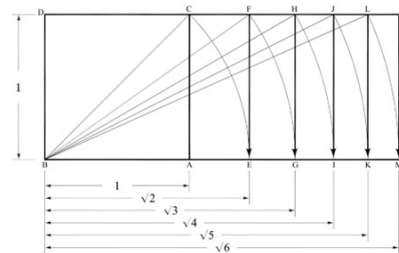


Figure 5. Dynamic rectangle

DYNAMIC RECTANGLE

$\sqrt{2}$ is the root of the number two, and just as in nature, the root of the plant is the cause of birth and reproduction, $\sqrt{2}$ is a symbol of duality, production, and multiplication [17], [18]. Combining the different proportions of this original number creates various regular forms such as octagonal, 16-sided, 24-sided, 32-sided, and 64-sided. According to Figure 6, the Root-2 rectangle corresponds to the Octagon. In the microcosm and macrocosm, the number 8 represents attributes, The number of cubes and musical notes [11], [13].

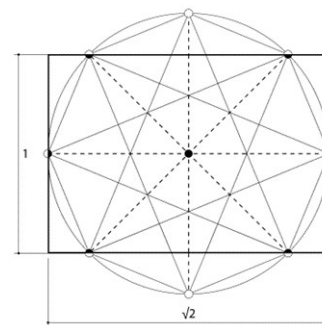


Figure 6. $\sqrt{2}$ rectangles (Root-2 rectangle)

On the other hand, there is a rectangle $\sqrt{3}$ known as the Platonic Rectangle. In ancient times, $\sqrt{3}$ ratio has been after $\sqrt{2}$. The result of drawing the diameter of a cube on one side is $\sqrt{3}$. This number was obtained through two overlapping circles centred on the other perimeter. The hexagon enclosed in a circle of unit diameter also creates this number. Combining the different proportions $\sqrt{3}$ creates regular forms such as equilateral triangles, hexagons, 12-sided, 24-sided, and 48-sided. According to Figure 7, the root-3 rectangle has been called the six-ton, and its short side and long side are proportionally equivalent to the side and diameter of a hexagon. In the macrocosm, the number 6 symbolizes the body. The microcosm symbolizes the six powers of motion in the six directions of coordinates, the first unequal number equal to the number of faces of a cube [11], [12]. Ancient Egyptian units of measure in the square are the root-2 and root-3 rectangles [14].

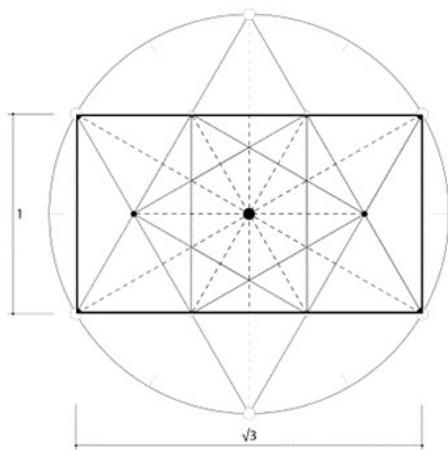


Figure 7. $\sqrt{3}$ Rectangle (Root-3 rectangle)

$\sqrt{5}$ is a proportional number that paves the way for the principle of golden ratios. This proportion may not affect the whole of nature, but the effect of the golden mean can be seen wherever there is desirable perfection, special beauty, or harmony in the form. Combining the $\sqrt{5}$ ratios creates the regular forms of a pentagon, 10-sided, 20-sided, and so on. The root-5 rectangle has been related to the golden ratio (ϕ). The longer side equals 1 plus two times $1/\phi$ (0.618). According to Figure 8, the Root-5 rectangle is a pentagon. The number 5 in the macrocosm symbolizes nature, and the microcosm symbolizes the five human senses. Number 5 is the first circular number in Persian [11], [13].

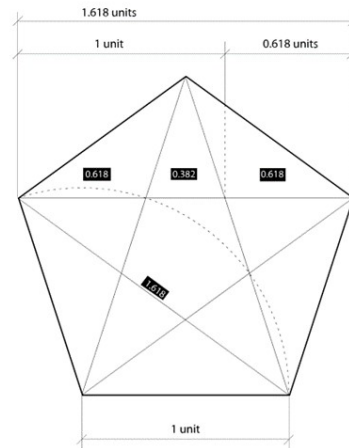


Figure 8. $\sqrt{5}$ Rectangle (Root-5 rectangle)

GOLDEN RECTANGLE

A golden rectangle is a rectangle whose side sizes are in the golden ratio (Figure 9). The second-century Italian architect Marcus Vitruvius Pollio coined the divine ratio (gold size or golden ratio). These ratios have been found throughout nature and in the human body. According to Vitruvius' treatise [20], there is a golden ratio when a line is divided into two unequal parts, and the relation of the smaller part to the larger part is the same as the relation of the larger part to the whole line. By using these geometric principles, the artist can express his desire and ideas [21].

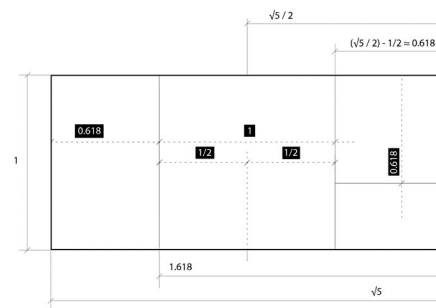


Figure 9. The relation between $\sqrt{5}$ rectangles and a golden rectangle

THE LUTE OF PYTHAGORAS

The lute of Pythagoras is a sequence diagram of five-pointed stars (Figure 10). The harmonic nature of the sequence in the lute of Pythagoras indicates that the relation between points 1 and A is the golden mean length ratio between 1 and 2. The distance from a to 2 is equal to 2-3. Maintaining the same proportion, the distance from A to x is greater than x to B since the total length of line AB is greater than A to x. This definition can be expressed as a fraction, the result of which, like other proportional fractions, is equal to a vague number. For example, a 12-sided with twelve to five faces (enclosed in a sphere) gives a 5-sided surface structure that, if five vertices are connected, the result is a five-pointed star. The larger the star tip

connections, the smaller the pentagons and five-pointed stars. This proportional set has been called the golden mean. According to Abu Rayhan al-Biruni, 12 faces with twelve to 5 faces symbolize the whole world [11].

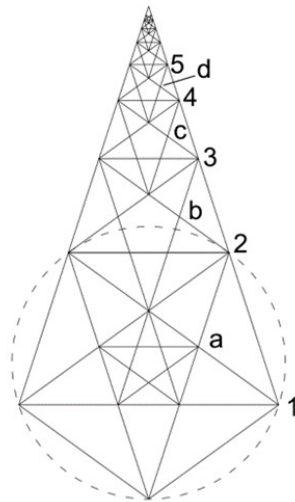


Figure 10. The lute of Pythagoras

The quadrilateral, or the number 4, is a symbol of matter. The square, the most obvious form of creation, like the Earth, represents the polar state of quantity, while the circle, like the sky, represents quality. A triangle does the combination of the two with both properties. Hence, the world's end has symbolically been represented by squaring a circle. The manifestation of such concepts can be seen in the architecture of the Achaemenid period and the works of the Sassanid period, it appears boldly, and later it was used in the Seljuk period. These concepts emerged in the design of the Jameh Mosque of Varamin, and the first combination of Iranian architecture with Islamic thought can be seen. The domes of mosques are the most obvious examples that clearly show the semantic role of numbers in the outline, details, and arrays. The dome is an image of the throne arch and an embodiment of the infinite world of the soul in which the circle is the most immediate geometric symbol. Terms such as arch Muqarnas, Kase-sarnegoun, Mina umbrella, and Kheyime-dahr all convey the esoteric meaning of the dome [11], [12]. by transforming squares hierarchically into 8, 16, and 32 sides in the corner. Finally, the circle in the dome and the Iranian domes' general geometry demonstrate the unity's multiplicity. Applying the principles mentioned above under a hierarchical system in the architecture of the Jameh Mosque of Varamin, a new layer of meaning has been formed in different levels, which emphasizes the

principle of unity as the cornerstone of Islamic art. Applying these geometric principles in such a coordinated external system also emphasizes the multiplicity in unity and vice versa. In the following, the mentioned geometric principles will be examined and applied to the mosque documents.

DATA ANALYSIS AND RESULTS

This section is concerned with the structure of the Jameh Mosque of Varamin in two dimensions: vertical (views and sections) and horizontal (plan). As shown in Figure 11, A is the main entrance to the building and the first vertical element that a person encounters when approaching and entering a mosque.

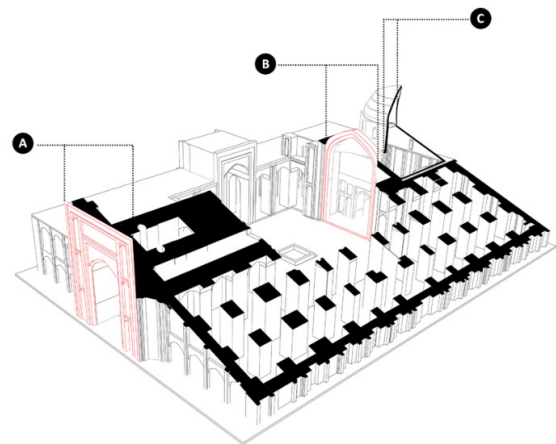


Figure 11. Vertical dimension elements

The input set contains the geometric details mentioned in Figure 12. According to Figure 12, the overall dimensions of the entrance door begin from the square (A-1). This square is a dynamic rectangle that turns into the second root, third root, fifth root, and so on in a hierarchy. Similarly, the second root rectangle in A-2, the third root rectangle in A-3, and the fifth root rectangle in A-4 appear in a hierarchy from the outermost to the innermost and central states.

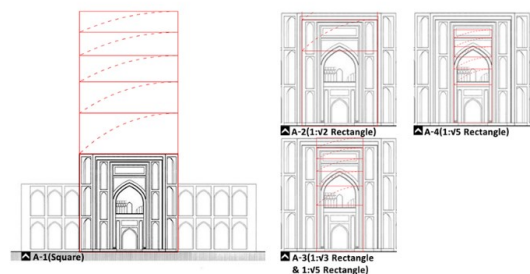


Figure 12. Dynamic rectangle in entrance gate (North view)

By entering the Mosque and passing through the entrance arch, one comes to the second stage of

the evolution of the designer's ideology, namely B, which is the north porch and the main entrance of the dome. In the porch mentioned, one sees that the height of the porch corresponds to the lute of Pythagoras, which is related to the golden ratio (Figure 13). According to B-1 (Figure 13), the mentioned hierarchy continues with the appearance of the lute of Pythagoras representing the golden ratio, and according to B-2, it has been repeated in smaller dimensions in the entrance arch to the dome.

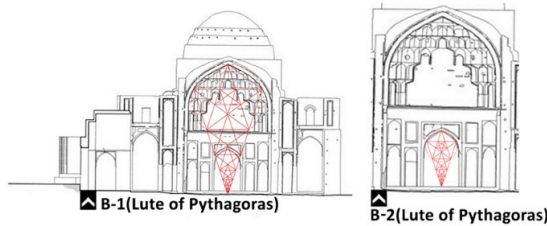


Figure 13. The lute of Pythagoras in the Mosque's dome entrance gate (East-west section)

The third stage in C (Figure 14) is the dome itself, one of the most important stages linking Iranian architecture with Islamic thought. The fifth root rectangle (C) associated with the golden rectangle 1: 1.618 forms the overall height and dimensions of the dome in the vertical dimension (section).

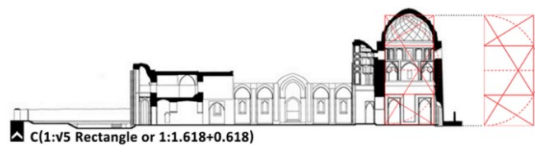


Figure 14. Golden ratio in the dome (North-south section)

As shown in Figure 15, the hierarchy mentioned before starts from the largest dimension, which is the general plan of the Mosque, and continues until the dome's location. The study will begin with a fixed consideration of the side of the north wall of the Mosque. According to D-1 (Figure 15), the general plan of the Mosque is enclosed in a second root rectangle. The location of columns A, 2, and 8 rows is determined based on the rectangle of the third root (D-2). D-3 ends in the fifth root rectangle, determined by the rows of subsequent columns C, 3, and 7. The overall location of the dome is determined by drawing a golden rectangle 1: 1.618 inside the fifth root rectangle of D-4. By drawing the quadrilateral of the golden rectangle, one of the sides of the square inside the rectangle can be seen, located along the axis of the dome.

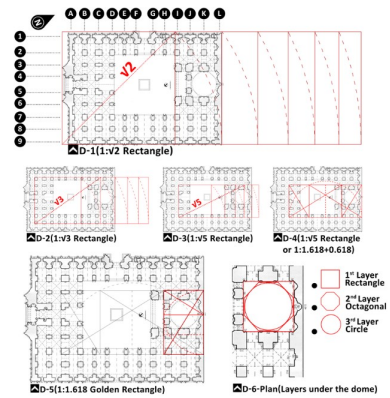


Figure 15. Horizontal dimension studied elements (Plan)

According to D-6 and 16, the dome has a square plan at the lowest level, which becomes octagonal as it rises above the ground. Here, the Octagon consists of 8 elastics (triangles) that transfer the dome's weight to the ground with a square plan. Finally, at the highest level of the ground, the octagonal plane becomes a circle, which is the starting point for the formation of the dome.

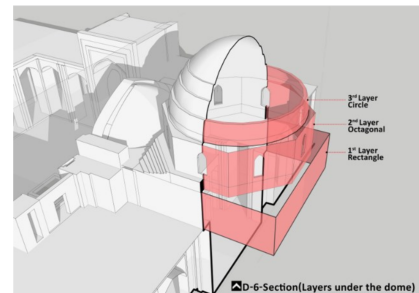


Figure 16. Mosque's dome layers (Section diagram)

CONCLUSION

A perception of proportional harmony is felt throughout Islamic architecture, which visitors usually experience intuitively. Based on previous explanations of the dynamic rectangle, the golden rectangle, and the lute of Pythagoras, the geometric principles and a pre-hierarchy design used in the architecture of the Jameh Mosque of Varamin emphasize the concept of monotheism. Monotheism is the first and most important principle in Islam, especially in Islamic architecture. According to the data analysis, the vertical and horizontal dimensions (views and plan, respectively) gradually lead to oneness and monotheism. Closely associated with the golden ratio, the rectangles and geometry of the dome were designed to bring one from the lowest to the highest level of perceiving the meaning of the Absolute (Allah, God). There are some relations between geometry and unity, as shown in Figure 17. The order of applying geometric principles in different dimensions of the Mosque seems to follow a hierarchy (Figure 17).

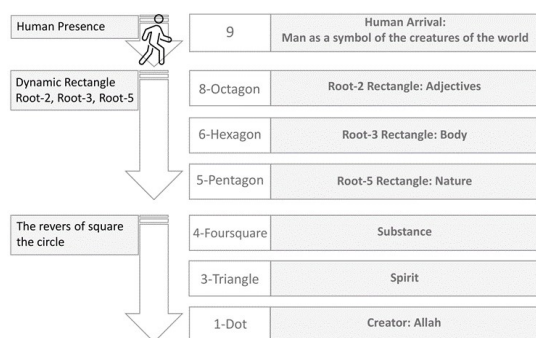


Figure 17. The relations between geometry and the unity concept

The monotheism concept hidden in the conceptual layers can be manifested in the first stage. Based on the order of rectangles (8-sided, 6-sided, and 5-sided cornerstone, respectively), it seems that the 8-sided rectangle is a symbol of human attributes, the 6-sided rectangle is a symbol of the body, and the 5-sided rectangle is a symbol of nature. By entering the dome, the stage continues to the four sides, the flexures, which are the same as the three sides, and finally, the circles are inverse of squaring the circle, symbolically referring to the end of the world. Such an event is rooted in Achaemenid architecture. The Jameh Mosque of Varamin is the intersection of Iranian architectural ideals adapted and integrated with Islamic principles. This stage demonstrates the ultimate objective of Islamic art to break one free from material bondage and drive his/him attention to the divine essence.

REFERENCES

- [1] M. M. Hejazi, F. M. Saradj, *Persian architectural heritage: Structure*, WIT Press, 2014.
- [2] A. N. Ebrahimi and M. Aliabadi, "The role of mathematics and geometry in formation of Persian architecture," *Asian Culture and History*, vol. 7, no. 1, pp. 220, 2014.
- [3] M. Hejazi, B. Hejazi, and S. Hejazi, "Evolution of Persian traditional architecture through the history," *Journal of Architecture and Urbanism*, vol. 39, no. 3, pp. 188–207, 2015. Doi: <https://doi.org/10.3846/20297955.2015.1088415>
- [4] M. Goudarzi, M. Bemanian, and M. Leylian, "Geometrical analysis of architectural drawings in the Shah-Mosque Isfahan," *Curved and Layered Structures*, vol. 7, no. 1, pp. 68–79, 2020. Doi: <https://doi.org/10.1515/cls-2020-0007>
- [5] F. A. E. Z. E. H. NABAVI and Y. A. H. A. Y. A. AHMAD, "Is there any geometrical golden ratio in traditional Iranian courtyard houses?," *International Journal of Architectural Research: ArchNet-IJAR*, vol. 10, no. 1, p. 143, 2016.
- [6] A. Tokhmechian and M. Gharehbaglou, "Music, architecture and mathematics in traditional Iranian architecture," *Nexus Network Journal*, vol. 20, no. 2, pp. 353–371, 2018. Doi: <https://doi.org/10.1007/s00004-018-0381-0>
- [7] M. Zandiyehvakili, I. Hojat, and M. Mahmudi, "The role of geometrical features in the architectural structural interaction: Some case studies of the Iranian ancient architecture," *Frontiers of Structural and Civil Engineering*, vol. 13, no. 3, pp. 716–724, Sep. 2018. Doi: <https://doi.org/10.1007/s11709-018-0508-0>
- [8] A. Dahar, R. Alipour, "Geometrical analysis of architecture of Sheik Lotfollah Mosque to find the geometrical relations between its prayer hall and the entrance," *The Monthly Scientific Journal of Bagh-e Nazar*, vol. 10(26), pp. 77–40, 2013.
- [9] A. A. Aziz, *Execution of contemporary Islamic architecture through design: The cyberjaya green platinum mosque project in Malaysia*, WIT Transactions on The Built Environment, 2016.
- [10] M. K. Pirnia, *Introduction to Iranian Islamic architecture*, Goljam Publications, 2018.
- [11] N. Ardalan and L. Bakhtiar, *The sense of unity: The Sufi tradition in Persian architecture*. Chicago: Ill. University of Chicago Press, 1973.
- [12] C. Gruber, "Architectural Dynamics in pre-revolutionary Iran," *Intellect Books*, 2019.
- [13] T. C. Parker, *A modern proportion: Dynamic symmetry in Minimalism*, Doctoral dissertation, Northern Illinois University, 2010.
- [14] R. Fletcher, "Dynamic root rectangles part one: The fundamentals," *Nexus Network Journal*, vol. 9, no. 2, pp. 327–362, 2007. doi: <https://doi.org/10.1007/s00004-007-0047-9>
- [15] P. S. Kassim, N. M. Nawawi, H. M. Sharif, and S. Hamat, "The identity of Islam in project development and the Public Realm," *Islamic Development Management*, pp. 299–315, 2019.
- [16] J. Wilson, "Dynamic symmetry: a history and analysis," *Journal of Mathematics and the Arts*, vol. 15, no. 1, pp. 19–32, Aug. 2020.
- [17] L. M. Dabbour, "Geometric proportions: The underlying structure of design process for Islamic geometric patterns," *Frontiers of Architectural Research*, vol. 1, no. 4, pp. 380–391, 2012. doi: <https://doi.org/10.1080/17513472.2020.1805157>
- [18] T. Barrie, *The sacred in-between: The mediating roles of Architecture*, Routledge: 2013.
- [19] P. Aicher, I. D. Rowland, and T. N. Howe, "Vitruvius: Ten Books on Architecture," *The Classical World*, vol. 94, no. 3, p. 303, 2001.
- [20] R. Fletcher, "The golden section," *Nexus Network Journal*, vol. 2, no. 5, pp. 67–89, 2006.
- [21] A. S. Chaitanya, G. R. Reddy, A. K. Reddy, M. R. Rachala, G. P. Reddy, and S. N. Reddy, "Divine Proportions in the Assessment of Facial Esthetics—Antiquity vs Contemporary: A Systematic Review," *Journal of Indian Orthodontic Society*, vol. 90, issue 7, pp. 216–226, 2022, DOI: [10.1177/03015742221107218](https://doi.org/10.1177/03015742221107218).