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Bühlmann's Credibility Model with Claims of Negative Binomial and 2-Poisson Distribution

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ABSTRACT

One technique for determining the premium is using the credibility theory. In this study, a credibility premium determination model was derived with the best accuracy approach in the form of Bühlmann's credibility premium. The approach used was a parameteric approach where the claim data is assumed to have a Negative Binomial and 2-Poisson distribution. The Bühlmann's credibility premium formula is given explicitly for these two data distributions. The obtained model is also applied to the correct data following these distributions. From the simulation results, it is obtained that the premium values are very close in value so that both models can be applied to the data and have a high level of credibility because they have a high credibility factor value. The results of this study provide a basic contribution to the development of actuarial science, especially in the technique of determining insurance premiums.

Keywords: 2-Poisson distribution; Bühlmann's credibility; Negative Binomial distribution

INTRODUCTION

In everyday life, humans are very vulnerable to risk. For example, the risk of accidents, property loss, illness, and even loss of life. This risk causes humans to lose their assets. Therefore, insurance comes with the aim of minimizing loss if the risk does occur. Insurance is an agreement between the insured (policyholder) and the insurer which requires the policyholder to pay a premium as compensation for the insurance benefits to be provided by the insurer in the event of a risk of failure to the policyholder [1].

One of the problems of insurance companies is how to determine the premium of a product. One of the techniques used is credibility theory. This theory predicts the amount of premium rates in the future based on experience data in the past. There are two predictive models that can be formed, namely a model for the number of claims and a model for the amount of claims (claim severity) that occur. This of course will be related to the type of data distribution used. The statistical approach that can be used in modeling the data is a parametric approach and a nonparametric approach. In this approach, we used a

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parametric approach where the claim data is assumed to follow a certain distribution [2].

One type of credibility that is widely used is the best accuracy credibility which consists of the Bühlmann model and the Bühlmann-Straub model [3]. In the Bühlmann model, policyholders are assumed to be the same number between time periods, while the Bühlmann-Straub model is a general form of the Bühlmann model where the number of policyholders may differ between time periods. Several studies related to the determination of premiums with Bühlmann and Bühlmann-Straub credibility parametrically can be seen in the research conducted by [4]–[7].

The distributions that are commonly used to model many claims are the Negative Binomial distribution and the Poisson distribution [8]. According to [9], mixed distribution is a distribution that can be considered in modeling the data. This is because, data modeling becomes more accurate. One of the mixed distributions used in this study is the 2-Poisson distribution [10]. The use of 2-Poisson distribution has not been found in previous studies. By using the assumption of a Negative Binomial and 2-Poisson distribution on the data, the Bühlmann's credibility model will be determined on the data that satisfies this distribution. The equation for determining Bühlmann's credibility parameter is given explicitly. In addition, the prediction results obtained through the application of the data are also compared. Applications on nonparametric data using R can be seen in [11].

METHODS

Poisson Distribution

The Poisson distribution is a distribution with one parameter (λ). The probability function for the Poisson distribution is

$$p_k = \frac{e^{-\lambda}\lambda^k}{k!}, \tag{1}$$

with $k = 0,1,2,\ldots$.

The expected value and variance of the Poisson distribution are

$$E(X) = \lambda, \tag{2}$$

$$Var(X) = \lambda \tag{3}$$

[12]. The other properties and the applications of this distribution can be seen in [13].

Gamma Distribution

A random variable is said to have a Gamma distribution with parameters α and β, if it has a probability density function

$$g_X(x;\alpha,\beta) = \frac{x^{\alpha-1}e^{-x/\beta}}{\Gamma(\alpha)\beta^\alpha}, x \in \mathbb{R}^+ \tag{4}$$

with $\alpha > 0$, $\beta > 0$, $\Gamma(\alpha) > 0$, and $\Gamma(\alpha) = \int_0^\infty y^{\alpha-1}\, e^{-y} dy$. The parameter α is called the shape parameter associated with the Gamma distribution and the parameter β is generally called the scale parameter because it multiplies the random variable with a Gamma distribution by a positive constant. The expected value and variance of the Gamma distribution are

$$E(X) = \frac{\alpha}{\tau}, \tag{5}$$

$$Var(X) = \frac{\alpha}{\tau^2}. \tag{6}$$

The evidence can be found at [14].

Negative Binomial Distribution

A Negative Binomial distribution is formed by an experiment that satisfies the following conditions:

1. An experiment consists of a series of independent experiments.
2. Each experiment can only produce one of two possible outcomes, failure and success.
3. The experiment continues until a total number of x successes.

The Negative Binomial distribution can be formed from a mixed distribution of the Poisson distribution and the Gamma distribution. By using Equation (1) and (4), it can be shown that this distribution has two parameters (α and τ). The probability density function is

$$p_X(x) = \binom{x+\alpha-1}{\alpha-1}(1-\tau)^x\tau^\alpha, \tag{7}$$

with $x = 0,1,2,\ldots$ [15].

2-Poisson Distribution

The 2-Poisson distribution is a distribution with three parameters (λ_1, λ_2, and p). The probability function for the 2-Poisson distribution is

$$p_X(x) = \frac{e^{-\lambda_1}{\lambda_1}^x}{x!}p + (1-p)\frac{e^{-\lambda_2}{\lambda_2}^x}{x!}, \tag{8}$$

with $x = 0,1,2,\ldots$ [16]. Some of the applications of this distribution can be seen in [17]-[18].

Bühlmann's Credibility

Bühlmann's credibility is a credibility model with the best accuracy approach. In this model, the number of policyholders observed is assumed to be the same every year. Premiums are determined based on a linear model between past data and theoretical premiums. The parameters used in the Bühlmann's credibility model are as follows:

1. The average value of individual claims or premiums and the expected values

$$\boldsymbol{\mu(\theta) = E(X|\Theta)}, \tag{9}$$

$$\boldsymbol{\mu = E\big(\mu(\theta)\big)}, \tag{10}$$

2. The variance of the hypothetical mean

$$\boldsymbol{a = Var\big(\mu(\theta)\big) = Var(\Theta)}, \tag{11}$$

3. Variance process and the expected value of variance

$$\boldsymbol{v(\theta) = Var(X|\Theta)}, \tag{12}$$

$$\boldsymbol{v = E\big(v(\theta)\big)}, \tag{13}$$

4. Credibility coefficient

$$\boldsymbol{K = \frac{v}{a}}, \tag{14}$$

5. Credibility factor

$$\boldsymbol{Z = \frac{N}{N+K}}, \tag{15}$$

6. Credibility premium

$$P_C = Z\bar{X} + (1-Z) \tag{16}$$

[19].

Goodnes-of-Fit Test

Suppose m is the largest value in the distribution of the observed data and r is the number of parameters to be estimated, then with several predicted parameters, statistics

$$\chi^2 = \sum_{x=0}^{m} \frac{(n_x - np_X(x))^2}{npx^{(x)}} \qquad (17)$$

asymptotically spread χ^2 with degrees of freedom $m - r$.

Hypothesis testing is one of the statistical tests carried out for testing the suitability of the parameter β_i which is made with the following hypothesis:
$H_0: \theta = \beta_i$, (data has a distribution that matches the distribution of the test)
$H_1: \theta \neq \beta_i$. (data does not have a distribution that matches the distribution of the test)
By using the values of the calculated chi-square and the table chi-square, the following decision rules apply: If $\chi^2_{calculated} \leq \chi^2_{table}$ then the null hypothesis is accepted and if $\chi^2_{calculated} > \chi^2_{table}$ then the null hypothesis is rejected, by setting the alpha value as well as the degrees of freedom of the Chi-Square distribution [20].

RESULTS AND DISCUSSION

Estimating Bühlmann's Credibility Parameters using Negative Binomial Distribution

Bühlmann's credibility model with claims of Negative Binomial distribution can be derived by providing an estimate of the credibility parameter assuming the frequency of claims with a Negative Binomial distribution. We know that the Negative Binomial distribution is a mixed distribution of the Poisson distribution and the Gamma distribution. Suppose $X|\Theta \backsim \text{Poisson}(\theta)$ and $\Theta \backsim \text{Gamma}(\alpha, \tau)$, it can be proven that X has a Negative Binomial distribution with parameters α and τ [21]. The following formula is given for the Bühlmann's credibility parameters using this distribution assumption.

Hypothetical Mean and the Expected Value for Negative Binomial Model

The hypothetical mean for Negative Binomial model can be determined using Equation (9). Since $X|\Theta \backsim \text{Poisson}(\theta)$ then $\mu(\Theta) = E(X|\Theta = \Theta) = \theta$. The expected value of the hypothetical mean or known as the individual premium (μ) can be determined by Equation (10) as follows:

$$\mu = E\big(\mu(\Theta)\big) = E(\Theta).$$

Since Θ has Gamma distribution (α, τ), then according to Equation (5),

$$\mu = \frac{\alpha}{\tau}. \qquad (18)$$

To determine the credibility coefficient, it is necessary to estimate the value of parameter a. The value of a is the variance of the hypothetical mean. Using Equation (11), the formula for the variance value of the hypothetical mean can be determined as follows:

$$a = Var\big(\mu(\theta)\big) = Var(\Theta) = E(\Theta^2) - \big(E(\Theta)\big)^2.$$

Since Θ has Gamma distribution, then according to Equation (6),

$$a = \frac{\alpha}{\tau^2}. \qquad (19)$$

Variance Process and the Expected Value for Negative Binomial Model

The variance process formula and the expected value can be determined using Equation (12) and (13). Since $X|\Theta \backsim \text{Poisson}(\theta)$ then $v(\theta) = Var(X|\Theta = \Theta) = \theta$. The expected value of the variance process (v) ishypothetical$v = E\big(v(\Theta)\big) = E(\Theta)$

$$v = \frac{\alpha}{\tau}. \qquad (20)$$

Bühlmann's Credibility Coefficient for Negative Binomial Model

The credibility coefficient is the ratio between the expected value of variance process and variance of the hypothetical mean. By using Equation (11), (19), and (20), it is obtained

$$K = \frac{v}{a} = \tau. \tag{21}$$

Bühlmann's Credibility Premium for Negative Binomial Model

After obtaining the formula from the credibility parameter for the Negative Binomial model, it can be formulated a formula for determining the premium with the Negative Binomial model using Equation (15) and (16) as follows:

$$P_C = Z\bar{X} + (1 - Z)\,\mu, \tag{22}$$

with

$$Z = \frac{N}{N+K} = \frac{N}{N+\tau},$$
$$\bar{X} = \frac{\sum_{i=1}^{N} x_i(n_{x_i})}{(n_{x_i})}.$$

$\bar{X}$ is the average of the observed data while μ is the individual premium which can be determined using Equation (18). The Z variable is also called the Bühlmann's credibility factor for the frequency of claims with a Negative Binomial distribution, where K is a credibility coefficient that satisfies Equation (21).

Estimating Bühlmann's Credibility Parameters using 2-Poisson Distribution

The following formula is given for the Bühlmann's credibility parameters using this distribution assumption.

Hypothetical Mean and the Expected Value for 2-Poisson Model

As before, the hypothetical mean for 2-Poisson model can be determined using Equation (9). The 2-Poisson distribution gives that $X|\Theta \backsim \text{Poisson}(\theta)$ and

$$\Theta \backsim u(\theta) = \begin{cases} p & : \theta = \lambda_1 \\ 1-p & : \theta = \lambda_2. \end{cases} \tag{23}$$

Since $X|\Theta \backsim \text{Poisson}(\theta)$, then according to Equation (2),

$$\mu(\theta) = E(X|\Theta = \theta) = \theta. \tag{24}$$

The expected value of the hypothetical mean (μ) is

$$\begin{aligned} \mu &= E(\mu(\Theta)) = E(\Theta) \\ &= p\lambda_1 + (1-p)\lambda_2 \\ &= p(\lambda_1 - \lambda_2) + \lambda_2. \end{aligned} \tag{25}$$

Furthermore, the variance of the hypothetical mean can be determined using Equation (11) and (23) as follows:

$$\begin{aligned} a = Var(\mu(\theta)) &= Var(\Theta) = E(\Theta^2) - (E(\Theta))^2 \\ &= \lambda_1{}^2 p + \lambda_2{}^2(1-p) - (p(\lambda_1 - \lambda_2) + \lambda_2)^2 \\ &= \lambda_1{}^2 p + \lambda_2{}^2(1-p) - (p^2(\lambda_1 - \lambda_2)^2 + 2p(\lambda_1 - \lambda_2)\lambda_2 + \lambda_2{}^2) \\ &= (\lambda_1{}^2 - \lambda_2{}^2)p + \lambda_2{}^2 - [(p(\lambda_1 - \lambda_2) + 2)p(\lambda_1 - \lambda_2) + \lambda_2{}^2] \\ &= \frac{(\lambda_1{}^2 - \lambda_2{}^2)p}{(\lambda_1 - \lambda_2)(\lambda_1 + \lambda_2)} - [(p(\lambda_1 - \lambda_2) + 2)p(\lambda_1 - \lambda_2)] \\ &= [(\lambda_1 + \lambda_2) - (p(\lambda_1 - \lambda_2) + 2)p(\lambda_1 - \lambda_2)] \\ a &= [(1-p)(\lambda_1 + \lambda_2) - 2]p(\lambda_1 - \lambda_2). \end{aligned} \tag{26}$$

Variance Process and the Expected Value for 2-Poisson Model

The variance process and the expected value can be determined using Equation (12) and (13). Since $X|\Theta \backsim \text{Poisson}(\theta)$ then according to Equation (3), we get $v(\theta) = Var(X|\Theta = \theta) = \theta$. The expected value of the variance process (v) can be obtained using Equation (23), which is as follows:

$$\begin{aligned} v &= E\big(v(\Theta)\big) = E(\Theta) \\ &= p\lambda_1 + (1-p)\lambda_2 \\ v &= p(\lambda_1 - \lambda_2) + \lambda_2. \end{aligned} \tag{27}$$

Bühlmann's Credibility Coefficient for 2-Poisson Model

The last part of determining Bühlmann's credibility premium is the need to assign a credibility coefficient value. By using Equation (14), (26), and (27), it is obtained

$$\begin{aligned} K &= \frac{v}{a} \\ K &= \frac{p\lambda_1 + (1-p)\lambda_2}{[(1-p)(\lambda_1+\lambda_2) - 2]p(\lambda_1 - \lambda_2)}. \end{aligned} \tag{28}$$

Bühlmann's Credibility Premium for 2-Poisson Model

Bühlmann's credibility premium is obtained by using Equation (15) and (28) so that the Bühlmann credibility value for the 2-Poisson model is

$$P_C = Z\bar{X} + (1-Z)\,\mu, \tag{29}$$

with

$$Z = \frac{N}{N+K} = \frac{N}{N + \frac{p\lambda_1+(1-p)\lambda_2}{[(1-p)(\lambda_1+\lambda_2)-2]p(\lambda_1-\lambda_2)}},$$

$$\bar{X} = \frac{\sum_{i=1}^{N} x_i (n_{x_i})}{(n_{x_i})}.$$

The Z variable is also known as the Bühlmann's credibility factor for the frequency of claims with 2-Poisson distribution. The value of μ can be determined by Equation (25) and $\bar{X}$ is the average of the observed data.

Application on Data

The data used for the application of the model is data on the distribution of claims (n_x) on the motor vehicle insurance portfolio in Singapore [22]. The number of claims occurred from 1993 to 2001 which can be seen in Table 1.

Table 1. Portfolio of the number of claims from observation results (millions of dollars)

x	n_x
0	178.080
1	19.224
2	1.859
3	177
4	11
5	1
>5	0
Total	n_x = 199.352

Before applying to the model, it is first tested whether the claim frequency data in Table 1 has a Negative Binomial and 2-Poisson distribution or not. Testing the distribution of data was carried out using the chi-square test.

Negative Binomial Distribution Test

The Negative Binomial distribution has two estimators for the parameters ($\hat{\alpha}$ and $\hat{\tau}$) based on Equation (7). Parameter estimates can be obtained using the moment method. Based on Table 1, the average value of the number of claims $\bar{X} = 0{,}1179923$, $\bar{X}^2 = 0{,}01392218$, and variance $S^2 = 0{,}12881027$. The values of $\hat{\alpha}$ and $\hat{\tau}$ are

$$\hat{\alpha} = \frac{\bar{X}^2}{S^2 - \bar{X}} = \frac{0{,}01392218}{0{,}01081797} = 1{,}28694966,$$

$$\hat{\tau} = \frac{\bar{X}}{S^2 - \bar{X}} = \frac{0{,}1179923}{0{,}01081797} = 10{,}9070648.$$

The distribution test steps carried out are as follows:

a. Hypothesis formulation.
   H_0 : data has Negative Binomial distribution.
   H_1 : data is not distributed Negative Binomial.

b. Calculates the probability for each claim frequency and the expected value.
   The probability is calculated for each claim frequency (p_x) for each x in the table data and based on the parameter estimator values and the Negative Binomial distribution formula, then for $x = 0$:

$$p_x = \binom{\alpha + x - 1}{x}\left(\frac{\tau}{1+\tau}\right)^{\alpha}\left(\frac{1}{1+\tau}\right)^{x}$$

$$p_0 = \binom{1{,}28694966 \ + 0 - 1}{0}\left(\frac{10{,}9070648}{1 + 10{,}9070648}\right)^{1{,}28694966}\left(\frac{1}{1 + 10{,}9070648}\right)^{0}$$

$$= 0{,}89324646.$$

   For the expected value (np_x):

$$np_0 = 199.352(0{,}89324646) = 178.070{,}47.$$

   In the same way, it will produce a portfolio in Table 2.

Table 2. Portfolio of the number of claims with Negative Binomial distribution

x	n_x	$np_X(x)$
0	178.080	178.070,47
1	19.224	19.246,38
2	1.859	1.848,29
3	177	170,07
4	11	15,31
5	1	1,36
>5	0	0,12
Total	199.352	199.352

c. Determine the value of the chi-square test statistic.
   The chi-square test statistic determined by Equation (17) is obtained $\chi^2_{calculated} = 1{,}6912$. With a 95% confidence interval, then $\alpha = 0{,}05$ and χ^2_{table} with degrees of freedom $m - r \ = \ 5 - 2 = 3$ is 7,8147. Based on the values of $\chi^2_{calculated}$ and χ^2_{table}, it can be concluded that $\chi^2_{calculated} < \chi^2_{table}$ and the null hypothesis is accepted. Thus, the data used in this study has met the requirements for a Negative Binomial distribution.

2-Poisson Distribution Test

There are three parameters that are assumed to have 2-Poisson distribution based on

Equation (8). By using the moment method, it is obtained that $\hat{p} = 0{,}77481$, $\widehat{\lambda_1} = 0{,}06191$, and $\widehat{\lambda_2} = 0{,}31092$. The distribution test steps carried out are as follows:

a. Hypothesis formulation.
   H_0 : data has 2-Poisson distribution.
   H_1 : data is not distributed 2-Poisson.

b. Calculates the probability for each claim frequency and the expected value.
   The probability is calculated for each claim frequency (p_x) for each x in the table data and based on the parameter estimator values and the 2-Poisson distribution formula, the portfolio is obtained in Table 3.

Table 3. Portfolio of the number of claims with 2-Poisson distribution

x	n_x	$np_X(x)$
0	178.080	178.081,52
1	19.224	19.217,82
2	1.859	1.868,36
3	177	170,53
4	11	12,89
5	1	0,79
>5	0	0,04
Total	199.352	199.352

c. Determine the value of the chi-square test statistic.
   The chi-square test statistic determined by Equation (17) is obtained $\chi^2_{calculated} = 0.6249634$. With a 95% confidence interval, then $\alpha = 0{,}05$ and χ^2_{table} with degrees of freedom $m - r = 4 - 3 = 1$ is 3,8414. Based on the values of $\chi^2_{calculated}$ and χ^2_{table}, it can be concluded that $\chi^2_{calculated} < \chi^2_{table}$ and the null hypothesis is accepted. Thus, the data used has met the requirements for a 2-Poisson distribution.

Parameter Estimation of Bühlmann's Credibility Premium

Based on the estimation of distribution parameter values that have been obtained, it can be determined the parameter estimation of Bühlmann's credibility premium. Determination of the estimated parameter value using Equation (18)-(29). The alleged results are presented in the Table 4.

Table 4. Parameter estimation of Bühlmann's credibility premium from the data used

Parameter Estimation	Negative Binomial	2-Poisson
$\bar{X}$	0,1179	0,1179
$\hat{\mu}$	0,1180	0,1179
$\hat{a}$	0,0108	0,3696
$\hat{v}$	0,1180	0,1179
$\hat{K}$	10,9071	0,3191
$\hat{Z}$	0,9999	0,9494
P_C	0,11790	0,11799

Based on the results in Table 4, it can be seen that the estimated frequency of claims in the next period assuming the data is Negative Binomial and 2-Poisson distribution is 0.11790 and 0.11799, respectively. This means that in the Negative Binomial model it is estimated that there will be 11.79% of policyholders who will make insurance claims in the next period, while in the 2-Poisson model it is estimated that there will be 11.799% of policyholders who will make insurance claims. The two models give fairly close results, this is because the Bühlmann's credibility factor for the two distribution models is quite large, namely each 0.9999 and 0.9494.

CONCLUSIONS

Bühlmann's credibility formula has been given for data with Negative Binomial and 2-Poisson distribution. Both distributions are mixed distributions. Mixed distributions are quite well used in determining premiums with credibility. This is because in a mixed distribution, the distribution of claim frequency often depends on the distribution of risk. The simulation on the data shows that the premium value obtained is very good with high credibility for both distributions modeled.

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