CAUCHY –Jurnal Matematika Murni dan Aplikasi
Volume 7(2) (2022), Pages 231-239
p-ISSN: 2086-0382; e-ISSN: 2477-3344

Average Based-FTS Markov Chain With Modifications to the Frequency Density Partition to Predict COVID-19 in Central Java

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ABSTRACT

Every day, new Covid-19 positive cases are discovered in Central Java. Many research has used various methodologies to try to forecast new positive instances. The fuzzy time series (FTS) approach is one of them. Many FTS are now in development, including the FTS Markov Chain. The duration of the gap in the FTS must be determined carefully because it will affect the FLR, which will be used to estimate the forecast value. The average-based method can be used to determine the optimum interval length; however, other research use frequency density partitioning to determine the optimal interval length in order to produce superior predicting values. The goal of this research is to improve the accuracy of forecasting values by modifying the frequency density partition on the average based-FTS Markov Chain. The approach utilized is average-based, with the length of the interval determined by the average, the forecast value determined by the FTS Markov Chain, and the frequency density partition modified to provide the ideal interval. The average-based FTS Markov Chain approach with adjustments to the frequency density partition achieves an accuracy rate of 89.3 percent, according to the findings of this study. Because changes to the frequency density partition can produce a good level of accuracy in forecasting new positive cases of Covid-19 in Central Java, it is hoped that this modification of the frequency density partition on the average-based FTS Markov chain can be used as a model for forecasting in fields other than new positive cases. Covid-19.

Keywords: Average Based; FTS Markov Chain; Modified frequency density partitioning; COVID19; MAPE

INTRODUCTION

COVID-19 first appeared in the city of Wuhan, Hubei Province, China, which spread almost all over the world, including Indonesia. At the beginning of 2020, Indonesia experienced a COVID-19 pandemic, which, to this day, new positive cases are still being found[1]. The government is still thinking about how to make Indonesia free from COVID-19.

Many experts have estimated the amount of new Covid-19 positive cases in Indonesia. Forecasting is the process of predicting what will happen in the future over a lengthy period of time[2]. However, no approach for accurately forecasting anything,

Submitted: September 18, 2021 Reviewed: December 08, 2021 Accepted: December 21, 2021
DOI: http://dx.doi.org/10.18860/ca.v7i1.13371

including new Covid-19 positive cases, has been developed to yet. The fuzzy time series (FTS) is one of the forecasting approaches for determining the number of new positive cases in Indonesia. The concept of fuzzy logic is used in the forecasting of FTS. Song and Chissom introduced the FTS in 1993[3], and it has since been widely developed, including the Markov method[4], Chen's method[5], Chen and Hsu's method[6], the weighted method[7], the multiple-attribute fuzzy time series method[8], the percentage change method[9], and Markov chain method[10].

Ruey Chyn Tsaur, developing fuzzy time series by merging the fuzzy time series method with the Markov chain concept [10]. Markov chain is a stochastic process in which future events only depend on today's events. Markov chain is used in the defuzzification process [11].

The determination of the length of the interval in the fuzzy time series does not have a definite formula, but the determination of the length of the interval in the fuzzy time series is based on the researcher. As a result, even if each researcher is utilizing the same data, the length of the interval will vary[3]. Even though the determination of the length of the interval is a very influential part in the formation of a fuzzy logical relationship (FLR).[12]. One method that can be used to determine the length of the interval is the average based. This average based uses an average-based method in determining the length of the interval [12].

Chen and Hsu in 2004 also developed a fuzzy time series. Chen and Hsu developed fuzzy time series by repartitioning based on frequency density. Chen and Hsu's frequency density repartitioning algorithm divides the interval with the highest frequency density into four sub-intervals, the interval with the second highest density into three sub-intervals, the interval with the third highest density into two sub-intervals, and the interval with the lowest density into one sub-interval.[6][13].

Based on the description above, the researcher is interested in modifying the frequency density partitioning algorithm used by Chen and Hsu, namely by exchanging the partition between the interval with the first densest frequency with the third densest interval, which was originally the first densest interval partitioned into 4 sub-intervals, the researcher changed the first densest was partitioned into 2 sub-intervals, and for the interval with the third densest frequency initially partitioned into 2 sub-intervals the researcher changed it to 4 sub-intervals.

Furthermore, the researcher will use the average-based method to determine the interval length in the fuzzy time series type Markov chain, and apply it to forecasting new positive cases of COVID-19 in Central Java.

METHODS

Fuzzy Time Series

The definition of fuzzy time series was first introduced by Song dan Chisom (1993). Let U *universe of discourse*, with $U = \{u_1, u_2, \dots, u_n\}$ on a fuzzy set A_i, defined as[3]:

$$A_i = \frac{f_A(u_1)}{u_1} + \frac{f_A(u_2)}{u_2} + \cdots + \frac{f_A(u_n)}{u_n} \quad (1)$$

where f_A is the membership of the fuzzy set A_i and u_k is an element of the fuzzy set A_i and $f_A(u_k)$ shows the degree of membership of u_k in A_i where $k = 1,2,3, \dots, n$.

Definition. If $F(t)$ is caused by $F(t-1)$, then the relation in the first order model $F(t)$ can be stated as follows: [5].

$$F(t) = F(t-1) \circ R(t, t-1) \quad (2)$$

where "○" is Max-Min composition operator, and $R(t, t-1)$ is a relation matrix to describe the fuzzy relationship between $F(t-1)$ dan $F(t)$.

Average-Based

Average based is an algorithm that can be used to set the interval length that is determined at the initial stage of forecasting when using fuzzy time series. The steps of the average based algorithm are as follows [12], [14]:

a. Determine the absolute difference (lag) between data $n+1$ and data n with the formula:

$$lagD_n = |(Data\ n+1) - (Data\ n)| \quad (3)$$

b. Determine the length of the interval

$$length\ of\ interval = \left(\frac{total\ lag}{numbers\ of\ data}\right) : 2 \quad (4)$$

c. Determine the basis value of the interval length according to Table 1. Following:

Table 1. Basis Mapping Table

Range	Basis
0,1 – 1,0	0,1
1,1 -10	1
11-100	10
101-1000	100

d. The length of the interval is then rounded up according to the interval basis table.

Modification Frequency Density Partition

In this study, modifications to the frequency density partition were used, the algorithm used is as follows:

a. The interval with the first densest frequency is divided into 2 subintervals.
b. The interval with the second densest frequency is divided into 3 subintervals.
c. The interval with the third densest frequency is divided into 4 subintervals
d. Eliminates intervals that have no frequency.

Fuzzy Time Series Markov Chain

Markov Chain's Fuzzy Time Series forecasting procedure is as follows[10]:

Step 1. Collecting historical data (Yt).

Step 2. Defines the U universe set of data, with D_1 and D_2 being the corresponding positive numbers.

$$U = [D_{min} - D1, D_{max} + D2] \quad (5)$$

Step 3. Specify the number of fuzzy intervals.

Step 4. Defining the fuzzy set in the universe of discourse U, the Fuzzy A_i set declares the linguistic variable of the share price by $1 \le i \le n$.

Step 5. Fuzzification of historical data. If a time series data is included in the u_i interval, then that data is fuzzification into A_i.

Step 6. Specifies fuzzy logical relationship (FLR) and Fuzzy Logical Relationships Group (FLRG).

Step 7. Calculate forecasting results

For time series data, using FLRG, a probability can be obtained from a state heading to the next state. In order to calculate the predicting value, a Markov probability transition matrix with a dimension of $n\ x\ n$ was used. If state A_i transition to a state A_j and pass the state $A_k, i, j = 1, 2, \ldots, n$, then we can obtain FLRG. The transition probability formula is as follows:

$$P_{ij} = \frac{M_{ij}}{M_i}, i, j = 1, 2, \ldots, n \quad (6)$$

with:
P_{ij} = probability of transition from state A_i to state A_j one step
M_{ij}= number of transitions from state A_i to state A_j one step
M_i = the amount of data included in the A_i
The probability matrix R of all states can be written as follows:

$$R = \begin{bmatrix} P_{11} & \cdots & P_{1n} \\ \vdots & \ddots & \vdots \\ P_{n1} & \cdots & P_{nn} \end{bmatrix} \quad (7)$$

Matrix R reflects the transition of the entire system. If $F(t-1) = A_i$, then the process will be defined in the A_i at the time of $(t-1)$, then the forecasting results $F(t)$ will be calculated using the $[P_{i1}, P_{i2}, \ldots, P_{in}]$ on the matrix R. Forecasting results $F(t)$ is the weighted average value of the m_1, m_2, ..., m_n (midpoint of u_1, u_2, ..., u_n). The forecasting output result value on $F(t)$ can be determined by using the following rules:
Rule 1: if fuzzy logical relationship group A_i is *one-to-one* (suppose $A_i \rightarrow A_k$ where $P_{ik} = 1$ and $P_{ij} = 0, j \neq k$) then the forecasting value of $F(t)$ is m_k the middle value of the u_k.

$$F(t) = m_k P_{ik} = m_k \quad (8)$$

Rule 2: if the FLRG A_i is one-to many (e.g. $A_j \rightarrow A_1, A_2, \ldots, A_n$. $j = 1, 2, \ldots, n$), when $Y(t-1)$ at time $(t-1)$ is included in state A_j then the forecasting $F(t)$, is:

$$F(t) = m_1 P_{j1} + m_2 P_{j2} + \ldots + m_{j-1} P_{j(j-1)} + Y(t-1) P_{jj} + m_{j+1} P_{j(j+1)} + \ldots + m_n P_{jn} \quad (9)$$

where $m_1, m_2, \ldots, m_{j-1}, m_{j+1}, \ldots, m_n$ is the middle value $u_1, u_2, \ldots, u_{j-1}, u_{j+1}, \ldots, u_n$, and $Y(t-1)$ are state values A_j at time $t-1$.
Rule 3: if the FLRG A_i is empty $(A_i \rightarrow \emptyset)$ forecast value $F(t)$ is m_i which is the middle value of u_i with the following equation:

$$F(t) = m_i \quad (10)$$

Step 8. Adjusting the trend of forecasting values with the following rules:

- If state A_i communicates with A_i, starting from state A_i at time $t-1$ expressed as $F(t-1) = A_i$, and undergoing an increasing transition to state A_j at the time t where $(i < j)$, then the adjustment value is:

  $$D_{t1} = \left(\frac{l}{2}\right) \quad (11)$$

  where l is the basis interval.
- If state A_i communicates with A_i, starting from state A_i at the time $t-1$ expressed as $F(t-1) = A_i$, and experiencing a decreasing transition to state A_j at the time t where $(i > j)$ the adjustment value is:

  $$D_{t1} = -\left(\frac{l}{2}\right) \quad (12)$$

- If state A_i at the time $t-1$ is expressed $F(t-1) = A_i$, and undergoes a jump forward transition to state A_{i+s} at the time t where $(1 \leq s \leq n-i)$ then the adjustment value is:
$$D_{t2} = (\frac{l}{2})s \tag{13}$$
where s is the number of forward jumps.
- If state A_i at the time $t-1$ is as $F(t-1) = A_i$, and undergoes a jump-backward transition to *state* A_{i-v} at the time t where $(1 \leq v \leq i)$ then the adjustment value is:
$$D_{t2} = -(\frac{l}{2})v \tag{14}$$
where v is the number of jumps backward.

Step 9. Determine the final forecast value based on the adjustment of the trend of the forecasting value

If FLRG A_i is one-to-many and state A_{i+1} can be accessed from state A_i where state A_i is related to A_i then the forecasting result becomes $'(t) = F(t) + D_{t1} + D_{t2} = F(t) + (\frac{l}{2}) + (\frac{l}{2})$. If FLRG A_i is one-too-many and state A_{i+1} can be accessed from A_i where state A_i is not related to A_i then the forecasting values becomes $F'(t) = F(t) + D_{t2} = F(t) + \left(\frac{l}{2}\right)$. If FLRG A_i is one to many and state A_{i-2} can be accessed from state A_i where A_i is not related to A_i then the forecasting result is $F'(t) = F(t) - D_{t2} = F(t) - (\frac{l}{2})\, x\, 2 = F(t) - l$. If v is jump step, the general form of the forecast is:
$$F'(t) = F(t) \pm D_{t1} \pm D_{t2} = F(t) \pm (\frac{l}{2}) \pm (\frac{l}{2})\, v. \tag{15}$$

Forecasting Error Measurement

The reliability of a forecast can be determined by looking at mean average percentage error (MAPE), this MAPE formulas[15]:
$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\frac{|Y(t) - F'(t)|}{Y(t)}\, x\, 100\% \tag{16}$$
with Y_t : actual data period t, F'_t: t period forecasting value, and n : the predictable amount of data.

RESULTS AND DISCUSSION

Forecasting with an average based-FTS Markov Chain with modified frequency density partitioning, the first step is to collect COVID-19 in the Central Java period June 25, 2021 until August 20, 2021 as a universe discourse (U). Next, determine the greatest value ($D_{max} = 5655$) and smallest values ($D_{min} = 1428$), and the value of D_1 = 8 and D_2 = 5, so it can be defined $U = [1428 - 8,\ 5655 + 5] = [1420,\ 5660]$. Then, calculate the absolute difference from historical data, the average absolute difference from 57 data points is 658,554, which is then divided by 2 to yield 329,277. The value of 329,277 is then determined using Table 1. The basis of the length of the interval is 100, so that U can be partitioned into the same interval length, namely $u_1, u_2, u_3, u_4, u_5, \dots, u_{39}, u_{40}, u_{41}, u_{42}, u_{43}$ successively the value for each interval is

Table 2. Universe Discourse of New Positive Cases of Covid-19

$u_1 = [1420, 1520]$	$\vdots$
$u_2 = [1520, 1620]$	$u_{41} = [5420, 5520]$
$u_3 = [1620, 1720]$	$u_{42} = [5520,5620]$
$u_4 = [1720, 1820]$	$u_{43} = [5620, 5720]$

The next step is to distribute the data to each interval and determine the frequency density, resulting in the densest interval, which is then repartitioned using a modified method. The following outcomes were achieved:

Table 3. Frequency Density and Repartition Interval

Interval	Frequency	Redivided interval
$u_{29} = [4220, 4320]$	5	$u_{29,1} = [4220, 4270]$, $u_{29,2} = [4270, 4320]$
$u_{28} = [4120, 4220]$	4	$u_{28,1} = [4120, 4153.33]$ $u_{28,2} = [4153.33, 4186.67]$ $u_{28,3} = [4186.67, 4220]$
$u_{16} = [2920, 3020]$	3	$u_{16,1} = [2920, 2945]$, $u_{16,2} = [2945, 2970]$, $u_{16,3} = [2970, 2995]$ $u_{16,3} = [2995, 3020]$
$u_{17} = [3020, 3120]$	3	$u_{17,1} = [3020, 3045]$, $u_{17,2} = [3045, 3070]$, $u_{17,3} = [3070, 3095]$, $u_{17,4} = [3095, 3120]$
$u_{19} = [3220, 3320]$	3	$u_{19,1} = [3220, 3245]$, $u_{19,2} = [3245, 3270]$, $u_{19,3} = [3270, 3295]$, $u_{19,4} = [3295, 3320]$
$u_{27} = [4020, 4120]$	3	$u_{27,1} = [4020, 4045]$, $u_{27,2} = [4045, 4070]$, $u_{27,3} = [4070, 4095]$, $u_{27,4} = [4095, 4120]$
$u_{32} = [4520, 4620]$	3	$u_{32,1} = [4520, 4545]$, $u_{32,2} = [4545, 4570]$, $u_{32,3} = [4570, 4590]$, $u_{32,4} = [4590, 4620]$
$u_{33} = [4620, 4720]$	3	$u_{33,1} = [4620, 4645]$, $u_{33,2} = [4645, 4670]$, $u_{33,3} = [4670, 4695]$, $u_{33,4} = [4695, 4720]$
$u_2, u_3, u_4, u_5, u_6, u_8, u_{11,}$ $u_{14}, u_{20}, u_{23}, u_{26}, u_{34}, u_{42}$	0	Removed

Next, look for the middle value (m_1) for each interval, we get:

Table 4. Middle Value

$\boldsymbol{u_i}$	$\boldsymbol{m_i}$	$\boldsymbol{u_i}$	$\boldsymbol{m_i}$
u_1	1470	$\vdots$	$\vdots$
u_7	2070	u_{39}	5270
u_9	2270	u_{40}	5370
u_{10}	2370	u_{41}	5470
u_{12}	2570	u_{43}	5670

Furthermore, defining fuzzy sets, fuzzy sets that can be formed from the universe conversation are 44 fuzzy sets. The fuzzy sets formed is as follows:

$$A_1 = \left\{ {}^{1}/_{u_1} + {}^{0,5}/_{u_7} + {}^{0}/_{u_9} + {}^{0}/_{u_{10}} + \cdots + {}^{0}/_{u_{40}} + {}^{0}/_{u_{41}} + {}^{0}/_{u_{43}} \right\}$$
$$A_2 = \left\{ {}^{0,5}/_{u_1} + {}^{1}/_{u_7} + {}^{0,5}/_{u_9} + {}^{0}/_{u_{10}} + \cdots + {}^{0}/_{u_{40}} + {}^{0}/_{u_{41}} + {}^{0}/_{u_{43}} \right\}$$
$$A_3 = \left\{ {}^{0}/_{u_1} + {}^{0,5}/_{u_7} + {}^{1}/_{u_9} + {}^{0,5}/_{u_{10}} + \cdots + {}^{0}/_{u_{40}} + {}^{0}/_{u_{41}} + {}^{0}/_{u_{43}} \right\}$$
$$\vdots$$
$$A_{49} = \left\{ {}^{0}/_{u_1} + {}^{0}/_{u_7} + {}^{0}/_{u_9} + {}^{0}/_{u_{10}} + \cdots + {}^{1}/_{u_{40}} + {}^{0,5}/_{u_{41}} + {}^{0}/_{u_{43}} \right\}$$
$$A_{50} = \left\{ {}^{0}/_{u_1} + {}^{0}/_{u_7} + {}^{0}/_{u_9} + {}^{0}/_{u_{10}} + \cdots + {}^{0,5}/_{u_{40}} + {}^{1}/_{u_{41}} + {}^{0,5}/_{u_{43}} \right\}$$
$$A_{51} = \left\{ {}^{0}/_{u_1} + {}^{0}/_{u_7} + {}^{0}/_{u_9} + {}^{0}/_{u_{10}} + \cdots + {}^{0}/_{u_{40}} + {}^{0,5}/_{u_{41}} + {}^{1}/_{u_{43}} \right\}$$

The next step is to perform fuzzification, the data from the fuzzification results are presented in the following table:

Table 5. Fuzzification Results

t	Actual Data	*fuzzy* Data	t	Actual Data	*fuzzy* Data
1	2311	A_3	⋮	⋮	⋮
2	2064	A_2	54	3263	A_{18}
3	3079	A_{14}	55	3078	A_{14}
4	2702	A_6	56	1428	A_1
5	2932	A_8	57	1432	A_1

The next step, determine the FLR and FLRG, as shown in Table 6. And Table 7:

Table 6. FLR

Data Order	FLR	Data Order	FLR
1-2	$A_3 \rightarrow A_2$	⋮	⋮
2-3	$A_2 \rightarrow A_{14}$	54-55	$A_{18} \rightarrow A_{14}$
3-4	$A_{14} \rightarrow A_6$	55-56	$A_{14} \rightarrow A_1$
4-5	$A_6 \rightarrow A_8$	56-57	$A_1 \rightarrow A_1$

Table 7. FLRG

Current State	*Next State*	*Current State*	*Next State*
A_1	$(1)A_1$	⋮	⋮
A_2	$(1)A_{14}$	A_{49}	$(1)A_{35}, (1)A_{42}$
A_3	$(1)A_2$	A_{50}	$(1)A_{48}$
A_4	$(1)A_6$	A_{51}	$(1)A_{43}$

The initial forecast will be calculated next. For example, for $t = 2$, June 26, 2021, the forecast computation based on formulas (8), (9), and (10) is $F(2) = m_k P_{ik} = m_k = 2070$. The summary of the initial forecasting results is as follows:

Table 8. Initial Forecasting Results ($F(t)$)

Period	Actual Data	$F(t)$	Period	Actual Data	$F(t)$
6/25/21	2311	Na	⋮	⋮	⋮
6/26/21	2064	2070	8/19/21	1428	2070
6/27/21	3078	3082,5	8/20/21	1432	1428

After we get the initial forecasting, the next step is to adjust the forecasting trend. For example, adjustment value for June 26, 2021, the next step is A_2 and the current state is A_3 then the adjustment calculation uses the forecast adjustment rule (14) we get $D_{t2} = -\left(\frac{l}{2}\right) v = -\left(\frac{100}{2}\right) 1 = -(50)$. Dor the calculation of other forecasting value adjustment using equations (11), (12), (13), and (14). The following is a forecast adjustment Table.

Table 8. Forecasting Trend Adjustment Value

Period	FLR	D_{tn}	Period	FLR	D_{t2}
6/25/21	$A_3 \rightarrow A_2$	Na	⋮	⋮	⋮
6/26/21	$A_2 \rightarrow A_{14}$	-50	8/19/21	$A_{14} \rightarrow A_1$	-650
6/27/21	$A_{14} \rightarrow A_9$	600	8/20/21	$A_1 \rightarrow A_1$	0

Calculate the final forecast value. The final forecast is the sum of the initial forecast value with the forecast adjustment value by following the equation (15). For example, the final forecast value for June 26, 2021 data is $F'_2 = F_2 \pm D_{t2} = 2070 + (-50) = 2020$. By doing the same way, the summary of the final forecasting result is as follows:

Table. 9. Final Forecast Value

Period	Y(t)	F'_t	Period	Y(t)	F'_t
6/25/21	2311	Na	⋮	⋮	⋮
6/26/21	2064	2020	8/19/21	1428	1907.5
6/27/21	3078	3682,5	8/20/21	1432	1428

Furthermore, the average based-FTS Markov chain is based on the modified frequency density partition to forecast data for new positive cases on August 21, 2021, the current state is A_1, from FLRG it is known that the next state of A_1 is A_1, then based on equations (8), (9), and (10) the forecasting result is 1 times the data of the previous new positive case $Y(t-1)$ is 1432.

The last step is to calculate the forecast accuracy value using MAPE. The MAPE values of average based-FTS based on a modified frequency density partitioning is 10,7%. For forecasting results using average based-FTS based on a modified frequency density partitioning, are presented in the following figure:

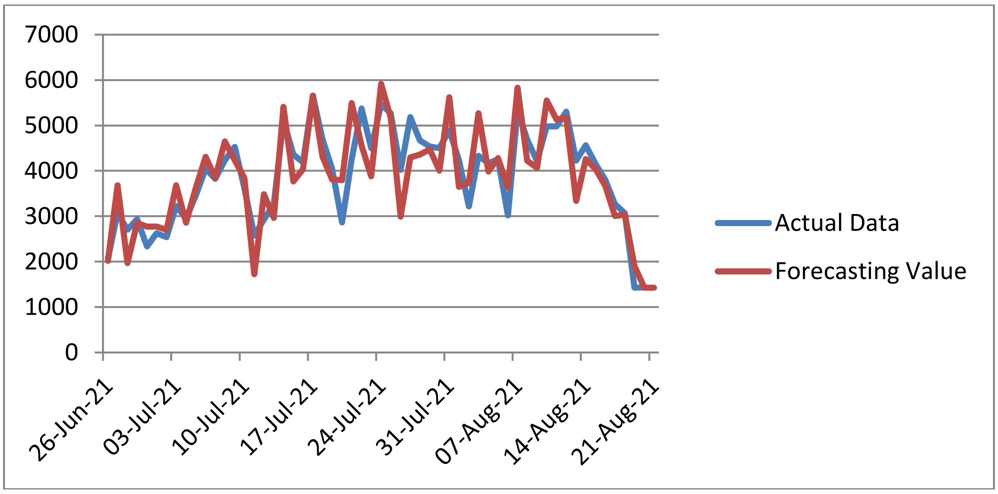

Figure 1. Graph of Forecasting Results Using Average Based-FTS Markov Chain Based on Modified Frequency Density Partitioning

CONCLUSIONS

Forecasting new positive cases of COVID-19 in Central Java using Average Based-FTS Markov Chain Based on Modified Frequency Density Partitioning has a good level of accuracy, this can be seen from the MAPE value obtained which is 10.7%. And for the predicted value of new positive cases on August 21, 2021, it is 1432 new positive cases of COVID-19 in Central Java.

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