CAUCHY –Jurnal Matematika Murni dan Aplikasi
Volume 7(2) (2022), Pages 195-206
p-ISSN: 2086-0382; e-ISSN: 2477-3344

Richards Curve Implementation for Prediction of Covid-19 Spread in Maluku Province

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ABSTRACT

The first case of COVID-19 in Maluku Province, Indonesia was reported at the end of March 2020 as many as 1 case and the total cumulative cases reported were 3.884 cases on November 4, 2020. The purpose of this study is to predict the spread of COVID-19 cases in Maluku Province by estimating the Richards function parameters are I is the population size, K is carrying capacity, k is the growth rate, a is the scaling parameter and t_m is the turning point using the nonlinear least-squares (NLS) method. The method use in this research is Richards Curve method. The results of this research found the estimation results, with RMSE = 75,1057, the peak of the spread of COVID-19 cases in the Maluku province is predicted to occur on October 22, 2020, with a total of 3.623 cases and ends on May 25, 2023, with a total of 9.451 cases. This research can provide an overview of the results of predictions for the development of Covid-19 for the government, making it easier for the government to make decisions in the future.

Keywords: Carrying Capacity; COVID-19; Prediction; Richards Curve; Turning Point

INTRODUCTION

Coronavirus is a group of viruses from the subfamily Orthocronavirinae in the Coronaviridae family and the order Nidovirales. This group of viruses can cause disease in birds and mammals, including humans [1]. In 2002, the SARS-CoV coronavirus (SARS Coronavirus) caused Severe Acute Respiratory Syndrome (SARS) in Guangdong, China [2]. In 2012 the type of Coronavirus MERS-CoV (MERS Coronavirus) caused Middle Eastern Respiratory Syndrome (MERS) which occurred in Saudi Arabia and the Middle East [3].

In early 2020, WHO (World Health Organization) received a report from China that there were 44 patients with severe pneumonia in Wuhan City, Hubei Province, China [4]. Subsequent research showed a close relationship with the Coronavirus that caused SARS in 2002 [5]. On February 11, 2020, WHO inaugurated the term COVID-19 (Coronavirus Disease 2019) which is an infectious disease similar to influenza caused by Severe Acute Respiratory Syndrome 2 (SARS-CoV-2) [6], [7]. The first COVID-19 was reported in Indonesia on March 23, 2020, with two cases. Data on March 31, 2020, showed that there were 1,528 confirmed cases and 136 deaths.

Submitted: September 10, 2021 Reviewed: December 22, 2021 Accepted: January 05, 2022
DOI: http://dx.doi.org/10.18860/ca.v7i1.13323

In 1839 Verhulst introduced the Logistics Equation to model population growth which became known as the Verhulst equation and was rediscovered in 1912 [8], [9]. in 1959 in research entitled: A Flexible Growth Function For Empirical Use, Richards modified the Verhulst Equation and became known as the Richards Curve [10] or Generalized Logistic Function [11] because it is an extension of the Logistic Model [12], [13] and in some literature, the Richards Curve is also called the Theta Logistic Model [14], [15] with parameters namely K (carrying capacity), k (growth rate), t_m (inflection point) and a (scaling parameter) The shape of the Richards Curve resembles the shape of the Exponential Curve [16]. Richards Curve is a model of a population growth curve in conditions where growth is not symmetrical with inflexion points [17], [18].

In 2004 the Richards Curve was used to predict the spread of SARS in Singapore, Hong Kong and Beijing [19] After estimation with the Richards Curve, the results obtained are that the spread of SARS in Beijing is predicted to end on 27 June 2003 with a total of 2.595 cases, in Hong Kong it is predicted to end on 29 June 2003 with a total of 1.748 cases and in Singapore it is predicted to end in May 28, 2003 with a total of 207 cases. The prediction results of the spread of SARS in Singapore, Hong Kong and Beijing using the Richards Curve were considered quite successful, because based on the data obtained, Singapore last reported cases of SARS on May 18, 2003 with a total of 206 cases, Hong Kong on June 11, 2003 with a total of cases of 1.755 cases and Beijing on June 11, 2003 with a total of 2.631 cases. Besides that, the Richards Curve was widely used in other studies [20]–[22] and in 2020, the Richards Curve was used to predict the spread of COVID-19 in the province of South Sulawesi, Indonesia, with the peak of the spread predicted to occur in mid-June 2020 - July 2020 with a total of 10,000-12,000 cases and the end of the spread is predicted to occur at the end of November 2020.

Based on the above background, where the Richards Curve is considered quite good in predicting the spread of SARS in Singapore, Hong Kong and Beijing in 2002, therefore in this study the Richards Curve will be used to predict the spread of COVID-19 in Maluku province.

METHODS

In general, the differential form of the Richards Curve is : [10], [23]

$$I'(t) = \left(\frac{dI}{dt}\right) = rI\left[1 - \left(\frac{I}{K}\right)^{a}\right] \tag{1}$$

Where I is the population size, K is carrying capacity, k is the growth rate and a is the scaling parameter. To find a solution to equation 1, the integration technique can be written as:

$$\int \left(\frac{K^a}{I\left[K^a - I^a\right]}\right) dI = \int r\, dt$$

Or it can be written:

$$\int\left(\frac{A\left(K^a-I^a\right)+B\left(I\right)}{I\left(K^a-I^a\right)}\right)dI=\int r\,dt$$

Based on the similarity of the two sides, the values of $A=1$ and $B=I^{a-1}$ are obtained so as to obtain :

$$\int\left(\frac{K^a}{I\left[K^a-I^a\right]}\right)dI=\int\left(\frac{A}{I}+\frac{B}{K^a-I^a}\right)dI$$

$$=\int\left(\frac{1}{I}\right)dI+\int\left(\frac{I^{a-1}}{K^a-I^a}\right)dI$$

So we get:

$$\int\left(\frac{K^a}{I\left[K^a-I^a\right]}\right)dI=\left(\ln\left|I\right|-\left(\ln\left|\left(K^a-I^a\right)^{a^{-1}}\right|\right)\right)$$

Since we get $\int r\,dt=rt+C$, we can write: :

$$\left(\ln\left|I\right|\right)-\left(\ln\left|\left(K^a-I^a\right)^{a^{-1}}\right|\right)=rt+C$$

Or it can be written :

$$\left(\ln\left|\frac{\left(K^a-I^a\right)^{a^{-1}}}{I}\right|\right)=-rt-C$$

So we get:

$$\left(\frac{\left(K^a-I^a\right)^{a^{-1}}}{I}\right)=e^{-rt-C}$$

To simplify the above form, both sides can be raised to the power of a so that we get: :

$$I^a=\left(\frac{K^a}{1+\left(e^{-art}\right)\left(e^{-aC}\right)}\right) \qquad (2)$$

From equation 2, since a, r and C are constants, it is assumed that k is the product of ar and Q is the product of $e^{(-aC)}$, so it can be written as:

$$I^a = \left(\frac{K^a}{1+Q\left(e^{-kt}\right)} \right)$$

So we get:

$$I(t) = \left(\frac{K}{\left[1+Q\left(e^{-kt}\right)\right]^{\frac{1}{a}}} \right) \quad (3)$$

Since the inflection point of equation 3 is $\left(\frac{K}{(a+1)^{\frac{1}{a}}} \right)$ [24] , let t_m be the parameter of the inflection point of equation 3 then it can be written as : [25]

$$I(t) = \left(\frac{K}{\left[1+ae^{-k(t-t_m)}\right]^{\frac{1}{a}}} \right) \quad (4)$$

Where I is the population size or the total number of cases that occurred at the time of t, K is the carrying capacity or total of the latest cases, k is the rate of growth of cases, t_m is the inflexion point or time of the peak of the spread of COVID-19 cases where $I(t_m) = \left(\frac{K}{\left[1+ae^{-k(t_m-t_m)}\right]^{\frac{1}{a}}} \right) = \left(\frac{K}{[1+a]^{\frac{1}{a}}} \right)$.

RESULTS AND DISCUSSION

COVID-19 cases in Maluku province have continued to increase since it was first reported on March 23, 2020, and as of November 4, 2020, the total cumulative cases of COVID-19 in Maluku province were reported as many as 3,884 cases, including 551 positive patient cases or with a percentage of 14.18%, 3,286 cases of patients cured or with a percentage of 84.6 and 47 cases of patients dying or with a percentage of 1.2%. Cumulative case developments and the addition of daily cases of COVID-19 in Maluku province from March 23, 2020 – November 4, 2020, can be described as follows:

Table 1. Cumulative and daily case data of COVID-19 in Maluku province

Date	Cumulative Cases	Daily Cases
March 23, 2020	1	0
March 24, 2020	1	0
March 25, 2020	1	0
March 26, 2020	1	0

Date	Cumulative Cases	Daily Cases
March 27, 2020	1	0
...	...	...
October 30, 2020	3849	59
October 31, 2020	3851	2
November 1, 2020	3863	12
November 2, 2020	3863	0
November 3, 2020	3877	14
November 4, 2020	3884	7

The development of cumulative COVID-19 cases in Maluku province from 23 March – 4 November 2020 can be described as follows:

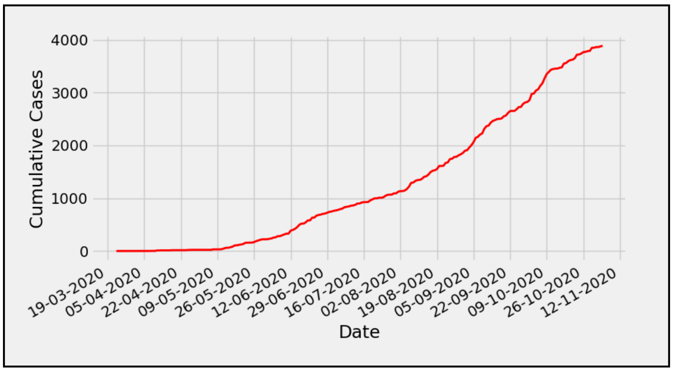

Figure 1. Cumulative case development

The first case of the spread of COVID-19 in Maluku province was reported on March 23, 2020, as many as 1 case and up to July 5, 2020 the total cumulative cases reported were 794 cases or with a growth rate of $755,23\%$. On July 6 to October 22, 2020 the average daily addition of cases increased to 26 cases with the average growth rate increasing significantly as much as $1864,953\%$ from the previous one, which was $2620,183\%$, and from October 23 to November 4, 2020, the average increase in cases The daily rate of COVID-19 in Maluku province decreased by 18 cases. The graph of the daily increase in cases can be seen in Figure 2, where the Maluku province experienced the highest number of cases on October 2, 2020, which was 117 cases.

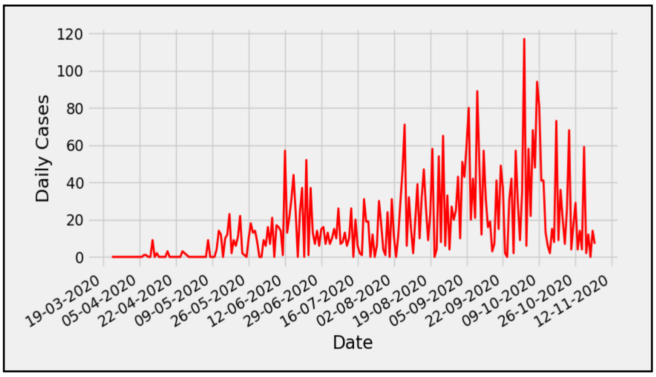

Figure 2. Development of daily cases of COVID-19 in Maluku province 23 March – 4 November 2020

Parameter Estimation Results

By using data on cumulative cases of COVID-19 in Maluku province from March 23 – November 4, 2020, an estimate was made with the Richards Function parameter using the nonlinear least square method in Python with the following script:

```
import scipy.optimize as optimize
from scipy.optimize import curve_fit
import numpy as np
import pandas as pd
def RichardsFunction(t,K,a,k,tm):
        return K/(1 + a*(np.exp(-k*(t-tm))))**(1/a)
df=pd.read_excel('CovidDate_Maluku.xlsx')
data=df[0:227]
y=data['CummulativeCases']
t=np.arange(1,228,1)
popt,pcov=optimize.curve_fit(RichardsFunction,t,y,bounds=(0.01,np.inf))
```

The results obtained are:

Table 2. Richards parameter estimation results (RMSE: 75,1057)

K	a	k	t_m
9.451,245	0,085	0,01	213,918

So by substituting the parameter values K, a, k and t_m in equation (4) obtained the Richards equation, namely :

$$I(t)=\left(\frac{9.451,245}{\left[1+0,085e^{-0.01(t-213,918)}\right]^{\frac{1}{0,0851}}}\right)$$

Then it can be illustrated that the comparison between the cumulative COVID- 19 case data from the Richards Function parameter estimation results with the actual data for $t=[1,227]$ is as follows:

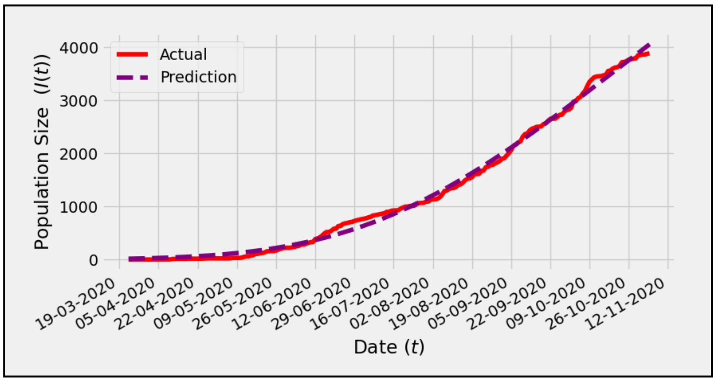

Figure 3. Comparison of the results of predictions of cumulative cases of covid-19 in Maluku province with actual data

In Figure 3. The cumulative comparison between the actual data and the predicted data using the Richards function at the time of t = 1 to t = 227, we get RMSE = 75.1057 while using the logistic function we get a larger RMSE value of 85.1813. The comparison of the error values between the predicted results and the actual data can be seen in the following Table 3:

Table 3. The error value of the predicted data with the actual data

t	Actual	Predict	Error
1	1	16.65457518205870	15.65457518205870
2	1	17.48844921400830	16.48844921400830
3	1	18.35884011574600	17.35884011574600
4	1	19.26706628665680	18.26706628665680
5	1	20.21448005719650	19.21448005719650
...	...	...	...
222	3849	3889.25714338151000	40.25714338151370
223	3851	3922.50525096264000	71.50525096263660
224	3863	3955.72658861666000	92.72658861666100
225	3863	3988.91835825547000	125.91835825547500
226	3877	4022.07779333937000	145.07779333936700
227	3884	4055.20215931066000	171.20215931065600

From Figure 3, the Richards Curve can be described from the estimation results as follows:

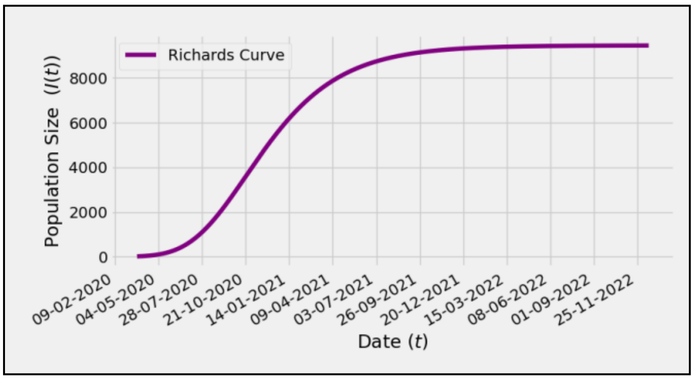

Figure 4. Richards Curve of estimation results

From Figure 4, suppose that $I(t_i)$ is the total cumulative cases on day i and $I(t_{i-1})$ is the total cumulative cases on day $i-1$, then the total addition of daily cases can be formulated as follows : [26]

$$J(t_i) = I(t_i) - I(t_{i-1}); i = 1, 2, 3, \ldots \quad (5)$$

So the comparison between the predicted data and actual data from daily COVID-19 cases in Maluku province can be described as follows:

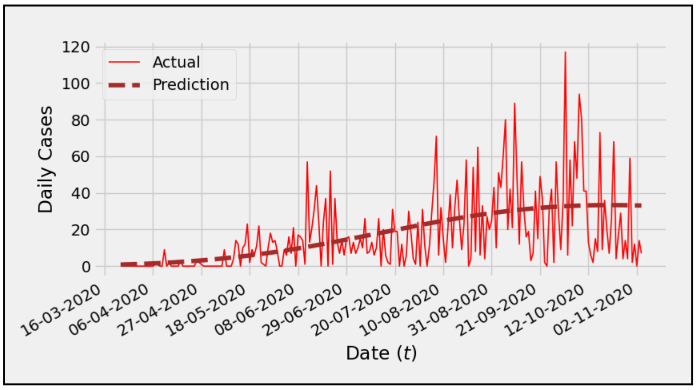

Figure 5. Comparison of the results of daily Covid-19 case predictions in Maluku province with actual data

From Figure 5, it can be seen that the results of daily case predictions for COVID- 19 in Maluku province are as follows:

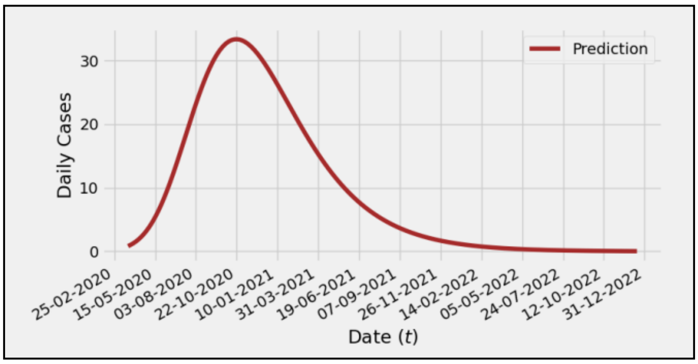

Figure 6. Daily cases of COVID-19 in Maluku province from estimated result

Turning Point of Case Deployment

From the results of Richards parameter estimation with data on COVID-19 cases in Maluku province, the parameter t_m value is 213.918, meaning that the time of the turning point for the spread of COVID-19 in Maluku province is predicted to occur on the 214th day, where the total cases on the 214th day are obtained. from the equation:

$$I(214) = \left(\frac{9.451,245}{\left[1+0,085e^{-0.01(214-213,918)}\right]^{\frac{1}{0,0851}}} \right)$$

which is 3.622,654 means that the total cases at the inflection point are 3.623 cases or can be described as follows:

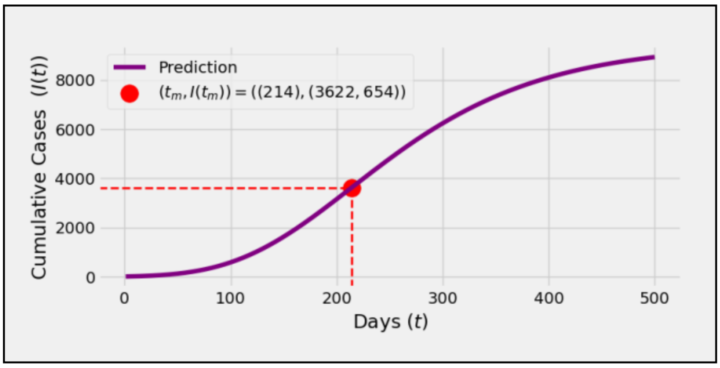

Figure 7. Inflection Point

For the addition of daily cases, the total addition of cases can be obtained at the inflexion point or when $t = t_m$ namely :

$$= I(214) - I(213)$$

$$= \left(\frac{9.451,245}{\left[1+0,085e^{-0.01(214-213,918)}\right]^{\frac{1}{0,0851}}} \right) - \left(\frac{9.451,245}{\left[1+0,085e^{-0.01(213-213,918)}\right]^{\frac{1}{0,0851}}} \right) = (33,358161)$$

So the total addition of daily cases at the inflection point is 33 cases, so it can be concluded that the turning point of the COVID-19 case in Maluku province is based on the estimation results, namely $(t, I(t)) = (214, 33)$ or can be described as follows :

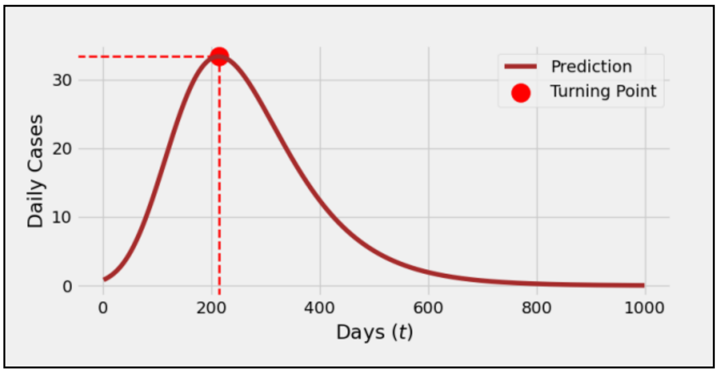

Figure 8. Turning Point

In figure 7, the point $(t, I(t)) = (214, 33)$ which is the turning point of the curve is also the peak of the curve, namely when t = 214.

End of Case Deployment

From Richards parameter estimation results with data on COVID-19 cases in Maluku province, the parameter K value is 9,451,245, meaning that the latest total cases for COVID-19 cases in Maluku province are predicted to be 9,451 cases. For example, if t_{end} is the end time of COVID-19 cases in Maluku province, with a total of 9,450.5 cases or can be written as $I(t_{end})$= 9.450,5 then the value of t_{end} can be obtained from the equation:

$$9.450,5 = \left(\frac{9.451,245}{\left[1+0,085e^{-0.01(t_{end}-213,918)}\right]^{\frac{1}{0,0851}}} \right)$$

That is t_{end} = 1.158,681 meaning that the time for the end of the COVID-19 case in Maluku province is predicted to occur on the 1.159th day.

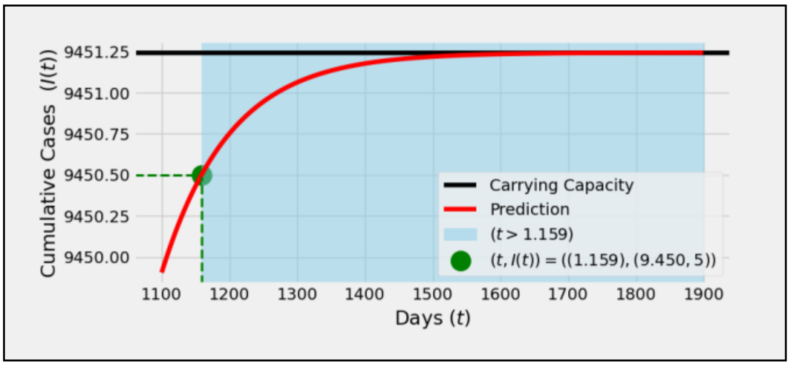

Figure 9. Total case when $t > 1.159$

From **Figure 9**, when $t > 1.159$ the population size will always be at number 1.159 and will only move towards the value of K or carrying capacity.

CONCLUSIONS

From the estimation results of the Richards function parameter with the cumulative case data of COVID-19 in the Maluku province, the Richards equation is obtained to predict the spread of COVID-19 in the Maluku province, namely:

$$I(t) = \left(\frac{9.451,245}{\left[1 + 0,085e^{-0.01(t-213,918)}\right]^{\frac{1}{0,0851}}} \right)$$

Where, the turning point or peak of the spread of COVID-19 in Maluku province is predicted to occur on October 22, 2020 with a total of 3.623 cases, while the time for the end of the spread of COVID-19 in Maluku province is predicted to occur on May 25, 2023 with 9.451 cases.

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