



Stability Analysis of HIV/AIDS Model with Educated Subpopulation

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ABSTRACT

We had constructed mathematical model of HIV/AIDS with seven compartments. There were two different stages of infection and susceptible subpopulations. Two stages in infection subpopulation were an HIV-positive with consuming ARV such that this subpopulation can survive longer and an HIV-positive not consuming ARV. The susceptible subpopulation was divided into two, uneducated and educated susceptible subpopulations. In this paper, we consider the transmission only from uneducated to infection subpopulations. In other hand, educated subpopulation did not have contact with the infected subpopulations. We investigated local stability of the equilibrium points according to the basic reproduction number (R_0) as a threshold of disease transmission. The disease-free and endemic equilibrium points were locally asymptotically stable when $R_0 < 1$ and $R_0 > 1$ respectively. To support the analytical results, numerical simulation was conducted.

Keywords: dynamical system; HIV/AIDS; educated subpopulations; local stability

INTRODUCTION

AIDS (Acquired Immune Deficiency Syndrome) is a disease of the immune system caused by HIV (human immunodeficiency virus) (HIV). AIDS is a threat in the world because people infected with HIV can cause death. WHO seeks to campaign for the dangers of this disease and provide various controls including the use of condoms or consume ARV (Antiretroviral Treatment).

Mathematical models have made a significant contribution to understanding the spread of HIV infection. Mathematical model HIV/AIDS have been studied by [1]-[4] where they formulated the mathematical model of HIV/AIDS with the treatment stated in the SIATR. In [4] and [5], they constructed and conducted a dynamic analysis of the HIV / AIDS epidemic model with different stages of infection and different stages of susceptible subpopulations respectively. [6] studied dynamical analysis the model SI_1I_2ATR locally and globally. The results, the disease-free and endemic equilibrium points were locally and globally asymptotically stable.

In this research, we propose mathematical model of HIV/AIDS with educated subpopulation. The proposed model is more realistic. We determined the disease-free and endemic equilibrium points as the solution of the model, the basic reproduction number (R_0), and analyzed the stability of equilibrium points locally following [7]-[14]. The disease-free equilibrium point is locally asymptotically stable when $R_0 < 1$ and the

endemic equilibrium point is locally asymptotically stable when $R_0 > 1$. Numerical simulations were performed using values of selected parameters to support the results of the analysis.

METHODS

To this end the research, we started by literature review. We modify the mathematical model of HIV/AIDS from [6] by adding educated subpopulation. Next, we analyzed the constructed model dynamically. Firstly, we should find the equilibrium points (disease-free and endemic equilibrium points). Then, we find the basic reproduction number used as a threshold endemic-occurred. Furthermore, we analyze local stability using the Routh-Hurwitz criteria. Numerical simulation is performed to see behavior of the model solution using the Runge-Kutta 4th order method.

RESULTS AND DISCUSSION

The model of HIV/AIDS with educated subpopulation consist of seven compartments (Figure 1) are $S(t)$, $E(t)$, $I_1(t)$, $I_2(t)$, $A(t)$, $T(t)$, and $R(t)$. $S(t)$ is susceptible/uneducated individuals; $E(t)$ is educated individuals, $I_1(t)$ is HIV-positive individuals consuming ARV; and $I_2(t)$ is HIV-positive individuals not consuming ARV; $A(t)$ is full-blown AIDS not receiving treatment; $T(t)$ is individuals receiving ARV treatment; $R(t)$ is recovered individuals who change and maintain their sexual habits for the rest of their lives.

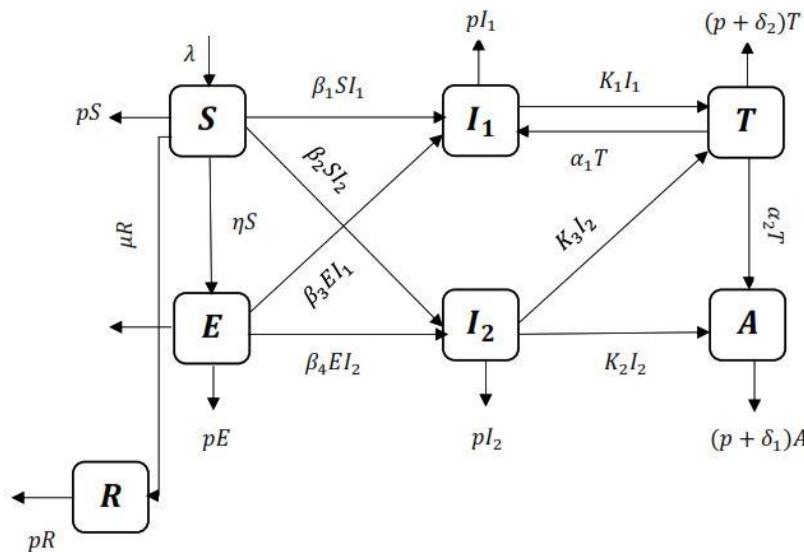


Figure 1. The compartment diagram of an HIV/AIDS model with educated subpopulation.

We establish an HIV/AIDS model with educated subpopulation in the form of a system of non-linear differential equations as follows.

$$\begin{aligned}
 \frac{dS}{dt} &= \lambda - \beta_1SI_1 - \beta_2SI_2 - aS, \\
 \frac{dE}{dt} &= \eta S - pE, \\
 \frac{dI_1}{dt} &= \beta_1SI_1 + \alpha_1T - bI_1, \\
 \frac{dI_2}{dt} &= \beta_2SI_2 - cI_2, \\
 \frac{dT}{dt} &= K_1I_1 + K_3I_2 - eT,
 \end{aligned}
 \tag{1}$$

$$\begin{aligned} \frac{dA}{dt} &= K_2 I_2 + \alpha_2 T - fA, \\ \frac{dR}{dt} &= \mu S - pR, \end{aligned}$$

where $a = \eta + \mu + p$, $b = K_1 + p$, $c = K_2 + K_3 + p$, $e = \alpha_1 + \alpha_2 + \delta_2 + d$, dan $f = \delta_1 + p$. Parameter λ is recruitment rate of the population, β_1 and β_2 are transmission coefficient from uneducated subpopulation to infection stage I_1 and I_2 respectively, β_3 and β_4 are transmission coefficient from educated subpopulation to infection stage I_1 and I_2 respectively, p is natural mortality rate, α_1 is the proportion of successful treatment, α_2 is the proportion of treatment failure, K_1 is progression rate from I_1 to T , K_2 is progression rate from I_2 to A , K_3 is progression rate from I_2 to T , δ_1 is the disease-related death rate of the AIDS, δ_2 is the disease-related death rate of being treated, μ is the rate of susceptible individuals who changed their habits, and η is the rate of educated individuals who received information of HIV/AIDS. The transmission coefficients from educated and uneducated subpopulations to infection stages were β_i where $i = 1, 2, 3, 4$ ($(\beta_1$ and $\beta_2) > (\beta_3$ and $\beta_4)$) where in this research we consider the case of β_3 and β_4 were zero. It means that $E(t)$ subpopulation is free of infection.

Positivity of solutions

We proof positivity of solutions of the model follows [6].

Theorem. If point $S(0) \geq 0, E(0) \geq 0, I_1(0) \geq 0, I_2(0) \geq 0, T(0) \geq 0, A(0) \geq 0, R(0) \geq 0$, then the solutions of system (1), S, E, I_1, I_2, T, A, R are positive for all $t > 0$.

Proof. From the first equation of system (1), we have

$$\frac{dS}{dt} = \lambda - \beta_1 S I_1 - \beta_2 S I_2 - aS = \lambda - Z_1(t)S, \tag{2}$$

where $Z_1(t) = \beta_1 I_1 + \beta_2 I_2 + a$. We multiply equation (2) with $e^{\int_0^t Z_1(r)dr}$ to give

$$\frac{dS}{dt} e^{\int_0^t Z_1(r)dr} = \{\lambda - Z_1(t)S\} e^{\int_0^t Z_1(r)dr}, \tag{3}$$

which implies

$$\frac{dS}{dt} e^{\int_0^t Z_1(r)dr} + S e^{\int_0^t Z_1(r)dr} = \lambda e^{\int_0^t Z_1(r)dr}. \tag{4}$$

Next, we write the left hand side of equation (4) as derivative of $S e^{\int_0^t Z_1(r)dr}$ with respect to t , to yield

$$\frac{d}{dt} \{S e^{\int_0^t Z_1(r)dr}\} = \lambda e^{\int_0^t Z_1(r)dr}, \tag{5}$$

then we integrate with respect to v from 0 to t , we get

$$S e^{\int_0^t Z_1(r)dr} - S(0) = \lambda \{ \int_0^t e^{\int_0^v Z_1(r)dr} dv \}. \tag{6}$$

We multiply equation (6) by $e^{-\int_0^v Z_1(r)dr}$ to give

$$S(t) = S(0)e^{-\int_0^t Z_1(r)dr} + \lambda e^{-\int_0^t Z_1(r)dr} \left\{ \int_0^t e^{\int_0^v Z_1(r)dr} dv \right\} \geq 0, \quad (7a)$$

which is said that the solution of the first equation of system (1) is positive.

Furthermore, the solutions of system (1) can be written as

$$E(t) = E(0)e^{-pt} + e^{-pt} \left\{ \int_0^t e^{pt} Z_2(t) dv \right\} \geq 0, \quad (7b)$$

$$I_1(t) = I_1(0)e^{\int_0^t Z_3(r)dr} + e^{\int_0^t Z_3(r)dr} \left\{ \int_0^t \alpha_1 T(t) e^{-\int_0^v Z_3(r)dr} dv \right\} \geq 0, \quad (7c)$$

$$I_2(t) = I_2(0)e^{Z_4 t} \geq 0, \quad (7d)$$

$$A(t) = A(0)e^{-et} + e^{-et} \left\{ \int_0^t e^{et} Z_5(t) dv \right\} \geq 0, \quad (7e)$$

$$T(t) = T(0)e^{-ft} + e^{-ft} \left\{ \int_0^t e^{ft} Z_6(t) dv \right\} \geq 0, \quad (7f)$$

$$R(t) = R(0)e^{-pt} + e^{-pt} \left\{ \int_0^t e^{pt} Z_7(t) dv \right\} \geq 0, \quad (7g)$$

where $Z_2(t) = \eta S(t)$, $Z_3(t) = \beta_1 S(t) - b$, $Z_4(t) = \beta_2 S(t) - c$, $Z_5(t) = K_1 I_1(t) + K_3 I_2(t)$, $Z_6(t) = K_2 I_2(t) + \alpha_2 T(t)$, and $Z_7(t) = \mu S(t)$. Hence, we can say that the solutions of system (1), $S(t), E(t), I_1(t), I_2(t), T(t), A(t)$, and $R(t)$ are positive for all $t > 0$.

Equilibrium points

We will find two equilibrium points, disease-free and endemic equilibrium points. The equilibrium points are obtained by solving the equations system (1) when $\frac{dS}{dt} = 0, \frac{dE}{dt} = 0, \frac{dI_1}{dt} = 0, \frac{dI_2}{dt} = 0, \frac{dT}{dt} = 0, \frac{dA}{dt} = 0, \text{ dan } \frac{dR}{dt} = 0$. The disease-free equilibrium point $K^0 = (S^0, E^0, I_1^0, I_2^0, T^0, A^0, R^0)$ is

$$K^0 = \left(\frac{\lambda}{a}, \frac{\eta\lambda}{pa}, 0, 0, 0, 0, \frac{\mu\lambda}{pa} \right). \quad (8)$$

The basic reproduction number (R_0) is obtained by using the next generation matrix method [15]. The constituent components of the next generation matrix method only consist of infected population groups, namely

$$\begin{aligned} \frac{dI_1}{dt} &= \beta_1 S I_1 + \alpha_1 T - b \\ \frac{dI_2}{dt} &= \beta_2 S I_2 I_1, -c I_2. \end{aligned} \quad (9)$$

Before we find the endemic equilibrium point, we will find the basic reproduction number. First, define $x_i' = (I_1', I_2')^T$, the system of equations (9) can be stated as

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} F_1 \\ F_2 \end{pmatrix} - \begin{pmatrix} V_1 \\ V_2 \end{pmatrix},$$

where

$$F = \begin{pmatrix} F_1 \\ F_2 \end{pmatrix} = \begin{pmatrix} \beta_1 S I_1 \\ \beta_2 S I_2 \end{pmatrix},$$

and

$$V = \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} b I_1 - \alpha_1 T \\ c I_2 \end{pmatrix}.$$

The partial derivative of V is

$$DF = \begin{pmatrix} \frac{\partial F_1}{\partial I_1} & \frac{\partial F_1}{\partial I_2} \\ \frac{\partial F_2}{\partial I_1} & \frac{\partial F_2}{\partial I_2} \end{pmatrix} = \begin{pmatrix} \beta_1 S & 0 \\ 0 & \beta_2 S \end{pmatrix}.$$

We substitute the point K^0 to DF matrix to get

$$DF(K^0) = \begin{pmatrix} \beta_1 S^0 & 0 \\ 0 & \beta_2 S^0 \end{pmatrix}.$$

Furthermore, the partial derivatives of matrix V are

$$DV = \begin{pmatrix} \frac{\partial V_1}{\partial I_1} & \frac{\partial V_1}{\partial I_2} \\ \frac{\partial V_2}{\partial I_1} & \frac{\partial V_2}{\partial I_2} \end{pmatrix} = \begin{pmatrix} b & 0 \\ 0 & c \end{pmatrix}.$$

We substitute point K^0 into DV to yield

$$DV(K^0) = \begin{pmatrix} b & 0 \\ 0 & c \end{pmatrix}.$$

The inverse of the matrix $DV(K^0)$ is

$$(DV(K^0))^{-1} = \frac{1}{bc} \begin{pmatrix} c & 0 \\ 0 & b \end{pmatrix} = \begin{pmatrix} \frac{1}{b} & 0 \\ 0 & \frac{1}{c} \end{pmatrix}.$$

The next generation matrix is obtained as follows

$$\begin{aligned} R &= (DF(K^0))(DV(K^0))^{-1}, \\ &= \begin{pmatrix} \frac{\beta_1 S^0}{b} & 0 \\ 0 & \frac{\beta_2 S^0}{c} \end{pmatrix}. \end{aligned}$$

Then, the eigenvalues of R matrix are obtained as follows.

$$r_1 = \frac{\beta_1 S^0}{b} \text{ dan } r_2 = \frac{\beta_2 S^0}{c}.$$

The basic reproduction number (R_0) is obtained from the spectral radius of the R or the largest modulus of the eigenvalues of the matrix R and the value of $\beta_1 < \beta_2$, then we get

$$R_0 = \frac{\beta_2 S^0}{c} = \frac{\beta_2 \lambda}{ac}. \tag{10}$$

Furthermore, the endemic equilibrium point $K^* = (S^*, E^*, I_1^*, I_2^*, T^*, A^*, R^*)$ can be written as follows

$$\begin{aligned} S^* &= \frac{c}{\beta_2} \\ E^* &= \frac{\eta c}{\beta_2 p} \\ I_1^* &= \frac{aca_1 K_3 (R_0 - 1)}{c\beta_1 \alpha_1 K_3 I_1 + c(\beta_2 b e - \alpha_1 \beta_2 K_1 - e\beta_1 c)} \\ I_2^* &= \frac{(\beta_2 b e - \alpha_1 \beta_2 K_1 - e\beta_1 c) I_1^*}{\alpha_1 \beta_2 K_3} \\ T^* &= \frac{(\beta_2 b - \beta_1 c) I_1^*}{\alpha_1 \beta_2} \\ A^* &= \frac{(K_2(\beta_2 b e - \alpha_1 \beta_2 K_1 - e\beta_1 c) + \alpha_2 K_3(\beta_2 b e - e\beta_1 c)) I_1^*}{f \alpha_1 \beta_2 K_3} \\ R^* &= \frac{\mu c}{\beta_2 p} \end{aligned} \tag{11}$$

Local stability analysis

The local stability of the equilibrium point is obtained by linearizing the system around the equilibrium point. We linearize the system (1) to get the Jacobi matrix

$$J(K) = \begin{pmatrix} -\beta_1 I_1 - \beta_2 I_2 - a & 0 & -\beta_1 S & -\beta_2 S & 0 & 0 & 0 \\ \eta & -p & 0 & 0 & 0 & 0 & 0 \\ \beta_1 I_1 & 0 & \beta_1 S - b & 0 & \alpha_1 & 0 & 0 \\ \beta_2 I_2 & 0 & 0 & \beta_2 S - c & 0 & 0 & 0 \\ 0 & 0 & K_1 & K_3 & -e & 0 & 0 \\ 0 & 0 & 0 & K_2 & \alpha_2 & -f & 0 \\ \mu & 0 & 0 & 0 & 0 & 0 & -p \end{pmatrix},$$

Theorem 1. *The free-disease equilibrium point K^0 is locally asymptotically stable when $R_0 < 1$ and unstable otherwise.*

Proof. The Jacobi matrix at the equilibrium point K^0 is

$$J(K^0) = \begin{pmatrix} -a & 0 & -\beta_1 \frac{\lambda}{a} & -\beta_2 \frac{\lambda}{a} & 0 & 0 & 0 \\ \eta & -p & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_1 \frac{\lambda}{a} - b & 0 & \alpha_1 & 0 & 0 \\ 0 & 0 & 0 & \beta_2 \frac{\lambda}{a} - c & 0 & 0 & 0 \\ 0 & 0 & K_1 & K_3 & -e & 0 & 0 \\ 0 & 0 & 0 & K_2 & \alpha_2 & -f & 0 \\ \mu & 0 & 0 & 0 & 0 & 0 & -p \end{pmatrix}.$$

The equation characteristic matrix $J(K^0)$ can be obtained by solving equation $|J(K^0) - rI| = 0$, i.e

$$\begin{aligned} & |J(K^0) - rI| \\ & \begin{vmatrix} -a-r & 0 & -\beta_1 \frac{\lambda}{a} & -\beta_2 \frac{\lambda}{a} & 0 & 0 & 0 \\ \eta & -p-r & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_1 \frac{\lambda}{a} - b - r & 0 & \alpha_1 & 0 & 0 \\ 0 & 0 & 0 & \beta_2 \frac{\lambda}{a} - c - r & 0 & 0 & 0 \\ 0 & 0 & K_1 & K_3 & -e-r & 0 & 0 \\ 0 & 0 & 0 & K_2 & \alpha_2 & -f-r & 0 \\ \mu & 0 & 0 & 0 & 0 & 0 & -p-r \end{vmatrix} \end{aligned} \tag{12} \\ & = 0 \end{aligned}$$

Determinant of equation (12) is

$$\varphi \begin{vmatrix} \beta_1 \frac{\lambda}{a} - b - r & \alpha_1 \\ K_1 & -e - r \end{vmatrix} = 0,$$

where $\varphi = (-p-r)(-f-r)(-p-r)(-a-r)(\beta_2 \frac{\lambda}{a} - c - r)$. The we get $r_1 = -p, r_2 = -f, r_3 = -p, r_4 = -a, r_5 = \beta_2 \frac{\lambda}{a} - c$, and $r_{6,7}$.

We can write $r_5 = \frac{\beta_2 \lambda - ca}{a}$ such that we get $r_5 = c(R_0 - 1)$, where $r_5 < 0$ when $R_0 < 1$. $r_{6,7}$

is obtained when satisfies $\begin{vmatrix} \beta_1 \frac{\lambda}{a} - b - r & \alpha_1 \\ K_1 & -e - r \end{vmatrix} = 0$.

For example, $g = \beta_1 \frac{\lambda}{a} - b$, we get quadratic polynomial

$$r^2 + (e - g)r - ge - \alpha_1 K_1 = 0. \tag{13}$$

The roots of equation (13) are negative when satisfies the properties: $D > 0, r_6 + r_7 < 0$, and $r_6 r_7 > 0$. The discriminant of equation (13) is

$$D = (e + g)^2 + 4\alpha_1 K_1 > 0,$$

where $r_6 + r_7 = -(e - g) < 0$ and $r_6 r_7 = -ge - \alpha_1 K_1 > 0$. Finally, the equilibrium point for the disease-free is asymptotically stable because we have all negative eigen values, when $R_0 < 1$, and unstable otherwise.

Theorem 2. The endemic equilibrium K^* is locally asymptotically stable when $R_0 > 1$ and unstable otherwise.

Proof: The Jacobian matrix at the K^* equilibrium point is

$$J(K^*) = \begin{pmatrix} -H_1 & 0 & -\beta_1 S^* & -\beta_2 S^* & 0 & 0 & 0 \\ \eta & -p & 0 & 0 & 0 & 0 & 0 \\ \beta_1 I_1^* & 0 & \beta_1 S^* - b & 0 & \alpha_1 & 0 & 0 \\ \beta_2 I_2^* & 0 & 0 & -\beta_2 S^* - c & 0 & 0 & 0 \\ 0 & 0 & K_1 & K_3 & -e & 0 & 0 \\ 0 & 0 & 0 & K_2 & \alpha_2 & -f & 0 \\ \mu & 0 & 0 & 0 & 0 & 0 & -p \end{pmatrix},$$

where $H_1 = \beta_1 I_1^* + \beta_2 I_2^* + a$. The matrix's characteristic equation $J(K^*)$ is obtained by solving equation $|J(K^*) - rI| = 0$, that is

$$|J(K^*) - rI| = \begin{vmatrix} \omega_1 & 0 & -\beta_1 S^* & -\beta_2 S^* & 0 & 0 & 0 \\ \eta & \omega_2 & 0 & 0 & 0 & 0 & 0 \\ \beta_1 I_1^* & 0 & \omega_3 & 0 & \alpha_1 & 0 & 0 \\ \beta_2 I_2^* & 0 & 0 & \omega_4 & 0 & 0 & 0 \\ 0 & 0 & K_1 & K_3 & \omega_5 & 0 & 0 \\ 0 & 0 & 0 & K_2 & \alpha_2 & \omega_6 & 0 \\ \mu & 0 & 0 & 0 & 0 & 0 & \omega_7 \end{vmatrix} = 0, \quad (14)$$

dimana $\omega_1 = -H_1 - r$, $\omega_2 = -p - r$, $\omega_3 = \beta_1 S^* - b - r$, $\omega_4 = \beta_2 S^* - c - r$, $\omega_5 = -e - r$, $\omega_6 = -f - r$, $\omega_7 = -p - r$. Determinant of equation (14) is

$$(-p - r)(-f - r)(-p - r) \begin{vmatrix} -H_1 - r & -\beta_1 \frac{c}{\beta_2} & -\beta_2 \frac{c}{\beta_2} & 0 \\ \beta_1 I_1^* & \beta_1 S^* - b - r & 0 & \alpha_1 \\ \beta_2 I_2^* & 0 & \beta_2 S^* - c - r & 0 \\ 0 & K_1 & K_3 & -e - r \end{vmatrix} = 0,$$

or we can write as follows

$$(-p - r)(-f - r)(-p - r)(\alpha_1 J_1 + (-e - r)J_2) = 0, \quad (15)$$

with

$$J_1 = \begin{vmatrix} H_1 - r & -\beta_1 \frac{c}{\beta_2} & -\beta_2 \frac{c}{\beta_2} \\ \beta_2 I_2^* & 0 & \beta_2 S^* - c - r \\ 0 & K_1 & K_3 \end{vmatrix},$$

and

$$J_2 = \begin{vmatrix} -H_1 - r & -\beta_1 \frac{c}{\beta_2} & -\beta_2 \frac{c}{\beta_2} \\ \beta_1 I_1^* & \beta_1 S^* - b - r & 0 \\ \beta_2 I_2^* & 0 & \beta_2 S^* - c - r \end{vmatrix}.$$

From equation (15), we obtained $r_1 = -p$, $r_2 = -f$, $r_3 = -p$ and $r_{4,5,6,7}$

$$r^4 + \left(H_1 + b + e - \frac{\beta_1 c}{\beta_2} \right) r^3 + \left(\frac{c\beta_1\beta_1 I_1^*}{\beta_2} + c\beta_2 I_2^* + H_1 b + H_1 e + H_1 b + eb - \frac{H_1\beta_1 c}{\beta_2} - \alpha_1 K_1 - \frac{\beta_1 ce}{\beta_2} \right) r^2 + \left(\frac{ce\beta_1\beta_1 I_1^*}{\beta_2} + cb\beta_2 I_2^* + ce\beta_2 I_2^* + H_1 be - \beta_1 ce I_1^* - H_1 \alpha_1 K_1 - \frac{H_1 ce\beta_1}{\beta_2} \right) r + (c\alpha_1 K_3 \beta_1 I_2^* + bce\beta_2 I_2^* - \alpha_1 \beta_2 c K_1 I_2^* - cce\beta_1 I_2^*) = 0,$$

or we can write as follows

$$r^4 + b_1 r^3 + b_2 r^2 + b_3 r + b_4 = 0, \quad (16)$$

with

$$\begin{aligned}
 b_1 &= H_1 + b + e - \frac{\beta_1 c}{\beta_2}, \\
 b_2 &= \frac{c\beta_1\beta_1 I_1^*}{\beta_2} + c\beta_2 I_2^* + H_1 b + H_1 e + H_1 b + eb - \frac{H_1\beta_1 c}{\beta_2} - \alpha_1 K_1 - \frac{\beta_1 ce}{\beta_2}, \\
 b_3 &= \frac{ce\beta_1\beta_1 I_1^*}{\beta_2} + cb\beta_2 I_2^* + ce\beta_2 I_2^* + H_1 be - \beta_1 ce I_1^* - H_1 \alpha_1 K_1 - \frac{H_1 ce \beta_1}{\beta_2}, \\
 b_4 &= c\alpha_1 K_3 \beta_1 I_2^* + bce\beta_2 I_2^* - \alpha_1 \beta_2 c K_1 I_2^* - cce\beta_1 I_2^*,
 \end{aligned}$$

Sometime, it is difficult to find roots of the characteristic polynomial, therefore the Routh-Hurwitz criteria can be used to find stability characteristic of K^* equilibrium. K^* equilibrium point is asymptotically stable if and only if it meets the following conditions.

- i. $b_1 > 0$,
- ii. $b_1 b_2 - b_3 > 0$,
- iii. $b_1 b_2 b_3 - b_3^2 - b_1^2 b_4 > 0$,
- iv. $b_1 b_2 b_3 b_4 - b_1^2 b_4^2 - b_3^2 b_4 > 0$,

NUMERICAL SIMULATION

We give numerical simulation to illustrate the main result. Numerical simulations are solved by using the 4th order Runge-Kutta method. Numerical simulation is conducted in order to understand the behavior of the proposed HIV/AIDS model and to confirm the stability analysis of the equilibrium points (disease-free and endemic equilibrium points) in the previous section. We will show that the disease-free equilibrium point is asymptotically stable when $R_0 < 1$ and the endemic equilibrium point is asymptotically stable when $R_0 > 1$. We use the parameter values for numerical simulation in Table 1.

Table 1. Parameter values for numerical simulation.

Symbol	Value	Symbol	Value
λ	0.55 year ⁻¹	K_3	0.05 year ⁻¹
β_1	0.23 year ⁻¹	μ	0.03 year ⁻¹
β_2	0.33 year ⁻¹	α_1	0.02
p	0.0196 year ⁻¹	α_2	0.05
η	0.1 year ⁻¹	δ_1	0.0909 year ⁻¹
K_1	0.0498 year ⁻¹	δ_2	0.0667 year ⁻¹
K_2	0.008 year ⁻¹		

We choose the parameter values in order to satisfies reproduction number $R_0 > 1$ for the endemic equilibrium point. According to Table 1, we get the basic reproduction number $R_0 = 15.6345 > 1$. The dynamics of subpopulations are shown in Figure 2. Figure 2 shows the solutions of HIV/AIDS model with initial values $NA = (30, 10, 25, 35, 20, 16, 50)$ lead to endemic equilibrium point $K^* = (0.02353, 1.2018, 3.1928, 4.4043, 1.4166, 2.4263, 0.3631)$. Using the parameters in Table 1, the condition of the Routh-Hurwitz criterias are satisfied

$$\begin{aligned}
 b_1 b_2 - b_3 &= 1.77347 > 0 \\
 b_1 b_2 b_3 - b_3^2 - b_1^2 b_4 &= 0.00309 > 0 \\
 b_1 b_2 b_3 b_4 - b_1^2 b_4^2 - b_3^2 b_4 &= 0.00002 > 0
 \end{aligned}$$

and

$$\begin{aligned}
 b_1 &= 2.33736 > 0 \\
 b_2 &= 0.71722 > 0 \\
 b_3 &= 0.02602 > 0
 \end{aligned}$$

$$b_4 = 0.00684 > 0$$

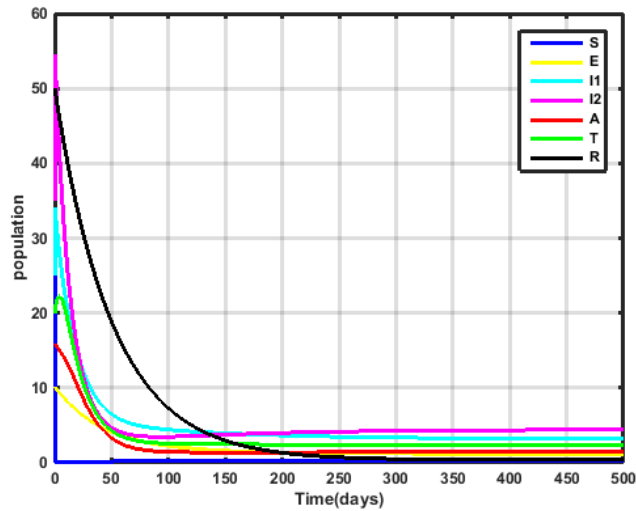


Figure 2. Numerical simulation of the endemic equilibrium point ($R_0 > 1$)

The simulation shows the endemic equilibrium point is asymptotically stable when $R_0 > 1$, and the numerical results support the analysis results. Based on the numerical simulation results above, it can be seen that over time the number of individuals infected with HIV with symptoms will go to 3.1928 and individuals infected with HIV without symptoms will go to 4.4043. Thus, it can be interpreted that individuals infected with HIV / AIDS will always exist, so that there will be the spread of HIV / AIDS infection in that environment.

Next, we simulate the stability of model solutions for the disease-free equilibrium point numerically. We choose the parameter values in order to satisfy the basic reproduction number $R_0 > 1$ as shown in Table 1 except the values $\beta_1 = 0.0023$, $\beta_2 = 0.0033$, and $\eta = 0.3$. We obtain $R_0 = 0.0069 < 1$. Figure 3 shows the solutions of HIV/AIDS model with initial values $NA = (30, 10, 25, 35, 20, 16, 50)$ lead to disease-free equilibrium point $K^0 = (1.5732, 24.0799, 0, 0, 0, 0, 2.4107)$ as in Figure 3.

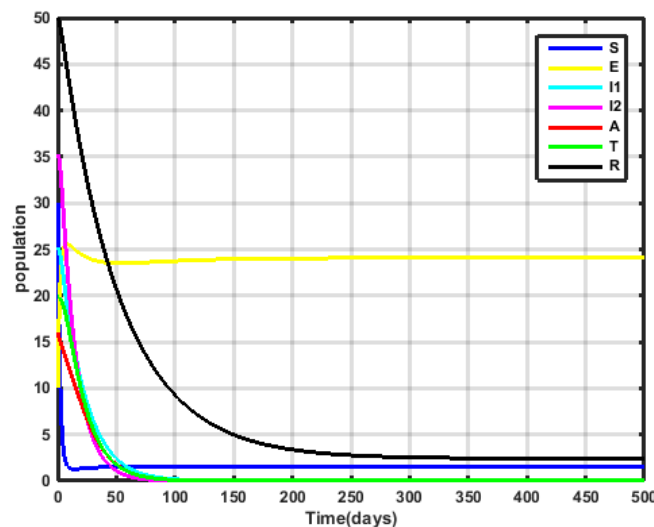


Figure 3. Numerical simulation for the disease-free equilibrium point, ($R_0 < 1$)

The numerical simulation results obtained support the results of the analysis in the previous section that if $\mathcal{R}_0 < 1$, then the HIV / AIDS disease-free equilibrium point K^0 , is asymptotically stable, which means that after quite a long time, the infected individual will vanish.

CONCLUSIONS

The mathematical model of HIV/AIDS with educated subpopulation have been established. The model consists of seven compartments (susceptible, educated, infected with and without treatment, AIDS, treatment, and recovered subpopulations). The infected subpopulation are in HIV-positive with consuming ARV I_1 such that this subpopulation can survive longer and an HIV-positive without consuming ARV I_2 . The susceptible subpopulation was divided into two, uneducated and educated susceptible subpopulations.

The stability analysis of HIV/AIDS model is determined according the basic reproduction number. The disease-free equilibrium is locally asymptotically stable when $R_0 < 1$ and unstable when $R_0 > 1$. The endemic equilibrium is locally asymptotically stable when $R_0 < 1$ and unstable otherwise. The endemic equilibrium is globally asymptotically stable when $R_0 > 1$ and unstable otherwise. Numerical simulation are performed using values of selected parameters to support the analytical results.

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