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Spatial Weight Matrix Comparison of SAR-X Model using Casetti Approach

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ABSTRACT

The Spatial Autoregressive Exogenous (SAR-X) model has the disadvantage of producing a beta estimator for the entire observation location. This was overcome by developing the SAR-X model into a SAR-X model with the Casetti approach, which produces a beta estimator for each observation location. The SAR-X model, with the Casetti approach, is a spatial model used for the description and prediction of location-based data. One of the characteristics of the SAR-X model is the spatial weighting matrix, which influences the parameter estimation results and prediction results. Therefore, this research will examine the SAR-X model with the Casetti approach using spatial contiguity and inverse distance weight matrices on location-based data, namely the habits and behavior of the people of Java Island towards culture. The analysis results show that the SAR-X model with the Casetti approach is the best, using an inverse square distance spatial weight matrix supported by a minimum Root Mean Square Error value and a fairly high coefficient of determination value.

Keywords: Spatial Autoregressive Exogenous; Casetti; Culture; Spatial Weight Matrix

INTRODUCTION

The order of data based on location is referred to as spatial data. The analysis used to analyze spatial data is called spatial analysis. One of the models used in spatial analysis is Spatial Autoregressive (SAR). SAR models are used for the description and prediction of location-based data. The SAR model with additional exogenous variables becomes the Spatial Autoregressive Exogenous (SAR-X) model. [1] conducted research on the SAR-X model and its application to WBTb determination data. This research shows that one β estimate is obtained for each location, this is a limitation in the SAR-X model. Therefore, the SAR-X model was developed using the Casetti approach. The SAR-X model with the Casetti approach involves longitude and latitude coordinates, which will produce an estimate of β for each sample location, so it is assumed that the SAR-X model with the Casetti approach will produce more accurate descriptions and predictions.

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One of the characteristics of the SAR-X model is the existence of a weight matrix called the spatial weight matrix. Determining the spatial weight matrix varies, including using contiguity and inverse distance. The spatial weight matrix is determined before starting the analysis. The selection of the spatial weight matrix contributes to the parameter estimation results, prediction results, and prediction error as measured by the Root Mean Square Error (RMSE). Therefore, studying the spatial weight matrix in the SAR-X model using the Casetti approach is interesting to investigate.

The SAR-X model with the Casetti approach was studied by [2] and [3]. [2] used the SAR-X model with the Casetti approach and applied it to educational data, the spatial weight matrix used was only the distance inverse spatial weight matrix. [3] used the SAR-X model with the Casetti approach and applied it to rainfall data, the spatial weight matrix used was the inverse distance weight matrix. Therefore, this research Gap research is a SAR-X model with a Casetti approach using a spatial contiguity weight matrix (rook contiguity, bishop contiguity, queen contiguity), inverse distance, and inverse squared distance on the habits and behavior of the people of Java Island towards culture.

People's habits and behaviors towards culture are part of intangible cultural heritage, which is defined as practices, representations, expressions, knowledge, skills, objects, artifacts, and cultural spaces in which the people involved are part of the world's cultural heritage [3]. The selection of data on the habits and behavior of the people of Java Island towards culture in the Casetti approach SAR-X model analysis was due to a shift in the function of culture in maintaining a harmonious relationship between society and nature [4]. This is an important basis in the process of selecting research data. This is confirmed by Law Number 5 of 2017, which states that culture is considered a source of investment that needs to be maintained and preserved from generation to generation. Therefore, it is necessary to describe and predict the behavioral habits of the people of Java Island towards culture in order to build cultural preservation.

The exogenous variables used in this research are the Percentage of Population Involved in Performing Arts (PPIPA), the Percentage of Households that Use Traditional Products (PHTP), and the Percentage of Attendance for Traditional Ceremonies by Households (PATCH). The selection of exogenous variables is based on the main aim of describing and analyzing the extent to which traditional culture is still alive and plays a role in modern society, so that it can provide deeper analysis results regarding the preservation and adaptation of traditional culture.

Research by [1] pointed out the shortcomings of the SAR-X model used in that the model does not involve latitude and longitude coordinates, which produce β estimators for each location. [2] and [3] only revealed the results of the description and prediction of the SAR-X model with the Casetti approach using an inverse distance spatial weight matrix. Therefore, this study uses the SAR-X model with the Casetti approach to overcome the β estimator results obtained for each location and the use of various spatial weight matrices. The spatial weight matrices used in this research are contiguity (rook contiguity, bishop contiguity, queen contiguity), inverse distance, and inverse squared distance. The results of the description and prediction of the SAR-X model using the Casetti approach using the spatial contiguity weight matrix and inverse distance of the habits and behaviors of the people of Java Island towards culture are expected to become recommendations for the Department of Tourism, Culture, Youth, and Sports (Disparbudpora) in determining strategic policies for preserving intangible cultural heritage on the island of Java.

METHODS

This research uses data on the habits and behavior of people on the island of Java towards (Y) culture in 2013–2023 obtained from the Ministry of Education and Culture. The exogenous variables used in 2021 are PPIPA (X_1), PHTP (X_2), and PATCH (X_3), which were obtained from the Central Statistics Agency. The SAR-X model, using the Casetti approach as written by Casetti in [2], is shown through Equation (1).

$$\mathbf{y} = \rho\mathbf{W}\mathbf{y} + \mathbf{XZJ}\boldsymbol{\beta_0} + \boldsymbol{\varepsilon}, \boldsymbol{\varepsilon} \overset{iid}{\sim} \mathrm{N}(0, \sigma^2\mathbf{I}), \tag{1}$$

with,
$\mathbf{y}$: vector of independent variables of size $n \times 1$,
ρ: spatial lag parameter coefficient,
$\mathbf{W}$: a spatial weight matrix of size$(n \times n)$,
$\mathbf{X}$: matrix of size-independent variables $(n \times nk)$,
$\boldsymbol{Z}$: a location-related matrix of entries that each reflect the latitude and longitude of an observation, each of varying size $(nk \times 2nk)$,
$\mathbf{J}$: expansion of the identity matrix $(2nk \times 2k)$,
$\boldsymbol{\beta_0}$: the parameter vector expressed by $\beta_{latitude}, \beta_{longitude}$ size $(2k \times 1)$,
$\boldsymbol{\varepsilon}$: vector error $(n \times 1)$.

Data analysis calculations were assisted by R programming version 4.2.2. The steps for data analysis using the SAR-X model with the Casetti approach to the habits and behavior of Javanese people towards culture are as follows:

1. Perform data exploration.
2. Create a weighted spatial matrix using inverse distance and contiguity. As per reference [5], the spatial weight matrix is utilized to determine the weights among observable places, taking into account their adjacent relationships. The spatial contiguity weight matrix is represented by queen contiguity, which defines neighbors based on sides and angles. The weight given is one for neighboring observation units and zero for non-adjacent units. The distance inverse spatial weight matrix is shown trough the inverse distance spastial weight matriks, and the inverse distance squared spatial weight matrix is calculated based on distance between locations [6]. Equations (2) and (3) show how to calculate the spatial weight matrices for inverse distance and inverse distance squared.

$$w_{ij} = \frac{1}{d_{ij}}, \tag{2}$$

$$w_{ijk} = \frac{1}{d_{ij}^2}, \tag{3}$$

with,

$$d_{ij} = \sqrt{(xi - xj)^2 + \left(y_i - y_j\right)^2}. \tag{4}$$

The spatial weight matrix is standardized so that its rows sum to one using Equation (5).

$$\mathbf{W} = \frac{c_{ij}}{c_i}. \tag{5}$$

3. Check the influence of location using a spatial dependence test. Spatial dependency

test is conducted to observe spatial autocorrelation among locations [7]. The hypothesis testing is:
H_0 : there is no spatial autocorrelation between locations.
H_1: there is spatial autocorrelation between locations.
Test statistics:

$$Z_{value} = \frac{I - \mathrm{E}(I)}{\sqrt{\mathrm{Var}\,(I)}}, \tag{6}$$

I: Moran indeks value,
$\mathrm{Var}(I)$: Moran index variance,
$\mathrm{E}(I)$: expected value of Moran index.
Decision:
H_0 rejected if $\mathrm{Z_{value}} \leq -Z_{\frac{\alpha}{2}}$ atau $\mathrm{Z_{value}} \geq Z_{\frac{\alpha}{2}}$.

4. Estimation of SAR-X model parameters using the Casetti approach using a weight matrix of spatial contiguity and inverse distance using the maximum likelihood method. The MLE approach will be used to estimate the parameters of the SAR-X model because the random error variable has a distribution and is presumed to be normal. According to [8], a likelihood function will be maximized to determine the unknown parameters in the MLE approach. Equation (1) can be written as Equation (7).

$$\boldsymbol{\varepsilon} = \mathbf{y} - \rho\mathbf{W}\mathbf{y} - \mathbf{XZJ}\boldsymbol{\beta}_0. \tag{7}$$

The probability density function used will be shown by Equation (8):

$$f(\varepsilon|\rho, \boldsymbol{\beta}_0) = \frac{1}{(2\pi)^{\frac{n}{2}}(\sigma^2)^{\frac{n}{2}}} \exp\left(-\frac{(\mathbf{y} - \rho\mathbf{W}\mathbf{y} - \mathbf{XZJ}\boldsymbol{\beta}_0)^{\mathrm{T}}(\mathbf{y} - \rho\mathbf{W}\mathbf{y} - \mathbf{XJZ}\boldsymbol{\beta}_0)}{2\sigma^2}\right). \tag{8}$$

Likelihood function:

$$L(\rho, \boldsymbol{\beta}_0|\varepsilon) = \frac{1}{(2\pi)^{\frac{n}{2}}(\sigma^2)^{\frac{n}{2}}} \exp\left(-\frac{(\mathbf{y} - \rho\mathbf{W}\mathbf{y} - \mathbf{XZJ}\boldsymbol{\beta}_0)^{\mathrm{T}}(\mathbf{y} - \rho\mathbf{W}\mathbf{y} - \mathbf{XZJ}\boldsymbol{\beta}_0)}{2\sigma^2}\right). \tag{9}$$

Log Likelihood Function:

$$\begin{aligned} \ln L(\rho, \boldsymbol{\beta}_0|\varepsilon) &= \ln\left(\frac{1}{(2\pi)^{\frac{n}{2}}(\sigma^2)^{\frac{n}{2}}} \exp\left(-\frac{(\mathbf{y} - \rho\mathbf{W}\mathbf{y} - \mathbf{XZJ}\boldsymbol{\beta}_0)^{\mathrm{T}}(\mathbf{y} - \rho\mathbf{W}\mathbf{y} - \mathbf{XZJ}\boldsymbol{\beta}_0)}{2\sigma^2}\right)\right), \\ &= -\frac{n}{2}\ln(2\pi) - \frac{n}{2}\ln\sigma^2 - \frac{(\mathbf{y} - \rho\mathbf{W}\mathbf{y} - \mathbf{XZJ}\boldsymbol{\beta}_0)^{\mathrm{T}}(\mathbf{y} - \rho\mathbf{W}\mathbf{y} - \mathbf{XZJ}\boldsymbol{\beta}_0)}{2\sigma^2}. \end{aligned} \tag{10}$$

Parameter estimates $\hat{\rho}$ and $\widehat{\boldsymbol{\beta}_0}$ are obtained by maximizing the log likelihood function,

$$\hat{\rho} = (\mathbf{y} - \mathbf{XZJ}\boldsymbol{\beta}_0)(\mathbf{W}\mathbf{y})^{\mathbf{T}}(\mathbf{W}\mathbf{y}(\mathbf{W}\mathbf{y})^{\mathbf{T}})^{-1}, \tag{11}$$

$$\widehat{\boldsymbol{\beta}_0} = (\mathbf{y} - \rho\mathbf{W}\mathbf{y})(\mathbf{XZJ})^{\mathbf{T}}(\mathbf{XZJ}(\mathbf{XZJ})^{\mathbf{T}})^{-1}. \tag{12}$$

5. Check the influence of exogenous variables using parameter significance tests. The parameter significance test is carried out to determine the function of the independent variables in the model [9]. The significant parameters of the test using Wald are shown by Equation (13). The hypothesis testing is:
H_0 : parameter is not significant.
H_1: parameter is significant.
Test statistics used:

$$\mathrm{Wald} = \frac{\widehat{\boldsymbol{\beta}}^2}{\mathrm{Var}(\widehat{\boldsymbol{\beta}})}, \tag{13}$$

with,

$$\mathrm{Var}(\widehat{\boldsymbol{\beta}}) = \mathrm{Var}\ (\mathbf{ZJ}(\mathbf{y} - \rho\mathbf{Wy})(\mathbf{XZJ})^{\mathbf{T}}(\mathbf{XZJXZJ}^{\mathbf{T}})^{-1}). \quad (14)$$

Decision:
H_0 rejected if Wald $> X^2_{(\propto,1)}$.

6. Selection the best model by Root Mean Square Error (RMSE) and coefficient determination value. RMSE is an evaluation of prediction methods used to measure the accuracy of a model's prediction results [10]. The lower the RMSE value, the more accurate the method accuracy. RMSE can be calculated using Equation (15).

$$\mathrm{RMSE} = \sqrt{\frac{1}{n}\sum_{i}^{n}(\hat{y}_i - y_i)^2}. \quad (15)$$

with,
y_i: actual data i-th,
$\hat{y}_i$: prediction data i-th,
n: data count.
The accuracy with which the independent variables in the regression model can account for the variability of the dependent variable is shown by a measurement called the coefficient of determination [11].
The general formula for the coefficient of determination can be written in Equation (13):

$$R^2 = 1 - \frac{\sum_i^n (y_i - \hat{y}_i)^2}{\sum_i^n (y_i - \bar{y})^2}. \quad (13)$$

The closer R^2 is to 1, the higher the variability of the dependent variable, the more it can be explained by the independent variable [12].

7. Interpretation of the SAR-X model with the Casetti approach based on the best model obtained.

RESULTS AND DISCUSSION

a. Data Exploration

Data exploration is carried out to find out general initial information from the data. The following shows the number of habits and behaviors of the people of Java Island towards culture from 2013–2023, which is shown in Figure 1.

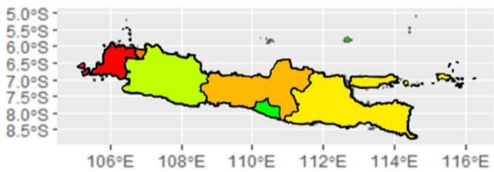

Figure 4. Map of the distribution of habits and behaviors of the people of Java Island towards culture from 2013–2023

Figure 4 shows the island of Java, which consists of six provinces: Banten, DKI Jakarta, West Java, Central Java, DI Yogyakarta, and East Java. Figure 4 depicts the mapping of the number of habits and behaviors of the people of Java Island towards culture in 2013–2023. The province with the highest number of habits and attitudes of people on the island of Java towards culture is shown in green, namely DI Yogyakarta. The province with the lowest number of Java Island people's habits and attitudes towards culture is shown in red, namely Banten Province.

b. Spatial Analysis

This research aims to predict the influence of location and exogenous variables (PPIPA, PHTP, and PATCH) on the habits and behavior of the people of Java Island towards culture. The spatial weighting matrices used in this research, namely queen contiguity, inverse distance, and inverse squared distance, are shown as follows:

$$\mathbf{W}^*_{\text{queen contiguity}} = \begin{bmatrix} 0 & 0,5 & 0 & 0 & 0 & 0,5 \\ 0,33 & 0 & 0,33 & 0,33 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0,5 & 0 & 0 & 0 & 0 & 0,5 \\ 0,5 & 0 & 0 & 0 & 0,5 & 0 \end{bmatrix}; \ \mathbf{W}^*_{\text{invers jarak}} = \begin{bmatrix} 0 & 0,17 & 0,09 & 0,14 & 0,34 & 0,24 \\ 0,15 & 0 & 0,17 & 0,47 & 0,10 & 0,08 \\ 0,13 & 0,29 & 0 & 0,34 & 0,11 & 0,10 \\ 0,13 & 0,48 & 0,20 & 0 & 0,09 & 0,08 \\ 0,29 & 0,10 & 0,06 & 0,09 & 0 & 0,44 \\ 0,23 & 0,09 & 0,06 & 0,08 & 0,51 & 0 \end{bmatrix};$$

$$\mathbf{W}^*_{\text{invers jarak kuadrat}} = \begin{bmatrix} 0 & 0,12 & 0,03 & 0,09 & 0,51 & 0,24 \\ 0,07 & 0 & 0,10 & 0,75 & 0,03 & 0,02 \\ 0,07 & 0,35 & 0 & 0,47 & 0,05 & 0,04 \\ 0,05 & 0,75 & 0,13 & 0 & 0,02 & 0,02 \\ 0,28 & 0,03 & 0,01 & 0,02 & 0 & 0,63 \\ 0,16 & 0,02 & 0,01 & 0,02 & 0,77 & 0 \end{bmatrix}$$

The spatial weight matrix is assumed to be more accurate than the inverse distance spatial weight matrix because the definition of neighbors is calculated based on the distance between observation areas, so that all observation locations are considered neighbors and have value.

The influence of location is described through a spatial dependence test. The results of the spatial dependency test using $\alpha = 0.1$ are shown in Table 1.

Table 1. Results of spatial dependency testing using the Moran index

Spatial Weight Matrix	Z_{value}	Autocorrelation
Queen Contiguity	2.3084	yes
Inverse Distance	2.4685	yes
Squared Inverse Distance	2.3827	yes

If the value of $Z_{\text{value}} \geq Z_{\frac{\alpha}{2}}$ (1.64), then there is an influence of location, which is shown through a positive correlation coefficient. This means that there is an influence of location on the habits and behavior of the people of Java Island regarding culture.

The estimated value of parameter ρ is obtained using Equation (11) which is shown in Table 2. The estimated value of parameter β is obtained based on Equation (12) which is shown in Table 3.

Table 2. Estimation $\hat{\rho}$ SAR-X model with Casetti approach

Spatial Weight Matrix	$\hat{\rho}$
Queen Contiguity	0.89
Inverse Distance	1.08
Squared Inverse Distance	0.99

The positive $\hat{\rho}$ value means that the behavior and habits of people in Java Island towards culture are influenced by the location of provinces that are close to the observed province.

Table 3. Estimation $\hat{\boldsymbol{\beta}}$ SAR-X model with Casetti approach

Province	Queen Contiguity Spatial Weight Matrix			Inverse Distance Spatial Weight Matrix			Square Inverse Distance Spatial Weight Matrix		
	$\hat{\boldsymbol{\beta}}_{X_1}$	$\hat{\boldsymbol{\beta}}_{X_2}$	$\hat{\boldsymbol{\beta}}_{X_3}$	$\hat{\boldsymbol{\beta}}_{X_1}$	$\hat{\boldsymbol{\beta}}_{X_2}$	$\hat{\boldsymbol{\beta}}_{X_3}$	$\hat{\boldsymbol{\beta}}_{X_1}$	$\hat{\boldsymbol{\beta}}_{X_2}$	$\hat{\boldsymbol{\beta}}_{X_3}$
West Java	36.18	-0.79	-0.04	12.14	-0.48	0.30	9.95	-0.39	0.23
Central Java	41.47	-1.08	0.63	13.14	-0.53	0.48	10.75	-0.43	0.37

Province	Queen Contiguity Spatial Weight Matrix			Inverse Distance Spatial Weight Matrix			Square Inverse Distance Spatial Weight Matrix		
	$\hat{\boldsymbol{\beta}}_{X_1}$	$\hat{\boldsymbol{\beta}}_{X_2}$	$\hat{\boldsymbol{\beta}}_{X_3}$	$\hat{\boldsymbol{\beta}}_{X_1}$	$\hat{\boldsymbol{\beta}}_{X_2}$	$\hat{\boldsymbol{\beta}}_{X_3}$	$\hat{\boldsymbol{\beta}}_{X_1}$	$\hat{\boldsymbol{\beta}}_{X_2}$	$\hat{\boldsymbol{\beta}}_{X_3}$
East Java	31.46	-0.44	-0.99	11.63	-0.44	0.07	9.58	-0.37	0.05
DI Yogyakarta	10.93	0.79	-4.03	8.19	-0.25	-0.69	6.86	-0.22	-0.55
DKI Jakarta	71.73	-2.98	5.42	17.87	-0.80	1.67	14.46	-0.63	1.32
Banten	60.80	-2.32	3.79	16.04	-0.70	1.26	13.02	-0.56	0.99

Positive $\hat{\boldsymbol{\beta}}$ value can be interpreted that there is a positive relationship between exogenous variables and people's habits and behavior towards culture. This means that if other variables are considered constant, when exogenous variables increase, it can increase the habits and behavior of people in Java Island towards culture, but if the value of $\hat{\boldsymbol{\beta}}$ is negative, it means the opposite.

The influence of exogenous variables is examined using parameter significance tests using Wald values. Using $\alpha = 0.1$, if the Wald value greater than 2.7055, then the exogenous variable is significant in the habits and behavior of the people of Java Island towards culture. The results of the parameter significance test are shown in Table 4.

Table 4. Parameter significance test results with Wald

Province	Queen Contiguity Spatial Weight Matrix			Inverse Distance Spatial Weight Matrix			Square Inverse Distance Spatial Weight Matrix		
	Significant $\hat{\boldsymbol{\beta}}_{X_1}$	Significant $\hat{\boldsymbol{\beta}}_{X_2}$	Significant $\hat{\boldsymbol{\beta}}_{X_3}$	Significant $\hat{\boldsymbol{\beta}}_{X_1}$	Significant $\hat{\boldsymbol{\beta}}_{X_2}$	Significant $\hat{\boldsymbol{\beta}}_{X_3}$	Significant $\hat{\boldsymbol{\beta}}_{X_1}$	Significant $\hat{\boldsymbol{\beta}}_{X_2}$	Significant $\hat{\boldsymbol{\beta}}_{X_3}$
West Java	yes	yes	yes	yes	yes	no	yes	yes	no
Central Java	yes	yes	yes	yes	yes	yes	yes	yes	yes
East Java	yes	yes	yes	yes	yes	no	yes	yes	no
DI Yogyakarta	yes	no	yes	yes	no	yes	yes	yes	yes
DKI Jakarta	yes	yes	no	yes	yes	yes	yes	yes	yes
Banten	yes	yes	yes	yes	yes	yes	yes	yes	yes

Illustrated by Table 4. that when the SAR-X model with the Casetti approach uses the queen contiguity spatial weight matrix, the exogenous variable X_1 has a significant effect on all Y observation locations, X_2 has a significant effect on Y observation locations except DI Yogyakarta, and X3 has a significant effect on Y observation locations except DKI Jakarta. If the SAR-X model with the Casetti approach uses the inverse distance spatial weight matrix, the exogenous variable X_1 has a significant effect on all Y observation locations, the variable X_2 has a significant effect on Y observation locations except DI Yogyakarta, and the variable X_3 has a significant effect on Y observation locations except East Java. If the SAR-X model with the casetti approach uses a squared inverse distance spatial weight matrix, the exogenous variables X_1 and X_2 have an effect on all Y observation locations, and X_3 has an effect on all Y observation locations except West Java and East Java.

The prediction results using the SAR-X model with the Casetti approach are best measured through the RMSE value and the coefficient of determination. shown in Table 5. the RMSE value and the coefficient of determination of the SAR-X prediction model with the Casetti approach on the habits and behavior of the people of Java Island towards culture.

Table 5. RMSE and coefficient determination value

Spatial Weight Matrix	RMSE	R^2
Queen Contiguity	2.490214	0.23019060
Inverse Distance	1.792024	0.36293045
Squared Inverse Distance	1.231756	0.69901231

Table 5 shows the lowest RMSE value obtained from the SAR-X model with the Casetti approach using the inverse distance squared spatial weight matrix, which is 1.23. The highest coefficient of determination is obtained from the SAR-X model with the Casetti approach using the square distance inverse spatial weight matrix, which is 70%. Therefore, the best SAR-X model with the Casetti approach to describe and predict the effect of location and exogenous variables (PPIPA, PHTP, and PATCH) on people's habits and behavior (Java Island) towards culture is using the inverse spatial weight matrix of squared distance.

CONCLUSIONS

The SAR-X model with the Casetti approach can describe and predict the influence of location and exogenous variables (PPIPA, PHTP, and PATCH) on the habits and behaviors of the people of Java Island towards culture. The results of the analysis show that there is an influence of location and exogenous variables (PPIPA, PTHP, and PATCH) on the habits and behavior of the people of Java Island towards culture. The estimated results of the SAR-X model with the Casetti approach provide different β parameter estimates for each exogenous variable in each province. The best SAR-X model with the Casetti approach to predict the habits and behavior of the people on Java Island was obtained using the inverse distance squared spatial weight matrix. This is supported by the minimum RMSE value and the highest coefficient of determination.

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