



# Multipolar Intuitionistic Fuzzy Positive Implicative Ideal in B-Algebras

Royyan Amigo<sup>1</sup>, Noor Hidayat<sup>2</sup>, Vira Hari Krisnawati<sup>3</sup>

<sup>1</sup>Department of Actuarial Science, University of Darunnajah, Jakarta, Indonesia

<sup>2,3</sup>Departement of Mathematics, University of Brawijaya, Malang, Indonesia

Email: [royyanamigo@darunnajah.ac.id](mailto:royyanamigo@darunnajah.ac.id)

## ABSTRACT

In this paper, we start with the concept of  $B$ -algebras, commutative  $B$ -algebras and fuzzy ideal in  $B$ -algebras. We also study about multipolar intuitionistic fuzzy ideal. We explain the notion of multipolar intuitionistic fuzzy positive implicative ideal in  $B$ -algebras and some characterizes. In addition, we examine some theorems and proportions which contain the conditions for a multipolar intuitionistic fuzzy set become a multipolar intuitionistic fuzzy positive implicative ideal in  $B$ -algebras. One of the result is a multipolar intuitionistic fuzzy set  $(\hat{\ell}, \hat{s})$  over commutative  $B$ -algebra  $X$  is a multipolar intuitionistic fuzzy positive implicative ideal  $(\hat{\ell}, \hat{s})$  over commutative  $B$ -algebra  $X$  if and only if  $(x * y) * z = 0$  implies  $\hat{\ell}(x) \geq \inf\{\hat{\ell}(y), \hat{\ell}(z)\}$  and  $\hat{s}(x) \leq \sup\{\hat{s}(y), \hat{s}(z)\}$ , for all  $x, y, z \in X$ .

**Keywords:** B-algebras; commutative B-algebras; multipolar intuitionistic fuzzy set; multipolar intuitionistic fuzzy ideal; multipolar intuitionistic fuzzy positive implicative ideal

Copyright © 2024 by Authors, Published by CAUCHY Group. This is an open access article under the CC BY-SA License (<https://creativecommons.org/licenses/by-sa/4.0/>)

## INTRODUCTION

One of the topics in mathematics is about algebraic structure. An essential idea of algebraic structure about  $BCK$ -algebras was proposed by Imai and Iseki [1] in 1966. In that year, Iseki [2] also give the new concept of  $BCI$ -algebras which is generalization from  $BCK$ -algebras. A  $B$ -algebras which satisfies some properties of  $BCK$ -algebras and  $BCI$ -algebras was introduced by Neggers and Kim in [3]. They also discussed some properties of  $B$ -algebras.

Not only algebraic structure, fuzzy is the most valuable topics in mathematics. The terminology of fuzzy set was proposed by Zadeh [4]. A bipolar fuzzy set as generalized of fuzzy set was introduced in [5]. By two concepts, many researchers combine that two concepts into a new research. Meng [6] constructed fuzzy implicative ideals in  $BCK$ -algebras. Jun et al studied fuzzy  $B$ -algebras [7]. Moreover, Ahn and Bang [8] discussed about fuzzy  $B$ -subalgebras in  $B$ -algebras.

The researchers also develop the concepts of fuzzy into bipolar fuzzy or multipolar fuzzy. In addition, Muhiuddin and Al-Kadi [9] studied bipolar fuzzy  $BCI$ -implicative ideals of  $BCI$ -algebras. In the next year, they also studied about bipolar fuzzy implicative ideals

in *BCK*-algebras [10]. They investigated the properties between a bipolar fuzzy ideal and bipolar fuzzy implicative ideal. A multipolar fuzzy sets which is the generalize of bipolar fuzzy set was introduced in [11]. Furthermore, the concept of *m*-polar fuzzy ideals of *BCK/BCI*-algebras were discussed in [12]. In the same year, Borzooei et al in [13] introduced about multipolar fuzzy *p*-ideals of *BCI*-algebras and investigated some properties.

In other hand, the researchers also studied about intuitionistic fuzzy set. Atanasov [14] introduced the new notion about intuitionistic fuzzy set. The researchers also combine this with the concept of multipolar fuzzy and algebraic structure. So, the concept of multipolar intuitionistic fuzzy set with finite degree and its application in *BCK/BCI*-algebras was introduced by [15]. Then, the concept of multipolar intuitionistic fuzzy *B*-algebras was proposed by Borzooei et al in [16]. They gived the concept of simple *m*-polar fuzzy set and *m*-polar intuitionistic fuzzy subalgebras of *B*-algebras. Recently, the new structure of multipolar intuitionistic fuzzy ideal in *B*-algebras was introduced by Amigo et al in [17]. They discussed some properties of that.

In this paper, we continue the last concept from Amigo et al [17]. So, we study the concept of multipolar intuitionistic fuzzy positive implicative ideal in *B*-algebras and investigated some related characterizes. Furthermore, we prove the conditions for multipolar intuitionistic fuzzy set to be a multipolar intuitionistic fuzzy positive implicative ideal.

In the first, we begin with definitions and propositions in *B*-algebra and commutative *B*-algebra which are helpful to our main results. Then, we give the definitions of fuzzy ideal in *B*-algebras too.

**Definition 1.1** [18] A *B*-algebra is a non empty set *X* with 0 as identity element (right) and a binary operations *\** satisfying the following axioms:

- i.  $a * a = 0$ , for all  $a \in X$ .
- ii.  $a * 0 = a$ , for all  $a \in X$ .
- iii.  $(a * b) * c = a * (c * (0 * b))$ , for all  $a, b, c \in X$ .

First, we define a partial ordering relation " $\leq$ " on *X* by  $a \leq b$  if and only if  $a * b = 0$  for all  $a, b \in X$  ([16]). Then, we give some example of *B*-algebra and proporsition of *B*-algebra.

**Example 1.2** [18] Let  $X = \{0, a, b, c\}$  be a set with Cayley table as follows:

Table 1: Cayley table for  $(X; *, 0)$

<i>*</i>	<b>0</b>	<b>a</b>	<b>b</b>	<b>c</b>
<b>0</b>	0	0	<i>b</i>	<i>b</i>
<b>a</b>	<i>a</i>	0	<i>c</i>	<i>b</i>
<b>b</b>	<i>b</i>	<i>b</i>	0	0
<b>c</b>	<i>c</i>	<i>b</i>	<i>a</i>	0

Then,  $(X; *, 0)$  is a *B*-algebra.

**Example 1.3** [18] Let  $(\mathbb{Z}; -, 0)$  with " $-$ " be a substraction operations of integers  $\mathbb{Z}$ . Then,  $(\mathbb{Z}; -, 0)$  is a *B*-algebra.

**Example 1.4** [17] Let  $(\mathbb{R}^+ - \{0\}; *, 1)$  with " $*$ " be a binary operation of  $\mathbb{R}^+ - \{0\}$  defined by

$$a * b = \frac{a}{b}.$$

Then,  $(\mathbb{R}^+ - \{0\}; *, 1)$  is a  $B$ -algebra.

**Proposition 1.5** [19] *If  $(X; *, 0)$  is a  $B$ -algebra, then*

- i.  $(a * b) * (0 * b) = a$ , for all  $a, b \in X$ .
- ii.  $a * (b * c) = (a * (0 * c)) * b$ , for all  $a, b, c \in X$ .
- iii. *If  $a * b = 0$  then  $a = b$ , for all  $a, b \in X$ .*
- iv.  $0 * (0 * a) = a$ , for all  $a \in X$ .
- v.  $(a * c) * (b * c) = a * b$ , for all  $a, b, c \in X$ .
- vi.  $0 * (a * b) = b * a$ , for all  $a, b \in X$ .
- vii.  $a * b = 0$  if and only if  $b * a = 0$ , for all  $a, b \in X$ .
- viii. *If  $0 * a = 0$  then  $X$  contains only 0, for all  $a \in X$ .*

After that, we also give the concept of commutative  $B$ -algebra such as the definition, example, and proposition.

**Definition 1.6** [19] A  $B$ -algebra  $(X; *, 0)$  is said commutative  $B$ -algebra if

$$a * (0 * b) = b * (0 * a)$$

for all  $a, b \in X$ .

**Example 1.7** [17] Let  $(\mathbb{Z}; -, 0)$  with " $-$ " be a subtraction operations of integers  $\mathbb{Z}$ . Then,  $(\mathbb{Z}; -, 0)$  is a commutative  $B$ -algebra.

**Proposition 1.8** [19] *If  $(X; *, 0)$  is a commutative  $B$ -algebra, then*

- i.  $(0 * a) * (0 * b) = b * a$ , for all  $a, b \in X$ .
- ii.  $(c * b) * (c * a) = a * b$ , for all  $a, b, c \in X$ .
- iii.  $(a * b) * c = (a * c) * b$ , for all  $a, b, c \in X$ .
- iv.  $(a * (a * b)) * b = 0$ , for all  $a, b \in X$ .
- v.  $(a * c) * (b * d) = (d * c) * (b * a)$ , for all  $a, b, c, d \in X$ .

Not only about  $B$ -algebra and commutative  $B$ -algebra, we explain some concept of ideal in  $B$ -algebra and subalgebras too.

**Definition 1.9** [18] Let  $(X; *, 0)$  be a  $B$ -algebra. A non empty subset  $I$  of  $X$  is called ideal of  $X$  if it satisfies:

- i.  $0 \in I$ .
- ii. If  $b \in I$  and  $a * b \in I$  then  $a \in I$ , for all  $a, b \in X$ .

**Example 1.10** [18] Let  $I = \mathbb{Z}^+ \cup \{0\}$  be a subset of  $B$ -algebra  $(\mathbb{Z}; -, 0)$ , then  $I$  is ideal of  $\mathbb{Z}$ .

**Definition 1.11** [20] Let  $(X; *, 0)$  be a  $B$ -algebra. A non empty subset  $I$  of  $X$  is called positive implicative ideal of  $X$  if it satisfies:

- i.  $0 \in I$ .
- ii. If  $((a * b) * c) \in I$  and  $(b * c) \in I$  then  $(a * c) \in I$ , for all  $a, b, c \in X$ .

Let  $(X; *, 0)$  be a  $B$ -algebras. A non empty subset  $I$  of  $X$  is called *subalgebras* ( $B$ -subalgebras) of  $X$  if  $0 \in I$  and  $a * b \in I$ , for all  $a, b \in I$  ([18]).

In  $B$ -algebras, we combine that with the concept of *fuzzy* and multipolar intuitionistic fuzzy. So, we study about fuzzy  $B$ -algebras, fuzzy ideal in  $B$ -algebra, and multipolar intuitionistic fuzzy ideal in  $B$ -algebra.

**Definition 1.12** [7] Let  $(X; *, 0)$  be a  $B$ -algebra. A fuzzy set  $A$  in  $X$  is called fuzzy  $B$ -algebras if it satisfies the inequality

$$\mu_A(a * b) \geq \min\{\mu_A(a), \mu_A(b)\},$$

for all  $a, b \in X$ .

Let  $(X; *, 0)$  be a  $B$ -algebra. A fuzzy set  $A$  in  $X$  is called *fuzzy ideal*  $B$ -algebra ([21]) if it satisfies

$$\begin{aligned} \mu_A(0) &\geq \mu_A(a), \\ \mu_A(a) &\geq \min\{\mu_A(a * b), \mu_A(b)\}, \end{aligned}$$

for all  $a, b \in X$ .

A  $B$ -algebra  $(X; *, 0)$  in the Example 1.2. If we define a fuzzy set  $A$  in  $X$  by  $\mu_A(0) = \mu_A(b) = 1$  and  $\mu_A(a) = \mu_A(c) = 0.5$ , then  $A$  is fuzzy ideal of  $X$ . Moreover, a  $B$ -algebra  $(\mathbb{R}^+ - \{0\}; *, 1)$  in the Example 1.4. If we define a fuzzy set  $A$  in  $\mathbb{R}^+ - \{0\}$  by

$$\mu_A(x) = \begin{cases} 1 & \text{if } x = 1, \\ 0.5 & \text{if } x \neq 1, \end{cases}$$

then  $A$  is fuzzy ideal of  $\mathbb{R}^+ - \{0\}$ . Let  $(X; *, 0)$  be a  $B$ -algebra. A multipolar intuitionistic fuzzy set over  $X$  is a mapping

$$\begin{aligned} (\hat{\ell}, \hat{s}) : X &\rightarrow ([0,1] \times [0,1])^m \\ a &\mapsto (\hat{\ell}(a), \hat{s}(a)), \end{aligned}$$

where  $\hat{\ell} : X \rightarrow [0,1]^m$  and  $\hat{s} : X \rightarrow [0,1]^m$  are multipolar fuzzy sets over  $X$  which is satisfies the condition

$$\hat{\ell}(a) + \hat{s}(a) \leq 1$$

where  $\pi_i : [0,1]^m \rightarrow [0,1]$  such that

$$(\pi_i \circ \hat{\ell})(a) + (\pi_i \circ \hat{s})(a) \leq 1$$

for all  $a \in X$  and  $i = 1, 2, \dots, m$ . (see [16]).

**Definition 1.13** [17] Let  $(X; *, 0)$  be a  $B$ -algebra. A multipolar intuitionistic fuzzy set  $(\hat{\ell}, \hat{s})$  over  $X$  is said multipolar intuitionistic fuzzy ideal in  $X$  if satisfies:

- i.  $(\forall a \in X) (\hat{\ell}(0) \geq \hat{\ell}(a) \text{ and } \hat{s}(0) \leq \hat{s}(a))$  such that  $(\pi_i \circ \hat{\ell})(0) \geq (\pi_i \circ \hat{\ell})(a)$  and  $(\pi_i \circ \hat{s})(0) \leq (\pi_i \circ \hat{s})(a)$ ,
  - ii.  $(\forall a, b \in X) (\hat{\ell}(a) \geq \inf\{\hat{\ell}(a * b), \hat{\ell}(b)\} \text{ and } \hat{s}(a) \leq \sup\{\hat{s}(a * b), \hat{s}(b)\})$  such that  $(\pi_i \circ \hat{\ell})(a) \geq \inf\{(\pi_i \circ \hat{\ell})(a * b), (\pi_i \circ \hat{\ell})(b)\}$  and  $(\pi_i \circ \hat{s})(a) \leq \sup\{(\pi_i \circ \hat{s})(a * b), (\pi_i \circ \hat{s})(b)\}$ ,
- for all  $a, b \in X$  and  $i = 1, 2, \dots, m$ .

**Example 1.14** [17] Let  $(X; *, 0)$  be a  $B$ -algebra based on Example 1.2. Given a multipolar intuitionistic fuzzy set  $(\hat{\ell}, \hat{s})$  over  $X$  by

$$\begin{aligned}
 & (\hat{\ell}, \hat{s}) : X \rightarrow ([0,1] \times [0,1])^5, \\
 x \mapsto & \begin{cases} ((0.7,0.3), (0.6,0.25), (0.7,0.15), (0.63,0.2), (0.8,0.18)) & \text{if } x \in \{0, b\}, \\ ((0.3,0.6), (0.4,0.5), (0.5,0.4), (0.2,0.7), (0.4,0.5)) & \text{if } x \in \{a, c\}. \end{cases}
 \end{aligned}$$

Then,  $(\hat{\ell}, \hat{s})$  is 5-polar intuitionistic fuzzy ideal of  $X$ .

## METHODS

In 2021, G. Muhiuddin and D. Al-Kadi [10] study about bipolar fuzzy implicative ideals of  $BCK$ -algebras and its characterizes. By the similiar way using analogy concepts, we obtain and investigated the definitions and related characterizes of multipolar intuitionistic fuzzy positive implicative ideal in  $B$ -algebras. The procedure in this study was carried out as follows.

1. We constructing a new structure, namely multipolar intuitionistic fuzzy positive implicative ideal in  $B$ -algebras and give the definition of this concept.
2. We give the example of multipolar intuitionistic fuzzy positive implicative ideal in  $B$ -algebras.
3. By using analogy concepts in [10], we investigated and proven some theorems or propositions of multipolar intuitionistic fuzzy positive implicative ideal in  $B$ -algebras.

## RESULTS AND DISCUSSION

Muhiuddin and Al-Kadi [10] studied about bipolar fuzzy implicative ideals in  $BCK$ -algebras and Amigo et al in [17] explained the new structure of multipolar intuitionistic fuzzy ideal in  $B$ -algebras. So, we continue the concept written by Amigo [17] with combining the concept from Muhiuddin and Al-Kadi [10]. In this part, we discuss about multipolar intuitionistic fuzzy positive implicative ideal in  $B$ -algebras. The section starts with the definition of new structure in  $B$ -algebras is multipolar intuitionistic fuzzy positive implicative ideal, then we give some examples and prove some properties of that.

**Definition 3.1** Let  $(X; *, 0)$  be a  $B$ -algebra. A multipolar intuitionistic fuzzy set  $(\hat{\ell}, \hat{s})$  over  $X$  is said multipolar intuitionistic fuzzy positive implicative ideal in  $X$  if satisfies:

- i.  $(\forall a \in X)(\hat{\ell}(0) \geq \hat{\ell}(a)$  and  $\hat{s}(0) \leq \hat{s}(a)$ ) such that  
 $(\pi_i \circ \hat{\ell})(0) \geq (\pi_i \circ \hat{\ell})(a)$  and  $(\pi_i \circ \hat{s})(0) \leq (\pi_i \circ \hat{s})(a)$ ,
- ii.  $(\forall a, b, c \in X)$   $(\hat{\ell}(a * c) \geq \inf\{\hat{\ell}((a * b) * c), \hat{\ell}(b * c)\})$  and  
 $\hat{s}(a * c) \leq \sup\{\hat{s}((a * b) * c), \hat{s}(b * c)\})$  such that  
 $(\pi_i \circ \hat{\ell})(a * c) \geq \inf\{(\pi_i \circ \hat{\ell})((a * b) * c), (\pi_i \circ \hat{\ell})(b * c)\}$  and  
 $(\pi_i \circ \hat{s})(a * c) \leq \sup\{(\pi_i \circ \hat{s})((a * b) * c), (\pi_i \circ \hat{s})(b * c)\}$ ,

for all  $a, b, c \in X$  and  $i = 1, 2, \dots, m$ .

**Example 3.2** Let  $(\mathbb{R}^+ - \{0\}; *, 1)$  be a  $B$ -algebra based on Example 1.4. Given a multipolar intuitionistic fuzzy set  $(\hat{\ell}, \hat{s})$  over  $\mathbb{R}^+ - \{0\}$  by

$$\begin{aligned}
 & (\hat{\ell}, \hat{s}) : X \rightarrow ([0,1] \times [0,1])^5, \\
 x \mapsto & \begin{cases} ((1,0), (1,0), (1,0), (1,0), (1,0)) & \text{if } x = 1, \\ ((0.5,0.5), (0.4,0.4), (0.3,0.3), (0.2,0.2), (0.1,0.1)) & \text{if } x \neq 1. \end{cases}
 \end{aligned}$$

Then,  $(\hat{\ell}, \hat{s})$  is 5-polar intuitionistic fuzzy positive implicative ideal of  $\mathbb{R}^+ - \{0\}$ .

Moreover, we explain the condition of the set  $I(\omega)$  to be an ideal of  $X$  for all  $\omega \in X$ . Furthermore, we give the condition for the set  $I(\omega)$  to be a positive implicative ideal of  $X$  and its example.

**Theorem 3.3** Let  $(X; *, 0)$  be a B-algebra and  $x \in X$ . If  $(\hat{\ell}, \hat{s})$  is a multipolar intuitionistic fuzzy positive implicative ideal of  $X$ , then  $I(\omega)$  is an ideal of  $X$  where

$$I(\omega) = \{a \in X \mid \hat{\ell}(a) \geq \hat{\ell}(\omega) \text{ and } \hat{s}(a) \leq \hat{s}(\omega)\}.$$

Proof. Let  $(\hat{\ell}, \hat{s})$  be a multipolar intuitionistic fuzzy ideal of  $X$  where

$$I(\omega) = \{a \in X \mid \hat{\ell}(a) \geq \hat{\ell}(\omega) \text{ and } \hat{s}(a) \leq \hat{s}(\omega)\}.$$

i. By using Definition 1.13 (i) we have that

$$\hat{\ell}(0) \geq \hat{\ell}(\omega) \text{ and } \hat{s}(0) \leq \hat{s}(\omega).$$

So,  $0 \in I(\omega)$ .

ii. Let  $a, b \in X$  such that  $a * b \in I(\omega)$  and  $b \in I(\omega)$ . Then,

$$\hat{\ell}(a * b) \geq \hat{\ell}(\omega) \text{ and } \hat{s}(a * b) \leq \hat{s}(\omega),$$

$$\hat{\ell}(b) \geq \hat{\ell}(\omega) \text{ and } \hat{s}(b) \leq \hat{s}(\omega).$$

By using Definition 3.1 (ii) with replacing  $c = 0$ , we have that

$$\hat{\ell}(a) \geq \inf\{\hat{\ell}(a * b), \hat{\ell}(b)\} \text{ and } \hat{s}(a) \leq \sup\{\hat{s}(a * b), \hat{s}(b)\},$$

such that

$$\hat{\ell}(a) \geq \hat{\ell}(\omega) \text{ and } \hat{s}(a) \leq \hat{s}(\omega).$$

So,  $a \in I(\omega)$ .

Therefore,  $I(\omega)$  is an ideal of  $X$ . ■

**Theorem 3.4** Let  $(X; *, 0)$  be a B-algebra and  $a \in X$ . If  $(\hat{\ell}, \hat{s})$  is a multipolar intuitionistic fuzzy positive implicative ideal of  $X$ , then  $I(\omega)$  is a positive implicative ideal of  $X$  where

$$I(\omega) = \{a \in X \mid \hat{\ell}(a) \geq \hat{\ell}(\omega) \text{ and } \hat{s}(a) \leq \hat{s}(\omega)\}.$$

Proof. Let  $(\hat{\ell}, \hat{s})$  be a multipolar intuitionistic fuzzy positive implicative ideal of  $X$  where

$$I(\omega) = \{a \in X \mid \hat{\ell}(a) \geq \hat{\ell}(\omega) \text{ and } \hat{s}(a) \leq \hat{s}(\omega)\}.$$

i. By using Definition 3.1 (i) we have that

$$\hat{\ell}(0) \geq \hat{\ell}(\omega) \text{ and } \hat{s}(0) \leq \hat{s}(\omega).$$

So,  $0 \in I(\omega)$ .

ii. Let  $a, b, c \in X$  such that  $(a * b) * c \in I(\omega)$  and  $b * c \in I(\omega)$ . Then,

$$\hat{\ell}((a * b) * c) \geq \hat{\ell}(\omega) \text{ and } \hat{s}((a * b) * c) \leq \hat{s}(\omega),$$

$$\hat{\ell}(b * c) \geq \hat{\ell}(\omega) \text{ and } \hat{s}(b * c) \leq \hat{s}(\omega).$$

By using Definition 3.1 (ii), we have that

$$\hat{\ell}(a * c) \geq \inf\{\hat{\ell}((a * b) * c), \hat{\ell}(b * c)\}$$

and

$$\hat{s}(a * c) \leq \sup\{\hat{s}((a * b) * c), \hat{s}(b * c)\},$$

such that

$$\hat{\ell}(a * c) \geq \hat{\ell}(\omega) \text{ and } \hat{s}(a * c) \leq \hat{s}(\omega).$$

So,  $a * c \in I(\omega)$ .

Therefore,  $I(\omega)$  is a positive implicative ideal of  $X$ . ■

**Example 3.5** Let  $(\mathbb{R}^+ - \{0\}; *, 1)$  be a  $B$ -algebra based on Example 1.4. Given a multipolar intuitionistic fuzzy ideal  $(\hat{\ell}, \hat{s})$  over  $\mathbb{R}^+ - \{0\}$  based on Example 3.2 where

$I(x) = \{1, x \mid \hat{\ell}(1) \geq \hat{\ell}(x) \text{ and } \hat{s}(1) \leq \hat{s}(x), \hat{\ell}(x) \geq \hat{\ell}(x) \text{ and } \hat{s}(x) \leq \hat{s}(x)\}$   
 with  $x \neq 1$  and  $x \in \mathbb{R}^+ - \{0\}$ . Then,  $I(x)$  is a positive implicative ideal of  $\mathbb{R}^+ - \{0\}$ .

In addition, we examine about properties of multipolar intuitionistic fuzzy positive implicative ideal in  $B$ -algebra such that make some following propositions and corollary.

**Proposition 3.6** Let  $(X; *, 0)$  be a  $B$ -algebra. Every multipolar intuitionistic fuzzy positive implicative ideal  $(\hat{\ell}, \hat{s})$  over  $X$  satisfies the following implication,

$$\text{if } a \leq b \text{ then } \hat{\ell}(a) \geq \hat{\ell}(b) \text{ and } \hat{s}(a) \leq \hat{s}(b),$$

for all  $a, b \in X$ .

Proof. Let  $a, b \in X$  such that  $a \leq b$ . So,  $a * b = 0$ . By using Definition 3.1 (i) and (ii) with replacing  $c = 0$ , we have that

$$\hat{\ell}(a) \geq \inf\{\hat{\ell}(a * b), \hat{\ell}(b)\} = \hat{\ell}(b)$$

and

$$\hat{s}(a) \leq \sup\{\hat{s}(a * b), \hat{s}(b)\} = \hat{s}(b). \quad \blacksquare$$

**Proposition 3.7** Let  $(X; *, 0)$  be a commutative  $B$ -algebra. For any multipolar intuitionistic fuzzy positive implicative ideal  $(\hat{\ell}, \hat{s})$  over  $X$ , if it satisfies

$$\hat{\ell}(a * b) \geq \hat{\ell}((a * b) * b) \text{ and } \hat{s}(a * b) \leq \hat{s}((a * b) * b),$$

for all  $a, b \in X$ , then it satisfies

$$\hat{\ell}((a * c) * (b * c)) \geq \hat{\ell}((a * b) * c) \text{ and } \hat{s}((a * c) * (b * c)) \leq \hat{s}((a * b) * c),$$

for all  $a, b, c \in X$ .

Proof. Let  $a, b, c \in X$  such that

$$((a * c) * (b * c)) * c \leq (a * b) * c.$$

By using Proposition 1.5 and 1.8, we have that

$$((a * (b * c)) * c) * c = ((a * c) * (b * c)) * c \leq (a * b) * c.$$

From Proposition 3.6, we have

$$\hat{\ell}(((a * (b * c)) * c) * c) \geq \hat{\ell}((a * b) * c)$$

and

$$\hat{s}(((a * (b * c)) * c) * c) \leq \hat{s}((a * b) * c).$$

So, from Proposition 1.8, we have that

$$\hat{\ell}((a * c) * (b * c)) \geq \hat{\ell}((a * b) * c) \text{ and } \hat{s}((a * c) * (b * c)) \leq \hat{s}((a * b) * c). \quad \blacksquare$$

**Proposition 3.8** Let  $(X; *, 0)$  is a  $B$ -algebra. For any multipolar intuitionistic fuzzy positive implicative ideal  $(\hat{\ell}, \hat{s})$  over  $X$ , if it satisfies

$$\hat{\ell}((a * c) * (b * c)) \geq \hat{\ell}((a * b) * c) \text{ and } \hat{s}((a * c) * (b * c)) \leq \hat{s}((a * b) * c),$$

for all  $a, b, c \in X$ , then it satisfies

$$\hat{\ell}(a * b) \geq \hat{\ell}((a * b) * b) \text{ and } \hat{s}(a * b) \leq \hat{s}((a * b) * b),$$

for all  $a, b \in X$ .

Proof. Let  $a, b, c \in X$ . By using Definition 1.1 (i) and (ii) with replacing  $c$  by  $b$ , we have  $\hat{\ell}(a * b) \geq \hat{\ell}((a * b) * b)$  and  $\hat{s}(a * b) \leq \hat{s}((a * b) * b)$ .

From Proposition 3.7 and Proposition 3.8, we have this following corollary.

**Corollary** *If we assume that  $X$  is a commutative  $B$ -algebra, then the statements in Proposition 3.7 and Proposition 3.8 are equivalent.*

Moreover, we gives an another condition of multipolar intuitionistic fuzzy positive implicative ideal in  $B$ -algebra such that make this following proposition and lemma.

**Proposition 3.9** *Let  $(X; *, 0)$  is a commutative  $B$ -algebra. A multipolar intuitionistic fuzzy set  $(\hat{\ell}, \hat{s})$  over  $X$  is a multipolar intuitionistic fuzzy positive implicative ideal  $(\hat{\ell}, \hat{s})$  over  $X$  if and only if  $(a * b) * c = 0$  implies*

$$\hat{\ell}(a) \geq \inf\{\hat{\ell}(b), \hat{\ell}(c)\} \text{ and } \hat{s}(a) \leq \sup\{\hat{s}(b), \hat{s}(c)\},$$

for all  $a, b, c \in X$ .

Proof. We assume that  $(\hat{\ell}, \hat{s})$  is a multipolar intuitionistic fuzzy positive implicative ideal over  $X$ . Let  $a, b, c \in X$  such that  $(a * b) * c = 0$ . So,  $a * b \leq c$ . By using Definition 3.1 (i) and (ii) with replacing  $c = 0$ , we have

$$\hat{\ell}(a) \geq \inf\{\hat{\ell}(a * b), \hat{\ell}(b)\} \geq \inf\{\inf\{\hat{\ell}((a * b) * c), \hat{\ell}(c)\}, \hat{\ell}(b)\} = \inf\{\hat{\ell}(b), \hat{\ell}(c)\}$$

and

$$\hat{s}(a) \leq \sup\{\hat{s}(a * b), \hat{s}(b)\} \leq \sup\{\sup\{\hat{s}((a * b) * c), \hat{s}(c)\}, \hat{s}(b)\} = \sup\{\hat{s}(b), \hat{s}(c)\}.$$

Conversely, we assume  $(a * b) * c = 0$  implies

$$\hat{\ell}(a) \geq \inf\{\hat{\ell}(b), \hat{\ell}(c)\} \text{ and } \hat{s}(a) \leq \sup\{\hat{s}(b), \hat{s}(c)\},$$

for all  $a, b, c \in X$ . Let  $a \in X$ . By using Definition 1.1 (ii) and Definition 1.12, we have

$$\hat{\ell}(0) = \hat{\ell}(a * a) \geq \inf\{\hat{\ell}(a), \hat{\ell}(a)\} = \hat{\ell}(a) \text{ and } \hat{s}(0) = \hat{s}(a * a) \leq \sup\{\hat{s}(a), \hat{s}(a)\} = \hat{s}(a).$$

Then, let  $a, b, c \in X$ . From Definition 1.1 (i), we have

$$((a * c) * (b * c)) * ((a * c) * (b * c)) = 0$$

$$((a * c) * ((a * c) * (b * c))) * (b * c) = 0$$

such that, by using Definition 1.13 (ii) and Proposition 3.7, we have

$$\begin{aligned} \hat{\ell}(a * c) &\geq \inf\{\hat{\ell}((a * c) * (b * c)), \hat{\ell}(b * c)\} \\ &\geq \inf\{\hat{\ell}((a * b) * c), \hat{\ell}(b * c)\} \end{aligned}$$

and

$$\begin{aligned} \hat{s}(a * c) &\leq \sup\{\hat{s}((a * c) * (b * c)), \hat{s}(b * c)\} \\ &\leq \sup\{\hat{s}((a * b) * c), \hat{s}(b * c)\}. \end{aligned}$$

So,  $(\hat{\ell}, \hat{s})$  is a multipolar intuitionistic fuzzy positive implicative ideal over  $X$ . ■



**Lemma 3.10** Let  $(X; *, 0)$  be a commutative B-algebra. If  $(\hat{\varphi}, \hat{\psi})$  is a multipolar intuitionistic fuzzy positive implicative ideal over  $X$  then

$$\hat{\varphi}((a * c) * (b * c)) \geq \inf \wedge \{\hat{\varphi}(\omega_i) | i = 1, 2, \dots, n\}$$

and

$$\hat{\psi}((a * c) * (b * c)) \leq \sup \wedge \{\hat{\psi}(\omega_i) | i = 1, 2, \dots, n\}$$

where

$$\prod_{i=1}^n ((a * b) * c) * \omega_i = 0,$$

for all  $a, b, c, \omega_1, \dots, \omega_n \in X$ .

Proof. Let  $a, b, c, \omega_1, \dots, \omega_n \in X$  such that

$$\prod_{i=1}^n ((a * b) * c) * \omega_i = 0.$$

By using Proposition 3.7 and Proposition 3.9, we have

$$\begin{aligned} \hat{\varphi}((a * c) * (b * c)) &\geq \hat{\varphi}((a * b) * c) \\ &\geq \inf \wedge \{\hat{\varphi}(\omega_i) | i = 1, 2, \dots, n\} \end{aligned}$$

and

$$\begin{aligned} \hat{\psi}((a * c) * (b * c)) &\leq \hat{\psi}((a * b) * c) \\ &\leq \sup \wedge \{\hat{\psi}(\omega_i) | i = 1, 2, \dots, n\}. \end{aligned}$$

■

## CONCLUSIONS

In this paper, we examine the concept of a multipolar intuitionistic fuzzy positive implicative ideal in B-algebras and investigated some its properties. We explain the conditions for a multipolar intuitionistic fuzzy set become a multipolar intuitionistic fuzzy positive implicative ideal and give some examples too. In our opinion, these definitions and main results of multipolar intuitionistic fuzzy positive implicative ideal can be applied with similarly in other algebraic structure such as BG-algebras, BF-algebras or BD-algebras because thats structure have some same basic concept with B-algebras. The future researchers can use this concept to investigate the relation with previous structure like multipolar intuitionistic fuzzy ideal in B-algebras too.

## REFERENCES

- [1] Imai, Y.; Iseki, K. 1966. On axiom system of propositional calculi, XIV. *Japan Acad* 42, 19-22.
- [2] Iseki, K. 1966. An algebra related with a propositional calculus. *Japan Acad* 42, 26-29.
- [3] Neggers, J.; Kim, H. S. 2002. On B-algebras. *Mate Vesnik*, no. 54, 21-29.
- [4] Zadeh, L.A. 1965. Fuzzy Sets. *Information and Control*, Vol. 8, no. 3, 338-353.

- 
- [5] Zhang, W.R.; Yang, Y. 1994. Bipolar fuzzy sets. *Proceedings of the 1998 IEEE International Conference on Fuzzy Systems*, 835–840. Anchorage, AK, USA.
- [6] Meng, J.; Jun, Y.B.; Kim, H.S. 1997. Fuzzy Implicative Ideals of BCK-Algebras. *Fuzzy Sets and Systems*, Vol. 89, 243-248.
- [7] Jun, Y.B.; Roh, E.H.; Kim, H.S. 2002. On fuzzy B-algebras. *Czechoslovak Mathematical Journal*, Vol. 52, no. 2, 375—384.
- [8] Ahn, S.S.; Bang, K. 2003. On fuzzy subalgebras in B-algebras. *Commun Korean Math. Soc*, Vol. 10, no. 3, 429-437.
- [9] Muhiuddin, G. and D. Al-Kadi (2020). Bipolar Fuzzy BCI-Implicative Ideals of BCI-Algebras. *Annals of Communication in Mathematics*, Vol. 3, no. 1, 88-96.
- [10] Muhiuddin, G.; Al-Kadi, D. 2021. Bipolar Fuzzy Implicative Ideals of BCK-Algebras. *Journal of Mathematics*.
- [11] Chen, J.; Li, S.; Ma, S.; Wang, X. 2014. m-polar fuzzy sets: An extension of bipolar fuzzy sets. *Sci. World J*.
- [12] Al-Masarwah, A.; Ahmad, A.G. 2019. m-Polar Fuzzy Ideals of BCK/BCI-Algebras. *Journal of King Saud University-Science*, Vol. 31, no. 4, 1220–1226.
- [13] Borzooei, R.A., M. M. Takallo, S. S. Ahn, and Y. B. Jun (2019). Multipolar Fuzzy p-ideals of BCI-Algebras. *MDPI*, 7, 1094.
- [14] Attanassov, K.T. 1999. Intuitionistic Fuzzy Sets Theory and Applications. Bulgaria.
- [15] Kang, K. T., S. Z. Song, and Y. B. Jun (2020). Multipolar Intuitionistic Fuzzy Set with Finite Degree and Its Application in BCK/BCI-Algebras. *MDPI*, 8, 177.
- [16] Borzooei, R.A.; Kim, H.S.; Jun, Y.B.; Ahn, S.S. 2020. On Multipolar Intuitionistic Fuzzy B-Algebras. *MDPI*, 8, 907.
- [17] Amigo, Royyan; Hidayat, Noor; Krisnawati, Vira Hari. 2022. Multipolar Intuitionistic Fuzzy Ideal in B-algebras. *Cauchy-Jurnal Matematika Murni dan Aplikasi*, Vol. 7, no. 2, 293-301.
- [18] Fitria, E.; Gemawati, S.; Kartini. 2017. Prime Ideals in B-algebras. *International Journal of Algebra*, Vol. 11, no. 7, 301-309.
- [19] Abdullah, H.K; Atshan, A.A. 2017. Complete Ideal and n-Ideal of B-algebras. *Applied Mathematical Sciences*, Vol. 11, no. 35, 1705-1713.
- [20] Muhiuddin, G; Al-Kadi, D; Mahboob, A; Shum, K.P. 2020. New Types of Bipolar Fuzzy Ideals of BCK-Algebras. *IJAA*.
- [21] Senapati, T.; Bhowmik, M.; Pal, M. 2011. Fuzzy Closed Ideals of B-Algebras. *IJCSET*, Vol. 1, no. 10, 669-673.