



η –Intuitionistic Fuzzy Soft Groups

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ABSTRACT

The fuzzy set theory was introduced by Zadeh in 1965 and the soft set theory was introduced by Molodtsov in 1999. Recently, many researchers have developed these two theories and combined the theory of fuzzy set and soft set became the fuzzy soft set. In this research, we present the idea of the η –intuitionistic fuzzy soft group defined on the η –intuitionistic fuzzy soft set. The main purpose of this research is to create a new concept, which is an η –intuitionistic fuzzy group. To achieve this, we combine the concept of η –intuitionistic fuzzy group and intuitionistic fuzzy soft group. As the main result, we prove the correlation between intuitionistic fuzzy soft group and η –intuitionistic fuzzy soft group along with some properties of η –intuitionistic fuzzy soft group. Also, we prove some properties of subgroup of an η –intuitionistic fuzzy soft group. An η –intuitionistic fuzzy soft homomorphism is also proved.

Keywords: intuitionistic fuzzy group; intuitionistic fuzzy soft group; η –intuitionistic fuzzy group; η –intuitionistic fuzzy soft group

INTRODUCTION

The theory of fuzzy has been studied by many researchers in various fields. Zadeh introduced the fuzzy set theory in [1] by defining a membership function that maps each member of a set to a closed interval of 0 and 1. Then Atanassov formed the intuitionistic fuzzy set that consist of membership function and nonmembership function in [2]. The theory of fuzzy set and intuitionistic fuzzy set was developed into group theory became fuzzy subgroup in [3] and intuitionistic fuzzy subgroup in [4]. The intuitionistic fuzzy subgroup was studied in various types. For example, the intuitionistic L-fuzzy subgroups formed in [5], the (s, t) –intuitionistic fuzzy subgroups defined in [6], definition of (α, β) cut of intuitionistic fuzzy subgroups in [7], and t-intuitionistic fuzzy subgroups in [8]. Doda and Sharma studied the finite groups of different orders and gave the idea of recording the count of intuitionistic fuzzy subgroups in [9]. Zhou and Xu extended the intuitionistic fuzzy sets based on the hesitant fuzzy membership in [10]. The concept of the (λ, μ) –intuitionistic fuzzy subgroups and normal subgroups were defined in [11]. Then the fundamental properties of t-intuitionistic fuzzy abelian subgroup along with the homomorphism of t-intuitionistic fuzzy abelian subgroup were studied in [12]. Latif, et al. studied the fundamental theorems of t-intuitionistic fuzzy isomorphism of t-intuitionistic fuzzy subgroup in [13]. Moreover, the concept of ξ –intuitionistic fuzzy subgroup, ξ –intuitionistic fuzzy cosets, and ξ –intuitionistic fuzzy normal subgroup were characterized in [14]. Based on those research, Shuaib, et al. in [15] formed a concept

called an η – intuitionistic fuzzy subgroup on an η – intuitionistic fuzzy set along with η – intuitionistic fuzzy homomorphism.

While the theory of soft set is introduced in [16] which is an ordered pair of parameter and function that maps each member of parameter to the power set of an empty set. The soft sets constructed in the form of membership function became fuzzy soft sets defined in [17]. Maji, et al. introduced the concept of intuitionistic fuzzy soft set which is a generalization of intuitionistic fuzzy set and soft set in [18]. Furthermore, the operation properties and algebraic structure of intuitionistic fuzzy soft set were discussed in [19]. Soft group on the soft set is defined in [16]. Then Aygünoglu and Aygun in [20] developed the concept of soft group in the form of membership function became fuzzy soft group. The concept of intuitionistic fuzzy soft set to semigroup was applied in [21] and intuitionistic fuzzy soft ideals over ordered ternary semigroup was defined in [22]. The concept of soft group is developed into an intersection called soft int-group in [23]. Moreover, Karaaslan, et al. in [24] applied the concept of soft int-group into intuitionistic fuzzy soft set by forming the intuitionistic fuzzy soft group and investigate some properties of intuitionistic fuzzy soft group.

Based on [15] and [24], we introduce the notion of the η – intuitionistic fuzzy soft group on the η – intuitionistic fuzzy soft set and give some basic properties. Moreover, we define the notion of the η – intuitionistic fuzzy subgroup and investigate the properties of homomorphism of an η – intuitionistic fuzzy soft group.

METHODS

The method of this research is literature review, data collecting techniques by conducting review studies of books, notes, and other scientific research results related to the object of the problem. In this paper, we begin by forming the definition of η – intuitionistic fuzzy soft set based on the definition of intuitionistic fuzzy soft set and η – intuitionistic fuzzy set. Then, we form the definition of η – intuitionistic fuzzy soft group based on the definition of intuitionistic fuzzy soft group. We continue to prove some properties of the η – intuitionistic fuzzy soft group along with subgroup of an η – intuitionistic fuzzy soft group. Moreover, we continue to define the definition of image and pre-image of an η – intuitionistic fuzzy soft group and prove the homomorphism of an η – intuitionistic fuzzy soft group. Here is shown the definition of intuitionistic fuzzy soft set, η – intuitionistic fuzzy set, and intuitionistic fuzzy soft group.

Definition 1 [18]. Let X be a non empty set and E be a set of parameter with $A \subseteq E$. Let $\mathcal{IF}(X)$ be a set of all intuitionistic fuzzy set of X . An intuitionistic fuzzy soft set of A over X is defined by

$$\Gamma_A = \{(a, \gamma_A(a)) : a \in A\},$$

where $\gamma_A(a) : E \rightarrow \mathcal{IF}(X)$ such as $\gamma_A(a) = \emptyset$ if $a \notin A$. Therefore, for all $a \in E$, $\gamma_A(a)$ is called intuitionistic fuzzy value set of a . $\gamma_A(a)$ can be written as

$$\gamma_A(a) = \{(x, \mu_{\gamma_A(a)}(x), \nu_{\gamma_A(a)}(x)) : x \in X\},$$

for all $a \in E$.

Definition 2 [15]. Suppose A is an intuitionistic fuzzy set of a non empty set X where μ_A be a membership function and ν_A be a nonmembership function in A . Let $\eta \in [0, 1]$. An η – intuitionistic fuzzy set is defined by

$$A^\eta = \{(x, \mu_{A^\eta}(x), \nu_{A^\eta}(x)) : x \in X\},$$

where $\mu_{A^\eta}(x) = \psi[\mu_A(x), \eta] = \sqrt{\mu_A(x) \cdot \eta}$ and $\nu_{A^\eta}(x) = \psi'[\nu_A(x), 1 - \eta] = \sqrt{\nu_A(x) \cdot (1 - \eta)}$.

Definition 3 [24]. Let G be an arbitrary group and Γ_G be an intuitionistic fuzzy soft set over universe X , then Γ_G is called intuitionistic fuzzy soft group on G over X if

1. $\gamma_G(xy) \supseteq \gamma_G(x) \cap \gamma_G(y)$,
 2. $\gamma_G(x^{-1}) = \gamma_G(x)$,
- for all $x \in G$.

RESULTS AND DISCUSSION

In this section, we introduce the notion of η – intuitionistic fuzzy soft group on η – intuitionistic fuzzy soft set. Inspiring from Definition 1 and Definition 2, we define the notion of η – intuitionistic fuzzy soft set.

Definition 4. Suppose Γ_A be an intuitionistic fuzzy soft set of parameter A over universe X and $\eta \in [0, 1]$. An η – intuitionistic fuzzy soft set of A over X is defined by

$$\Gamma_A^\eta = \{(a, \gamma_A^\eta(a)) : a \in A\},$$

where $\gamma_A^\eta(a)$ is an η – intuitionistic fuzzy set of X defined as

$$\gamma_A^\eta(a) = \Psi[\gamma_A(a), \eta],$$

for all $a \in A$, where

$$\Psi[\gamma_A(a), \eta] = \{(x, \psi[\mu_{\gamma_A(a)}(x), \eta], \psi[v_{\gamma_A(a)}(x), 1 - \eta]) : x \in X\}.$$

The value of $\psi[\mu_{\gamma_A(a)}(x), \eta] = \sqrt{\mu_{\gamma_A(a)}(x) \cdot \eta}$ and $\psi[v_{\gamma_A(a)}(x), 1 - \eta] = \sqrt{\mu_{\gamma_A(a)}(x) \cdot (1 - \eta)}$.

Some basic properties such as intersection and union of η – intuitionistic fuzzy soft set is proved by the following proposition.

Proposition 1. The intersection of any two η – intuitionistic fuzzy soft sets is an η – intuitionistic fuzzy soft set.

Proof. Let $\Gamma_A^\eta = \{(a, \gamma_A^\eta(a)) : a \in A\}$ and $\Gamma_B^\eta = \{(b, \gamma_B^\eta(b)) : b \in B\}$, respectively be two η – intuitionistic fuzzy soft sets of A and B over X . For any $c \in A \cap B$, then

$$\begin{aligned} \gamma_{(A \cap B)^\eta}(c) &= \Psi[\gamma_{A \cap B}(c), \eta] \\ &= \left\{ \left(x, \psi[\min[\mu_{\gamma_A(c)}(x), \mu_{\gamma_B(c)}(x)], \eta], \psi[\max[v_{\gamma_A(c)}(x), v_{\gamma_B(c)}(x)], 1 - \eta] \right) : x \in X \right\} \\ &= \left\{ \left(x, \psi[\min[\mu_{\gamma_A(c)}(x), \eta], \min[\mu_{\gamma_B(c)}(x), \eta]], \psi[\max[v_{\gamma_A(c)}(x), 1 - \eta], \max[v_{\gamma_B(c)}(x), 1 - \eta]] \right) : x \in X \right\} \\ &= \left\{ \left(x, \min[\psi[\mu_{\gamma_A(c)}(x), \eta], \psi[\mu_{\gamma_B(c)}(x), \eta]], \max[\psi[v_{\gamma_A(c)}(x), 1 - \eta], \psi[v_{\gamma_B(c)}(x), 1 - \eta]] \right) : x \in X \right\} \\ &= \Psi[\gamma_A(c), \eta] \cap \Psi[\gamma_B(c), \eta] \\ &= \gamma_{A \cap B}^\eta(c). \end{aligned}$$

Hence $\Gamma_A^\eta \cap \Gamma_B^\eta$ is an η – intuitionistic fuzzy soft set. ■

Remark 1. The union of any two η – intuitionistic fuzzy soft set is an η – intuitionistic fuzzy soft set.

The notion of η – intuitionistic fuzzy soft group is defined based on Definition 3. An η – intuitionistic fuzzy soft group must satisfy 2 axioms to be an η – intuitionistic fuzzy soft group.

Definition 5. Let G be a group and Γ_G^η be an η – intuitionistic fuzzy soft set over universe U , then Γ_G^η is called an η – intuitionistic fuzzy soft group over U if

1. $\gamma_G^\eta(xy) \supseteq \gamma_G^\eta(x) \cap \gamma_G^\eta(y)$,
 2. $\gamma_G^\eta(x^{-1}) = \gamma_G^\eta(x)$,
- for all $x, y \in G$.

The property of the identity element of the group on an η –intuitionistic fuzzy soft group is proved by the following proposition.

Proposition 2. Let G be a group and Γ_G^η be an η –intuitionistic fuzzy soft group over universe U , then $\gamma_G^\eta(e) \supseteq \gamma_G^\eta(x)$ for all $x \in G$.

Proof. Let $\Gamma_G^\eta = \{(x, \gamma_G^\eta(x)): x \in G\}$ be an η –intuitionistic fuzzy soft group and e be an identity element of G . For $e \in G$, then $\gamma_G^\eta(e) = \gamma_G^\eta(xx^{-1})$. Since Γ_G^η is an η –intuitionistic fuzzy soft group, then

$$\begin{aligned} \gamma_G^\eta(e) &\supseteq \gamma_G^\eta(x) \cap \gamma_G^\eta(x^{-1}) \\ &= \gamma_G^\eta(x) \cap \gamma_G^\eta(x) \\ &= \gamma_G^\eta(x). \end{aligned}$$

So, $\gamma_G^\eta(e) \supseteq \gamma_G^\eta(x)$, for all $x \in G$. ■

An η –intuitionistic fuzzy soft set is called an η –intuitionistic fuzzy soft group if it satisfies 2 axioms in Definition 5. Here we prove the alternative way of η –intuitionistic fuzzy soft group.

Theorem 1. An η –intuitionistic fuzzy soft set is called η –intuitionistic fuzzy soft group if and only if $\gamma_G^\eta(xy^{-1}) \supseteq \gamma_G^\eta(x) \cap \gamma_G^\eta(y)$ for all $x, y \in G$.

Proof.

(\Rightarrow) Let $\Gamma_G^\eta = \{(x, \gamma_G^\eta(x)): x \in G\}$ be an η –intuitionistic fuzzy soft group. From Definition 5, then for all $x, y \in G$ we have

$$\gamma_G^\eta(xy^{-1}) \supseteq \gamma_G^\eta(x) \cap \gamma_G^\eta(y^{-1}) = \gamma_G^\eta(x) \cap \gamma_G^\eta(y).$$

(\Leftarrow) Since $\gamma_G^\eta(xy^{-1}) \supseteq \gamma_G^\eta(x) \cap \gamma_G^\eta(y)$ for all $x, y \in G$, then

$$\gamma_G^\eta(x^{-1}) = \gamma_G^\eta(ex) \supseteq \gamma_G^\eta(e) \cap \gamma_G^\eta(x) \supseteq \gamma_G^\eta(x) \cap \gamma_G^\eta(x) = \gamma_G^\eta(x),$$

and

$$\gamma_G^\eta(x) = \gamma_G^\eta(e(x^{-1})^{-1}) \supseteq \gamma_G^\eta(e) \cap \gamma_G^\eta(x^{-1}) \supseteq \gamma_G^\eta(x)^{-1} \cap \gamma_G^\eta(x^{-1}) = \gamma_G^\eta(x^{-1}).$$

Hence $\gamma_G^\eta(x^{-1}) = \gamma_G^\eta(x)$ for all $x \in G$. Then for all $x, y \in G$ we have

$$\begin{aligned} \gamma_G^\eta(xy) &= \gamma_G^\eta(x(y^{-1})^{-1}) \\ &\supseteq \gamma_G^\eta(x) \cap \gamma_G^\eta(y^{-1}) \\ &= \gamma_G^\eta(x) \cap \gamma_G^\eta(y). \end{aligned}$$

Hence $\gamma_G^\eta(xy) \supseteq \gamma_G^\eta(x) \cap \gamma_G^\eta(y)$ for all $x, y \in G$. Therefore, Γ_G^η is an η –intuitionistic fuzzy soft group. ■

Since η –intuitionistic fuzzy soft group is defined based on intuitionistic fuzzy soft group, so there is a correlation between those concepts. The following theorem, we prove that every intuitionistic fuzzy soft group is an η –intuitionistic fuzzy soft group.

Theorem 2. If Γ_G is an intuitionistic fuzzy soft group, then Γ_G^η is an η –intuitionistic fuzzy soft group for all $\eta \in [0, 1]$.

Proof. Let $\Gamma_G = \{(x, \gamma_G(x)): x \in G\}$ be an intuitionistic fuzzy soft group over the universe U where $\gamma_G(x) = \{(u, \mu_{\gamma_G(x)}(u), \nu_{\gamma_G(x)}(u)): u \in U\}$. For any $x, y \in G$ and $\eta \in [0, 1]$, then

$$\gamma_G^\eta(xy^{-1}) = \Psi[\gamma_G(xy^{-1}), \eta].$$

Since Γ_G is an intuitionistic fuzzy soft group, then

$$\begin{aligned} \gamma_G^\eta(xy^{-1}) &\supseteq \Psi[(\gamma_G(x) \cap \gamma_G(y)), \eta] \\ &= \left\{ \left(u, \psi[\min[\mu_{\gamma_G(x)}(u), \mu_{\gamma_G(y)}(u)], \eta], \right. \right. \\ &\quad \left. \left. \psi[\max[\nu_{\gamma_G(x)}(u), \nu_{\gamma_G(y)}(u)], 1 - \eta] \right) : u \in U \right\} \\ &= \left\{ \left(x, \psi[\min[\mu_{\gamma_G(x)}(u), \eta], \min[\mu_{\gamma_G(y)}(u), \eta]], \right. \right. \\ &\quad \left. \left. \psi[\max[\nu_{\gamma_G(x)}(u), 1 - \eta], \max[\nu_{\gamma_G(y)}(u), 1 - \eta]] \right) : u \in U \right\} \end{aligned}$$

$$\begin{aligned}
 &= \left\{ \left(x, \min[\psi[\mu_{\gamma_G(x)}(u), \eta], \psi[\mu_{\gamma_G(y)}(u), \eta]], \right. \right. \\
 &\quad \left. \left. \max[\psi[\nu_{\gamma_G(x)}(u), 1 - \eta], \psi[\nu_{\gamma_G(y)}(u), 1 - \eta]] \right) : u \in U \right\} \\
 &= \Psi[\gamma_G(x), \eta] \cap \Psi[\gamma_G(y), \eta] \\
 &= \gamma_G^\eta(x) \cap \gamma_G^\eta(y).
 \end{aligned}$$

Therefore, Γ_G^η is an η – intuitionistic fuzzy soft group. ■

Here we define the η – intuitionistic fuzzy subgroup of an η – intuitionistic fuzzy group. The properties of η – intuitionistic fuzzy subgroup are proved in the following theorem.

Definition 6. Suppose G be a group and H be a subgroup of G . Let Γ_G^η be an η – intuitionistic fuzzy soft group over universe U , then Γ_H^η is called an η – intuitionistic fuzzy soft subgroup of Γ_G^η if Γ_H^η is an η – intuitionistic fuzzy soft group over U .

Theorem 3. Let Γ_G^η be an η – intuitionistic fuzzy soft group over universe U . Suppose Γ_H^η and Γ_N^η be two η – intuitionistic fuzzy soft subgroups of Γ_G^η , then $\Gamma_H^\eta \tilde{\cap} \Gamma_N^\eta$ is an η – intuitionistic fuzzy soft subgroup of Γ_G^η .

Proof. Defined $\Gamma_H^\eta \tilde{\cap} \Gamma_N^\eta = \{(x, \gamma_{H\tilde{\cap}N}^\eta(x)) : x \in H \cap N\}$. For any $x, y \in G$, then

$$\begin{aligned}
 \gamma_{H\tilde{\cap}N}^\eta(xy^{-1}) &= \gamma_H^\eta(xy^{-1}) \cap \gamma_N^\eta(xy^{-1}) \\
 &\supseteq (\gamma_H^\eta(x) \cap \gamma_H^\eta(y)) \cap (\gamma_N^\eta(x) \cap \gamma_N^\eta(y)) \\
 &= (\gamma_H^\eta(x) \cap \gamma_N^\eta(x)) \cap (\gamma_H^\eta(y) \cap \gamma_N^\eta(y)) \\
 &= \gamma_{H\tilde{\cap}N}^\eta(x) \cap \gamma_{H\tilde{\cap}N}^\eta(y).
 \end{aligned}$$

Hence $\Gamma_H^\eta \tilde{\cap} \Gamma_N^\eta$ is an η – intuitionistic fuzzy soft subgroup of Γ_G^η . ■

Theorem 4. Let Γ_G^η be an η – intuitionistic fuzzy soft group over universe U and e be an identity element of G . Then $\Gamma_G^\eta|_e = \{(x, \gamma_G^\eta(x)) : \gamma_G^\eta(x) = \gamma_G^\eta(e), x \in G\}$ is an η – intuitionistic fuzzy subgroup of G .

Proof. Since $(e, \gamma_G^\eta(e)) \in \Gamma_G^\eta|_e$, then $\Gamma_G^\eta|_e \neq \emptyset$. Let $(x, \gamma_G^\eta(x)), (y, \gamma_G^\eta(y)) \in \Gamma_G^\eta|_e$, we have $\gamma_G^\eta(x) = \gamma_G^\eta(y) = \gamma_G^\eta(e)$ for $x, y \in G$. Then

$$\begin{aligned}
 \gamma_G^\eta(xy^{-1}) &\supseteq \gamma_G^\eta(x) \cap \gamma_G^\eta(y) \\
 &= \gamma_G^\eta(e) \cap \gamma_G^\eta(e) \\
 &= \gamma_G^\eta(e).
 \end{aligned}$$

Since $\gamma_G^\eta(e) \supseteq \gamma_G^\eta(xy^{-1})$, then $\gamma_G^\eta(xy^{-1}) = \gamma_G^\eta(e)$. Thus $(xy^{-1}, \gamma_G^\eta(xy^{-1})) \in \Gamma_G^\eta|_e$. Therefore $\Gamma_G^\eta|_e$ is an η – intuitionistic fuzzy subgroup of G . ■

Here notioned the definition of image and pre-image of an η – intuitionistic fuzzy soft group.

Definition 7. Let Γ_A^η and Γ_B^η be two η – intuitionistic fuzzy soft sets over universe U , and let $\phi: A \rightarrow B$, then

1. Image of Γ_A^η under the map ϕ , denoted by $\phi(\Gamma_A^\eta)$ defined by

$$\phi(\Gamma_A^\eta) = \{(b, \phi(\gamma_A^\eta)(b)) : b \in B\},$$

where for all $b \in B$, then

$$\phi(\gamma_A^\eta)(b) = \begin{cases} \cap \{\gamma_A^\eta(a) : a \in A, \phi(a) = b\}, & \text{if } \phi(a) \in \phi(A), \\ \gamma_\emptyset^\eta, & \text{others.} \end{cases}$$

2. Pre-image of Γ_B^η under ϕ , denoted by $\phi^{-1}(\Gamma_B^\eta)$ defined by

$$\phi^{-1}(\Gamma_B^\eta) = \{(a, \phi^{-1}(\gamma_B^\eta)(a)) : a \in A\},$$

where for all $a \in A$, then

$$\phi^{-1}(\gamma_B^\eta)(a) = \gamma_B^\eta(\phi(a)).$$

Lemma 1. Let Γ_A^η and Γ_B^η be two η – intuitionistic fuzzy soft sets over U , then $\phi(\Gamma_A^\eta)$ and $\phi^{-1}(\Gamma_B^\eta)$ are η – intuitionistic fuzzy soft sets over universe U .

Proof. Let $\Gamma_A^\eta = \{(a, \gamma_A^\eta(a)): a \in A\}$ and $\Gamma_B^\eta = \{(b, \gamma_B^\eta(b)): b \in B\}$ respectively be two η –intuitionistic fuzzy soft sets of A and B over U . Let $\phi: A \rightarrow B$. From Definition 7, there is $b \in \phi(A)$ such that $\phi(a) = b$, then

$$\phi(\gamma_A^\eta)(b) = \cap \{\gamma_A^\eta(a): a \in A, \phi(a) = b\}.$$

From proposition 1, we have $\phi(\gamma_A^\eta)(b)$ is an η –intuitionistic fuzzy set.

If $\phi(a) \notin \phi(A)$, then $\phi(\gamma_A^\eta)(b) = \gamma_\emptyset^\eta$ is an η –intuitionistic fuzzy set. Then $\forall a \in A$, we have $\phi^{-1}(\gamma_B^\eta)(a) = \gamma_B^\eta(\phi(a))$ is an η –intuitionistic fuzzy set. Therefore, $\phi(\Gamma_A^\eta)$ and $\phi^{-1}(\Gamma_B^\eta)$ are η –intuitionistic fuzzy soft sets over U . ■

The map $f: G_1 \rightarrow G_2$ of a group G_1 to a group G_2 is called homomorphism if for all $x, y \in G_1$ then $f(xy) = f(x)f(y)$. The following theorems, we prove that every image and pre-image of the η –intuitionistic fuzzy group under the homomorphism function are the η –intuitionistic fuzzy soft group.

Theorem 5. Let $\Gamma_{G_1}^\eta$ is an η –intuitionistic fuzzy soft group over universe U and $\phi_{\text{hom}}: G_1 \rightarrow G_2$, then $\phi(\Gamma_{G_1}^\eta)$ is an η –intuitionistic fuzzy soft group.

Proof. Let G_1 and G_2 are two groups, $\phi_{\text{hom}}: G_1 \rightarrow G_2$, and $\Gamma_{G_1}^\eta$ is an η –intuitionistic fuzzy soft group over U . Let $u, v \in G_2$, $u \notin \phi(G_1)$ or $v \notin \phi(G_1)$, then

$$\phi(\gamma_{G_1}^\eta)(u) \cap \phi(\gamma_{G_1}^\eta)(v) = \gamma_\emptyset^\eta,$$

means

$$\phi(\gamma_{G_1}^\eta)(uv) \supseteq \phi(\gamma_{G_1}^\eta)(u) \cap \phi(\gamma_{G_1}^\eta)(v).$$

Since $u \notin \phi(G_1)$, then $u^{-1} \notin \phi(G_1)$, so

$$\phi(\gamma_{G_1}^\eta)(u^{-1}) = \phi(\gamma_{G_1}^\eta)(u) = \gamma_\emptyset^\eta.$$

Suppose $\phi(x) = u$ and $\phi(y) = v$ for $x, y \in G_1$. Let $z = xy$, then

1. $\phi(\gamma_{G_1}^\eta)(uv) = \cap \{\gamma_{G_1}^\eta(z): z \in G_1, \phi(z) = uv\}$
 $= \cap \{\gamma_{G_1}^\eta(xy): x, y \in G_1, \phi(xy) = uv\}$
 $\supseteq \cap \{\gamma_{G_1}^\eta(x) \cap \gamma_{G_1}^\eta(y): x, y \in G_1, \phi(x) = u, \phi(y) = v\}$
 $= (\cap \{\gamma_{G_1}^\eta(x): x \in G_1, \phi(x) = u\}) \cap (\cap \{\gamma_{G_1}^\eta(y): y \in G_1, \phi(y) = v\})$
 $= \phi(\gamma_{G_1}^\eta)(u) \cap \phi(\gamma_{G_1}^\eta)(v).$
2. $\phi(\gamma_{G_1}^\eta)(u) = \cap \{\gamma_{G_1}^\eta(x): x \in G_1, \phi(x) = u\}$
 $= \cap \{\gamma_{G_1}^\eta(x^{-1}): x \in G_1, \phi(x^{-1}) = u^{-1}\}$
 $= \phi(\gamma_{G_1}^\eta)(u^{-1}).$

Therefore, $\phi(\Gamma_{G_1}^\eta)$ is an η –intuitionistic fuzzy soft group over U . ■

Theorem 6. Let $\Gamma_{G_2}^\eta$ is an η –intuitionistic fuzzy soft group over universe U and $\phi_{\text{hom}}: G_1 \rightarrow G_2$, then $\phi^{-1}(\Gamma_{G_2}^\eta)$ is an η –intuitionistic fuzzy soft group.

Proof. Let G_1 and G_2 are two groups, $\phi_{\text{hom}}: G_1 \rightarrow G_2$, and $\Gamma_{G_2}^\eta$ is an η –intuitionistic fuzzy soft group over U . For all $x, y \in G_1$ we have

1. $\phi^{-1}(\gamma_{G_2}^\eta)(xy) = \gamma_{G_2}^\eta(\phi(xy))$
 $= \gamma_{G_2}^\eta(\phi(x)\phi(y))$
 $\supseteq \gamma_{G_2}^\eta(\phi(x)) \cap \gamma_{G_2}^\eta(\phi(y))$
 $= \phi^{-1}(\gamma_{G_2}^\eta)(x) \cap \phi^{-1}(\gamma_{G_2}^\eta)(y).$
2. $\phi^{-1}(\gamma_{G_2}^\eta)(x^{-1}) = \gamma_{G_2}^\eta(\phi(x^{-1}))$
 $= \gamma_{G_2}^\eta((\phi(x))^{-1})$
 $= \gamma_{G_2}^\eta(\phi(x))$
 $= \phi^{-1}(\gamma_{G_2}^\eta)(x).$

Therefore, $\phi^{-1}(\Gamma_{G_2}^\eta)$ is an η – intuitionistic fuzzy soft group over U . ■

Based on Theorem 5 and Theorem 6, we know that image and pre-image of an η – intuitionistic fuzzy soft group under η – intuitionistic fuzzy soft homomorphism are also an η – intuitionistic fuzzy soft group.

CONCLUSIONS

Based on the result, it is concluded that η – intuitionistic fuzzy soft group depends on intuitionistic fuzzy soft group. It is proved that every intuitionistic fuzzy soft group is an η – intuitionistic fuzzy soft group. The definition of η – intuitionistic fuzzy soft subgroup and its properties are presented. The η – intuitionistic fuzzy soft homomorphism show that image and pre-image of an η – intuitionistic fuzzy soft group are also η – intuitionistic fuzzy soft groups. For the future research, it is suggested to notion the η – intuitionistic fuzzy soft ring along with the properties.

REFERENCES

- [1] L. A. Zadeh, "Fuzzy sets," *Inf. Control*, vol. 8, no. 3, pp. 338–353, 1965, doi: 10.1016/S0019-9958(65)90241-X.
- [2] K. T. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets Syst.*, vol. 20, no. 1, pp. 87–96, 1986, doi: 10.1016/S0165-0114(86)80034-3.
- [3] A. Rosenfeld, "Fuzzy groups," *J. Math. Anal. Appl.*, vol. 35, no. 3, pp. 512–517, 1971, doi: 10.1016/0022-247X(71)90199-5.
- [4] R. Biswas, "Intuitionistic Fuzzy Subgroups," *Math. Forum*, vol. 10, pp. 37–46, 1989.
- [5] N. Palaniappan, S. Naganathan, and K. Arjunan, "A study on intuitionistic L-fuzzy subgroups," *Appl. Math. Sci.*, vol. 3, no. 53, pp. 2619–2624, 2009.
- [6] X. Yuan, H. Li, and E. . Lee, "On The Definition of The Intuitionistic Fuzzy Subgroups," *Comput. Math. with Apl.*, vol. 59, pp. 3117–3129, 2010.
- [7] P. K. Sharma, " (α, β) Cut of Intuitionistic Fuzzy Subgroups," *Int. Math. Forum*, vol. 6, no. 53, pp. 2605–2614, 2011.
- [8] P. K. Sharma, "t- Intuitionistic Fuzzy Subgroups," *Int. J. Fuzzy Math. Syst.*, vol. 2, no. 3, pp. 233–243, 2012.
- [9] N. Doda and P. K. Sharma, "Counting the number of intuitionistic fuzzy subgroups of finite Abelian groups of different order," *Notes Intuitionistic Fuzzy Sets*, vol. 19, no. 4, pp. 42–47, 2013.
- [10] W. Zhou and Z. Xu, "Extended Intuitionistic Fuzzy Sets Based on the Hesitant Fuzzy Membership and their Application in Decision Making with Risk Preference," *Int. J. Intelegence Syst.*, vol. 00, pp. 1–27, 2017, doi: 10.1002/int.
- [11] S. Sun and C. Liu, " (λ, μ) -Intuitionistic Fuzzy Subgroups of Groups with Operators," *Int. J. Math. Comput. Sci.*, vol. 10, no. 9, pp. 456–462, 2016.
- [12] M. Gulzar, D. Alghazzawi, M. H. Mateen, and N. Kausar, "A certain class of T-intuitionistic fuzzy subgroups," *IEEE Access*, vol. 8, pp. 163260–163268, 2020, doi: 10.1109/ACCESS.2020.3020366.

- [13] L. Latif, U. Shuaib, H. Alolaiyan, and A. Razaq, "On Fundamental Theorems of t-Intuitionistic Fuzzy Isomorphism of t-Intuitionistic Fuzzy Subgroups," *IEEE Access*, vol. 6, pp. 2169–3536, 2018.
- [14] U. Shuaib, M. Amin, S. Dilbar, and F. Tahir, "On Algebraic Attributes of ξ - Intuitionistic Fuzzy Subgroups," *Int. J. Math. Comput. Sci.*, vol. 15, no. 1, pp. 395–411, 2020.
- [15] U. Shuaib, H. Alolaiyan, A. Razaq, D. Saba, and F. Tahir, "On some algebraic aspects of η -intuitionistic fuzzy subgroups," *J. Taibah Univ. Sci.*, vol. 14, no. 1, pp. 463–469, 2020, doi: 10.1080/16583655.2020.1745491.
- [16] H. Aktaş and N. Çağman, "Soft sets and soft groups," *Inf. Sci. (Ny)*, vol. 177, no. 13, pp. 2726–2735, 2007, doi: 10.1016/j.ins.2006.12.008.
- [17] P. . Maji, A. . Roy, and R. Biswas, "Fuzzy Soft Sets," *J. Fuzzy Math.*, vol. 9, no. 3, pp. 589–602, 2001.
- [18] P. . Maji, R. Biswas, and A. . Roy, "Intuitionistic Fuzzy Soft Set," *J. Fuzzy Math.*, vol. 9, no. 3, pp. 677–692, 2001.
- [19] Y. Yin, H. Li, and Y. Bae, "On algebraic structure of intuitionistic fuzzy soft sets," *Comput. Math. with Appl.*, vol. 64, no. 9, pp. 2896–2911, 2012, doi: 10.1016/j.camwa.2012.05.004.
- [20] A. Aygünoglu and H. Aygun, "Introduction to Fuzzy Soft Groups," *Comput. Math. with Appl.*, vol. 58, pp. 1279–1286, 2009, doi: 10.1016/j.camwa.2009.07.047.
- [21] J. Zhou, "Intuitionistic fuzzy soft semigroups," *Math. Aeterna*, vol. 1, no. 03, pp. 173–183, 2011.
- [22] M. Akram and N. Yaqoob, "Intuitionistic fuzzy soft ordered ternary semigroups," *Int. J. Pure Appl. Math.*, vol. 84, no. 2, pp. 93–107, 2013, doi: 10.12732/ijpam.v84i2.8.
- [23] N. Cagman, F. Citak, and H. Aktas, "Soft int-group and its applications to group theory," *Neural Comput Applic*, vol. 21, no. 1, pp. S151–S158, 2012, doi: 10.1007/s00521-011-0752-x.
- [24] F. Karaaslan, K. Kaygisiz, and N. Cagman, "On Intuitionistic Fuzzy Soft Groups," *J. New Results Sci.*, vol. 2, no. 3, pp. 72–86, 2013.