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Systematic Literature Review on Adjustable Robust Shortest Path Problem

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ABSTRACT

Optimization model with parameter uncertainty can be solved using the Adjustable Robust Counterpart (ARC). The ARC method has two solutions from Robust Optimization (RO) in dealing with the problem of data uncertainty caused by errors in data measurement. By assuming the constraint function's uncertainty using the Robust Counterpart (RC) methodology, it is possible to add uncertainty that only exists in the constraint function into the linear optimization problem model. This paper discusses the Systematic Literature Review (SLR) on ARC for Shortest Path Problem (SPP) using the Reporting Method of Choice for Systematic Review and Meta-Analysis (PRISMA) by presents database mining algorithms for previous articles and related topics sourced from Scopus, Science Direct, Dimensions, and Google Scholar. Four stages of the algorithm are used, namely Identification, Screening, Eligibility, and Included. The analysis is performed on two datasets, Dataset obtained at the Eligibility and Include stage. The output may then be analyzed using the "bibliometrix" package in the RStudio software, and a link to the "glossy web interface" is then formed using the "biblioshiny()" command. In conclusion, there has been no research that discusses the ARC optimization model for SPP specifically. This can be a novelty for further research on this topic.

Keywords: mathematics; operation research; shortest path problem; uncertainty; adjustable robust counterpart; robust optimization

INTRODUCTION

Network problems are frequently seen in real-world situations, such as in distribution, telecommunications, transportation and logistics, and routing algorithms. The complexity of network problems, costs, and time must therefore be considered while solving network problems. Optimization problem solving can use operations research. Churchman, Arkoff, and Arnoff in 1950 AD, argued that operations research is the application of scientific methods, techniques and equipment in dealing with problems that arise in company operations with the aim of finding optimal solutions.

As an explanation, the shortest path problem in classic operations research can be used to solve optimization problems (SPP). Referring to [1] that SPP is the simplest and most studied model for combinatorial optimization problems. However, in fact, in the real world, there must be uncertainty factors that are often encountered and must be faced.

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For example, in the distribution of logistics transportation networks, such as accidents, traffic jams at certain hours, and weather conditions that can affect time and cost are obstacles that cause uncertainty in the length of the route.

The uncertainty factors can be formulated using an indeterminate optimization model. By assuming parameters that are not known with certainty, but are in an indeterminate set U. From this general model of indeterminate optimization, it can be developed into a formulation for the optimization of indeterminate SPP by determining the parameters of uncertainty first from the data obtained. Then, Robust Optimization (RO) method can be applied which has computationally tractable properties. Referring to [2] we get an indeterminate Robust Counterpart optimization model for linear programming from an indeterminate linear program which is equivalent to an explicitly computationally tractable problem where the indeterminate set U must be computationally tractable. Then from the Robust Counterpart model which refers to [3] it is discussed about the SPP which was developed into an indeterminate SPP to apply the Robust method to the SPP.

This article discusses the Systematic Literature Review (SLR) which uses a two-stage optimization model using Adjustable Robust Counterpart (ARC) for Robust Optimization development on SPP. Two-stage optimization has a decision variable whose value must be determined immediately without waiting for more or complete information from the uncertainty parameter or is called here-and-now. While "wait and see" refers to the decision variable whose value can be waited and decided after knowing all the data information from the uncertainty parameter. The purpose of the SLR is to obtain comprehensive objectives and summaries and results of critical analysis of previous research relevant to the topic being studied. The lack of research on ARC optimization models, especially for SPP problems, supports SLR on this topic. The research in this article applies the SLR by using the Preferred Reporting Items for Systematic Review and Meta-Analysis (PRISMA) method. The PRISMA method presents algorithms and procedures for selecting articles that are ready for review, analysis of bibliometric maps based on certain linkages, an approach to theme evolution, and determining research gaps and recommendations for further research.

To be precise, the PRISMA method in this article mainly refers to [4]. The PRISMA method provides an accurate standard methodology as a protocol for describing article selection criteria, search strategies, data extraction, and data analysis procedures [5]. For writing this article, we apply the PRISMA method which includes the determination of bibliometric map analysis using RStudio software with the "R-bibliometrix" command, which will be explained in more detail later. This article is structured as follows: Part 2 presents a brief discussion of a material and the steps of the method used, namely the PRISMA Method. Section 3 presents the results of research and discussion in the results of bibliometric maps, relation visualization using RStudio software, evolution analysis, results of SLR analysis. Finally, Section 4 presents the conclusion from the explanation in the previous section.

METHODS

Shortest Path Problem

According to [1], the Shortest Path Problem (SPP) is one of the simplest and most studied problems using combinatorial optimization problems. Suppose there is a network $G = (V, A))$ with a set of vertices V and a set of arcs A. An example of a case for SPP is to find a directed path from vertex s to a single vertex t with a minimum total length, with respect to the given length function $\mathbf{c}: \mathbf{A} \rightarrow \mathbb{R}_+$.The binary optimization model for the shortest path problem is

$$\min\{\mathbf{c}^{\mathbf{T}}\mathbf{x} : \mathbf{A}\mathbf{x} = \mathbf{b}, \mathbf{x} \in \{0,1\}^{A}\} \quad (1)$$

where A is the vertex-arc incident matrix, b is the vector in $\mathbb{R}^V$ with entries $b_s = 1$, $b_t = -1$ and $b_j = 0$ for each setiap $j \in V\backslash\{s,t\}$, and c is the arc length vector. Note that the constraint matrix is totally unimodular or every sub-square matrix has a determinant 0, $+1, -1$ therefore it can be considered a linear optimization problem equally.

Robust Optimization (RO)

Uncertainty is often encountered and must be faced in the application of optimization in the real world. These uncertainty factors can be formulated using an indefinite optimization model, assuming parameters that are not known for certain but are in an indeterminate set that has been discussed in research [6]. Problems with uncertainty in mathematical optimization can be solved using Robust Optimization (RO). RO is a methodology in dealing with the problem of data uncertainty on parameters. Referring to [2] [7], the general model of the linear optimization problem can be formulated as follows.

$$\min_{x}\{\mathbf{c}^{\mathbf{T}}\mathbf{x}: \mathbf{A}\mathbf{x} \leq \mathbf{b}\} \quad (2)$$

where $\mathbf{c} \in \mathbb{R}^n, \mathbf{x} \in \mathbb{R}^n, \mathbf{A} \in M_{m,n}(\mathbb{R}), \mathbf{b} \in \mathbb{R}^m$ assuming the parameter $(\mathbf{c}, \mathbf{A}, \mathbf{b})$ is not known with certainty, but it is in an indeterminate set U. Referring to [8] the general model of the indeterminate linear optimization problem is as follows.

$$\min_{x} \mathbf{c}^{\mathbf{T}}\mathbf{x}: \mathbf{A}\mathbf{x} \leq \mathbf{b} | (\mathbf{c}, \mathbf{A}, \mathbf{b}) \in \mathcal{U} \quad (3)$$

There are three assumptions related to decision making in Robust Optimization problems, namely as follows.

1. All decision variables $\mathbf{x} \in \mathbb{R}^n$ represent "here and-now" decisions, that is, all decision variables are decided simultaneously before the actual data appears.
2. The decision maker is responsible for the decisions made when the uncertainty parameters in the actual data are in the indeterminate set $\mathcal{U}$.
3. Constraints in the Robust Optimization problem are "hard", ie all constraints should not be violated, even though the decisions of the uncertainty parameters used are in the indeterminate set $\mathcal{U}$.

In the study [8] there are four additional assumptions related to the general model of the Robust Optimization problem as follows.

1. If there is uncertainty in the objective function, then the problem can be changed so that the uncertainty appears in the constraint function. From the general model (3), the objective function $\mathbf{c}^{\mathbf{T}}\mathbf{x}$ is replaced by an additional variable t $\in \mathbb{R}$ with t $\geq \mathbf{c}^{\mathbf{T}}\mathbf{x}$, so that the reformulation form in (4) shows that the uncertainty no longer exists in the objective function.
2. If there is uncertainty in $\mathbf{b}$, then add the variable $x_{n+1} = 1$ on the right side.

$$\min_{x,t}\{\mathrm{t}: \mathbf{c}^{\mathbf{T}}\mathbf{x} \leq \mathrm{t}, \mathbf{A}\mathbf{x} \leq \mathbf{b} | (\mathbf{c}, \mathbf{A}, \mathbf{b}) \in \mathcal{U}\}$$

or $\quad (4)$

$$\min_{x,t}\{\mathrm{t}: \mathbf{c}^{\mathbf{T}}\mathbf{x} - \mathrm{t} \leq 0, \mathbf{A}\mathbf{x} \leq \mathbf{b} | (\mathbf{c}, \mathbf{A}, \mathbf{b}) \in \mathcal{U}\}$$

3. The set of uncertainty $\mathcal{U}$ can be replaced by convex hull $\mathcal{U}$, which is the smallest convex set of $\mathcal{U}$. Finding a feasible solution of $\mathcal{U}$ is equivalent to obtaining the supremum of the

left-hand side constraints $\mathcal{U}$, where the objective function will produce an optimal value if the convex hull $\mathcal{U}$ has a maximum value.

4. Robustness to $\mathcal{U}$ can be formulated constraint-wise, namely robustness in Robust optimization problems can be seen in each constraint and the indeterminate set $\mathcal{U}$ is closed and convex.

Robust Counterpart (RC)

Referring to research [9], The uncertainty in the indeterminate linear optimization problem model can be formed by loading the uncertainty that only exists in the constraint function by assuming its uncertainty and can be solved using the Robust Counterpart (RC) methodology. RC is a single deterministic problem that has removed its uncertainty based on assumptions.

All the uncertainties that exist in the model can be collected in the constraint matrix **A** with $\mathbf{A} \in \mathcal{U}$ based on assumptions, so that the Robust Optimization problem becomes:

$$\min_{x}\{\mathbf{c}^{\mathrm{T}}\mathbf{x}: \mathbf{Ax} \le \mathbf{b} \mid \mathbf{A} \in \mathcal{U}\} \tag{5}$$

where $\mathbf{x} \in \mathbb{R}^n$, $\mathbf{b} \in \mathbb{R}^n$, $\mathbf{A} \in M_{m,n}(\mathbb{R})$, and $\mathcal{U}$ is an indefinite set. Then the constraint matrix **A** can be expressed in terms of a primitive indefinite parameter $\zeta \in \mathcal{Z}$, where $\mathcal{Z} \in \mathbb{R}^L$ is a primitive indefinite set, so that it can be expressed as:

$$\min_{x}\{\mathbf{c}^{\mathrm{T}}\mathbf{x}: \mathbf{A}(\zeta)x \le \mathbf{b} \mid \zeta \in \mathcal{U}\} \tag{6}$$

Remember that the nature of the uncertainty assumption Because Robust Optimization works on a constraint-by-constraint basis, the model can be centered on a single constraint as shown below.

$$(\bar{\mathbf{a}} + \boldsymbol{P}\zeta)^T\mathbf{x} \le \mathbf{b}, \forall \zeta \in \mathcal{Z} \tag{7}$$

where $(\bar{\mathbf{a}} + P\zeta)$ is an affine function over the primitive indeterminate parameters $\zeta \in \mathcal{Z}$, $\bar{\mathbf{a}} \in \mathbb{R}^n$, dan $\mathbf{P} \in M_{m,n}(\mathbb{R})$. Given that in dealing with data uncertainty, the solution obtained from the Robust Optimization problem is the "best worst-case" solution, that is, the solution obtained is the best solution of all the worst possible conditions. In constraint (7), it is necessary to reformulate the worst-case as follows.

$$\begin{aligned} &\max_{\zeta}(\bar{\mathbf{a}} + \mathbf{P}\zeta)^{\mathrm{T}}\mathbf{x} \le \mathbf{b}, \forall \zeta \in \mathcal{Z}, \\ &\max_{\zeta} \bar{\mathbf{a}}^{\mathrm{T}}\mathbf{x} + (\mathbf{P}\zeta)^{\mathrm{T}}\mathbf{x} \le \mathbf{b}, \forall \zeta \in \mathcal{Z}. \end{aligned} \tag{8}$$

Since $(\bar{\mathbf{a}}^{\mathrm{T}}\mathbf{x})$ does not contain ζ, the constraint can be formulated as:

$$\bar{\mathbf{a}}^{\mathrm{T}}\mathbf{x} + \max_{\zeta}(\mathbf{P}^{\mathrm{T}}\mathbf{x})^{\mathrm{T}}\zeta \le \mathbf{b}, \forall \zeta \in \mathcal{Z} \tag{9}$$

The problem in optimization with uncertainty is calculating how and when an indeterminate optimization problem can be reformulated so that its RC is a computationally tractable optimization problem and calculating the approximate RC with a problem that has been shown to be computationally tractable.

Adjustable Robust Counterpart (ARC)

Referring to research [6] [10], the two-stage optimization of the basic paradigm of Robust Optimization, namely having a decision variable whose value must be determined immediately without waiting for more or complete information from the uncertainty parameter or called "here-and-now", while the decision variable whose value can be postponed and determined after knowing all data information from the uncertainty parameter is called wait-and-see. Some of the decision variables can be adjusted at a later time according to the decision rule, which is a function of some or all of the indeterminate pieces of data. The general model of the Adjustable Robust Counterpart (ARC) is as follows:

$$\min_{x,y(\zeta)} \{\mathbf{c}^{\mathrm{T}}\mathbf{x}: \mathbf{A}(\zeta)\mathbf{x} + \mathbf{B}\mathbf{y}(\zeta) \leq \mathbf{b}, \forall \zeta \in \mathcal{Z}\} \qquad (10)$$

where $\mathbf{x} \in \mathbb{R}^n$ is a "here-and-now" first-stage decision made before $\zeta \in \mathbb{R}^L$is realized, $\mathbf{y} \in \mathbb{R}^k$ is a wait-and-see second-stage decision that can be adapted to the actual data , and $\mathbf{B} \in \mathbb{R}^{mxk}$is a definite coefficient matrix.

However, Adjustable Robust Counterpart (ARC) is a complex problem unless it limits the $\mathbf{y}(\zeta)$ function to a certain class. In practice, $\mathbf{y}(\zeta)$ is often approached with an affine or linear decision rule:

$$\mathbf{y}(\zeta) = \mathbf{y}^0 + \mathbf{Q}\zeta \qquad (11)$$

where $\mathbf{y}^0 \in \mathbb{R}^k$ and $\mathbf{Q} \in \mathbb{R}^{kxL}$are coefficients in the decision rule, that is, to be optimized. Thus, the reformulation of equation (12) is as follows:

$$\min_{x,y(\zeta)} \{\mathbf{c}^{\mathrm{T}}\mathbf{x}: \mathbf{A}(\zeta)\mathbf{x} + \mathbf{B}\mathbf{y}^0 + \mathbf{B}\mathbf{Q}\zeta \leq \mathbf{b}, \forall \zeta \in \mathcal{Z}\} \qquad (12)$$

Adjustable Robust Optimization is less conservative than the classic Robust Optimization approach, as it results in more flexible decisions that can be adapted to the realization of the amount of data at a given stage. More precisely, the Adjustable Robust Counterpart (ARC) Optimization yields an optimal objective value that is at least as good as the standard Robust Optimization approach.

Preferred Reporting Items for Systematic review and Meta Analyzes (PRISMA)

In this study, the literature search was conducted using the Preferred Reporting Items for Systematic review and Meta Analyzes (PRISMA), but before discussing the PRISMA method to be used, it is necessary to determine what materials are needed to support the SLR process in this article [11] [12].

The first stage is choosing keywords that are relevant to the research topic of the review article, namely the Shortest Path Problem using Adjustable Robust Counterpart. Then these keywords will be grouped into seven groups as shown in Table 1. General keywords such as Shortest Path Problem, Optimization Model, Robust Optimization, and Adjustable Robust Counterpart are discussed in Keyword A, B, and C. Then it is devoted again to keywords D, E, F, and G which are a combination of keywords A, B, and C by combining all three.

Table 1. Keywords Classification

Code	Keywords
A	"shortest path problem" AND "uncertainty"
B	("robust counterpart" OR "robust optimization") AND ("uncertainty" OR "polyhedral uncertainty set")
C	("adjustable robust counterpart" OR "adjustable robust optimization") AND ("uncertainty" OR "polyhedral uncertainty set")

D	A AND B ("shortest Path Problem" AND "uncertainty") AND ("robust counterpart" OR "robust optimization") AND ("uncertainty" OR "polyhedral uncertainty set")

Code	Keywords
E	A AND C ("shortest Path Problem" AND "uncertainty") AND ("adjustable robust counterpart" OR "adjustable robust optimization") AND ("uncertainty" OR "polyhedral uncertainty set")
F	B AND C ("robust counterpart" OR "robust optimization") AND ("uncertainty" OR "polyhedral uncertainty set") AND ("adjustable robust counterpart" OR "adjustable robust optimization") AND ("uncertainty" OR "polyhedral uncertainty set")
G	A AND B AND C ("shortest Path Problem" AND "uncertainty") AND ("robust counterpart" OR "robust optimization") AND ("uncertainty" OR "polyhedral uncertainty set") AND ("adjustable robust counterpart" OR "adjustable robust optimization") AND ("uncertainty" OR "polyhedral uncertainty set")

After selecting keywords, the next step is to determine the source of the article database. The six keywords in Table 1 are entered into the database, in this study, four database sources will be used, namely Scopus, Science Direct, Dimensions and Google Scholar. This database serves as an ingredient in the PRISMA method obtained by mining data from four database sources with a collection under several conditions: (1). The database is contained in the form of publication of research articles or conference papers (2). This type of database is open source, has unlimited access via the internet and is conferencing (3). The search database is contained in article titles, abstracts, and keywords. (4) The database is a publication for the last 6 years, namely from 2012 to 2022 (5). The database is research in mathematics, mathematical sciences, or applied mathematics (this research area is only used in Scopus, Science Direct, and Dimensions) (6). The database uses all types of publications, journal names, aid, funding, country/region, and lastly (7). The database uses English.

Then the results of the total number of databases filtered under these conditions will be shown in Table 2.

Table 2. Database mining results from four sources

Code	Scopus	Science Direct	Dimensions	Google Scholar (using Publish or Perish)	Total
A	24	1	25	6	56
B	479	50	481	450	1460
C	16	5	24	18	63
D	15	1	11	0	27
E	16	0	24	0	40

F	0	5	0	17	22
G	0	-*	0	0	0
Total	550	62	565	491	

Note: *Database mining cannot be done because it has to use fewer boolean connectors (max 8 per field)

Based on Table 2, it can be seen that the more specific the keywords entered, the less number of databases obtained, even for keyword G, not a single article was identified.

After obtaining the supporting material for the SLR process in this article, a literature search can be carried out directly using the Preferred Reporting Items for Systematic review and Meta Analyzes (PRISMA). Referring to research [13] PRISMA provides clear guidelines in conducting a systematic literature review. In addition, PRISMA has also been proven to be able to improve the quality of literature reviews, both in terms of the methodology used and the results obtained [14]. PRISMA consists of four stages, namely identification, screening, eligibility, and inclusion. Throughout the stages in PRISMA, several criteria can be applied such as the type of writing desired (research articles, review articles, books, etc.), year of publication, language of instruction, discussion topics, and various other criteria [15] [16].

RESULTS AND DISCUSSION

Mining Data using PRISMA Method

After mining the article database and obtaining keywords with codes A to G as shown in table 2, the database is saved in the ".bib" file format then through the PRISMA Method algorithm. In the first stage or the first stage, the article database is identified using software Mendeley, the articles released at this stage are articles that cannot be identified because of the ".bib" file.

The second stage in the PRISMA method is screening. Where at this stage duplication is carried out with the help of Mendeley software, so that it will cause a change in the number of articles. Then, an assessment of categories based on and abstract was also carried out with several title checks. First, the article discusses the Shortest Path Problem in general and is specific to a particular problem. Second, the article uses the ARC methodology to solve the problem. After performing duplication for keywords A to C, we will discuss the relevance of the keywords to each other to see the potential for further research novelties, which will be discussed later.

Table 3. The results of selected articles using the PRISMA method

Identification(1)			Screening(2)		Eligibility(3)		Included(4)
Code	**Total**	**Indentified by Mendeley**	**Duplicate Check**	**Title and Abstract Screening**	**3 Stars***	**4 Stars****	**Fulltext Screening/ 5 Stars*****
D	27	27	27	15	13	8	4
E	40	40	39	16	10	5	1
F	22	5	5	16	12	5	4
		TOTAL		47	35	18	9

Note: *not/less relevant, **Dataset 1 for bibliometric mapping, ***Dataset 2 for literature review

The third stage in the PRISMA method, is feasibility. In this stage, categorization is carried out based on the number of stars, namely three stars and four stars. Three stars are articles with additional checks, namely articles that discuss the shortest path problem and eliminate articles that are not/less relevant. While the four stars are articles that discuss the shortest path problem using an optimization model with Robust Optimization, so that 35 articles are obtained for three stars and 18 articles that are more specific for four stars. The database of 18 articles at this stage is called Dataset 1, which is used for bibliometric mapping and using RStudio software.

The fourth or final stage of the PRISMA method is the Included stage. Selected articles called Dataset 2 (five stars) will be used to analyze SLR in the form of state-of-the-art results tables. There are nine final articles selected with additional examination, namely articles that discuss the Shortest Path Problem using Adjustable Robust Counterparts in dealing with several different problems.

Mining Data using RStudio Software

Eighteen articles in Dataset 1 in the eligibility stage above, have a time span between 2013 to 2021 with 5 years from publication, 21.3 average citations per document, 2,878 average citations per year per document, and 280 references. Main information about the data obtained in the RStudio software which is used to support the emergence of bibliometric mapping that discusses the relationship between each author's keywords as shown in Figure 1.

In the process in the RStudio software, we use the 'biblioshiny()' command to create a link to the “shiny web interface”. Louvain Cluster algorithm is used with a minimum number of fifty nodes. We set the minimum number of edges to 1, meaning that the bibliometric map that will appear has at least one relationship between one node and another.

Figure 1 is a bibliometric map of the author's keyword column. 22 keywords appeared, and three groups were based on color. Clusters provide information on keyword grouping based on eighteen research topics in Dataset 1. The eight red clusters have the most relevant impact because they generate the keyword “Robust Optimization”, the word most relevant to the seven keywords in the abstract. Furthermore, the three blue clusters became the second most relevant keyword group, namely "Shortest Path Problem", "Graph Theory" and "Decision Making". Then, the eleven green clusters became the keyword group that appeared the most. In other words, the larger the size of the words/phrases that appear, the more those keywords are used in the research.

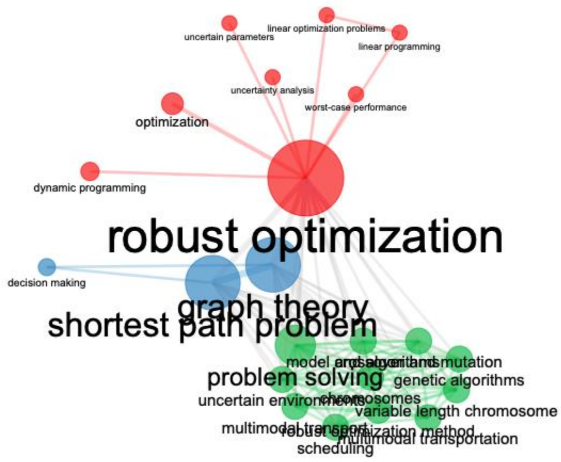


Figure 1. Bibliometrix Map of Author’s Keywords

Figure 2 is a bibliometric map of the titles. 13 titles with two groups classified by color. The six red clusters are titles related to the methods used in solving research problems, while the seven blue clusters are titles that discuss objects that need to be solved. There are large clusters including "Robust", "Shortest" and "Path". ” which are the three main highlights discussed in the research.

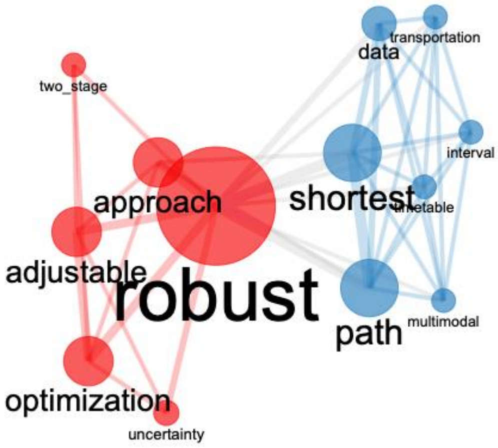


Figure 2. Bibliometrix of The Titles

Figure 3 is a bibliometric map of the abstract. 46 abstract keywords with 2 groups classified by color. 25 abstract keywords in red are abstract keywords related to the methods used in solving research problems, while 21 abstract keywords in blue are abstract keywords that discuss objects that need to be solved. It can be seen that in the red abstract keyword group there are several abstract keywords with larger circles, namely "optimization" "robust" and "uncertainty", this indicates that there have been many studies related to the topic of uncertainty using robust optimization.

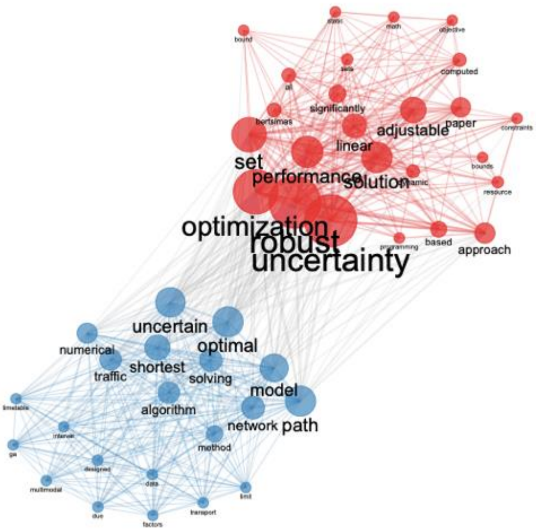


Figure 3. Bibliometrix Map of The Abstract

Evolution Analysis

The topic evolution analysis can be determined to provide important information regarding the differences in subtopics by the article authors based on the clusters obtained in the period 2013 to 2021. In this section, Dataset 2, which contains 9 article databases, is analyzed based on their evolution. The annual scientific production for dataset 2 can be seen in Figure 4. According to the annual scientific production output by the RStudio software, the result is that the most article production occurred in 2013, 2018 and 2019 which managed to publish two articles, respectively. Then followed by 2014, 2015, 2020 and 2021, each of which managed to publish one article. This means that research related to the topic of this article is fluctuating or experiencing increases and decreases from year to year.

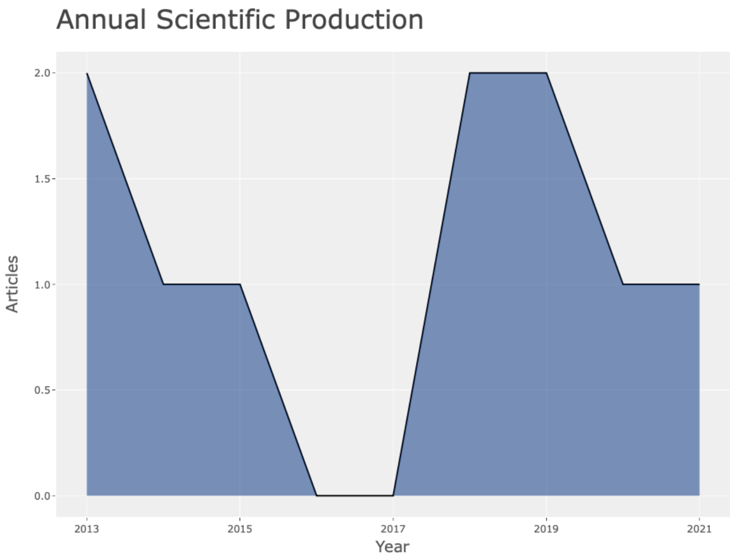


Figure 4. Annual Scientific Production

The author's keyword field for the author was selected since it is one of the most accurate representations of the keywords. The thematic evolution map discusses about the magnitude of relevance (centrality) plotted on the X-axis and the level of development (density) plotted on the Y-axis were used to identify these subjects, which were then used to study the development of themes in four different quadrants. The degree of inter-cluster interaction is determined by centrality. More specifically, centrality analyzes how closely a topic is related to other problems, or the degree of inter-cluster interaction. Furthermore, density represents the period of time that a topic has been developing between the terms in a certain cluster.

In Figure 5, based on the explanation of the X and Y axes, it is known that the upper right quadrant contains topics with high centrality and density, which means these topics can influence research and develop well. The lower right quadrant includes issues with strong centrality (able to control other problems) but weak density (not well developed). The lower left quadrant contains topics that are prone to centrality and density, meaning that these topics are less able to influence research and are not well developed. Finally, the upper left quadrant contains the opposite problem to the lower right quadrant.

Through this explanation, it can be seen that there are no topics that appear in the upper right quadrant, this means that there are no topics that so influence the research. In short, the ARC topic for the Shortest Path Problem specifically has not been studied much or even exists and is open for further related research. This can be a contribution opportunity for further research.

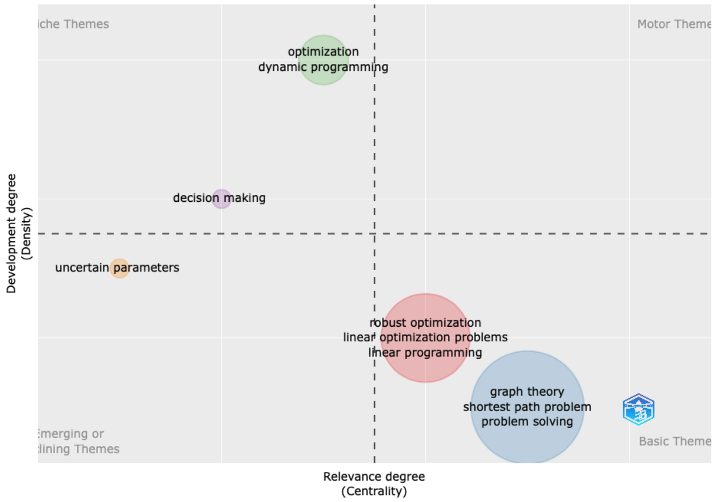
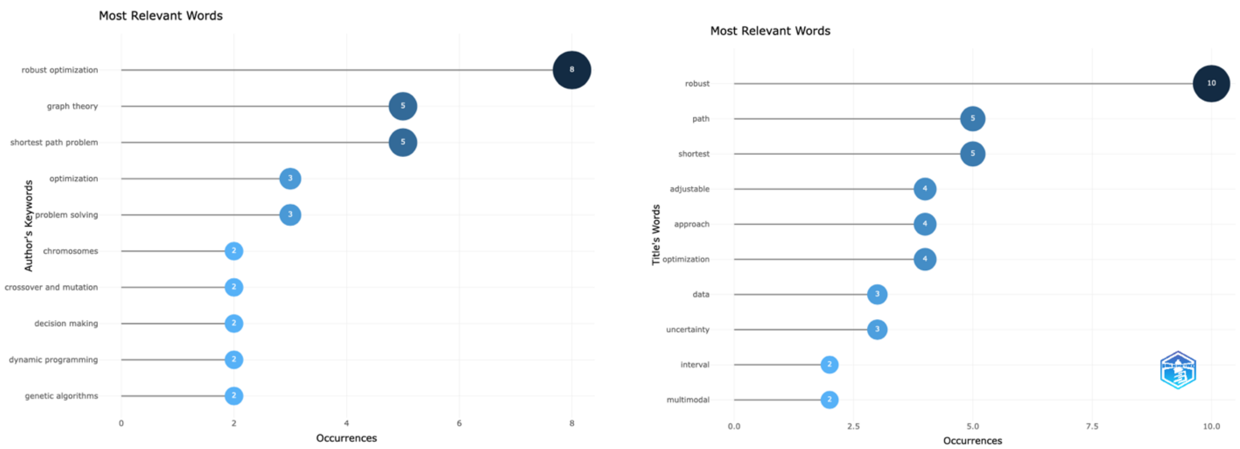


Figure 5. Thematic Map in Author's Keywords Field

Critical follow-up analysis is concerned with developing the most relevant words. The output of the most relevant words generated by the RStudio software has three types, namely unigram (maximum one word appears), bigram (maximum two words appear), and trigram (maximum three words appear). This study looks for the most relevant word trigrams in the abstract and keywords. This word relevance analysis serves to see which keywords often and rarely appear, examine which keywords still have the opportunity to be developed in research, and which keywords have been widely researched.

Figure 6a provides the ten most relevant words for various topics in the keyword field. The keyword field ranked first in "Robust Optimization" with eight relevant words, in the title field where "Robust" ranked first with ten relevant words. Based on the explanation of Figures 7a and 7b, it can be concluded that the ten relevant words produced do not represent the topic of this research simultaneously, namely Shortest Path Problem using Adjustable Robust Counterpart.



(a) (b)

Figure 6. (a) Most relevant words in the keywords field, (b) Most relevant words in titles field

The next important thing in the discussion of the analysis of the evolution of the article is a word-growth map which is one of the outputs. In the period 2013 to 2021, a word-growth plot map can be visualized, as shown in Figure 7. The Y-axis graph shows the number of articles written, the X-axis shows the year, and word growth is represented by a color plot. There are 10 classifications of word growth topics visualized with 10 different color plots.

For example, we can see that from 2013 to 2021 the topic of "robust optimization" experienced an increase in word growth, and from 2017 to 2019 it experienced a significant increase. It can also be seen that the topic "shortest path problem" has increased from 2017 to 2021. This means that research on these two topics is growing, but for the topic "shortest path problem" and the topic "adjustable robust optimization" has not been done much.

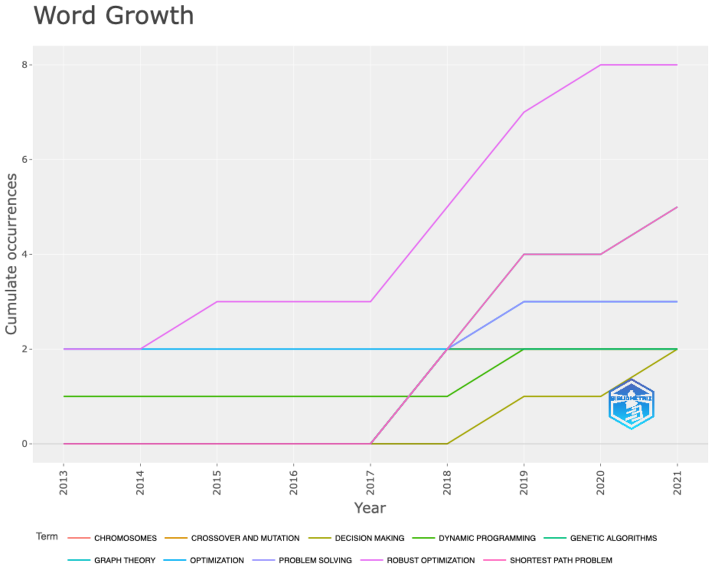


Figure 7. Word Growth Map

The Result of Systematic Literature Review

This section presents the study results from Dataset 2, namely a database containing six articles selected to the final stage as can be seen in Table 4. Articles in Dataset 2 were published within the 2013-2021 timeframe.

Based on the results of Dataset 2, it is known that the nine articles, when viewed in general, discuss network problems, especially the shortest path problem and use the Robust Optimization method and several developments of robust optimization such as Robust Shortest Path Problem (RSPP), Distributionally Robust Shortest Path Problem (DRSPP), Adjustable Robust Optimization, Two-stage Adjustable Robust Linear Optimization, Adjustable Robust Convex Optimization, etc.

Table 4. State-of-the-art of Dataset 2

No	Authors	Method	Problem	Application Method
1	[17]	Resource Constrained Shortest Path Problem (RCSPP)	Shortest Path Problem (SPP)	Modeling several applications in the field of transportation and communication using the Resource Constrained Shortest Path Problem (RCSPP). In particular, this paper calculates robust bounds on the

				resource consumption and cost by solving the robust shortest path problem and the dual robust Lagrangian relaxation.
2	[18]	Robust Optimization	Vehicle Routing Problem	This paper addresses the robust vehicle routing problem with time windows and propose two new formulations for the robust problem, each based on a different robust approach. The model only allows routes that are feasible for all values of the travel times in a predetermined uncertainty polytope.
3	[19]	Distributionally Robust Shortest Path Problem (DRSPP)	Distributional Shortest Path Problem (SPP)	This paper consider the shortest path problem, where the arc costs are subject to distributional uncertainty. The robust reformulation is shown to be NP-hard, but polynomially solvable and demonstrate that the MIP reformulation of DRSPP can be solved effectively using off-the-shelf solvers.
4	[20]	Robust Optimization	Shortest Path Problem (SPP)	This paper develop a new solution algorithm using robust shortest path problem. The algorithm is based on real-world traffic measurements provided by the City of Chicago.
5	[21]	Adjustable Robust Optimization	Network Design Problem	This paper develops an adjustable robust optimization approach for a network design problem explicitly incorporating traffic

				dynamics and demand uncertainty. The numerical results using one network from the literature demonstrate the modeling advantage of the adjustable robust approach. It has provided strategic managerial insights for enacting capacity expansion policies under demand uncertainty.
6	[22]	Two-stage Adjustable Robust Linear Optimization	Designing piecewise affine policies problem	This paper consider the problem of designing piecewise affine policies for two-stage adjustable robust linear optimization problems under right-hand side uncertainty and significantly better than the affine policy for many important uncertainty sets, such as ellipsoids and norm-balls.
7	[23]	Adjustable Robust Convex Optimization	The symmetry of the uncertainty set problem	This paper discuss about the performance of the optimal static robust solution is related to a fundamental property, namely, the symmetry of the uncertainty set menggunakan adjustable robust convex optimization under uncertainty.
8	[24]	Two-stage Adjustable Robust Linear Optimization	The symmetry of the uncertainty set problem	In this paper, performance of static solutions for two-stage adjustable robust linear optimization problems. A static solution is a single (here and now) solution that is feasible for all possible realizations of the uncertain parameters and also show that our bound is at least as good

				as the bound given by the symmetry of the uncertainty set.
9	[25]	Robust Shortest Path Problem (RSPP)	Multimodal transportation problem	This paper seeks an optimal transport scheme that considers both the uncertainty of the multimodal transport network and the timetable limit. Genetic algorithm (GA) is designed to solve the problem. The GA provides an encoding method for variable-length chromosomes applicable to shortest path problem solving.

Based on the discussion regarding the acquisition of Dataset 1 and 2 using the PRISMA Method, then conducting an evolutionary analysis, it can be seen that the research topic in the article we discussed provides several search gaps as follows. First, based on the bibliometric analysis in Figure 1, Figure 2, and Figure 3 which contains articles on dataset one and in Table 4 which contains articles on dataset two, there has been no research on optimization models for SPP using Adjustable Robust Counterpart (ARC) specifically. This is a gap in previous research which is an opportunity for something new that must be developed in future research. Second: there is no research on ARC optimization model with Polyhedral Uncertainty Set using ARC regarding SPP. Based on Table 4, four articles use Robust Optimization to solve specific SPP network problems, and five other articles use ARC to solve network problems but none are specific to solving SPP.

CONCLUSIONS

In this review article, we present a Systematic Literature Review (SLR) with the topic of Shortest Path Problem using Adjustable Robust Counterpart. The Preferred Reporting Items for Systematic Review and Meta-Analysis (PRISMA) method was used as a protocol to describe article selection criteria, search strategies, data extraction, and data analysis procedures that were proven to improve the quality of the literature review. We used six types of keyword combinations with 72 articles identified from four digital libraries, namely Scopus, Science Direct, Dimensions, and Google Scholar. After all the PRISMA Method protocols were performed, eighteen articles were obtained as Dataset 1, which was used for bibliometric mapping and evolutionary analysis. For comparison, the last nine articles as Dataset 2 were analyzed to support SLR.

The results of Dataset 1 analysis show that the sixteen articles have a time span between 2013 and 2021 with an average year of publication 5, an average of citations per year per document 21.3, and a total of 280 references. Furthermore, it can be concluded that the relevant words generated in the bibliometric results based on the most relevant keywords, titles and abstracts do not represent the topic of this research, namely the

Shortest Path Problem using the Adjustable Robust Counterpart. In short, the topic of ARC has not been studied much and is open for further related research. This is our contribution to the following research.

The results of the Dataset 2 analysis provide a research gap. There has been no research on ARC optimization model with Polyhedral Uncertainty Set using ARC regarding SPP. Based on Table 4, four articles use Robust Optimization to solve SPP specific network problems, and five other articles use ARC to solve network problems but none are specific to solving SPP. This becomes the main reference for further research on this topic.

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