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Optimization Modeling of Investment Portfolios Using The Mean-VaR Method with Target Return and ARIMA-GARCH

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ABSTRACT

This research develops a portfolio optimization model using the Mean-Value at Risk (Mean-VaR) approach with a target return constraint, addressing the gap in models that specific return objectives. The ARIMA-GARCH model is utilized to predict stock returns and volatility, offering precise inputs for optimization. By applying the Lagrange method and Kuhn-Tucker conditions, the model determines optimal portfolio weights that balance risk and return. Using data from infrastructure stocks on the Indonesia Stock Exchange (January 2019-September 2024), the model's effectiveness is validated through numerical simulations. The results illustrate efficient frontiers for target returns of 5×10^{-6}, 0.001, and 0.0019, revealing that higher return targets proportionally increase risk. ARIMA-GACRH's advantage lies in its ability to capture both mean and variance dynamics, ensuring reliable volatility estimates for informed decision-making. This study contributes to portfolio optimization literature by emphasizing target return constraints and demonstrating the practical utility of volatility modeling. The findings provide a robust framework for investors to align portfolios with financial goals and risk tolerance. Future work could explore broader market contexts or integrated additional constraints for enhanced applicability.

Keywords: Stocks; Portfolio Optimization; Mean-VaR; Target Return; ARIMA-GARCH

INTRODUCTION

Investment refers to the allocation of capital into a company or project with the aim of achieving positive returns in the future [1]. Investments can serve both short-term and long-term goals, providing flexibility for investors to align with their financial objectives [2]. In selecting investment instruments, return and risk are critical factors frequently evaluated by individuals and institutions [3]. Among these instruments, the stock market is particularly attractive due to its high potential returns [4]. However, this potential comes with significant risks, primarily stemming from market uncertainties. To achieve optimal investment outcomes, investors must construct a portfolio that not only seeks to maximize returns but also manages risk within acceptable limits. Successful portfolio management necessitates a strategic approach, encompassing asset diversification, balanced allocation, and the implementation of effective risk management

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strategies [5]. Diversification plays a key role in mitigating risk by spreading investments across various asset classes [6]. Such strategies are essential for achieving a balance between returns and a tolerable level of risk [7], requiring careful planning and precise asset selection [8].

The principles of portfolio optimization and diversification are central to the evolution and understanding of financial markets, especially in efforts to enhance returns while minimizing risks [9]. Portfolio selection focuses on constructing an asset combination that reflects an investor's risk preferences [10]. In this context, the Value at Risk (VaR) framework, an extension of the Mean-Variance methodology, is employed to quantify risk. VaR estimates the maximum potential loss over a specific time frame at given confidence level [9], offering a clearer perspective on risk by pinpointing the percentile of a loss or gain distribution [11]. The stock return data utilized in this study consists of time series data, characterized by fluctuating values over time. A primary challenge in financial data analysis is heteroskedasticity, where the variance of residuals varies over time. Commonly referred to as volatility, this phenomenon amplifies variability across different periods. The Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model addresses this issue effectively [12], offering a flexible framework to model dynamic variances and capture changing volatility in financial data. While ARIMA models are commonly used for trend analysis in time series data, they are less effective in managing heteroskedasticity [13]. Integrating ARIMA and GARCH provides a robust approach, with ARIMA modeling trends and GARCH capturing volatility [14].

Several studies have explored portfolio optimization using similar approaches. Portfolio optimization using VaR and univariate GARCH models has been shown to achieve a superior Mean-VaR trade-off compared to multivariate GARCH and historical VaR models across volatility scenarios [15]. Further studies estimated VaR and minimum variance for portfolios with non-constant volatility by employing ARMA for return estimation and GARCH for volatility modeling, yielding minimum variance values and optimal asset weight allocations to minimize risk [16]. The Mean-VaR approach has also been enhanced by incorporating asymmetric volatility and investor risk preferences through ARMA-GJR-GARCH models, demonstrating its effectiveness in designing portfolios with minimal asymmetric effects [13].

Despite these advancements, existing studies have not explicitly addressed the incorporation of target return in portfolio optimization. Target return is essential for investment decision-making, as it defines the expected return objectives of investors and ensures alignment between portfolio strategies and financial goals. To address this gap, this study aims to develop an investment portfolio optimization model using the Mean-VaR method. The model integrates ARIMA-GARCH for precise mean and variance predictions and introduces target return as a central constraint. The primary objective is to create a systematically approach that balances risk and return while meeting specific return goals. This research contributes to the stock investment literature by offering a practical framework for portfolio optimization, assisting investors in achieving tailored financial objectives through informed decision-making.

METHODS

This research utilizes daily closing prices data from selected infrastructure stocks listed on the Indonesia Stock Exchange (IDX). These data were obtained from the publicly accessible financial platform Yahoo Finance (www.finance.yahoo.com), covering the period from January 1, 2019, to September 30, 2024. This five-year dataset ensures a comprehensive analysis of stock return behavior over time. Model development and numerical analysis was conducted using this data to validate the performance of the proposed model. The numerical analysis employed R software version 4.3.2. for ARIMA-GARCH modeling and Microsoft Excel for portfolio optimization. The research follows these key steps:

1. Development of the Portfolio Optimization Model
   A Mean-VaR portfolio optimization model with a target return was developed to determine the optimal portfolio weights by balancing return and risk, with risk measured using Value at Risk (VaR). The objective function of the optimization problem is to maximize the adjusted portfolio return based on investor risk tolerance. The optimization problem is formulated as [17]:

$$\begin{gathered}\text{Maximize } \left(2\tau\mu_{pt} - VaR_{pt}\right), \\ s.t. \sum_{i=1}^{N} w_i = 1,\ w_i \geq 0,\end{gathered} \tag{1}$$

   where $\tau \geq 0$ represents the risk tolerance factor, $\mu_{pt}$is the portfolio return, and VaR_{pt} is the portfolio's Value at Risk. The VaR is calculated assuming a normal distribution as [5]:

$$VaR_{pt} = -W_0\left(\mathbf{w}^{\mathrm{T}}\boldsymbol{\mu} + z_\alpha(\mathbf{w}^{\mathrm{T}}\boldsymbol{\Sigma}\mathbf{w})^{\frac{1}{2}}\right), \tag{2}$$

   Where W_0 is the initial investment, $\mathbf{w}$ is the portfolio weight vector, $\boldsymbol{\mu}$ is the expected return vector, and $\boldsymbol{\Sigma}$ is the convariance matrix of returns. For simplicity, $W_0 = 1$ is assumed, allowing the optimization problem to be reformulated as:

$$\begin{gathered}\text{Maximize}\left(2\tau\boldsymbol{\mu}^{\mathrm{T}}\mathbf{w} + \left(\boldsymbol{\mu}^{\mathrm{T}}\mathbf{w} + z_\alpha(\mathbf{w}^{\mathrm{T}}\boldsymbol{\Sigma}\mathbf{w})^{\frac{1}{2}}\right)\right), \\ s.t.\ \mathbf{e}^{\mathrm{T}}\mathbf{w} = 1, w_i \geq 0,\end{gathered} \tag{3}$$

   where $\mathbf{e}$ is a vector of ones with dimensions $1 \times n$. The model was further enhanced by introducing a target return constraint [18]:

$$\sum_{i=1}^{n} \mu_i w_i = R_p. \tag{4}$$

   This Equation indicates that the portfolio's target return $\left(R_p\right)$ is derived from the weighted sum of the expected returns (μ_i) of individual stocks, with weights (w_i) representing their respective proportions in the portfolio. Consequently, achieving a desired target return requires investors to select appropriate combination of stocks and their corresponding weights.

2. Data Transformation and Stationary Testing
   Stock returns were calculated using the log-return formula [19]:

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right). \tag{5}$$

The log-return approach was chosen for its additive properties, enabling cumulative returns to be expressed as the sum of shorter periods. Stationarity of the data was verified using the Augmented Dickey-Fuller (ADF) test, and the Box-Cox transformation was applied to stabilize variance.

3. ARIMA Modeling
   Stationary data were used to identify the best-fitting ARIMA model. The ARIMA parameters (p, d, q) were selected using ACF and PACF plots, along with the ADF test [20]. The ARIMA model was formulated as [20]:

$$\phi_p(B)(1-B)^d r_t = c + \theta_q(B)\varepsilon_t, \tag{6}$$

   where $\phi_p(B) = 1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p$ represents the AR component of order p, $\theta_q(B) = +\theta_1 B + \theta_2 B^2 + \cdots + \theta_q B^q$ represents the MA component of order q, and c is a constant capturing any mean shift in the series. the error term $\varepsilon_t \sim N(0, \sigma^2)$ is assumed to follow a normal distribution with mean 0 and variance σ^2.

4. Heteroskedasticity Detection and GARCH Modeling
   The ARCH-LM test was conducted to detect conditional heteroskedasticity in the ARIMA residuals [21]. If heteroskedasticity was present, GARCH modeling was applied to capture time-varying variance. The GARCH model is expressed as [20]:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \cdots + \alpha_m \varepsilon_{t-m}^2 + \beta_1 \sigma_{t-1}^2 + \cdots + \beta_s \sigma_{t-s}^2, \tag{7}$$

   where σ_t^2 represents the conditional variance at time t, ε_{t-m}^2 are the past squared residuals (errors) from the mean Equation, and σ_{t-s}^2 are the past variances. The parameters α_0, α_m, and β_s are estimated from the data, with α_m representing the impact of past squared residuals and β_s representing the impact of past variances on the current variance.

5. Portfolio Optimization and Efficient Frontier
   Portfolio optimization using the Mean-VaR model was conducted for varying target returns. Initial target return values were set at the smallest mean forecasted return to minimize risk. The process iteratively increased the target return until a series of efficient portfolios was identified, each representing the minimum risk for a given level of return. The efficient frontier was visualized to depict the trade-off between return and risk.

RESULTS AND DISCUSSION

Formulation of the Mean-VaR Investment Portfolio Optimization Model with a Target Return

This subsection outlines the detailed formulation of the investment portfolio optimization model using the Mean-VaR approach. The target return is incorporated as an additional constraint to achieve an optimal portfolio that aligns with the investor's preferences. The formulation includes identifying key variables, constructing the Lagrange function, and solving the system of Equations that satisfies the Kuhn-Tucker conditions. The reformulation process begins by identifying the essential components of the portfolio optimization model. The model includes:

- Expected Return Vector ($\boldsymbol{\mu}$): Representing the average returns of each stock in the portfolio, as defined in Equation (8), with dimensions $1 \times n$.
- Unit Vector ($\mathbf{e}$): A vector with all elements equal to 1, ensuring that the sum of the portfolio weights equals one, as shown in Equation (8).

$$\boldsymbol{\mu} = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_n \end{pmatrix}, \mathbf{e} = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}. \tag{8}$$

- Covariance Matrix ($\boldsymbol{\Sigma}$): Representing the volatility correlation among stocks, as defined in Equation (9). This matrix is a key element in measuring portfolio risk.

$$\boldsymbol{\Sigma} = \begin{pmatrix} \sigma_{11}^2 & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22}^2 & \cdots & \sigma_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \cdots & \sigma_{nn}^2 \end{pmatrix}. \tag{9}$$

- Portfolio Risk (VaR): The VaR approach measures the maximum potential loss at a specific confidence level, as described in Equation (2).
- Target Return (R_p): The return desired by the investor, included as an additional constraint, as defined in Equation (4).

Once these elements are defined, the Mean-VaR portfolio optimization model is reformulated with the target return constraint. The base optimization model, shown in Equation (3), is augmented with the target return constraint (Equation 4), resulting in the complete model expressed in Equation (10):

$$\begin{gathered} \max\left(2\tau\mathbf{w}^T\boldsymbol{\mu} + \left(\mathbf{w}^T\boldsymbol{\mu} + z_\alpha(\mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w})^{\frac{1}{2}}\right)\right), \\ s.t. \qquad \mathbf{w}^T\mathbf{e} = 1, \\ \mathbf{w}^T\boldsymbol{\mu} = R, \\ w_i \geq 0, R \geq 0, \end{gathered} \tag{10}$$

where τ is the risk tolerance parameter. As the optimization model is quadratic, the Lagrange function and Kuhn-Tucker conditions are employed to ensure optimality. By adding the Lagrange multipliers (λ), the Lagrange function for Equation (10) is:

$$L = (2\tau + 1)\mathbf{w}^{\mathrm{T}}\boldsymbol{\mu} + z_\alpha(\mathbf{w}^{\mathrm{T}}\boldsymbol{\Sigma}\mathbf{w})^{\frac{1}{2}} + \lambda_1(\mathbf{w}^{\mathrm{T}}\mathbf{e} - 1) + \lambda_2(\mathbf{w}^{\mathrm{T}}\boldsymbol{\mu} - R). \tag{11}$$

Theorem 1 [22]. *If $f(\mathbf{X})$ has an extreme point (maximum or minimum) at $\mathbf{X} = \mathbf{X}^*$, and the first partial derivatives of $f(\mathbf{X})$ exist at $\mathbf{X}^*$, then*

$$\frac{\partial f}{\partial x_i}(\mathbf{X} = \mathbf{X}^*) = 0, \qquad i = 1,2,3,\dots,n. \tag{12}$$

Based on **Theorem 1**, the necessary conditions for optimality are derived as follows:

$$\frac{\partial L}{\partial \mathbf{w}} = (2\tau + 1)\,\boldsymbol{\mu} + \frac{z_\alpha\boldsymbol{\Sigma}\mathbf{w}}{(\mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w})^{\frac{1}{2}}} + \lambda_1\mathbf{e} + \lambda_2\,\boldsymbol{\mu} = 0, \tag{13}$$

$$\frac{\partial L}{\partial \lambda_1} = \mathbf{w}^{\mathrm{T}}\mathbf{e} - 1 = 0, \tag{14}$$

$$\frac{\partial L}{\partial \lambda_2} = \mathbf{w}^{\mathrm{T}}\,\boldsymbol{\mu} - R = 0. \tag{15}$$

The optimization process begins by deriving the portfolio weights that satisfy the Mean-VaR model constraints. From Equation (13), multiplying by $\frac{\boldsymbol{\Sigma}^{-1}}{z_\alpha}$, yields the expression for normalized portfolio weights as presented in Equation (16):

$$\frac{\mathbf{w}}{(\mathbf{w}^{\mathrm{T}}\boldsymbol{\Sigma}\mathbf{w})^{\frac{1}{2}}} = \frac{-(2\tau+1)\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} - \lambda_1\boldsymbol{\Sigma}^{-1}\mathbf{e} - \lambda_2\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}}{z_\alpha}, \tag{16}$$

since the total portfolio weights must sum to 1, i.e., $\mathbf{e}^{\mathrm{T}}\mathbf{w} = \mathbf{w}^{\mathrm{T}}\mathbf{e} = 1$, the portfolio weights vector can then be expressed as show in Equation (17):

$$\mathbf{w} = \frac{-(2\tau+1)\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} - \lambda_1\boldsymbol{\Sigma}^{-1}\mathbf{e} - \lambda_2\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}}{-(2\tau+1)\mathbf{e}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} - \lambda_1\mathbf{e}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\mathbf{e} - \lambda_2\mathbf{e}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}}. \tag{17}$$

Equation (17) represents the final expression for the portfolio weights, balancing the trade-off between risk and return under the constraints of the Mean-VaR optimization model. Here λ_1 and λ_2 are the Lagrange multipliers corresponding to the constraints on total portfolio weights and the target return, respectively.

The Lagrange multipliers λ_1 and λ_2 can be determined by solving a system of linear Equations derived from the constraints. Using the conditions, the following expressions for λ_1 and λ_2 are obtained, as shown in Equation (18) and (19):

$$\lambda_1 = \frac{(\boldsymbol{\mu}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} - R\mathbf{e}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu})\lambda_2 + (2\tau+1)(-R\mathbf{e}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} + \boldsymbol{\mu}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu})}{(R\mathbf{e}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\mathbf{e} - \boldsymbol{\mu}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\mathbf{e})}, \tag{18}$$

$$\lambda_2 = \frac{(2\tau+1)\left(-R + \mathbf{e}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} + \frac{(-R\mathbf{e}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} + \boldsymbol{\mu}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu})(1-\mathbf{e}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\mathbf{e})}{(R\mathbf{e}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\mathbf{e} - \boldsymbol{\mu}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\mathbf{e})}\right)}{\left(R - \mathbf{e}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} + \frac{(\boldsymbol{\mu}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} - R\mathbf{e}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu})(1-\mathbf{e}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\mathbf{e})}{(R\mathbf{e}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\mathbf{e} - \boldsymbol{\mu}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\mathbf{e})}\right)}. \tag{19}$$

The optimal portfolio weights w, along with the Lagrange multipliers λ_1 and λ_2, has been successfully derived to address the constraints of target return and risk within the Mean-VaR portfolio optimization model. These solutions serve as the foundation for determining optimal investment strategies based on an investor's risk tolerance and return objectives. By providing a systematic approach to balancing risk and return, this model offers practical insights for informed decision-making in portfolio management. Future implementations of this model can further refine its application across diverse investment scenarios, enhancing its utility in real-world financial markets.

Numerical Simulations

This section explains the data processing and application of the Mean-VaR with target return model to optimize the investment portfolio of infrastructure stocks. The ARIMA-GARCH approach was utilized to determine the mean and variance of stock returns.

Descriptive Statistics

The stock data from the infrastructure sector, six stocks were selected based on specific criteria outlined in the research methodology. These criteria were designed to ensure that the chosen stocks reflect a diverse representation of the sector while maintaining data quality and consistency. The selected stocks are presented in Table 1.

Table 1. Selected Infrastructure Stocks

No.	Code	Company Names
1	ISAT	Indosat Ooredoo Hutchison Tbk.
2	SSIA	Surya Semesta Internusa Tbk.
3	JSMR	Jasa Marga (Persero) Tbk.
4	CASS	Cardig Aero Services Tbk.
5	AKRA	AKR Corporindo Tbk.
6	TBIG	Tower Bersama Infrastructure Tbk.

The daily stock returns were calculated from the closing price data using Equation (5), which defines return as the percentage change in prices between consecutive trading days. The variability in daily returns across the six selected stocks is visualized in Figure 1, providing an initial insight into their fluctuation patterns.

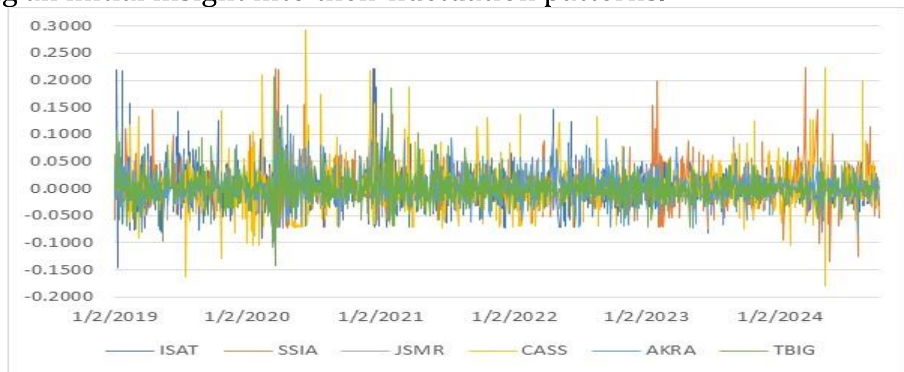


Figure 1. Daily Returns of Selected Stocks

To better understand the characteristics of the data, descriptive statistics were calculated using R software version 4.3.2. These statistics, including the mean, minimum, maximum, skewness, kurtosis, and standard deviation, provide a summary of the distribution and variability of daily stock returns. The results are presented in Table 2.

Table 2. Descriptive Statistics of Stock Returns

	ISAT	SSIA	JSMR	CASS	AKRA	TBIG
Min	-0.1456	-0.1348	-0.1421	-0.1791	-0.0963	-0.1413
Mean	0.0013	0.0006	0.0001	0.0005	0.0004	0.0006
Max	0.2213	0.2231	0.1361	0.2935	0.1537	0.2062
Skewness	1.3791	1.3765	0.2876	1.5332	0.5365	0.9171
Kurtosis	10.020	11.657	6.4617	14.527	6.1814	10.070
Std Dev	0.0339	0.0315	0.0231	0.0342	0.0257	0.0259

From Table 2, all stocks exhibit positive average returns, suggesting their potential profitability over the analyzed period. However, the standard deviations are noticeably higher than the mean values, indicating significant variability and risk associated with these stocks. Stocks with high kurtosis (e.g., CASS and TBIG) indicate a greater likelihood of extreme returns, both positive and negative, compared to stocks with lower kurtosis.

To validate the suitability of the data for further modeling, stationarity tests were conducted using the Augmented Dickey-Fuller (ADF) test and the Box-Cox transformation. These tests examine the stationarity of stock returns in terms of their mean and variance. The results are summarized in Table 3.

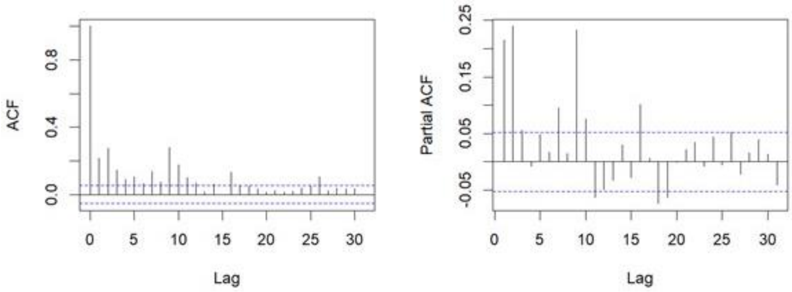
Table 3. Stationary Test Results

Stocks	ADF Test				Box-Cox Transformation	
	t	$t_{(0.05,1406)}$	p-value	Stationary (Mean)	λ	Stationary (Variance)
ISAT	-36.8	-1.65	0.01	Yes	1	Yes
SSIA	-36.3		0.01		1	
JSMR	-35.1		0.01		1	
CASS	-39.7		0.01		1	
AKRA	-37.2		0.01		$0.98 \approx 1$	
TBIG	-39.8		0.01		$0.99 \approx 1$	

The ADF test results confirm that the stock return data is stationary in terms of mean, as the test statistics exceed the critical values $\left(|t| > \left|t_{(0,05,1406)}\right|\right)$ and p-values are below 0.05. Furthermore, the Box-Cox transformation results, with rounded λ values close to 1, indicate that the data is also stationary in terms of variance. These findings validate the appropriateness of using the ARIMA-GARCH model for modeling and forecasting stock return volatility.

ARIMA-GARCH Model Identification

As previously explained, the identification of the ARIMA model requires examining the Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) plots to determine the appropriate orders of autoregressive (AR), differencing (I), and moving average (MA) components for each stock. The ACF and PACF plots for the six selected infrastructure stocks are presented in Figure 2. These visual tools provide insights into the dependencies and lag structures within the time series data.



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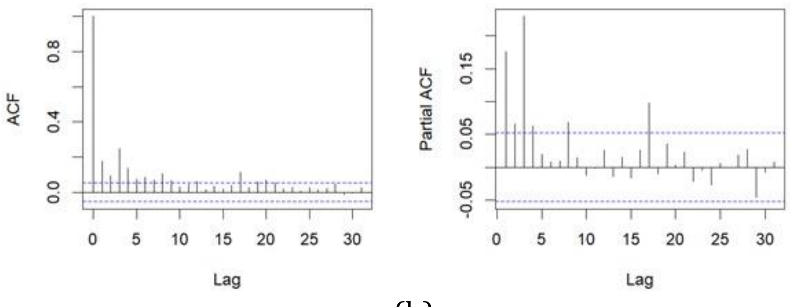

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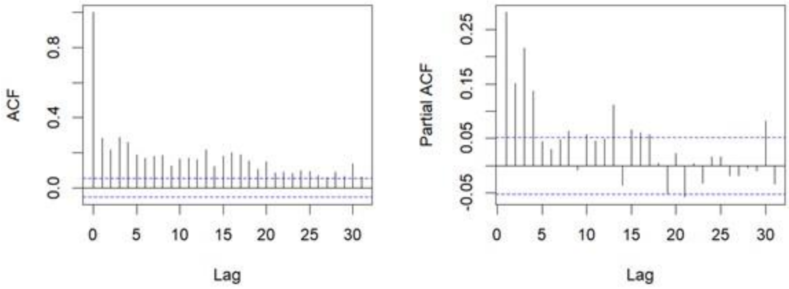

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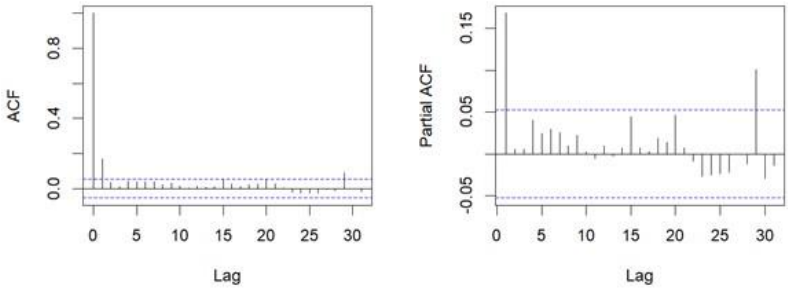

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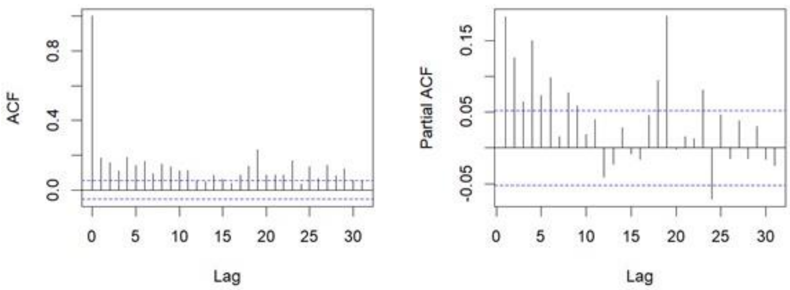

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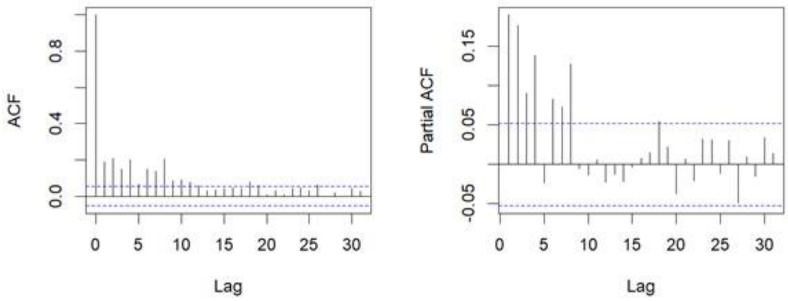


(f)

Figure 2. ACF and PACF Plots for the Selected Stocks:
ISAT (a), SSIA (b), JSMR (c), CASS (d), AKRA (e), TBIG (f).

Based on the patterns observed in the ACF and PACF plots, the optimal ARIMA model for each stock was identified. The ARIMA orders for each stock were selected based on patterns in the ACF and PACF plots (Figure 2), ensuring stationarity and minimal residual errors. Table 4 summarizes best ARIMA models and their corresponding equations.

Table 4. Best ARIMA Models for Each Stock

Stocks	ARIMA Orde	ARIMA Model
ISAT	ARIMA (5,0,3)	$\hat{r}_t = 0.0014 + 0.6136r_{t-1} + 0.2065r_{t-2} - 0.7410r_{t-3}$ $-0.0382r_{t-4} + 0.0321r_{t-5} - 0.6028e_{t-1} + 0.2166e_{t-2}$ $+0.8173e_{t-3}$
SSIA	ARIMA (0,0,3)	$\hat{r}_t = 0.0006 + 0.0357e_{t-1} - 0.0350e_{t-2} + 0.1097e_{t-3}$

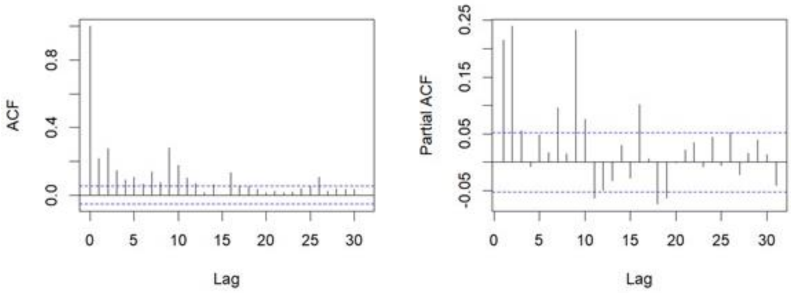
Stocks	ARIMA Orde	ARIMA Model
JSMR	ARIMA (0,0,1)	$\hat{r}_t = 0.0001 + 0.0688e_{t-1}$
CASS	ARIMA (5,0,0)	$\hat{r}_t = 0.0005 - 0.0589r_{t-1} + 0.0006r_{t-2} - 0.0231r_{t-3} + 0.0465r_{t-4} + 0.0644r_{t-5}$
AKRA	ARIMA (0,0,2)	$\hat{r}_t = 0.0004 + 0.0132e_{t-1} - 0.0800e_{t-2}$
TBIG	ARIMA (2,0,2)	$\hat{r}_t = 0.0006 + 0.0563r_{t-1} - 0.6637r_{t-2} - 0.1196e_{t-1} + 0.6160e_{t-2}$

Based on Table 4, stock returns can be modeled using ARIMA with varying orders, influenced by constants, stock returns, and return residuals. After determining the best ARIMA model for each stock, the next step involved assessing the presence of conditional heteroscedasticity in the residuals of the fitted ARIMA models. This was achieved using the ARCH-LM test, which detects volatility clustering, a common phenomenon in financial time series data. The ARCH-LM test results are presented in Table 5.

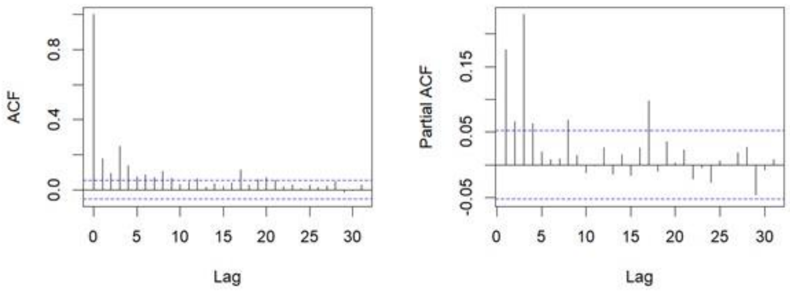
Table 5. ARCH-LM Test Results for Conditional Heteroscedasticity

Stocks	ARIMA Orde	p-value
ISAT	ARIMA (5,0,3)	$< 2.2 \times 10^{-16}$
SSIA	ARIMA (0,0,3)	$< 2.2 \times 10^{-16}$
JSMR	ARIMA (0,0,1)	$< 2.2 \times 10^{-16}$
CASS	ARIMA (5,0,0)	9.46×10^{-6}
AKRA	ARIMA (0,0,2)	$< 2.2 \times 10^{-16}$
TBIG	ARIMA (2,0,2)	$< 2.2 \times 10^{-16}$

The significantly low p-values for all stocks indicate the presence of conditional heteroscedasticity in the residuals of the ARIMA models. The ARCH-LM test confirmed the presence of conditional heteroscedasticity for all stocks, necessitating the incorporation of GARCH components to account for volatility clustering. To determine the appropriate GARCH order, the ACF and PACF plots of the squared residuals from the ARIMA models were analyzed (Figure 3).



(a)



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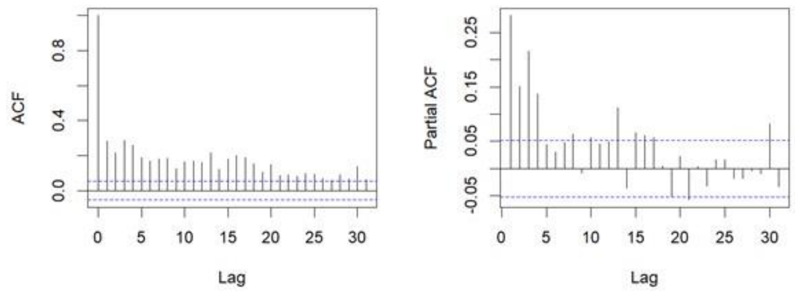

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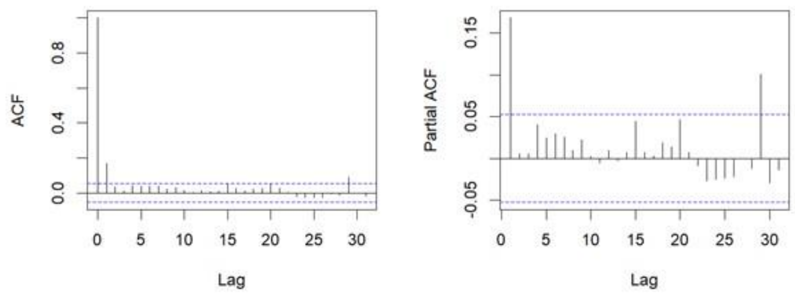
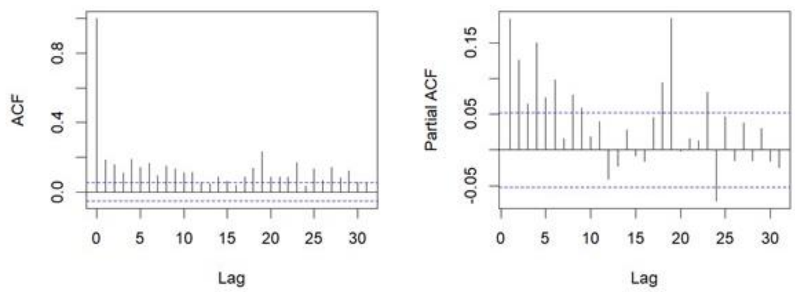
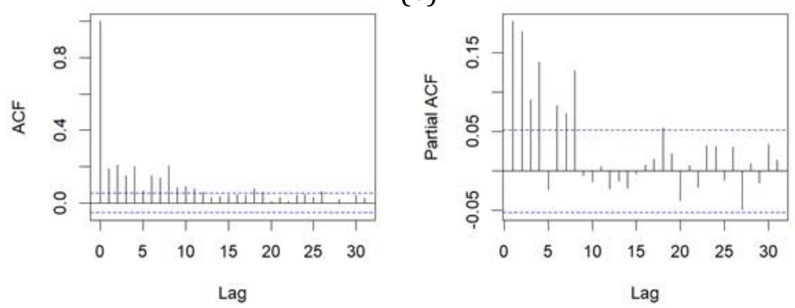


(d)



(e)



(f)

Figure 3. ACF and PACF Plots of the squared residuals for the Selected Stocks:
ISAT (a), SSIA (b), JSMR (c), CASS (d), AKRA (e), TBIG (f).

Based on Figure 3, the ACF and PACF plots generally show intersections at lags 1, 2, and 3. Therefore, this study restricts the GARCH model to low orders, specifically $m, s \leq 2$. Using these plots, the optimal GARCH model parameters were selected for each stock. Table 6 summarized combined ARIMA-GARCH models and their equations.

Table 6. Best ARIMA-GARCH Models

Stocks	ARIMA-GARCH Orde	ARIMA-GARCH Model
ISAT	ARIMA (5,0,3) – GARCH (1,1)	$\hat{r}_t = 0.0010 - 0.3240r_{t-1} + 0.6638r_{t-2} + 0.5436r_{t-3}$ $+0.0448r_{t-4} + 0.0598r_{t-5} + 0.2777e_{t-1} - 0.7093e_{t-2}$ $-0.5809e_{t-3}$ $\hat{\sigma}_t^2 = 0.00002 + 0.1126e_{t-1}^2 + 0.8601\sigma_{t-1}^2$
SSIA	ARIMA (0,0,3) – GARCH (1,1)	$\hat{r}_t = -0.0001 - 0.0626e_{t-1} - 0.0019e_{t-2} - 0.0970e_{t-3}$ $\hat{\sigma}_t^2 = 0.0001 + 0.2303e_{t-1}^2 + 0.6431\sigma_{t-1}^2$
JSMR	ARIMA (0,0,1) – GARCH (1,1)	$\hat{r}_t = 0.000002 - 0.0782e_{t-1}$ $\hat{\sigma}_t^2 = 0.00006 + 0.1582e_{t-1}^2 + 0.7070\sigma_{t-1}^2$
CASS	ARIMA (5,0,0) – GARCH (1,1)	$\hat{r}_t = 0.0001 + 0.0087r_{t-1} - 0.0113r_{t-2} - 0.0011r_{t-3}$ $-0.0881r_{t-4} - 0.0225r_{t-5}$ $\hat{\sigma}_t^2 = 0.0004 + 0.3921e_{t-1}^2 + 0.2517\sigma_{t-1}^2$
AKRA	ARIMA (0,0,2) – GARCH (1,2)	$\hat{r}_t = 0.00007 - 0.0061r_{t-1} - 0.0053r_{t-2}$ $\hat{\sigma}_t^2 = 0.00002 + 0.0624e_{t-1}^2 + 0.2819\sigma_{t-1}^2 + 0.6209\sigma_{t-2}^2$
TBIG	ARIMA (2,0,2) – GARCH (1,2)	$\hat{r}_t = -0.0002 + 1.4900r_{t-1} - 0.5255r_{t-2} - 1.5274e_{t-1} + 0.5558e_{t-2}$ $\hat{\sigma}_t^2 = 0.00003 + 0.1183e_{t-1}^2 + 0.3791\sigma_{t-1}^2 + 0.4533\sigma_{t-2}^2$

Table 6 summarizes the best ARIMA-GARCH models for each stock, selected based on ACF and PACF patterns of squared residuals and validated using the ARCH-LM test, which confirmed conditional heteroscedasticity ($p - value < 0.05$). These models were further optimized by minimizing the Akaike Information Criterion (AIC) and ensuring residuals resembled white noise. The inclusion of GARCH components effectively captured volatility clustering, enhancing variance prediction. For example, the ARIMA (5,0,3) – GARCH (1,1) model for ISAT show that future volatility ($\hat{\sigma}_t^2$) is influenced by past volatility ($\hat{\sigma}_{t-1}^2$) and lagged shocks ($\hat{\sigma}_{t-1}^2$), demonstrating the persistence of volatility over time. Similarly, the GARCH components in models for other stocks accurately reflect the unique volatility patterns of their return series.

These models provide the foundation for forecasting the mean and variance of stock returns, as shown in Table 7. The forecasts will serve as inputs for portfolio optimization, enhancing the accuracy of risk-return estimations.

Table 7. Forecasting Results of Mean and Variance of Stock Return

Stocks	Mean	Variance
ISAT	0.0014	0.00214
SSIA	0.0019	0.03532
JSMR	1.271×10^{-5}	0.01610
CASS	2.898×10^{-3}	0.02765
AKRA	5.564×10^{-6}	0.01916
TBIG	1.295×10^{-4}	0.01537

The forecasting results reveal significant variations in the mean and variance of stock returns across the six selected infrastructure stocks. SSIA exhibits the highest mean return (0.0019) but also the largest variance (0.03532), indicating a high-risk, high-reward profile. In contrast, ISAT shows a relatively high mean return (0.0014) with the lowest variance (0.00214), suggesting a more stable risk-return trade-off. Stocks such as JSMR and AKRA report low mean returns with moderate variances, while CASS and TBIG demonstrate a balanced risk-return profile. These findings highlight the diverse risk and return characteristics among the stocks, offering insights for strategic portfolio allocation and risk management.

Portfolio Optimization Results

Following the application of the ARIMA-GARCH model, the mean and variance for each stock were calculated to be used in the portfolio optimization process. The results are summarized in Table 7. The mean values were arranged into an expected return vector, as formulated in Equation (8). The resulting vector is as follows:

$$\boldsymbol{\mu} = \begin{bmatrix} 1.40\text{E}-03 \\ 1.91\text{E}-03 \\ 1.27\text{E}-05 \\ 2.90\text{E}-03 \\ 5.56\text{E}-06 \\ 1.30\text{E}-04 \end{bmatrix}. \tag{20}$$

The variance values were further utilized to compute the covariance values, which were subsequently arranged into a covariance matrix as formulated in Equation (9). The resulting matrix is as follows:

$$\boldsymbol{\Sigma} = \begin{bmatrix} 2.14\text{E}-02 & 0.005087 & 0.004637 & -0.000078 & 0.003429 & 0.003030 \\ 0.005087 & 3.53\text{E}-02 & 0.005531 & -0.000048 & 0.004939 & 0.003689 \\ 0.004637 & 0.005531 & 1.61\text{E}-02 & 0.000795 & 0.005018 & 0.003490 \\ -0.000078 & -0.000048 & 0.000795 & 2.77\text{E}-02 & 0.001147 & 0.000522 \\ 0.003429 & 0.004939 & 0.005018 & 0.001147 & 1.92\text{E}-02 & 0.002721 \\ 0.003030 & 0.003689 & 0.003490 & 0.000522 & 0.002721 & 1.54\text{E}-02 \end{bmatrix}. \tag{21}$$

The portfolio optimization process was then conducted to determine the optimal weights for the portfolio composition as formulated in Equation (17). The primary objective was to achieve maximum return while satisfying the specified target return levels. The target return was initialized based on the expected return vector obtained from Equation (4). Specifically, the following values were used as benchmarks in this study. The value of 5×10^{-6} represents the lowest expected return among the individual stocks in the portfolio. The value of 0.001 corresponds to the average expected return across all stocks in the portfolio. Additionally, the value of 0.0019 indicates the maximum target return. Tables 8, 9, and 10 summarize the optimization results for portfolios with different target returns, highlighting changes in asset weights, portfolio risks, and efficiency measures across varying levels of the parameter τ. The results provide insights into the relationship between target return levels, portfolio risk management, and performance efficiency.

Table 8. Portfolio Optimization for Target Return 5×10^{-6}

τ	λ_1	λ_2	$\mathbf{w}^{\mathrm{T}}$						μ_p	σ_p	VaR_p	Ratio
			ISAT	SSIA	JSMR	CASS	AKRA	TBIG				
0	8.91E-03	-1.8561	0.1299	0.0375	0.2125	0.1455	0.1913	0.2833	0.0007	0.0766	0.1253	0.0057
0.05	8.02E-03	-1.6705	0.1366	0.0449	0.2021	0.1583	0.1832	0.2748	0.0008	0.0762	0.1246	0.0062
0.1	7.13E-03	-1.4849	0.1446	0.0536	0.1899	0.1735	0.1736	0.2647	0.0008	0.0759	0.1239	0.0068
0.15	6.24E-03	-1.2992	0.1543	0.0641	0.1753	0.1916	0.1622	0.2526	0.0009	0.0758	0.1237	0.0075
0.2	5.35E-03	-1.1136	0.1660	0.0768	0.1573	0.2139	0.1481	0.2378	0.0010	0.0759	0.1239	0.0083

0.25	4.46E-03	-0.9280	0.1807	0.0928	0.1349	0.2417	0.1306	0.2193	0.0012	0.0768	0.1251	0.0093
0.3	3.56E-03	-0.7424	0.1996	0.1134	0.1060	0.2774	0.1080	0.1955	0.0013	0.0787	0.1281	0.0104
0.35	2.67E-03	-0.7053	0.2249	0.1408	0.0675	0.3252	0.0779	0.1637	0.0015	0.0826	0.1344	0.0115
0.37	2.32E-03	-0.6682	0.2375	0.1546	0.0482	0.3491	0.0628	0.1478	0.0017	0.0852	0.1384	0.0119
0.4	1.78E-03	-0.3712	0.2603	0.1793	0.0134	0.3921	0.0357	0.1191	0.0019	0.0905	0.1470	0.0126

As shown in Table 8, the portfolio composition adjusts dynamically with increasing τ (ranging from 0 to 0.4). At $\tau = 0$, the portfolio favors high-risk stocks, such as TBIG and AKRA, with allocations of 28.33% and 19.13%, respectively. This allocation suggests that achieving lower target returns often relies on leveraging high-risk assets. However, as τ increase, the portfolio gradually shifts towards more stable assets, such as JSMR and ISAT. This shift reflects a strategy aimed at balancing risk and return as target return increase. The portfolio risk (σ_p) remains within a controlled range of 0.0766 to 0.0905, demonstrating effective risk management throughout the optimization process. Furthermore, the return-to-risk ratio improves significantly, rising from 0.0057 at $\tau = 0$ to 0.0126 at $\tau = 0.4$, indicating increased portfolio efficiency. Notably, the optimization process achieves a maximum expected return portfolio (μ_p) of 0.0019 when $\tau = 0.4$, highlighting the portfolio's capability to meet higher return objectives under controlled risk conditions.

Table 9. Portfolio Optimization for Target Return 0.001

τ	λ_1	λ_2	$\mathbf{w}^{\mathrm{T}}$						μ_p	σ_p	VaR_p	Ratio
			ISAT	SSIA	JSMR	CASS	AKRA	TBIG				
0.521	2.28E-03	0.0393	0.2684	0.1881	0.0011	0.4074	0.0260	0.1089	0.0019	0.0926	0.1504	0.0128
0.525	2.71E-03	0.0468	0.2582	0.1770	0.0167	0.3881	0.0382	0.1218	0.0018	0.0899	0.1462	0.0126
0.53	3.25E-03	0.0561	0.2479	0.1658	0.0324	0.3687	0.0505	0.1347	0.0017	0.0875	0.1421	0.0123
0.535	3.79E-03	0.0655	0.2396	0.1568	0.0450	0.3529	0.0604	0.1452	0.0017	0.0856	0.1391	0.0121
0.537	4.01E-03	0.0692	0.2367	0.1537	0.0494	0.3475	0.0638	0.1488	0.0016	0.0849	0.1381	0.0119
0.54	4.33E-03	0.0749	0.2328	0.1494	0.0555	0.3401	0.0685	0.1538	0.0016	0.0842	0.1368	0.0118
0.545	4.88E-03	0.0842	0.2270	0.1432	0.0642	0.3292	0.0753	0.1609	0.0016	0.0830	0.1350	0.0116
0.547	5.09E-03	0.0879	0.2249	0.1409	0.0673	0.3254	0.0778	0.1636	0.0015	0.0826	0.1344	0.0115
0.55	5.42E-03	0.0936	0.2222	0.1379	0.0716	0.3200	0.0811	0.1671	0.0015	0.0821	0.1336	0.0114
0.555	5.96E-03	0.1029	0.2179	0.1333	0.0780	0.3121	0.0861	0.1724	0.0015	0.0814	0.1324	0.0112

Table 9 presents the optimization results for a higher target return of 0.001, with τ ranging from 0.521 to 0.555. Despite the higher target return, the portfolio demonstrates stable weight allocations and controlled risk levels. At $\tau = 0.521$, the portfolio return (μ_p) reaches its highest value of 0.0019, marking the optimal performance compared to other τ values. This is supported by a balanced weight allocation, where CASS (40.74%) and ISAT (26.84%) dominate, followed by SSIA (18.81%) and smaller contributions from AKRA (2.6%) and JSMR (0.11%). The risk level, reflected portfolio risk $(\sigma_p = 0.00926)$ and $VaR_p = 0.1504$, remains well-managed despite the ambitious target return.

As τ increase beyond 0.521, the the portfolio gradually shifts towards a more diversified composition. For instance, at $\tau = 0.555$, μ_p decreases to 0.0015, with portfolio weights shifting more toward high-risk stocks such as CASS (31.21%) and TBIG (17.24%). Although the risk level $(\sigma_p = 0.0814)$ and $VaR_p = 0.1324$, decrease at higher τ values, the trade-off between risk and return becomes apparent. The decision to highlight $\tau =$

0.521 as the optimal point is supported by its competitive return-to-risk ratio (0.0128), achieved alongside the maximum portfolio return. This demonstrates the portfolio's ability to balance return and risk effectively, making $\tau = 0.521$ the most desirable choice for performance optimization under the given conditions.

Table 10. Portfolio Optimization for Target Return 0.0019

τ	λ_1	λ_2	$\mathbf{w}^{\mathrm{T}}$						μ_p	σ_p	VaR_p	Ratio
			ISAT	SSIA	JSMR	CASS	AKRA	TBIG				
13	3.22E-02	2.8112	0.2691	0.1889	0.00003	0.4088	0.0252	0.1080	0.00194	0.09282	0.15074	0.01285
13.5	3.35E-02	2.9236	0.2689	0.1887	0.0003	0.4085	0.0254	0.1082	0.00194	0.09278	0.15067	0.01285
14	3.48E-02	3.0361	0.2688	0.1885	0.0005	0.4082	0.0255	0.1084	0.00193	0.09274	0.15061	0.01285
14.5	3.61E-02	3.1485	0.2686	0.1884	0.0007	0.4079	0.0257	0.1086	0.00193	0.09270	0.15055	0.01284
15	3.74E-02	3.2609	0.2685	0.1883	0.0009	0.4077	0.0259	0.1088	0.00193	0.09267	0.15049	0.01284
15.5	3.87E-02	3.3734	0.2684	0.1881	0.0011	0.4075	0.0260	0.1089	0.00193	0.09263	0.15044	0.01284
16	4.00E-02	3.4859	0.2683	0.1880	0.0013	0.4072	0.0261	0.1091	0.00193	0.09260	0.15039	0.01284
16.5	4.12E-02	3.5983	0.2682	0.1879	0.0014	0.4070	0.0263	0.1092	0.00193	0.09258	0.15034	0.01283
17	4.25E-02	3.7108	0.2681	0.1878	0.0016	0.4068	0.0264	0.1093	0.00193	0.09255	0.15030	0.01283

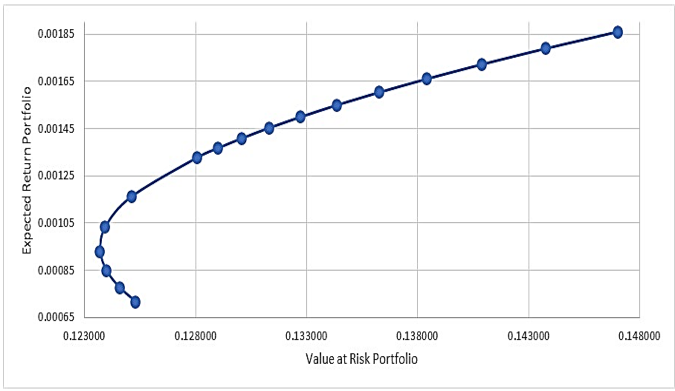
Table 10 explores the optimization process for the highest target return of 0.0019 reveals that the portfolio achieves its maximum return $\left(\mu_p = 0.00194\right)$ when $\tau = 13$. This value is the optimal performance in terms of return compared to other τ values. At this point, the weight allocation is as follows: ISAT holds 26.91%, CASS contributes 40.88%, TBIG holds 10.80%, and SSIA makes up 18.89%. The portfolio demonstrates a well-managed risk profile with a portfolio risk $\left(\sigma_p = 0.09282\right)$ and $VaR_p = 0.15074$ remaining within acceptable levels despite the higher target return. The return-to-risk ratio at $\tau = 13$ is 0.01285, which is stable across the τ values ranging from 13 to 17, further confirming the portfolio's efficiency in balancing return and risk.

In comparison, as the τ values increase (e.g., $\tau = 13.5, \tau = 14$, etc.), the portfolio return slightly decreases, with μ_p reducing to 0.00193. Despite this, the portfolio maintains its stability with small variations in the weight allocations and risk metrics. Notably, the return-to-risk ratio stabilizes between 0.01283 and 0.01285, indicating that the portfolio continues to perform efficiently under these higher return constraints. Thus, $\tau = 13$ represents the point of highest return and an optimal trade-off between risk and return, making it the most desirable value for portfolio performance optimization under these conditions.

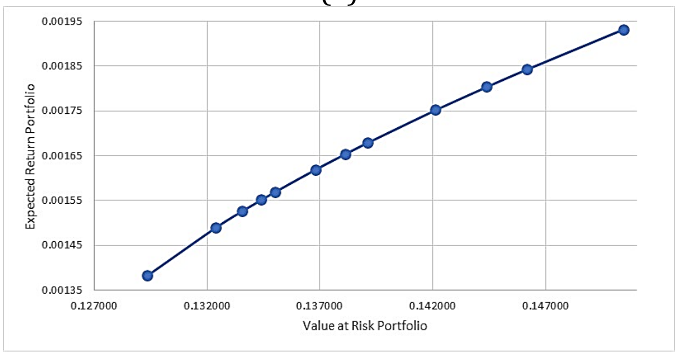
The results across Tables 8-10 illustrate a clear trend: as target returns increase, the portfolio transitions from favoring high-risk assets to incorporating a greater proportion of stable stocks. This dynamic adjustment underscores the importance of diversification for investors, especially those seeking to mitigate risk while pursuing higher returns. The analysis highlights that portfolio efficiency can be maintained even as return targets become more demanding, provided that appropriate weight adjustments are made.

Efficient Frontier Analysis

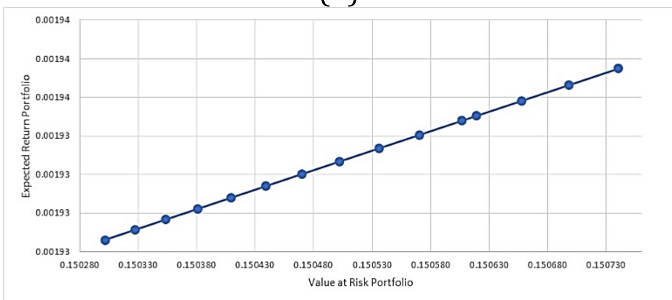
The following visualizations illustrate the efficient frontiers corresponding to the optimization results under different target return scenarios. These efficient frontiers represent the set of portfolios that achieve the lowest risk (measured as Value at Risk, VaR) for a given level of return. Each curve highlights the trade-off between risk and return, providing insights into the impact of target return constraints on portfolio composition and performance metrics.



(a)



(b)



(c)

Figure 4. Efficient Frontier Portfolio Chart for Each Target Return: 5×10^{-6} (a), 0.001 (b), 0.0019 (c)

From the efficient frontier graphs (Figure 4), it is evident that higher target returns generally shift the frontier upward. This indicates that achieving higher returns requires investors to accept a greater level of risk, as measured by VaR. This relationship reflects the fundamental principle in finance that higher returns often come with increased volatility and potential losses. The use of VaR on the x-axis provides practical insights by directly linking portfolio outcomes to the likelihood of extreme losses, offering a risk-focused perspective for decision-making. Efficient frontiers provide a visual representation of the trade-off between risk and expected return. For instance, moving from a target return of 5×10^{-6} to 0.001 results in approximately a 20% increase in portfolio reisk, as measured by VaR. Similarly, increasing the target return further to 0.0019 pushes the frontier to even higher risk levels. These insights allow investors to better align their portfolio strategies with their risk tolerance and return objectives, enabling more informed decision-making.

These visualizations emphasize the dynamics of portfolio diversification and the trade-offs faced when optimizing for specific return objectives. They highlight the critical role of target returns in shaping portfolio strategies, providing a framework for understanding how varying risk levels can impact portfolio performance and investor preferences.

CONCLUSIONS

This study developed and applied the Mean-VaR portfolio optimization model with a target return constraint, offering a systematic approach to balancing risk and return using financial metrics like expected return, covariance matrix, and VaR. By integrating ARIMA-GARCH for volatility modeling, the model enabled precise return predictions and reliable risk estimates. The optimization results demonstrated that the Mean-VaR model effectively adjusts portfolio weights to meet target return objectives. At lower return levels (5×10^{-6}), high-risk stocks dominated allocations, while diversification improved as target returns rose to 0.001 and 0.0019. Efficient frontiers visually highlighted these trade-offs, providing a robust framework for aligning strategies with investor risk tolerance and financial goals.

The findings have practical implications, offering investors a tool to manage portfolio risk dynamically while pursuing tailored return objectives. For instance, raising the target return from 0.001 to 0,0019 increased portfolio risk by 20%, demonstrating the proportional trade-off. Limitations of this study include its focus on infrastructure stocks in Indonesia, which may limit its generalizability. Future research could validate the model in broader markets, incorporate real-world constraints such as transaction costs and liquidity, and extend to multi-asset portfolios. Exploring the model's performance under varying economic conditions and risk measures could further enhance its applicability.

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REFERENCES

[1] D. G. Balamurugan and V. Sivanesan, "Financial Investment Pattern and Preference of College Professors at Trichy City," *Int. J. Eng. Manag. Res.*, vol. 12, no. 3, pp. 187–194, 2022, doi: 10.31033/ijemr.12.3.28.

[2] P. Gupta, M. K. Mehlawat, and A. Z. Khan, "Multi-period portfolio optimization using coherent fuzzy numbers in a credibilistic environment," *Expert Syst. Appl.*, vol. 167, 2021, doi: 10.1016/j.eswa.2020.114135.

[3] T. Stoilov, K. Stoilova, and M. Vladimirov, "Financial investments by Portfolio optimization," in *IOP Conference Series: Materials Science and Engineering*, 2019. doi: 10.1088/1757-899X/618/1/012030.

[4] A. K. Yusup, "Mean-Variance and Single-Index Model Portfolio Optimisation: Case in the Indonesian Stock Market," *Asian J. Bus. Account.*, vol. 15, no. 2, pp. 79–109, 2022, doi: 10.22452/ajba.vol15no2.3.

[5] Sukono, P. Sidi, A. T. Bin Bon, and S. Supian, "Modeling of Mean-VaR Portfolio Optimization by Risk Tolerance When the Utility Function is Quadratic," *AIP Conf.*

Proc., vol. 1827, 2017, doi: 10.1063/1.4979451.
[6] M. R. Insan Baihaqqy, "Strategi optimalisasi portofolio berbasis model markowitz pada jenis investasi reksadana," *JPPI (Jurnal Penelit. Pendidik. Indones.*, vol. 8, no. 1, p. 152, 2022, doi: 10.29210/020221382.
[7] T. Stoilov, K. Stoilova, and M. Vladimirov, "Modeling and Assessment of Financial Investments by Portfolio Optimization on Stock Exchange," in *Studies in Computational Intelligence*, 2021, pp. 340–356. doi: 10.1007/978-3-030-55347-0_29.
[8] A. D. Salsabilla, D. A. I. Maruddani, and A. Rusgiyono, "Measurement of risk value with mean-value at risk optimization model in stocks portfolio (case study: Stocks listed in the IDX30 index for evaluation of August 2020 - January 2021 during the 2020 period)," in *AIP Conference Proceedings*, 2023. doi: 10.1063/5.0140570.
[9] S. A. Heryanti, "Perhitungan Value at Risk Pada Portofolio Optimal: Studi Perbandingan Saham Syariah dan Saham Konvensional," *J. Islam. Econ. Bus.*, vol. 2, no. 1, pp. 75–84, 2017, doi: 10.24042/febi.v2i1.943.
[10] Y. Hu, W. B. Lindquist, and S. T. Rachev, "Portfolio Optimization Constrained by Performance Attribution," *J. Risk Financ. Manag.*, vol. 14, no. 5, 2021, doi: 10.3390/jrfm14050201.
[11] Y. Liu, H. Ahmadzade, and M. Farahikia, "Portfolio selection of uncertain random returns based on value at risk," *Soft Comput.*, vol. 25, no. 8, pp. 6339–6346, 2021, doi: 10.1007/s00500-021-05623-6.
[12] S. Geissel, H. Graf, J. Herbinger, and F. T. Seifried, "Portfolio optimization with optimal expected utility risk measures," *Ann. Oper. Res.*, vol. 309, no. 1, pp. 59–77, 2022, doi: 10.1007/s10479-021-04403-7.
[13] Y. Hidayat, T. Purwandari, I. G. Prihanto, R. A. Hidayana, and R. A. Ibrahim, "Mean-Value-at-Risk Portfolio Optimization Based on Risk Tolerance Preferences and Asymmetric Volatility," *Mathematics*, vol. 11, no. 23, 2023, doi: 10.3390/math11234761.
[14] T. Li, Y. S. Kim, Q. Fan, and F. Zhu, "Aumann–Serrano index of risk in portfolio optimization," *Math. Methods Oper. Res.*, vol. 94, no. 2, pp. 197–217, 2021, doi: 10.1007/s00186-021-00753-x.
[15] V. Ranković, M. Drenovak, B. Urosevic, and R. Jelic, "Mean-univariate GARCH VaR portfolio optimization: Actual portfolio approach," *Comput. Oper. Res.*, vol. 72, pp. 83–92, 2016, doi: 10.1016/j.cor.2016.01.014.
[16] Sukono, E. Lesmana, H. Napitupulu, Y. Hidayat, J. Saputra, and P. L. B. Ghazali, "Mean-var portfolio optimisations: An application of multiple index models with non-constant volatility and long memory effects," *Int. J. Innov. Creat. Chang.*, vol. 9, no. 12, pp. 364–381, 2019.
[17] Sukono, K. Parmikanti, Lisnawati, S. H. Gw, and J. Saputra, "Mean-var investment portfolio optimization under capital asset pricing model (capm) with nerlove transformation: An empirical study using time series approach," *Ind. Eng. Manag. Syst.*, vol. 19, no. 3, pp. 498–509, 2020, doi: 10.7232/iems.2020.19.3.498.
[18] L. Chin, E. Chendra, and A. Sukmana, "Analysis of portfolio optimization with lot of stocks amount constraint: Case study index LQ45," *IOP Conf. Ser. Mater. Sci. Eng.*, vol. 300, no. 1, 2018, doi: 10.1088/1757-899X/300/1/012004.
[19] L. Yang, L. Zhao, and C. Wang, "Portfolio optimization based on empirical mode decomposition," *Phys. A Stat. Mech. its Appl.*, vol. 531, 2019, doi: 10.1016/j.physa.2019.121813.
[20] W. W. S. Wei, "Time Series Analysis: Univariate and Multivariate Methods,"

Technometrics, vol. 33, no. 1. pp. 108–109, 2006. doi: 10.1080/00401706.1991.10484777.
[21] K. Autchariyapanitkul, S. Sriboonchitta, and S. Chanaim, "Portfolio optimization of stock returns in high-dimensions: A copula-based approach," *Thai J. Math.*, vol. 2014, pp. 11–23, 2014.
[22] S. S. Rao, *Engineering optimization: Theory and practice*, Fifth. John Wiley & Sons, Inc., 2019. doi: 10.1002/9781119454816.