



LQR and Fuzzy-PID Control Design on Double Inverted Pendulum

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ABSTRACT

Double inverted pendulum is a non-linear and unstable system. Double inverted pendulum can be stabilized in the upright position by providing control to the system. In this research we compare two types of controllers namely Linear Quadratic Regulator (LQR) and Fuzzy-PID. The objective is to determine the control strategy that provides better performance on the position of the cart and pendulum angle. We modelled the system which is then linearized and given control. From the simulation results, it is proven that LQR and Fuzzy-PID controllers have been successfully designed to stabilize the double inverted pendulum. However, when given a disturbance in the form of noise step, the LQR controller has not been able to achieve the desired reference for up to 20 seconds. In another hand, the Fuzzy-PID controller is able to achieve the desired reference after 8 seconds. Therefore, it can be concluded that the Fuzzy-PID controller when applied to the Double Inverted pendulum system has better performance than the LQR controller.

Keywords: Control; Double Inverted Pendulum; LQR; Fuzzy; PID

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INTRODUCTION

In recent years, technology has developed quickly, so with the advance of technology, nonlinear systems have also developed in life. Therefore, an accurate control system is needed for a system in order to produce a good response. The inverted pendulum is a classical control theory problem. The concept of inverted pendulum is widespread around the world, both in human-made structures and natural objects. One example of an observable inverted pendulum is a human. To stand properly, the human body must always balance itself [1]. Another example is on a cart, where a system whose pendulum mass is above its fulcrum. In this case, the fulcrum is placed at the top center of a cart that can be moved horizontally. Inverted pendulum has unstable and nonlinear properties. So a control is needed to keep the pendulum upright, even though the pendulum has a tendency to fall in all directions. [2].

The double inverted pendulum is a modification of the inverted pendulum, which adds one more pendulum that is assembled in connection with and mounted on a cart that can be stabilized and controlled by applying a force. [3]. The double inverted pendulum is more complicated than the inverted pendulum and it is challenging to maintain a stable

upright position in the event of disturbances when installed on an oscillating cart. If the oscillating cart is completely upright and the frequency is high, the double pendulum can be stabilized. Without an appropriate force on the double pendulum, the pendulum will fall over [4].

Linear Quadratic Regulator (LQR) is one of the optimal control system methods. LQR controls the process/plant using a linear combination of system state variables. The controller in this method is to stabilize the pendulum that is on top of the cart to keep it in an inverted position and perpendicular to the cart. The LQR method can be applied if the system is qualified with controlled and observable characteristics [5]. The principle of using the LQR method is to obtain the optimal control signal from the state feedback. The feedback matrix is obtained from the Riccati equation [6].

Fuzzy-PID controller is a combined control method of 2 kinds of controllers, which is PID with fuzzy algorithm [7]. The function of fuzzy is to help improve the efficiency of systems that have classical controller designs and how fuzzy can contribute in applications where controllers are initially projected poorly due to some situations such as complex systems [8].

In this research, a mathematical model will be obtained using the Lagrangian equation which will be studied related to the double inverted pendulum system. Furthermore, the system will be analyzed. LQR and PID-Fuzzy control are performed to stabilize the double inverted pendulum system. To test the robustness of the controller, the system will be given a noise to know that the controller can still maintain the stability. Then the two controllers are compared to find out the better performance in controlling the double inverted pendulum system.

METHODS

Linear Quadratic Regulator

Linear Quadratic Regulator (LQR) is one of the optimal control system methods. This method can be applied if the system meets the conditions of controllable and observable characteristics. Suppose the following state space equations are given [9]:

$$\dot{X} = AX + Bu \tag{1}$$

with feedback signal $u = -KX$, then obtained

$$\dot{X} = (A - BK)X \tag{2}$$

So that the performance index can be obtained that reaches the following minimum values:

$$J = \int_0^{\infty} (X^T Q X + u^T R u) dt \tag{3}$$

where Q is a positive semi-definite matrix and R is a positive definite symmetric matrix. Based on the results of the system state space equation and the performance index (J), the value of the optimal K matrix is obtained to minimize the selected performance index. The value of K can be obtained from

$$K = R^{-1} B^T P \tag{4}$$

where P is a positive definite matrix obtained from the solution of the following Riccati equation:

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \tag{5}$$

By using the LQR method, the impact of optimal control depends on the selection of matrices Q and R . One of the obstacles in solving the LQR method is solving the Riccati Equation manually. So that the Riccati equation can be solved using the Matlab application

[10].

Fuzzy-PID

The PID controller is a combination of three types of controllers, namely proportional, integral and derivative controllers. The objective of combining the three controllers is to improve system performance, where each controller complements and covers the weakness and strengths of each. While fuzzy is a term used by Lotfi A. Zadeh in July 1964 to describe a group / set that can be differentiated from other sets based on the degree of membership and has unclear boundaries (vague), unlike classical sets, which divide set membership into two parts, namely members and non-members. The control of the Fuzzy algorithm is carried out in three stages, namely fuzzification, fuzzy rules and defuzzification [11].

Fuzzy-PID controller is a combined control method of two types of controllers, which is PID with fuzzy algorithm [7]. Fuzzy-PID uses two inputs, named error (e) and derivative of the error (\dot{e}) and provides three outputs K_{pf} , K_{if} , and K_{df} which is determined by the fuzzy rule set. The fuzzy-PID controller output is used to determine the K_p , K_i and K_d gains of the PID controller which are shown in the equation (6), where $e(t)$ is error and $y(t)$ is the system output.

$$y(t) = (K_p + K_{pf})e(t) + (K_i + K_{if}) \int_0^t e(\tau) d\tau + (K_d + K_{df}) \frac{de(t)}{dt} \quad (6)$$

RESULTS AND DISCUSSION

Model of Double Inverted Pendulum System

An inverted pendulum is a system in which the pendulum's mass is above its fulcrum. In this case, the fulcrum is placed at the top middle of a cart that can be moved horizontally. Inverted pendulum has unstable and nonlinear characteristics. So a control is needed to keep the pendulum upright. Double inverted pendulum is a modification of the inverted pendulum, which adds one more pendulum that is assembled in connection with each other and mounted on a cart. Figure 1 shows the double inverted pendulum [4].

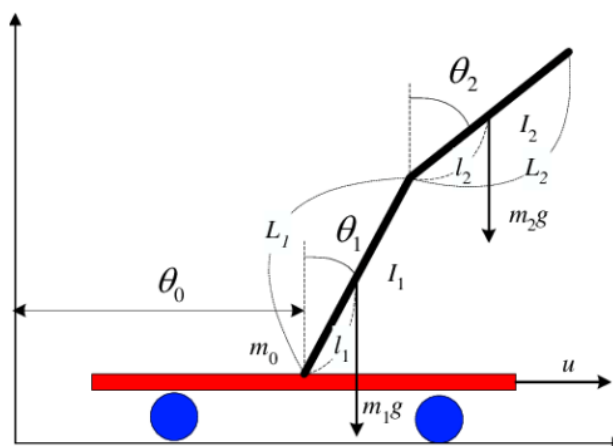


Figure 1. Double inverted pendulum system

The following is a table of parameters for the double inverted pendulum system [3]:

Parameter	Description	Value	Unit
m_0	Cart mass	0.8	Kg

m_1	Mass of the pendulum 1	0.5	Kg
m_2	Mass of the pendulum 2	0.3	Kg
l_1	Length of pendulum rope 1	0.3	m
l_2	Length of pendulum rope 2	0.2	m
J	moment of inertia of pendulum ($J_1 = J_2$)	0.006	Kgm ²
G	gravity force	9.8	m/s ²

Derivation of Double Inverted Pendulum Model

The mathematical model of double inverted pendulum can be obtained by using the Lagrangian equation for mechanical motion:

$$L = T - V \quad (7)$$

where L, T , dan V are the Lagrange Function, kinetic energy and potential energy, respectively [12].

The physical model of the double inverted pendulum system can be derived based on the principles of mechanics, as can be seen in Figure 1. Controls are necessary to keep the double inverted pendulum upright, even though the pendulum has a tendency to fall in all directions and is unstable. In this case, the motion of the double inverted pendulum is only limited in two dimensions, so that the double inverted pendulum only moves in two directions, the motion of the cart to the left and right and the motion of the pendulum to the left and right.

A kinetic energy will appear in the cart and also in the pendulum, if the cart is given a force of F . The kinetic energy of an object is defined as the effort required to move an object of a given mass from rest to a given speed. It is the energy that an object possesses due to its motion [13]. the cart only moves in the horizontal direction, so the kinetic energy on the cart (T_0) is:

$$T_0 = \frac{1}{2} m_0 \dot{x}^2 \quad (8)$$

While the pendulum can move horizontally, vertically, and rotationally, then the kinetic energy of inverted pendulum 1 (T_1) is:

$$T_1 = \frac{1}{2} m_1 (\dot{x} + l_1 \dot{\theta}_1 \cos \theta_1)^2 + \frac{1}{2} m_1 (l_1^2 \dot{\theta}_1^2 \sin^2 \theta_1) + \frac{1}{2} J_1 \dot{\theta}_1^2 \quad (9)$$

Kinetic energy of inverted pendulum 2 (T_2) has the same process as inverted pendulum 1 which moves horizontally, vertically and rotationally as follows:

$$T_2 = \frac{1}{2} m_2 (\dot{x} + l_1 \dot{\theta}_1 \cos \theta_1 + l_2 \dot{\theta}_2 \cos \theta_2)^2 + \frac{1}{2} m_2 (l_1 \dot{\theta}_1 \sin \theta_1 + l_2 \dot{\theta}_2 \sin \theta_2)^2 + \frac{1}{2} J_2 \dot{\theta}_2^2 \quad (10)$$

So the total kinetic energy obtained in the double inverted pendulum is as follows:

$$T = T_0 + T_1 + T_2$$

$$T = \frac{1}{2} \dot{x}^2 (m_0 + m_1 + m_2) + \frac{1}{2} \dot{\theta}_1^2 (m_1 l_1^2 + m_2 l_1^2 + J_1) + \frac{1}{2} \dot{\theta}_2^2 (m_2 l_2^2 + J_2) + \dot{x} \dot{\theta}_1 \cos \theta_1 (m_1 l_1 + m_2 l_1) + m_2 l_2 \dot{x} \dot{\theta}_2 \cos \theta_2 + m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 - \theta_2) \quad (11)$$

Next, we will calculate the potential energy of the double inverted pendulum system. Potential energy is the energy possessed by an object due to the effect of the place or position of the object. Potential energy on the cart is denoted by V_0 which is $V_0 = 0$.

The potential energy of inverted pendulum 1 is:

$$V_1 = m_1 g h = m_1 g l_1 \cos \theta_1 \quad (12)$$

As for the potential energy of inverted pendulum 2, its altitude is affected by inverted pendulum 1, so obtained:

$$\begin{aligned} V_2 &= m_1 g(h_1 + h_2) \\ &= m_2 g(l_1 \cos \theta_1 + l_2 \cos \theta_2) \end{aligned} \quad (13)$$

So the total potential energy obtained in the double inverted pendulum is as follows:

$$\begin{aligned} V &= V_0 + V_1 + V_2 \\ &= (m_1 l_1 + m_2 l_1) g \cos \theta_1 + m_2 g l_2 \cos \theta_2 \end{aligned} \quad (14)$$

Then substituting Equations (11) and (14) into the Lagrange equation so as to obtain

$$\begin{aligned} L &= \frac{1}{2} \dot{x}^2 (m_0 + m_1 + m_2) + \frac{1}{2} \dot{\theta}_1^2 (m_1 l_1^2 + m_2 l_1^2 + J_1) + \frac{1}{2} \dot{\theta}_2^2 (m_2 l_2^2 + J_2) \\ &\quad + \dot{x} \dot{\theta}_1 \cos \theta_1 (m_1 l_1 + m_2 l_1) + m_2 l_2 \dot{x} \dot{\theta}_2 \cos \theta_2 \\ &\quad + m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 - \theta_2) - (m_1 l_1 + m_2 l_1) g \cos \theta_1 \\ &\quad - m_2 l_2 g \cos \theta_2 \end{aligned} \quad (15)$$

To generalize the coordinates, it is necessary to consider the translational motion of the cart (x), the oscillating motion of inverted pendulum 1, and the oscillating motion of inverted pendulum 2 as three outputs that change whenever a force (u) is applied. By considering the vertical and horizontal components (x, θ_1, θ_2), then the inverted pendulum on the cart has degrees of freedom, named (x, θ_1, θ_2). So x, θ_1 dan θ_2 is the general coordinate, where $x = q_1, \theta_1 = q_2, \theta_2 = q_3$, therefore the Lagrange equation for this equation is [5]:

- Translation motion

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}} \right) - \frac{\partial L}{\partial x} = u \quad (16)$$

$$\begin{aligned} \ddot{x} (m_0 + m_1 + m_2) + \ddot{\theta}_1 \cos \theta_1 (m_1 l_1 + m_2 l_1) + m_2 l_2 \ddot{\theta}_2 \cos \theta_2 \\ - \sin \theta_1 (\dot{\theta}_1)^2 (m_1 l_1 + m_2 l_1) - m_2 l_2 (\dot{\theta}_2)^2 \sin \theta_2 = u \end{aligned} \quad (17)$$

- Oscillating motion of inverted pendulum 1

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}_1} \right) - \frac{\partial L}{\partial \theta_1} = 0 \quad (18)$$

$$\begin{aligned} \ddot{\theta}_1 (m_1 l_1^2 + m_2 l_1^2 + J_1) + \ddot{x} \cos \theta_1 (m_1 l_1 + m_2 l_1) + m_2 l_1 l_2 \ddot{\theta}_2 \cos(\theta_1 - \theta_2) \\ + m_2 l_1 l_2 \dot{\theta}_2^2 \sin(\theta_1 - \theta_2) - (m_1 l_1 + m_2 l_1) g \sin \theta_1 = 0 \end{aligned} \quad (19)$$

- Oscillating motion of inverted pendulum 2

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}_2} \right) - \frac{\partial L}{\partial \theta_2} = 0 \quad (20)$$

$$\begin{aligned} \ddot{\theta}_2 (m_2 l_2^2 + J_2) + m_2 l_2 \ddot{x} \cos \theta_2 + m_2 l_1 l_2 \ddot{\theta}_1 \cos(\theta_1 - \theta_2) - m_2 l_1 l_2 \dot{\theta}_1^2 \sin(\theta_1 - \theta_2) \\ - m_2 l_2 g \sin \theta_2 = 0 \end{aligned} \quad (21)$$

Based on Equation (17), (19) dan (21) which has been obtained, it can be assumed as follows:

$$\begin{aligned} a_0 &= m_0 + m_1 + m_2, & a_3 &= m_2 l_2 \\ a_1 &= m_1 l_1 + m_2 l_1, & a_4 &= m_2 l_1 l_2 \\ a_2 &= m_1 l_1^2 + m_2 l_1^2 + J_1, & a_5 &= m_2 l_2^2 + J_2 \end{aligned}$$

These assumptions are substituted into the Lagrange equation, thus obtaining:

$$\begin{aligned} a_0 \ddot{x} + a_1 \ddot{\theta}_1 \cos \theta_1 + a_3 \ddot{\theta}_2 \cos \theta_2 - a_1 \dot{\theta}_1^2 \sin \theta_1 - a_3 \dot{\theta}_2^2 \sin \theta_2 = u \\ a_2 \ddot{\theta}_1 + a_1 \ddot{x} \cos \theta_1 + a_4 \ddot{\theta}_2 \cos(\theta_1 - \theta_2) + a_4 \dot{\theta}_2^2 \sin(\theta_1 - \theta_2) - a_1 g \sin \theta_1 = 0 \\ a_5 \ddot{\theta}_2 + a_3 \ddot{x} \cos \theta_2 + a_4 \ddot{\theta}_1 \cos(\theta_1 - \theta_2) - a_4 \dot{\theta}_1^2 \sin(\theta_1 - \theta_2) - a_3 g \sin \theta_2 = 0 \end{aligned} \quad (22)$$

Because the controller can only work with linear functions, the nonlinear Equation (22) must first be linearized in the balanced position, so that it is obtained, $\theta_1 = \theta_2 = 0, \dot{\theta}_1 = \dot{\theta}_2 = 0, \sin \theta_1 = \theta_1, \cos \theta_1 = 1, \sin \theta_2 = \theta_2, \cos \theta_2 = 1, \sin(\theta_1 - \theta_2) = \theta_1 - \theta_2, \cos(\theta_1 - \theta_2) = 1$

So the linear equation changes to the following equation:

$$\begin{aligned} a_0\ddot{x} + a_1\ddot{\theta}_1 + a_3\ddot{\theta}_2 &= u \\ a_2\ddot{\theta}_1 + a_1\ddot{x} + a_4\ddot{\theta}_2 - a_1g\theta_1 &= 0 \\ a_5\ddot{\theta}_2 + a_3\ddot{x} + a_4\ddot{\theta}_1 - a_3g\theta_2 &= 0 \end{aligned} \tag{23}$$

State Space

Based on the mathematical model that has been applied before, to choose the state variable, it can be assumed as follows:

- x : cart movement
- \dot{x} : cart movement velocity
- θ_1 : angle of pendulum 1
- $\dot{\theta}_1$: angular velocity of pendulum 1
- θ_2 : angle of pendulum 2
- $\dot{\theta}_2$: angular velocity of pendulum 2

Then put the parameter values in Table 1 into Equation (23), so that using Matlab the double inverted pendulum system can be written state space and output as follows:

$$\dot{X} = AX + Bu \text{ dan } Y = Cx + Du$$

where

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\theta}_1 \\ \ddot{\theta}_1 \\ \dot{\theta}_2 \\ \ddot{\theta}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -8.2047 & 0 & -0.2279 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 63.814 & 0 & -9.1163 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -36.4651 & 0 & 42.5426 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \theta_1 \\ \dot{\theta}_1 \\ \theta_2 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1.1628 \\ 0 \\ -3.4884 \\ 0 \\ -0.3876 \end{bmatrix} u$$

$$Y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \theta_1 \\ \dot{\theta}_1 \\ \theta_2 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} u$$

System Analysis

• Stability Analysis

Determining the stability of the system can be performed by finding the roots of the characteristic equation. In this system, to find the roots of the characteristic is to use the eigenvalue of matrix A which can be shown as follows [14]:

$$|\lambda I - A| = 0$$

Based on the calculation results using Matlab, we obtained the following::

$$\lambda^6 - 106.36\lambda^4 + 2382.39\lambda^2 = 0 \tag{24}$$

Then we obtained $\lambda_1 = 0, \lambda_2 = 0, \lambda_3 = -8.6189, \lambda_4 = -5.6631, \lambda_5 = 8.6189$ and $\lambda_6 = 5.6631$. Because there is an eigenvalue greater than or equal to zero, the double inverted pendulum system is unstable.

• **Controllability analysis**

A system is said to be controlled if the matrix:

$$M_c = [B \quad AB \quad A^2B \quad \dots \quad A^{(n-1)}B] \tag{25}$$

has the same rank as the system order(n) [15].

By using Matlab, the rank $M_c = 6$ is obtained, because it has the same rank as the system order (n) which is 6, so that double inverted pendulum system is controlled.

• **Observability Analysis**

It is a necessary and sufficient condition for a system to be observed if the observability matrix:

$$M_o = [C \quad CA \quad CA^2 \quad \dots \quad CA^{(n-1)}]^T \tag{26}$$

has the same rank as the system order(n) [15].

By using Matlab, the rank $M_o = 6$ is obtained, because it has the same rank as the system order (n) which is 6, so that double inverted pendulum system is observed.

Simulation Result

Open loop system is a system which output signal does not affect the controller. This simulation is needed to determine the characteristics of the double inverted pendulum plant before designing the controller. Simulation is done by giving reference signal of cart position, pendulum angle 1 and pendulum angle 2 without giving feedback signal. The simulation is using Simulink in Matlab and resulted in the following system output response:



Figure 2. Response on open loop system

The next step is to control the double inverted pendulum system. The first method used is LQR. With Matlab, to solve the LQR method, we can use the command $[K, P, E] = lqr(A, B, Q, R)$. With the matrix selection,

$Q = [1 \ 0 \ 0 \ 0 \ 0; 0 \ 1 \ 0 \ 0 \ 0; 0 \ 0 \ 1 \ 0 \ 0; 0 \ 0 \ 0 \ 1 \ 0; 0 \ 0 \ 0 \ 0 \ 1]$ and matrix $R = 0.01$.

So the value of $K = [10 \ 19.2392 \ 341.451 \ -18.2577 \ 459.2919 \ 71.2329]$,

$$P = \begin{bmatrix} 1.9239 & 1.3507 & -1.8258 & 0.2938 & 7.1233 & 1.1498 \\ 1.3507 & 2.0602 & -3.8064 & 0.4085 & 12.5548 & 2.0082 \\ -1.8258 & -3.8064 & 54.6078 & 1.1667 & -82.9906 & -13.1104 \\ 0.2938 & 0.4085 & 1.1667 & 0.1861 & 0.1049 & 0.0217 \\ 7.1233 & 12.5548 & -82.9906 & 0.1049 & 158.0221 & 24.8707 \\ 1.1498 & 2.0082 & -13.1104 & 0.0217 & 24.8707 & 3.9913 \end{bmatrix}$$

$$E = \begin{bmatrix} -1.0136 + 0.0000i \\ -2.0889 + 1.3707i \\ -2.0889 - 1.3707i \\ -7.4189 + 2.5004i \\ -7.4189 + 2.5004i \\ -38.4222 + 0.0000i \end{bmatrix}$$

And resulted in the output response of the double inverted pendulum system as follows:

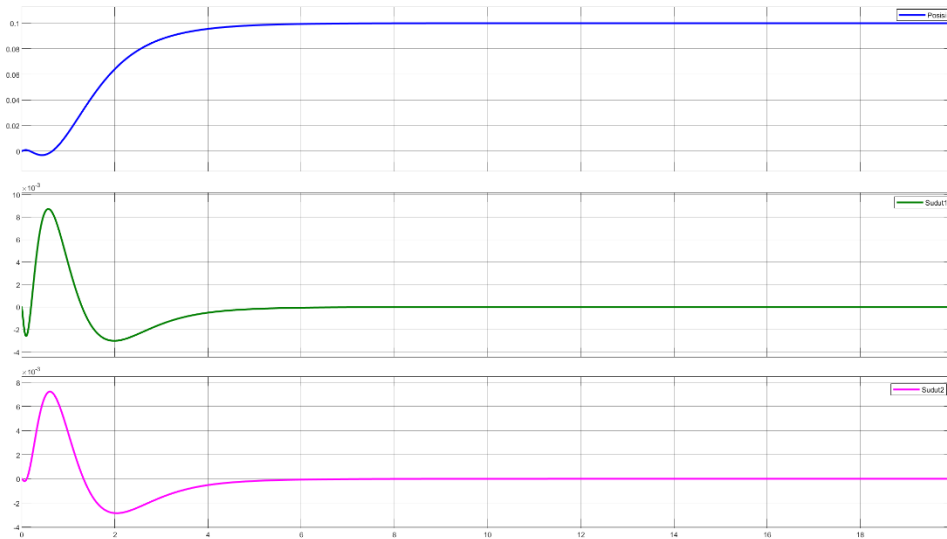


Figure 3. System response using LQR controller without disturbance

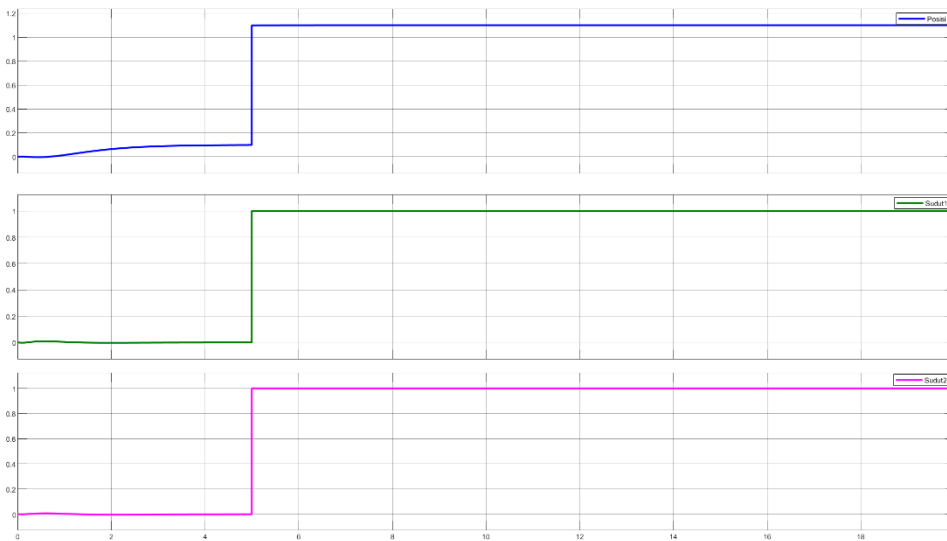


Figure 4. System response using LQR controller with disturbance

Figure 3 shows that the system is able to get to the reference model that has been determined. This means that the system will be stable with the LQR controller. However, in Figure 4 if given a disturbance to the system with a step noise, the system cannot reach the desired reference until 20 seconds..

The next method is the Fuzzy-PID method. Fuzzy-PID uses two inputs, error (e) and the derivative of error (\dot{e}) and gives three outputs K_{pf} , K_{if} , dan K_{df} which is determined by the set of fuzzy rules. The first step in designing a fuzzy algorithm is the definition of fuzzy sets and the construction of membership functions. In the duoble inverted pendulum system, seven fuzzy subsets are required. The fuzzy sets are defined as: NB,

NM, NS, Z, PS, PM and PB which represent Negative Big, Negative Medium, Negative Small, Zero, Positive Small, Positive Medium and Positive Big. Figure 5 to Figure 7 shows the membership functions used.

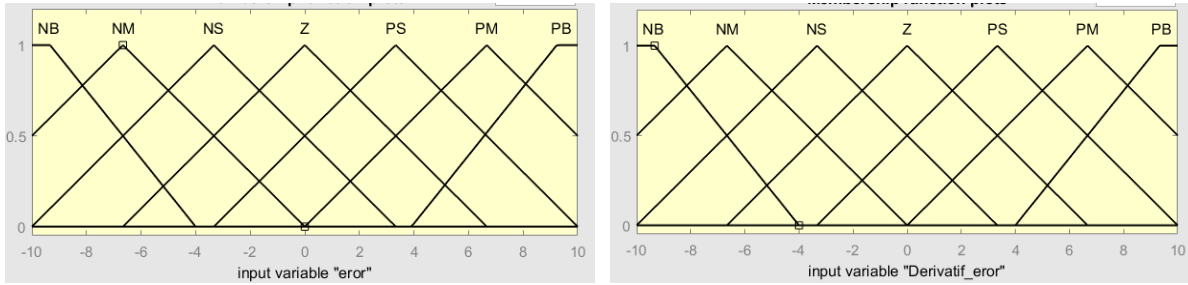


Figure 5. (a) Membership function for the input error (e). (b) Membership function for the input derivative error (\dot{e})

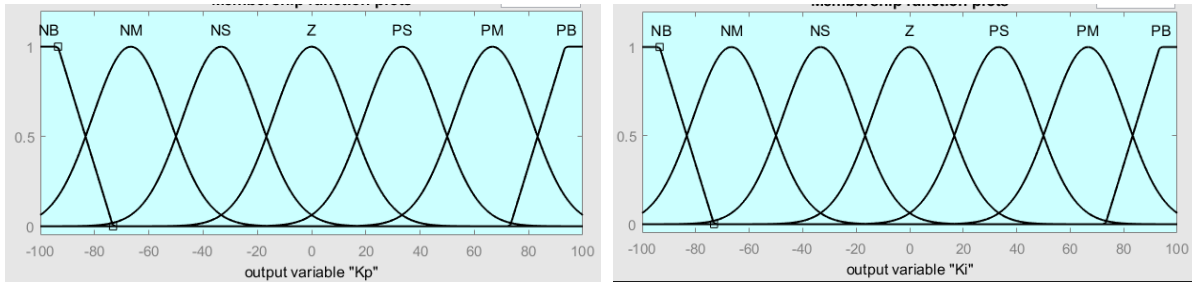


Figure 6. (a) Membership function for the output K_p . (b) Membership function for the output K_i

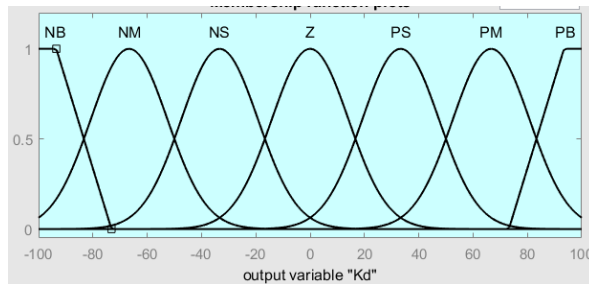


Figure 7. Membership function for the output K_d

Table 2 to Table 3 shows the fuzzy rules used for simulation of fuzzy-PID control on double inverted pendulum.

Table 2. (a) Fuzzy rule for K_p . (b) Fuzzy rule for K_i

\dot{e}/e	NB	NM	NS	ZO	PS	PM	PB	\dot{e}/e	NB	NM	NS	ZO	PS	PM	PB
NB	PB	PB	PM	PM	PS	ZO	ZO	NB	NB	NB	NM	NM	NS	ZO	ZO
NM	PB	PM	PM	PS	PS	ZO	NS	NM	NB	NM	NM	NS	NS	ZO	PS
NS	PM	PM	PS	PS	ZO	NS	NS	NS	NM	NM	NS	NS	ZO	PS	PS
ZO	PM	PS	PS	ZO	ZO	NS	NM	ZO	NM	NS	NS	ZO	ZO	PS	PM
PS	PS	PS	ZO	ZO	NS	NM	NM	PS	NS	NS	ZO	ZO	PS	PM	PM
PM	PS	ZO	ZO	NS	NS	NM	NB	PM	NS	ZO	ZO	PS	PS	PM	PB
PB	ZO	ZO	NS	NS	NM	NB	NB	PB	ZO	ZO	PS	PS	PM	PB	PB

Table 3. Fuzzy rule for K_d

\dot{e}/e	NB	NM	NS	ZO	PS	PM	PB
NB	NS	PS	PB	PB	PB	PM	NS
NM	NS	PS	PB	PM	PM	PS	ZO
NS	ZO	PS	PM	PM	PS	PS	ZO
ZO	ZO	PS	PS	PS	PS	PS	ZO
PS	ZO	ZO	ZO	ZO	ZO	ZO	ZO
PM	NB	PS	NS	NS	NS	NS	NB
PB	NB	NM	NM	NS	NS	NS	NB

The fuzzy-PID controller is implemented with a fuzzy algorithm. The fuzzy self-tuning controller is determined by two inputs, error and derivative of error and three outputs namely PID parameters K_p , K_i and K_d . These parameters are forwarded to the PID controller and then the system is controlled. The output response results of the double inverted pendulum system are shown in Figure 7 to Figure 8 below:

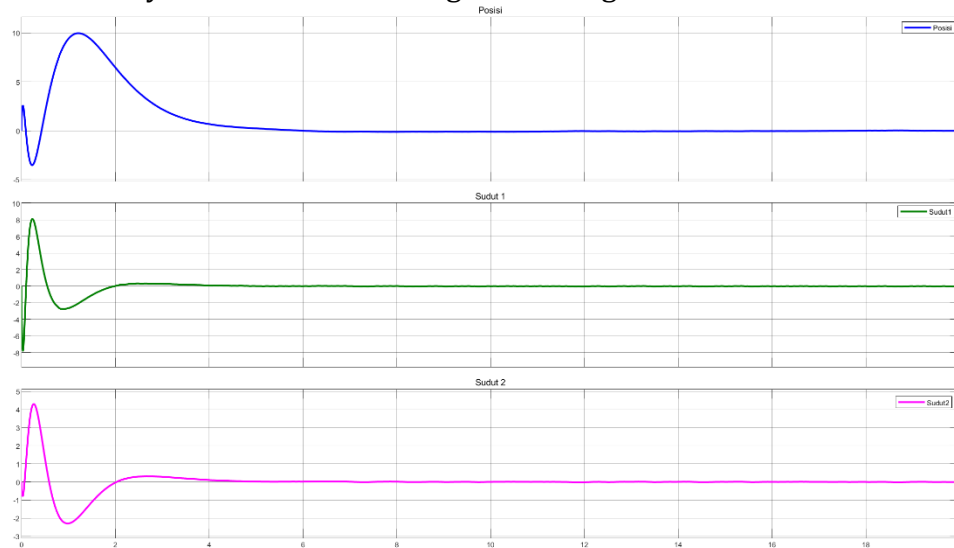


Figure 8. System response using Fuzzy-PID controller without disturbance

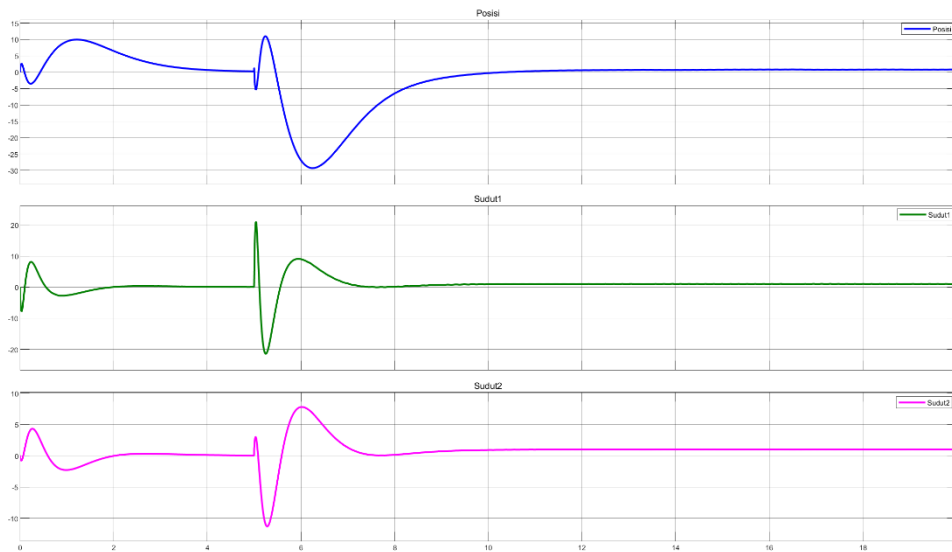


Figure 9. System response using Fuzzy-PID controller with disturbance

Figure 8 shows that the system is able to get to the reference model that has been determined. This means that the system will be stable with the Fuzzy-PID controller. While in Figure 9 if given a disturbance to the system in the form of a step noise, the system can also still reach the desired reference after 8 seconds.

CONCLUSIONS

In this research, we construct a double inverted pendulum system using Lagrangian equations. We linearize the system at the equilibrium point so that a linear system is obtained. Then, we stabilize the system using LQR and fuzzy-PID controllers. The simulation process was conducted on the system without disturbance and with disturbance. As a result, LQR and fuzzy-PID controllers can stabilize the given system, but

the LQR method has not been able to stabilize the disturbance system to the desired reference after 20 seconds. While the Fuzzy-PID controller is able to reach the desired reference after 8 seconds. Therefore, we conclude that Fuzzy-PID control system has better performance than LQR control on Double Inverted pendulum system. As a future work, we would like to try estimating the LQR controller with noise estimation methods such as Kalman filter so that the LQR controller can also be used for noise systems.

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