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Elliptical Orbits Mode Application for Approximation of Fuel Volume Change

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ABSTRACT

At a 45.507.21 Candirejo Tuntang gas station, it is difficult to ensure the stock of fuel supplies because there is always a difference between calculations using dipsticks and fuel dispensers. Because the calculation method used by gas stations throughout Indonesia is linear interpolation which is not smooth, then by using the pertalite (pertamina fuel products) measuring book data a smooth volume change approximation function will be formed. This article presents the Elliptical Orbits Mode (EOM) as a proposed method in approximating the function that describes the volume change of fuel with respect to fuel height in Underground Tank (UT). Since the calculation by the gas station is not smooth, it is necessary for a smoother data fitting by considering Residual Square Error (RSS) and Mean Square Error (MSE). The results of the Elliptical Orbits Mode approximation will be compared with the circle orbits mode and least square data fitting. The result show that EOM(θ) method with elliptical height control produces smaller RSS and MSE compared to using COM, EOM, Least Square degree two and three. In next research, the approximation results will be applied to the fuel dispenser data.

Keywords: Orbits Mode; Data Fitting; Ellipse; Fuel; Approximation

INTRODUCTION

Based on the assumptions given in [1] and [2] the previous Orbits Mode Data Fitting research which was used to calibrate the Dipstick measuring instrument that converts the height to the volume of fuel in the buried tank, it is explained that the approximation function of the change in fuel volume in the tank is only based on height.

In [1] and [2] it is explained that the Orbits Mode Data Fitting-based calibration is limited by several assumptions and field conditions, including the following:

1. The resulting approximation function is the change in the volume of fuel in the UT which only depends on the variable height of the fuel in the UT.
2. Orbits Mode Data Fitting proposed by the author is used only for the distribution of data that forms a semicircle or ellipse in the first quadrant.
3. The data to be approximated is the fuel measurement manual in the UT from the Semarang Regency Metrology Agency.

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4. It is assumed that UT is not tilted or flat during measurement, including tank trucks that deliver fuel to gas stations or are filling supplies at gas stations.
5. UT for the right and left have the same shape alias symmetrical, due to the second assumption.

In [3] data fitting is applied to approximate the shape of an island on the map, then in [4] the curve fitting which is used to detect enlarged and shrinking eye retina, research in [3] and [4] has their additional algorithm to approach the desired result, which will also be applied to the Orbits Mode Data Fitting starting from the proposed method and the object applied to calibration is something new.

In [5] Hyper Least Square or HyperLS was also introduced and [6] also calibrated data in the form of curves but using an orthogonal matrix where the more data the more complicated, so the method will be difficult for large data.

From [7] there is a design drawing of a buried tank where the tank is in the form of a capsule tube with a cross section that is not flat or protruding so that according to [8] also, changes in volume in the tank tend to form a semicircle or half an ellipse or a parabola.

The approximation function used by gas stations throughout Indonesia is linear interpolation which is not smooth, then by using the pertalite (pertamina fuel products) measuring book data a smooth volume change approximation function with Elliptical Orbits Mode will be formed, and then will be any improvement on ellips height control to minimize Residual Sum of Square (RSS) and Mean Square Error (MSE), where the data used is the change in the volume of fuel in the tank based on changes in the height of the fuel in the UT in units (cm) and will be converted to fuel volume (liter).

Therefore, the author proposes method because the calculation is simpler for small and large data and is smoother, although only for data that tends to be semicircular or elliptical, to approximate the fuel volume with minimized errors.

Orbits Mode Data Fitting is a method proposed by the author in approximating the function of the data which tends to be in the form of a semi-circle or half an ellipse. In [1] and [2] the author introduced the Orbits Mode Data Fitting method only in a circle shape, then compared it with Cubic Spline Interpolation and Least Square Data Fitting, but this time the authors made the Orbits Mode Data Fitting method in the shape of an ellipse too, because in the value approach there is a volume of fuel which has not been detected in the function.

Definition 1 (Ellipse Equation)

In [9] the ellipse equation is presented in equation (1) which (p, q) is the center point of the ellipse with the major and minor axes adjusting a and b,

$$\frac{(x-p)^2}{a^2} + \frac{(y-q)^2}{b^2} = 1 \tag{1}$$

Definition 2 (A_i set for Ellipse Mode)

A set A_i of points formed from two ellipse equations is defined as follows,

$$A_i = \left\{(x, y) \mid d_{i1} < \left(\frac{x_{\frac{W}{2}}}{\left(\frac{W}{2}\right)}\right)^2 + \left(\frac{y}{\left(\frac{L}{2}\right)}\right)^2 < d_{i2}\right\}, \tag{2}$$

with $i = 1,2, \dots, k$. the set (2) it can be visualized as follows,

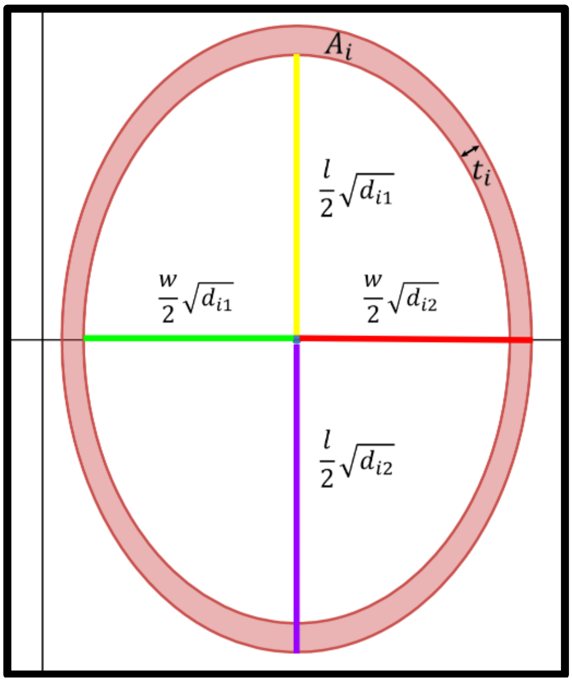

Figure 1. Set Visualization A_i (2)

with $x_{\frac{w}{2}} = \left(x - \frac{w}{2}\right)$ and $w =$ height of UT and $l =$ a half of maximum fuel change in UT

Defined $\frac{w}{2}\left(\sqrt{d_{i2}} - \sqrt{d_{i1}}\right) = \frac{l}{2}\left(\sqrt{d_{i2}} - \sqrt{d_{i1}}\right) = t_i$ the thickness of the ellipse from the partition interval taken from the maximum and minimum values of the volume change $[\Delta V(h)_{min}, \Delta V(h)_{max}]$ will be divided by several partitions where the thickness with the most points is sought, then

$$[\Delta V(h)_{min}, \Delta V(h)_{max}] = [d_{11}, d_{12}] \cup [d_{21}, d_{22}] \cup [d_{31}, d_{32}] \cup \ldots \cup [d_{k1}, d_{k2}] \qquad (3)$$

with $d_{i2} = d_{(i+1)1}$ and the intersection of the respective sub-blankets of the minimum and maximum volume intervals in equation (3) is denoted for each $[d_{i1}, d_{i2}] \cap [d_{(i+1)1}, d_{(i+1)2}]$ is equal to d_{i2} or $d_{(i+1)1}$, where $i = 1,2,\ldots,k$ with k is the number of blankets dividing the maximum and minimum intervals of the volume change.

Definition 3 (Partition of A_i set)

Partition of A_i set that divide A_i set to become partitions or sets of points between 2 ellipses equations, defined as follows,

$$\begin{aligned}
A_1 &= \left\{(x,y) \mid d_{11} < \left(\frac{x_{\frac{w}{2}}}{\left(\frac{w}{2}\right)}\right)^2 + \left(\frac{y}{\left(\frac{l}{2}\right)}\right)^2 < d_{12}\right\} \\
A_2 &= \left\{(x,y) \mid d_{21} < \left(\frac{x_{\frac{w}{2}}}{\left(\frac{w}{2}\right)}\right)^2 + \left(\frac{y}{\left(\frac{l}{2}\right)}\right)^2 < d_{22}\right\} \\
A_3 &= \left\{(x,y) \mid d_{31} < \left(\frac{x_{\frac{w}{2}}}{\left(\frac{w}{2}\right)}\right)^2 + \left(\frac{y}{\left(\frac{l}{2}\right)}\right)^2 < d_{32}\right\} \\
&\;\;\ldots
\end{aligned} \qquad (4)$$

$$A_k = \left\{ (x,y) \mid d_{k1} < \left(\frac{x_{\frac{w}{2}}}{\left(\frac{w}{2}\right)} \right)^2 + \left(\frac{y}{\left(\frac{l}{2}\right)} \right)^2 < d_{k2} \right\}$$

set of partitions (4) can be described as follows,

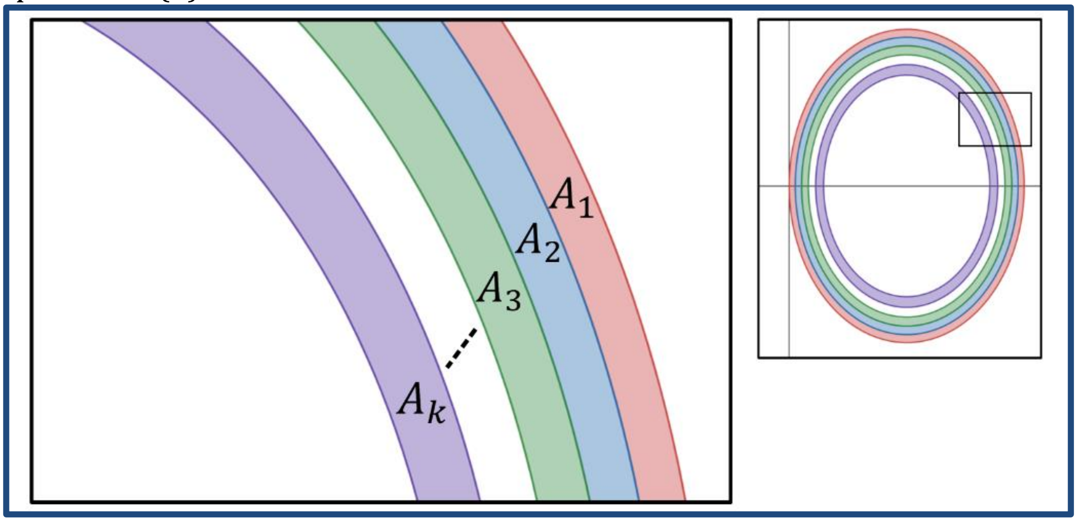

Figure 2. The visualization of partitions set A_1, A_2, A_3 up to A_k (4)

Definition 4 (Elliptical Orbits Mode)

Elliptical Orbits Mode was choose partition has the most points, defined as follows:

$$Max\big(n(A_i)\big) = Max\big(n(A_1), n(A_2), n(A_3), \dots, n(A_k)\big) \quad (5)$$

with $i = 1,2,\dots,k$. If there is a condition where $Max\big(n(A_i)\big) = n\big(A_{k_1}\big) = \cdots = n\big(A_{k_m}\big)$, then the average ellipses scale $A_{k_1}, A_{k_2}, \dots, A_{k_m}$ is taken so $Max\big(n(A_i)\big) = n(\overline{A_m})$, Therefore, the inequality whose ellipse will change is defined $\overline{A_m}$ as follows,

$$\overline{A_m} = \left\{ (x,y) \mid \left(\frac{d_{k_1 1} + \cdots + d_{k_m 1}}{m} \right) < \left(\frac{x_{\frac{w}{2}}}{\left(\frac{w}{2}\right)} \right)^2 + \left(\frac{y}{\left(\frac{l}{2}\right)} \right)^2 < \left(\frac{d_{k_1 2} + \cdots + d_{k_m 2}}{m} \right) \right\}$$

The next step, because we have obtained A_i or $\overline{A_m}$, then we approximate ellipse equation, divided which can be devide into two cases:

Case 1, $\left[Max\big(n(A_i)\big) = n(A_m) \right]$

Based on the set with the maximum number of points between 2 ellipses, choose $A_m = \left\{ (x,y) \mid d_{m1} < \left(\frac{x_{\frac{w}{2}}}{\left(\frac{w}{2}\right)} \right)^2 + \left(\frac{y}{\left(\frac{l}{2}\right)} \right)^2 < d_{m2} \right\}$, so that we get:

$$d_{m1} < \left(\frac{x_{\frac{w}{2}}}{\left(\frac{w}{2}\right)} \right)^2 + \left(\frac{y}{\left(\frac{l}{2}\right)} \right)^2 < d_{m2}$$

$$\Rightarrow \left(\frac{x_{\frac{w}{2}}}{\left(\frac{w}{2}\right)} \right)^2 + \left(\frac{y}{\left(\frac{l}{2}\right)} \right)^2 = \left(\frac{d_{m1} + d_{m2}}{2} \right) \quad (6)$$

$$\Rightarrow y = \left(\frac{l}{2}\right) \sqrt{\left(\frac{d_{m1} + d_{m2}}{2} \right) - \left(\frac{x_{\frac{w}{2}}}{\left(\frac{w}{2}\right)} \right)^2}$$

The steps in (6) can be visualized as follows,

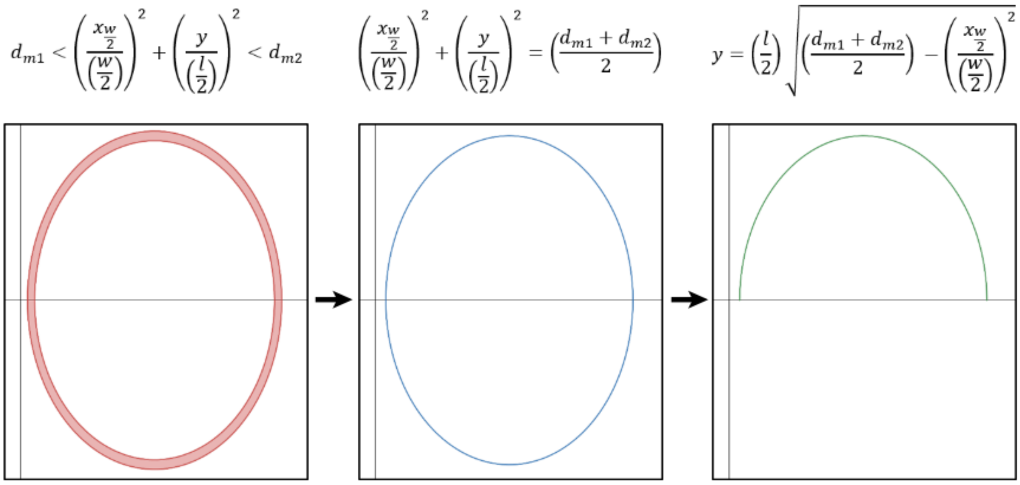

Figure 3. Visualization Steps in (6)

Therefore, from (6) the result of the elliptical orbital mode is a semicircular function by substituting $x_{\frac{w}{2}} = x_{d_m}$, as follows:

$$y = \left(\frac{l}{2}\right)\sqrt{\left(\frac{d_{m1}+d_{m2}}{2}\right) - \left(\frac{x_{d_m}}{\left(\frac{w}{2}\right)}\right)^2} \tag{7}$$

with $x =$ the fuel level in UT and $x_{d_m} = x - \left(\frac{w}{2}\right)\sqrt{\frac{d_{m1}+d_{m2}}{2}}$.

Theorem 1 (Defined Intervals for Elliptical Orbits Mode Approximation Function)

If $x_{\frac{w}{2}} = x_{d_m}$ is substituted to $y = \left(\frac{l}{2}\right)\sqrt{\left(\frac{d_{m1}+d_{m2}}{2}\right) - \left(\frac{x_{\frac{w}{2}}}{\left(\frac{w}{2}\right)}\right)^2}$ then y is defined in $\mathbb{R}$ in the interval $0 \le x \le \frac{w}{2} - \left(\frac{w}{2}\right)\sqrt{\frac{d_{m1}+d_{m2}}{2}}$.

Proof:

Substitute $x_{\frac{w}{2}} = x_{d_m}$ to $y = \left(\frac{l}{2}\right)\sqrt{\left(\frac{d_{m1}+d_{m2}}{2}\right) - \left(\frac{x_{\frac{w}{2}}}{\left(\frac{w}{2}\right)}\right)^2}$ so that it is obtained:

$$y = \left(\frac{l}{2}\right)\sqrt{\left(\frac{d_{m1}+d_{m2}}{2}\right) - \left(\frac{x_{d_m}}{\left(\frac{w}{2}\right)}\right)^2} \Leftrightarrow y = \left(\frac{l}{2}\right)\sqrt{\left(\frac{d_{m1}+d_{m2}}{2}\right) - \left(\frac{x - \left(\frac{w}{2}\right)\sqrt{\frac{d_{m1}+d_{m2}}{2}}}{\left(\frac{w}{2}\right)}\right)^2}$$

$$\Leftrightarrow y = \left(\frac{l}{2}\right)\sqrt{\left(\frac{d_{m1}+d_{m2}}{2}\right) - \left(\frac{x^2 - wx\sqrt{\frac{d_{m1}+d_{m2}}{2}} + \left(\frac{w}{2}\right)^2\left(\frac{d_{m1}+d_{m2}}{2}\right)}{\left(\frac{w}{2}\right)^2}\right)}$$

$$\Leftrightarrow y = \left(\frac{l}{2}\right)\sqrt{\left(\frac{d_{m1}+d_{m2}}{2}\right) - \left(\frac{d_{m1}+d_{m2}}{2}\right) - \left(\frac{x^2 - wx\sqrt{\frac{d_{m1}+d_{m2}}{2}}}{\left(\frac{w}{2}\right)^2}\right)}$$

$$\Leftrightarrow y = \left(\frac{l}{2}\right)\sqrt{\frac{x\left(w\sqrt{\frac{d_{m1}+d_{m2}}{2}} - x\right)}{\left(\frac{w}{2}\right)^2}}$$

$$\Leftrightarrow y = \left(\frac{l}{w}\right)\sqrt{x\left(w\sqrt{\frac{d_{m1}+d_{m2}}{2}} - x\right)}$$

with $w, d_{m1}, d_{m2} \in \mathbb{R}^+$. Obviously y is defined in $\mathbb{R}$ since $x\left(w\sqrt{\frac{d_{m1}+d_{m2}}{2}} - x\right) \geq 0$, then x must be on both interval $0 \leq x \leq w\sqrt{\frac{d_{m1}+d_{m2}}{2}}$ and $0 \leq x \leq \frac{w}{2} - \left(\frac{w}{2}\right)\sqrt{\frac{d_{m1}+d_{m2}}{2}}$, because $w\sqrt{\frac{d_{m1}+d_{m2}}{2}} > \frac{w}{2} - \left(\frac{w}{2}\right)\sqrt{\frac{d_{m1}+d_{m2}}{2}}$. ■

Substitute $x_{\frac{w}{2}} = x_{d_m}$ so that the value y is defined in $0 \leq x \leq \frac{w}{2} - \left(\frac{w}{2}\right)\sqrt{\frac{d_{m1}+d_{m2}}{2}}$, so that (7) the function of volume change to fuel level in the UT can be visualized as follows,

$$y = \left(\frac{l}{2}\right)\sqrt{\left(\frac{d_{m1}+d_{m2}}{2}\right) - \left(\frac{x_{\frac{w}{2}}}{\left(\frac{w}{2}\right)}\right)^2} \qquad y = \left(\frac{l}{2}\right)\sqrt{\left(\frac{d_{m1}+d_{m2}}{2}\right) - \left(\frac{x_{d_m}}{\left(\frac{w}{2}\right)}\right)^2}$$

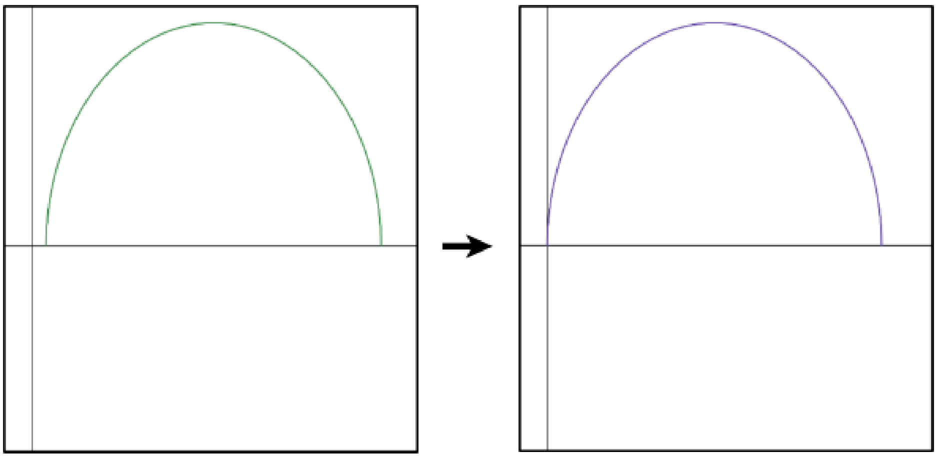

Figure 4. Visualization of Steps in (6) to (7)

The function is formed from the half ellipse selected for the calibration of the buried tank whose cross-sectional area is circular but convex, so that the volume of UT will be calculated as a function of the change in volume with respect to height, which then the coordinates are taken from the change in units (cm) that will be converted to volume per centimeter (liter/cm).

Case 2, $\left[\boldsymbol{Max}\left(\boldsymbol{n}(\boldsymbol{A_i})\right) = \boldsymbol{n}\left(\boldsymbol{A_{k_1}}\right) = \cdots = \boldsymbol{n}\left(\boldsymbol{A_{k_m}}\right)\right]$

Based on the set with the maximum number of points between two ellipses equations, $\overline{A_m} = \left\{(x,y) \mid \left(\frac{d_{k_1 1}+\cdots+d_{k_m 1}}{m}\right) < \left(\frac{x_{\frac{w}{2}}}{\left(\frac{w}{2}\right)}\right)^2 + \left(\frac{y}{\left(\frac{l}{2}\right)}\right)^2 < \left(\frac{d_{k_1 2}+\cdots+d_{k_m 2}}{m}\right)\right\}$, then:

$$\left(\frac{d_{k_1 1}+\cdots+d_{k_m 1}}{m}\right) < \left(\frac{x_{\frac{w}{2}}}{\left(\frac{w}{2}\right)}\right)^2 + \left(\frac{y}{\left(\frac{l}{2}\right)}\right)^2 < \left(\frac{d_{k_1 2}+\cdots+d_{k_m 2}}{m}\right)$$

$$\Rightarrow \left(\frac{x_{\frac{w}{2}}}{\left(\frac{w}{2}\right)}\right)^2 + \left(\frac{y}{\left(\frac{l}{2}\right)}\right)^2 = \left(\frac{\left(\frac{d_{k_1 1}+\cdots+d_{k_m 1}}{m}\right) + \left(\frac{d_{k_1 2}+\cdots+d_{k_m 2}}{m}\right)}{2}\right) \quad (8)$$

$$\Rightarrow \left(\frac{x_{\frac{w}{2}}}{\left(\frac{w}{2}\right)}\right)^2 + \left(\frac{y}{\left(\frac{l}{2}\right)}\right)^2 = \left(\frac{d_{m1}+d_{m2}}{2}\right).$$

The steps in (8) are visualized similarly in Figure 11 with $d_{m1} = \left(\frac{d_{k_1 1}+\cdots+d_{k_m 1}}{m}\right)$ and $d_{m2} = \left(\frac{d_{k_1 2}+\cdots+d_{k_m 2}}{m}\right)$, then

$$\Rightarrow y = \left(\frac{l}{2}\right)\sqrt{\left(\frac{d_{m1}+d_{m2}}{2}\right) - \left(\frac{x_{\frac{w}{2}}}{\left(\frac{w}{2}\right)}\right)^2}. \quad (9)$$

Therefore, the result of the orbital mode ellipse is a half-ellipse function, as follows:

$$y = \left(\frac{l}{2}\right)\sqrt{\left(\frac{d_{m1}+d_{m2}}{2}\right) - \left(\frac{x_{d_m}}{\left(\frac{w}{2}\right)}\right)^2}, \quad (10)$$

with $x =$ is the fuel level in UT and $x_{d_m} = x - \left(\frac{w}{2}\right)\sqrt{\frac{d_{m1}+d_{m2}}{2}}$.

As stated Theorem 1 about defined interval for (10) substituted $x_{\frac{w}{2}} = x_{d_m}$ so that y is defined values since $0 \le x \le \frac{w}{2} - \left(\frac{w}{2}\right)\sqrt{\frac{d_{m1}+d_{m2}}{2}}$, we get (10) the function of the change in volume to the fuel level in the UT which is visualized similarly in Figure 4.

METHODS

Research Steps

1. Construction of mathematical model Elliptical Orbits Mode methods with the following steps:
   a) Construction of mathematical model Elliptical Orbits Mode.
      - Review for Ellipse Equation
      - Define A_i set for Ellipse Mode
      - Make Partition for A_i set (Definition)
      - Choose Partitions of set A_i with $n(A_i)$ is the maximum value
      - Divide onto two cases singular and plural maximum value

- Create function $y = f(x)$ from chosen A_i sets
- Translate $y = f(x)$ so that y is defined on 0 and so on, and
- controlling height to find ellipse's height or vertical axis that minimized Residual Sum of Square (RSS) and Mean Square Error (MSE).

2. Construction of mathematical model Elliptical Orbits Mode will be applied on Data from Candirejo Gas Station, measuring book from Government Metrology Agency specific for Pertalite (Fuel Product of Pertamina) only and visualize it.
3. Measuring Performance using RSS [10] and MSE [11,12] and compare with Circle Orbits Mode from [1], Least Square with n = 2 and n = 3 from [13], and Elliptical with Height Control.

RESULTS AND DISCUSSION

Pertalite Measuring Book Data

A calculation of gas station 45.507.21 Candirejo using a measuring book from Government Metrology Agency to determine the volume of fuel in the buried tank, so the authors are just obtained the following data which is not proceed by authors, we need this data from Metrologi Agency as constructor of Approximation function, as follows:

Table 1. Fuel Volume Measuring Book Data for Pertalite Tanks from Metrology Agency

Height (x)	Volume	Diff (y)	Height (x)	Volume	Diff (y)	Height (x)	Volume	Diff (y)
0	0.0	0.0	75	7085.9	117.7	150	16124.4	111.1
1	237.1	237.1	76	7203.5	117.6	151	16235.6	111.2
2	294.3	57.2	77	7321.2	117.7	152	16346.7	111.1
3	351.4	57.1	78	7438.8	117.6	153	16457.8	111.1
4	409.4	58.0	79	7556.5	117.7	154	16568.9	111.1
5	468.2	58.8	80	7674.1	117.6	155	16680.0	111.1
6	527.1	58.9	81	7791.8	117.7	156	16791.1	111.1
7	586.1	59.0	82	7909.4	117.6	157	16902.2	111.1
8	646.7	60.6	83	8027.1	117.7	158	17013.3	111.1
9	707.3	60.6	84	8144.7	117.6	159	17124.4	111.1
10	767.9	60.6	85	8262.4	117.7	160	17232.6	108.2
11	831.6	63.7	86	8380.0	117.6	161	17337.9	105.3
12	896.1	64.5	87	8497.6	117.6	162	17443.2	105.3
13	960.6	64.5	88	8615.3	117.7	163	17548.4	105.2
14	1026.7	66.1	89	8732.9	117.6	164	17653.7	105.3
15	1093.3	66.6	90	8855.0	122.1	165	17758.9	105.2
16	1160.0	66.7	91	8980.0	125.0	166	17864.2	105.3
17	1228.3	68.3	92	9105.0	125.0	167	17969.5	105.3
18	1297,2	68.9	93	9230.0	125.0	168	18065.7	96.2
19	1366.2	69.0	94	9355.0	125.0	169	18161.0	95.3
20	1439.3	73.1	95	9480.0	125.0	170	18256.2	95.2
21	1513.3	74.0	96	9605.0	125.0	171	18351.4	95.2
22	1588,0	74.7	97	9730.0	125.0	172	18446.7	95.3
23	1668.0	80.0	98	9855.0	125.0	173	18541.9	95.2

Height (x)	Volume	Diff (y)	Height (x)	Volume	Diff (y)	Height (x)	Volume	Diff (y)
24	1748.0	80.0	99	9980.0	125.0	174	18637.1	95.2
25	1830.0	82.0	100	10105.0	125.0	175	18732.4	95.3
26	1913,3	83.3	101	10230.0	125.0	176	18825.5	93.1
27	1997,4	84.1	102	10355.0	125.0	177	18916.4	90.9
28	2084,3	86.9	103	10480.0	125.0	178	19007.3	90.9
29	2171.3	87.0	104	10605.0	125.0	179	19098.2	90.9
30	2261.8	90.5	105	10730.0	125.0	180	19189.1	90.9
31	2352.7	90.9	106	10855.0	125.0	181	19280.0	90.9
32	2446.7	94.0	107	10980.0	125.0	182	19370.9	90.9
33	2541.9	95.2	108	11105.0	125.0	183	19458.3	87.4
34	2637.1	95.2	109	11230.0	125.0	184	19545.2	86.9
35	2732.4	95.3	110	11355.0	125.0	185	19632.2	87.0
36	2830.0	97.6	111	11480.0	125.0	186	19719.1	86.9
37	2930.0	100.0	112	11605.0	125.0	187	19804.0	84.9
38	3030.0	100.0	113	11730.0	125.0	188	19884.0	80.0
39	3130.0	100.0	114	11855.0	125.0	189	19964.0	80.0
40	3230.0	100.0	115	11980.0	125.0	190	20044.0	80.0
41	3330.0	100.0	116	12105.0	125.0	191	20124.0	80.0
42	3430.0	100.0	117	12230.0	125.0	192	20202.2	78.2
43	3530.0	100.0	118	12355.0	125.0	193	20276.3	74.1
44	3630.0	100.0	119	12480.0	125.0	194	20350.4	74.1
45	3730.0	100.0	120	12605.0	125.0	195	20420.0	69.6
46	3832.6	102.6	121	12730.0	125.0	196	20486.7	66.7
47	3937.9	105.3	122	12855.0	125.0	197	20553.3	66.6
48	4043,2	105.3	123	12980.0	125.0	198	20617.5	64.2
49	4148.4	105.2	124	13105.0	125.0	199	20680.0	62.5
50	4257.8	109.4	125	13227.1	122.1	200	20742.5	62.5
51	4368.9	111.1	126	13344.7	117.6	201	20802.9	60.4
52	4480.0	111.1	127	13462.4	117.7	202	20860.0	57.1
53	4591.1	111.1	128	13580.0	117.6	203	20917.1	57.1
54	4702.2	111.1	129	13697.6	117.6	204	20974.3	57.2
55	4813.3	111.1	130	13815.3	117.7	205	21028.6	54.3
56	4924.4	111.1	131	13932.9	117.6	206	21082.7	54.1
57	5035,6	111.2	132	14050.6	117.7	207	21136.8	54.1
58	5146.7	111.1	133	14168,2	117.6	208	21187.1	50.3
59	5257.8	111.1	134	14285.9	117.7	209	21222.9	35.8
60	5368.9	111.1	135	14403.5	117.6	210	21258.6	35.7
61	5480.0	111.1	136	14521.2	117.7	211	21294.3	35.7
62	5591.1	111.1	137	14638.8	117.6	212	21330.0	35.7
63	5702.2	111.1	138	14756.5	117.7	213	21365,7	35.7
64	5813.3	111.1	139	14874.1	117.6	214	21387.1	21.4
65	5924.4	111.1	140	14991.8	117.7	215	21399.1	12.0
66	6035.6	111.2	141	15109.4	117.6	216	21411.0	11.9
67	6146.7	111.1	142	15227.1	117.7	217	21422.9	11.9
68	6262.4	115.7	143	15344.7	117.6	218	21434.8	11.9

69	6380.0	117.6	144	15457.8	113.1	219	21446.7	11.9
70	6497.6	117.6	145	15568.9	111.1	220	21458.6	11.9
71	6615.3	117.7	146	15680.0	111.1	221	21470.5	11.9
72	6732.9	117.6	147	15791.1	111.1	222	21482.4	11.9
73	6850.6	117.7	148	15902.2	111.1	222.3	21486.0	3.6
74	6968.2	117.6	149	16013.3	111.1			

Approximation using Elliptical Orbits Mode (EOM)

Table 1 is used as sample data; we derive Elliptical Orbits Mode approach as an approximation to the change of fuel volume. To obtain a half-ellipse function from the smallest to the largest abscissa, choose,

$$\frac{w}{2}=\frac{\text{maximum height on underground tank}}{2}=\frac{222.3}{2}=111.15$$

$$\frac{l}{2}=\text{maximum volume change on undergorund tank}=125$$

After that, select the difference d_{i1} and d_{i2} for the prefix of the A_i set which is $t_i = 0.2$ defined as follows,

$$A_i=\left\{(x,y)\ |0.8<\frac{(x-111.15)^2}{(111.15)^2}+\frac{y^2}{(125)^2}<1\right\}$$

with step-size = 0.02 then the number of partitions is obtained $\frac{t_i}{step-size}=10$, so that the partitions are obtained from the $A_i$set with $i = 1,2,\dots,10$,

$$A_1=\left\{(x,y)\ |\ 0.98<\frac{(x-111.15)^2}{(111.15)^2}+\frac{y^2}{(125)^2}<1\right\}$$
$$A_2=\left\{(x,y)\ |\ 0.96<\frac{(x-111.15)^2}{(111.15)^2}+\frac{y^2}{(125)^2}<0.98\right\}$$
$$A_3=\left\{(x,y)\ |\ 0.94<\frac{(x-111.15)^2}{(111.15)^2}+\frac{y^2}{(125)^2}<0.96\right\}$$
$$A_4=\left\{(x,y)\ |\ 0.92<\frac{(x-111.15)^2}{(111.15)^2}+\frac{y^2}{(125)^2}<0.94\right\}$$
$$A_5=\left\{(x,y)\ |\ 0.90<\frac{(x-111.15)^2}{(111.15)^2}+\frac{y^2}{(125)^2}<0.92\right\}$$
$$A_6=\left\{(x,y)\ |\ 0.88<\frac{(x-111.15)^2}{(111.15)^2}+\frac{y^2}{(125)^2}<0.90\right\}$$
$$A_7=\left\{(x,y)\ |\ 0.86<\frac{(x-111.15)^2}{(111.15)^2}+\frac{y^2}{(125)^2}<0.88\right\}$$
$$A_8=\left\{(x,y)\ |\ 0.84<\frac{(x-111.15)^2}{(111.15)^2}+\frac{y^2}{(125)^2}<0.86\right\}$$
$$A_9=\left\{(x,y)\ |\ 0.82<\frac{(x-111.15)^2}{(111.15)^2}+\frac{y^2}{(125)^2}<0.84\right\}$$
$$A_{10}=\left\{(x,y)\ |\ 0.80<\frac{(x-111.15)^2}{(111.15)^2}+\frac{y^2}{(125)^2}<0.82\right\}$$

Accordingly, the value of $n(A_i)$ for $i = 1,2,\dots,10$ is provided in Table 2.

Table 2. Calculation of Elliptical Orbits Mode$n(A_i)$

A_i	$n(A_i)$
A_1	11
A_2	15
A_3	16
A_4	26
A_5	25
A_6	19
A_7	9
A_8	3
A_9	0
A_{10}	0

According to Table 2 it is obtained that $Max(n(A_i)) = n(A_4)$, with $i = 1,2,\dots,10$ the selected A_4 set , after that from the A_4 set the following functions $y = f(x)$ will be formed,

$$0.92 < \frac{(x-111.15)^2}{(111.15)^2} + \frac{y^2}{(125)^2} < 0.94 \Rightarrow \frac{(x-111.15)^2}{(111.15)^2} + \frac{y^2}{(125)^2} = \left(\frac{0.92+0.94}{2}\right)$$

$$\Rightarrow \frac{(x-111.15)^2}{(111.15)^2} + \frac{y^2}{(125)^2} = 0.93$$

$$\Rightarrow y = 125\sqrt{0.93 - \frac{(x-111.15)^2}{(111.15)^2}}.$$

Then the translation $y = f(x)$ to be defined at $0 \le x \le 111.15 - 111.15\sqrt{0.93}$ or around $0 \le x \le 3{,}96$, substitution $x_{\frac{w}{2}} = (x - 111.5)$ with $x_{d_m} = \left(x - 111.15\sqrt{0.93}\right)$, so that we get:

$$y = 125\sqrt{0.93 - \frac{\left(x - 111.15\sqrt{0.93}\right)^2}{(111.15)^2}} \tag{11}$$

with y = change in the volume of fuel with respect to fuel height, x., for visualization of the graph of changes in the volume of fuel obtained:

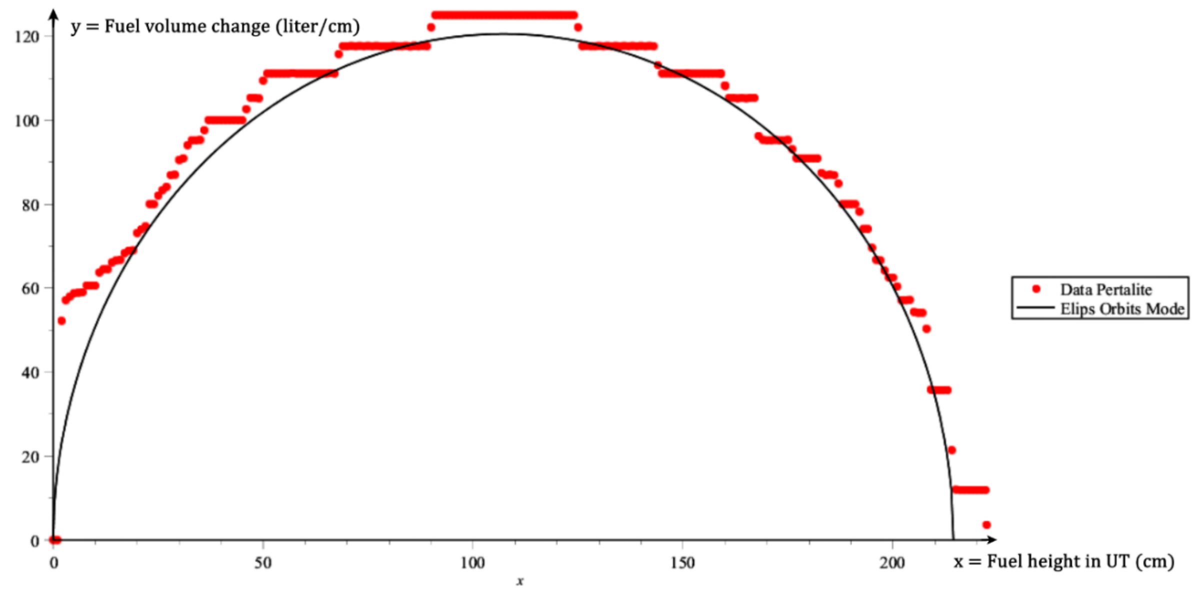

Figure 5. Graph of Change in Fuel Volume by EOM

Based on Figure 5, the EOM results produce a function that is fit to the pertalite volume change data, the volume as function of height (h) is then obtained by the following integration:

$$V(h) = \int_0^h 125\sqrt{0.93 - \frac{\left(x - 111.15\sqrt{0.93}\right)^2}{(111.15)^2}}\,dx$$

Using the help of Maple 2015 the integral results are obtained as follows,

$$\begin{aligned} V(h) &= \frac{535945891}{8000000000000000}\sqrt{18792746045928961} \\ &+ \frac{116250000}{17040701}\sqrt{896879}\arcsin\left(\frac{10182971929}{93000000000000}\sqrt{896879}\sqrt{93}\right) \\ &+ \frac{1}{160000000000}(-809433297500000000000\,h^2 \\ &+ 173524979933482229000000\,h + 18792746045928961)^{\frac{1}{2}}\,h \\ &- \frac{535945891}{8000000000000000}(-809433297500000000000\,h^2 \\ &+ 173524979933482229000000\,h + 18792746045928961)^{\frac{1}{2}} \\ &+ \frac{116250000}{17040701}\sqrt{896879}\arcsin\left(\frac{19}{93000000000000}\sqrt{896879}\sqrt{93}(5000000\,h \right. \\ &\left. - 535945891)\right); \end{aligned} \tag{12}$$

where $V(h)$ is defined on the interval $0 \le h \le 222.3\sqrt{0.93}$. The EOM version of the fuel volume calculation uses (12) with $V_{max} = 20.296.55$ liters.

Elliptical Orbits Mode with elliptical height control on Data Pertalite

Based on Figure 5, it can be seen that the Volume change function according to EOM will regress more pertalite data if the ellipse height is higher, so it is necessary to adjust the ellipse height.

The EOM result at (11) has an elliptical height 125 which represents the equation so that it has a volume change function with respect to the fuel level in the UT, with the general form of (11):

$$EOM(\theta, x) = \theta \cdot \sqrt{0.93 - \frac{\left(x - 111.15\sqrt{0.93}\right)^2}{(111.15)^2}} \tag{13}$$

With θ is the height of the ellipse, defined on the interval $0 \le x \le 222.3\sqrt{0.93}$. Choose θ the one that minimizes the residual sum square

$$E = \sum_{i=1}^{\lfloor 222.3\sqrt{0.93} \rfloor} \left(y_i - EOM(\theta, x_i)\right)^2$$

$$E = \sum_{i=1}^{214} \left(y_i^2 - 2EOM(\theta, x_i)y_i + EOM^2(\theta, x_i)\right)$$

$$E=\sum_{i=1}^{214}{y_i}^2-2\sum_{i=1}^{214}EOM(\theta,x_i)y_i+\sum_{i=1}^{214}\left(EOM(\theta,x_i)\right)^2$$

$$E=\sum_{i=1}^{214}{y_i}^2-2\sum_{i=1}^{214}y_i\cdot\theta\sqrt{0.93-\frac{\left(x-111.15\sqrt{0.93}\right)^2}{(111.15)^2}}+\sum_{i=1}^{214}\left(\theta\sqrt{0.93-\frac{\left(x-111.15\sqrt{0.93}\right)^2}{(111.15)^2}}\right)^2$$

$$E=\sum_{i=1}^{214}{y_i}^2-2\sum_{i=1}^{214}y_i\cdot\theta\sqrt{0.93-\frac{\left(x-111.15\sqrt{0.93}\right)^2}{(111.15)^2}}+\sum_{i=1}^{214}\theta^2\left(0.93-\frac{\left(x-111.15\sqrt{0.93}\right)^2}{(111.15)^2}\right)$$

Then, to find θ the minimization of E, find the solution of the equation $\frac{\partial E}{\partial\theta}=0$, we get:

$$\Leftrightarrow -2\sum_{i=1}^{214}y_i\sqrt{0.93-\frac{\left(x-111.15\sqrt{0.93}\right)^2}{(111.15)^2}}+2\theta\sum_{i=1}^{214}\left(0.93-\frac{\left(x-111.15\sqrt{0.93}\right)^2}{(111.15)^2}\right)=0$$

$$\Leftrightarrow \frac{-2\sum_{i=1}^{214}y_i\sqrt{0.93-\frac{\left(x-111.15\sqrt{0.93}\right)^2}{(111.15)^2}}+2\theta\sum_{i=1}^{214}\left(0.93-\frac{\left(x-111.15\sqrt{0.93}\right)^2}{(111.15)^2}\right)}{2\sqrt{0.93-\frac{\left(x-111.15\sqrt{0.93}\right)^2}{(111.15)^2}}}=0$$

$$\Leftrightarrow -\sum_{i=1}^{214}y_i+\theta\sum_{i=1}^{214}\sqrt{0.93-\frac{\left(x-111.15\sqrt{0.93}\right)^2}{(111.15)^2}}=0$$

$$\Leftrightarrow \theta=\frac{\sum_{i=1}^{214}y_i}{\sum_{i=1}^{214}\sqrt{0.93-\frac{\left(x-111.15\sqrt{0.93}\right)^2}{(111.15)^2}}}$$

By using the data in Table 1, it is obtained $\theta\approx 130{,}37$ that from (27) it is obtained:

$$EOM(130{,}37,x)=130{,}37\cdot\sqrt{0.93-\frac{\left(x-111.15\sqrt{0.93}\right)^2}{(111.15)^2}}$$

and visualized the graph of the function of the change in fuel volume and the actual fuel volume change in the reservoir as follows:

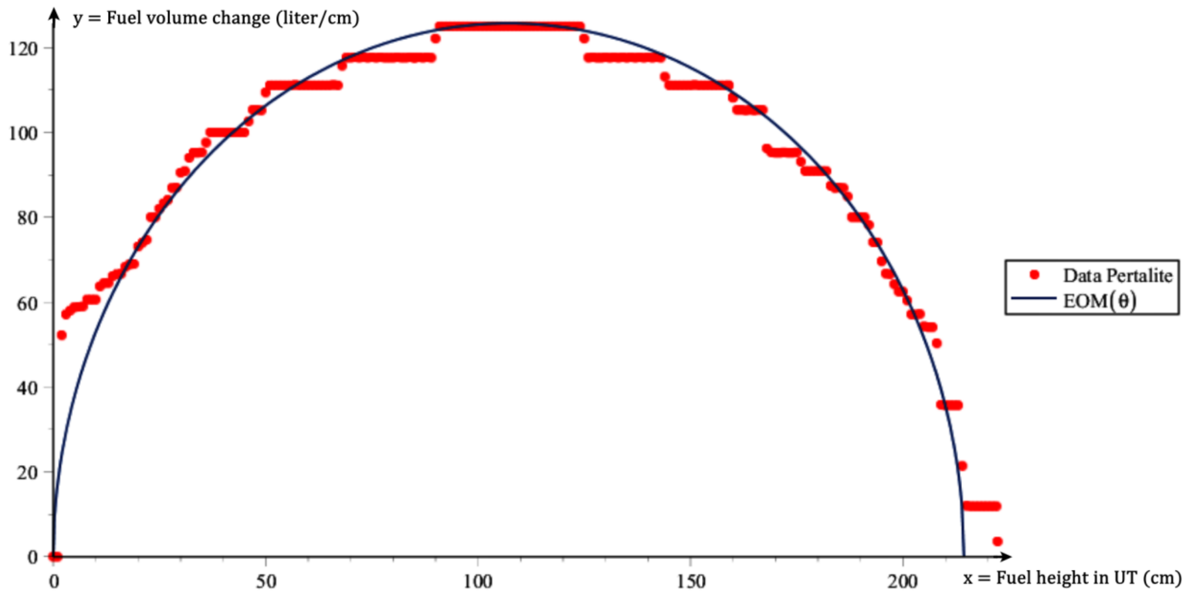

Figure 6. Graph of Changes in Fuel Volume by $EOM(\theta)$

Based on Figure 6 $y = f(x)$ the $EOM(\theta)$results produce a function that is more fit to the pertalite data than the EOM results in Figure 5, then we calculate the volume function $V(h)$ with integral and with (24) obtained:

$$V_\theta(h) = \frac{130{,}37}{125} \cdot V(h)$$

where .$V_\theta(h)$ is defined on the interval $0 \le h \le 222.3$. Calculation of the volume of the fuel $EOM(\theta)$ version has $V_{max} = 21.166{,}71$ liters.

Comparison of approximate visualization results

Data visualization results from Circle Orbits Mode, Elliptical Orbits Mode, $EOM(\theta)$, Least Square Data Fitting $n = 2$, and Least Square Data Fitting $n = 3$, as follows:

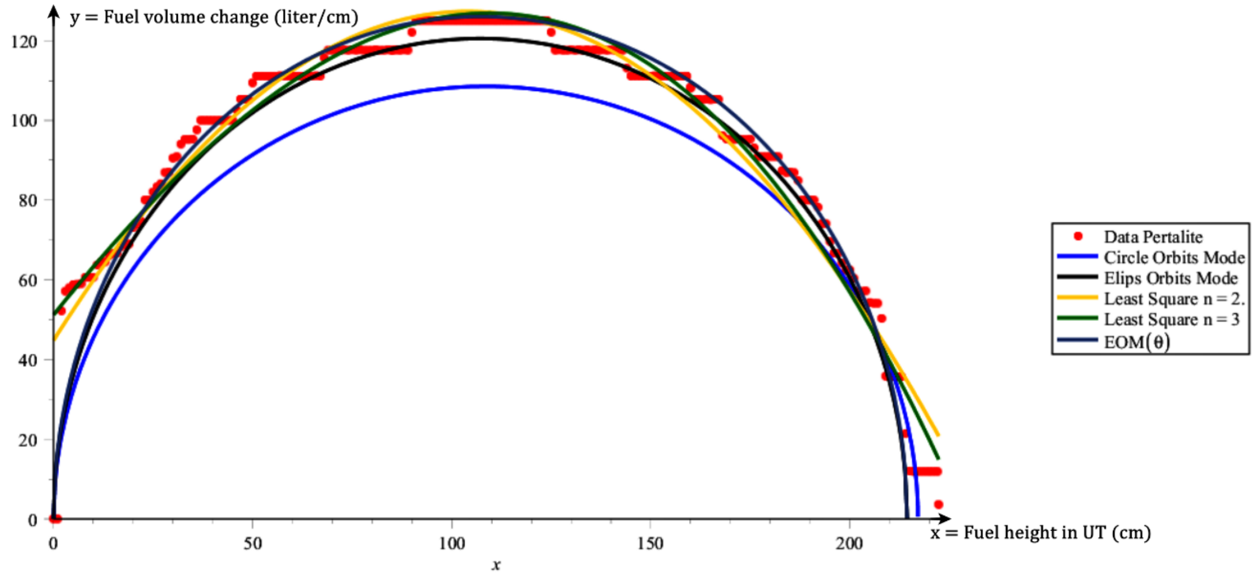

Figure 7. Comparison Graph of Approximation Method Results

The results of the calculation of changes in the volume of fuel based on the height of the fuel in the UT will be applied to the Pertalite Data to search for RSS and MSE from each result.

Pertalite Data Approximation RSS and MSE Calculation

Comparison of the results between the proposed method and other methods can be seen from the calculation of RSS and MSE with liter unit in Table 3 below:

Table 3. Pertalite Data Approximation RSS and MSE Calculation

Method	*RSS*	*MSE*
COM	40.390,49	185,28
EOM	8.529,37	39,67
$LS(n = 2)$	8.980,63	40,09
$LS(n = 3)$	7.574,51	33,81
$EOM(\theta)$	6.415,32	29,84

Based on Table 3 the calculation of COM which has the largest RSS and MSE, for EOM has RSS and MSE which is slightly below $LS(n = 2)$, but still above $LS(n = 3)$, then by controlling the height of the ellipse to find θ that minimizes the RSS we obtain the minimum i.e. 6.415,32 and its MSE 29,84. The smallest value RSS and MSE are obtained when using $EOM(\theta)$. Pertalite data approximation is not compared to the calculation of gas stations and Cubic Spline Interpolation because the RSS and MSE are definitely 0 and have unsmooth approximation.

Defined domain interval and maximum volume

In the Orbits Mode Data Fitting construction, there is a reduction in the BBM altitude domain in the UT, so that it is only defined at a certain height. The results of the comparison of the defined domain height and the maximum volume of each approximation method are as follows:

Table 4. Domain Height Intervals and maximum volume approximation method

Approximation Method	Domain Height (cm)	V_{max} maximum volume (liter)
Gas Station Calculation	$0 \leq h \leq 222{,}3$	$V_{max} = 21.486.00$ liter.
Circle Orbits Mode	$0 \leq h \leq 217{,}1$	$V_{max} = 18.508{,}85$ liter.
Elliptical Orbits Mode	$0 \leq h \leq 222{,}3\sqrt{0{,}93}$	$V_{max} = 20.296{,}55$ liter.
$EOM(\theta)$	$0 \leq h \leq 222{,}3\sqrt{0{,}93}$	$V_{max} = 21.166{,}04$ liter.
Least Square $n = 2$	$0 \leq h \leq 222{,}3$	$V_{max} = 21.248{,}90$ liter.
Least Square $n = 3$	$0 \leq h \leq 222{,}3$	$V_{max} = 21.256{,}66$ liter.

The calculation result of Circle Orbits Mode is only defined to altitude 217,1 cm there is a reduction of 5,2 cm and Elliptical Orbits Mode is only defined to a height of $222{,}3\sqrt{0{,}93}$ cm or about 214,382 cm, there is a reduction of $(222{,}3 - 222{,}3\sqrt{0{,}93})$ cm or about 7,92 cm. For further research, this has no effect on the application of Daily Sales Data (According to Dispenser) if the maximum height of fuel data is below 214,38 cm. So that the value is defined for all data as well as each Approximation Method as well. Approximation results will be validated by measuring Mean Average Deviation (MAD) based on [14] and then Mean Absolute Percentage Error (MAPE) based on [15]. If Aproximation Results has MAPE below on 10% then Aproximation Methods is very feasible.

CONCLUSIONS

Based on the results and discussion, it can be concluded that the method of approximating the pertalite data with the smallest RSS and MSE is $EOM(\theta)$ by $\theta \approx 130{,}37$, resulting in RSS and MSE respectively are 6.415,32 and 29,81. $EOM(\theta)$ also produces a more fit half-ellipse function than other approximation methods. The results of the comparison of the approximation of the pertalite data are compared with COM, EOM, $LS(n = 2)$, and $LS(n = 3)$

Although $EOM(\theta)$ produces RSS and MSE, which are smaller than other methods, there is a reduction in the altitude domain and has a different maximum volume compared to the calculation of gas stations. According to the Gas Station Metrology Measurement Book, the height of the UT is 222,3 cm and has a maximum volume of 21.486 liters, but $EOM(\theta)$ only detects the volume of fuel up to a height of about 214,1 cm and the maximum volume is below the calculation of the gas station.

The author hopes for the development of this research, applied to different types of fuel such as Pertamax and Dexlite. As well as for a more real problem under study, use data on changes in the height and volume of BBM based on Daily Sales according to the BBM Dispenser which must first be tested for the accuracy of the BBM Dispenser used. As well as calculating errors using MAPE, MAD, and other error calculations.

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