CAUCHY: Jurnal Matematika Murni dan Aplikasi
Volume 10(1) (2025), Pages 224-247
p-ISSN: 2086-0382; e-ISSN: 2477-3344

Optimization of investment portfolio weights using the Mean-Entropic-VaR model on the Top Ten Stocks from LQ45 in the Indonesian Capital Market

Nurnisaa binti Abdullah Suhaimi*, Herlina Napitupulu, Sukono

Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Padjadjaran, Sumedang 45363, Indonesia

Email: nurnisaa23001@mail.unpad.ac.id

ABSTRACT

In an investment portfolio, investors certainly choose a portfolio according to their preferences for return and risk. The problem is the allocation of investment weights in forming a portfolio, if the risk is in the form of Entropic-Value-at-Risk (EVaR). The purpose of this study is to determine the allocation of investment weights that maximize returns and minimize portfolio risk. The method used in this study is through investment portfolio optimization in the form of Mean-EVaR. The stages carried out are selecting the ten best stocks in the LQ45 index, estimating and testing the suitability of the return distribution, determining expectations, variance, and covariance between stock returns, and optimizing the allocation of investment portfolio weights using the Mean-EVaR model. This study uses daily return data for the period from August 2, 2021, to July 31, 2024, with the distribution of returns tested using the Anderson-Darling test. The evaluation of the optimal portfolio includes the average return, variance, and standard deviation, while the impact of the risk tolerance factor (τ) on portfolio allocation is also considered. Based on the results of the analysis, it was obtained that the optimal portfolio weight allocation is 0.01073, 0.23284, 0.04617, 0.08052, 0.00470, 0.09021, 0.14669, 0.00427, 0.22672, and 0.15715, to be allocated successively to the stocks ACES, BBRI, EXCEL, ITMG, PTBA, ADRO, BBTN, GGRM, KLBF, and AKRA. In this optimal portfolio, the average portfolio return is obtained at 0.00055 with an EVaR risk of 0.01632. These findings highlight the effectiveness of the Mean-EVaR model in balancing risk and return, providing practical insights for investors in making informed decisions. Future studies could explore broader datasets and alternative risk measures to enhance the robustness and applicability of the model.

Keywords: EVaR; investment; optimization; portfolio; return

INTRODUCTION

Optimization models involve constructing representations of systems that reflect real-world conditions, which are then converted into mathematical models by isolating key elements. Their main purpose is to maximize or minimize an objective function, such as profit, revenue, or efficiency, while adhering to specific constraints, which may be

Submitted: November 16, 2024 Reviewed: December 19, 2024 Accepted: February 17, 2025
DOI: http://dx.doi.org/10.18860/ca.v10i1.30794

physical, financial, or regulatory. These models are essential tools for identifying optimal solutions in decision-making by balancing trade-offs among different objectives. Their application significantly enhances efficiency, productivity, and decision-making across various sectors, including manufacturing, transportation, logistics, and services [1]. In an era marked by rapid changes and uncertainty, the ability to quickly develop solutions is crucial. Optimization models not only help address immediate challenges but also support long-term adaptability and resilience, offering a sustainable approach to future problems. By providing a structured framework for solving complex issues, these models play a vital role in enabling efficient and informed decision-making [2]. As the complexity of decision-making processes increases, optimization models continue to evolve, incorporating advanced computational techniques and algorithms to address multi-dimensional problems effectively [3].

An investment portfolio refers to a collection of valuable assets, including stocks, foreign currencies, bonds, gold, deposits, cash, properties, land, and more. Research by Cheng [4] highlights the high liquidity of the stock market, which explains why many individuals prefer stocks as a popular investment choice [5]. Stocks not only offer the potential for high returns but also provide flexibility and opportunities for diversification, making them a key element in many investment strategies. As financial instruments, stocks represent ownership or a stake in a company, and they are often seen as an ideal starting point for building a well-balanced portfolio. Hubbard [6] emphasizes that incorporating stocks into an investment portfolio is a common approach to increasing potential returns. However, changes in market conditions and economic cycles necessitate regular portfolio evaluation and optimization to maintain long-term investment stability. Therefore, understanding the dynamics of market trends and economic cycles is crucial for maintaining a resilient and well-optimized investment portfolio.

In investment portfolio determination, optimization models play a crucial role in managing risk effectively and achieving the best possible investment outcomes. According to Abuselidze & Slobodianyk [7], investment portfolios may consist of diverse financial instruments, including bonds, mutual funds, and real estate. By creating a diversified portfolio, investors can capitalize on growth opportunities while securing returns with minimal risk. In today’s dynamic and complex financial markets, investors face numerous options, each with varying levels of risk and return. Optimization models help investors allocate resources efficiently by considering key factors such as investment goals, risk tolerance, and existing constraints. Therefore, portfolio optimization models must integrate risk management strategies to ensure that portfolios are not only profitable but also capable of withstanding market fluctuations. Risk metrics like Value-at-Risk (VaR), Conditional Value-at-Risk (CVaR), and Entropic Value-at-Risk (EVaR) are useful for assessing the risk levels of portfolios. These metrics serve as foundational tools in modern financial analysis, enabling investors to quantify and manage uncertainties effectively. However, despite the increasing importance of portfolio optimization, limited research has systematically addressed the integration of advanced risk measures, such as EVaR, into comprehensive mathematical frameworks. This highlights the need for a deeper exploration of how such measures can enhance decision-making in complex investment scenarios.

Investors have access to various methods for measuring risk, with one widely used approach being Value-at-Risk (VaR). VaR serves as a tool to identify the underlying causes of risk and to establish effective policies for mitigating those risks [7,8]. Additionally, Conditional Value-at-Risk (CVaR) has been acknowledged as a coherent risk measurement method, as highlighted by Rockafellar & Uryasev [10] and Rockafellar [11]. While CVaR shares similarities with VaR, it offers distinct advantages. According to Markowitz, when losses are normally distributed, the optimal portfolios derived from CVaR, VaR, and Minimum Variance are essentially equivalent. This study incorporates the Entropic Value-at-Risk (EVaR), a more advanced risk measure developed by Ahmadi-Javid [12,13]. EVaR not only possesses coherent properties but also exhibits several unique characteristics. For example, EVaR is highly monotonic, setting it apart from other monotonic risk measures like VaR and CVaR. Through a model-based approach that utilizes input data, EVaR facilitates efficient portfolio optimization across a broad spectrum of return levels with known distributions [14]. The novelty of EVaR lies in its derivation from Chernoff's inequality, offering computational advantages and robustness in handling a wide range of stochastic optimization problems. Despite its potential, the literature still lacks a comprehensive evaluation of EVaR's performance in diverse financial contexts, particularly when compared to traditional risk measures.

Notably, in scenarios where return levels are independent and randomly distributed, traditional measures like VaR and CVaR are inadequate, as elaborated in Section 4 of Ahmadi-Javid [8]. EVaR's adaptability makes it especially valuable for managing risks in dynamic market conditions where conventional models may fall short. As a risk measure that serves as the upper bound for both CVaR and VaR, EVaR provides a coherent framework for addressing risks. Therefore, in cases involving independent and random return distributions, EVaR becomes a crucial tool for risk assessment and portfolio management. This study aims to fill this gap by systematically analyzing the effectiveness of EVaR in optimizing portfolios under varying market conditions and distributional assumptions, providing a robust alternative to traditional measures.

By employing optimization models that incorporate the EVaR risk measure, investors can effectively balance risk and return, which is crucial for achieving long-term financial objectives and safeguarding investments from market volatility. Such approaches are particularly important in navigating complex investment environments and uncertain market conditions. Portfolio weight optimization, for instance, serves as a powerful tool for making informed and efficient investment decisions [15,16]. According to Bodie et al. [17], the application of optimization models in constructing investment portfolios enables investors to efficiently align their portfolios with specific financial goals, risk tolerance, and individual preferences, ultimately improving long-term investment outcomes. However, the precise mechanisms through which EVaR contributes to enhanced portfolio performance remain underexplored, necessitating further investigation into its practical applications and theoretical underpinnings.

Ahmadi-Javid [8] introduced EVaR, a novel coherent risk measure designed as the tightest upper bound for Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR), derived from Chernoff's inequality. In further research, Ahmadi-Javid [2] demonstrated that EVaR significantly enhances computational efficiency for solving a broad range of stochastic optimization problems, many of which are impractical to address using CVaR. Building on this, Ahmed et al. [9] investigated the development and application of EVaR as a risk measure that acts as an upper bound for both VaR and CVaR. Their findings highlighted the superior computational efficiency of the EVaR-based portfolio optimization approach compared to the CVaR-based method, particularly as the sample size increases, where EVaR either matches or outperforms CVaR. Additionally, Cajas [18] extended the scope of EVaR by introducing Relativistic Value-at-Risk (RL-VaR), a generalized risk measure that lies between EVaR and the essential supremum (ess sup). Cajas' research [18] provided comprehensive theoretical advancements, mathematical formulations, and numerical implementations for both RL-VaR and EVaR, broadening the understanding and application of these risk measures in optimization contexts. While these studies underscore the potential of EVaR, they do not address its integration with advanced mathematical tools, such as vector and matrix equations, which could further refine portfolio optimization strategies. This research seeks to bridge this gap by incorporating these sophisticated techniques into the EVaR framework.

Previous studies have often emphasized minimizing variance, yet they have not fully explored optimization through the application of specific formulations involving vector and matrix equations. Addressing this limitation, the current research seeks to bridge the gap by advancing a portfolio optimization model based on the Mean-EVaR approach. To realize this objective, the study develops a comprehensive and detailed model that incorporates sophisticated mathematical techniques, particularly vector and matrix representations. These representations enhance the precision and robustness of the optimization process, offering a more solid and specific foundation for designing optimal investment portfolio strategies. Consequently, this research not only addresses the gaps identified in earlier studies but also makes a substantial contribution by introducing an innovative and mathematically grounded approach to portfolio optimization.

The benefit of this approach is rooted in the thoughtful incorporation of the EVaR risk measure into the process of portfolio weight optimization. By integrating these methods, a more robust framework for investment portfolio management is established, offering investors the flexibility to respond more effectively to market fluctuations. This increases the likelihood of meeting long-term financial objectives while reducing the potential for unforeseen losses. Additionally, this method allows for a more in-depth understanding of the risks involved, giving investors a competitive advantage when it comes to making swift adjustments to their portfolios in reaction to market shifts. The focus of this study is on investment portfolios, with an emphasis on developing optimization models that enhance the efficiency of portfolio management. Through the optimization model created in this research, an analysis is conducted to demonstrate how the integration of the Mean-EVaR method can improve risk management strategies within investment portfolios.

METHODS

This study involves the optimization of an investment portfolio based on the Mean-EVaR model, applied to the ten best-performing stocks from the LQ45 index traded on the Indonesian stock market. The objective is to determine the allocation weight composition for these ten stocks to achieve an optimal portfolio. It is assumed that the returns of these ten stocks are random numbers following a specific distribution pattern. To achieve this objective, the steps to be taken are outlined as follows:

Expected Return, Variance, and Covariance of stocks

In this step, the calculation of stock returns, expected return, variance, and covariance of stocks is analized. Let $P_{A,t}$ represent the price of stock (asset) A at time t, and $r_{A,t}$ return asset A at time t. Return asset $r_{A,t}$ can be calculated using equation (1).

$$r_{A,t} = ln\left(\frac{P_{A,t}}{P_{A,t-1}}\right). \tag{1}$$

Let $R_{A,t}$ represent the random variable of $r_{A,t}$. The following are formulas for the mean, variance, covariance, and correlation coefficient under the probabilistic approach:

The mean return of a stock A can be defined by equation (2).

$$\mu_A = E[R_{A,t}] = \int_{-\infty}^{+\infty} r_{A,t} f(r_{A,t}) dr_{A,t}, \tag{2}$$

where $r_t = r_1, r_2, r_3, \dots, r_N$, μ_A: is the expected return of stock A, $R_{A,t}$: is the random variable of the return of stock A at time t, $r_{A,t}$ is the value of the return of stock A at time t, and $f(r_{A,t})$ is the density function of R_t [19].

The variance of a stock A can be defined by equation (3).

$$\sigma_A^2 = E\left[(R_{A,t} - \mu_A)^2\right] = \int_{-\infty}^{+\infty} (r_{A,t} - \mu_A)^2 f(r_{A,t}) dr_{A,t} = E[R_{A,t}^2] - \{E[R_{A,t}]\}^2. \tag{3}$$

The covariance between stock A and B can be defined by equation (4).

$$\sigma_{AB} = E[(r_{A,t} - \mu_A)(r_{B,t} - \mu_B)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (r_{A,t} - \mu_A)(r_{B,t} - \mu_B) f(r_{A,t}, r_{B,t}) dr_{A,t} dr_{B,t}. \tag{4}$$

The correlation coefficient between the returns of stocks A and B is defined by equation (5) [19].

$$\rho_{AB} = \frac{\sigma_{AB}}{\sigma_A . \sigma_B}. \tag{5}$$

Distribution for Stock Returns

In this research, stock returns are assumed to follow a specific distribution, such as the log-logistic and Burr (4P) distributions, which are commonly used in risk analysis and economics due to their ability to capture heavy tails and asymmetry in empirical data [20]. The Burr (4P) distribution, introduced by Irving W. Burr in 1942, is highly flexible and can handle distributions with heavy tails and asymmetry [21], making it suitable for capturing market phenomena that often exhibit extreme volatility. On the other hand, the log-logistic distribution, known for its similarity to the log-normal distribution but with longer tails, is frequently used in duration analysis and survival modeling, making it a relevant approach for modeling long-term risks in the analyzed stocks.

However, while these distributions are widely used in the literature related to risk and economics, the assumption that stock returns follow these distributions needs to be validated through further statistical approaches, such as distribution fitting tests to ensure their suitability for the data. In this study, the chosen distributions will be tested using relevant distribution fitting methods and evaluated for their accuracy with historical stock return data from the top ten LQ45 stocks.

The Anderson-Darling test will be used to assess the goodness of fit of the chosen distributions. The importance of these distribution tests is to ensure that the portfolio optimization model, which uses stock return distributions as input, provides valid and reliable results. Therefore, selecting the appropriate distribution is crucial for applying the Mean-EVaR model to portfolio optimization in a way that is more accurate and aligned with the characteristics of the Indonesian capital market data. This will be further discussed in the following subsection.

Distribution Model Fit Test using Anderson Darling

In this step, a goodness-of-fit test is conducted to determine whether stock returns can be assumed to follow a specific distribution model. The goodness-of-fit test is performed using the Anderson-Darling method. Let $f(r_{A,t})$ represent the density function of R_t which is assumed to follow a certain distribution model. According to Bello et al. [22], the Anderson-Darling method is often used to verify whether sample data comes from a population with the expected distribution. Suppose, if there is data $x_1, x_2, x_3, \dots, x_n$ whose distribution will be tested at the confidence level θ, then the Anderson-Darling test is carried out using the following equation (6).

$$A = -n - S \,. \tag{6}$$

with S given by equation (7).

$$S = \frac{1}{n}\sum_{i=1}^{n}[2i-1]\left[\ln\left(F(Z_i)\right) + \ln(1 - F(Z_{n+1-i}))\right], \tag{7}$$

And determined using equation (8).

$$Z_i = \frac{x_i - \bar{x}}{s}. \tag{8}$$

Substitute equation (7) and equation (8) into equation (6) and we get the following equation (9).

$$A = -n - \frac{1}{n}\sum_{i=1}^{n}[2i-1]\left[\ln\left(F(Z_i)\right) + \ln(1 - F(Z_{n+1-i}))\right], \tag{9}$$

with,

A : test statistical value for the Anderson-Darling method,
x_i : i-th data that has been sorted,
Z_i : standardized x_i data,
$\bar{x}$: average data,
s : standard deviation of data,
$F(Z_i)$: value of the standard normal cumulative distribution function at Z_i.

a) Hypothesis: Testing using the Anderson-Darling method is carried out by determining the hypothesis as follows;
   H_0 : stock returns follow a specified distribution,
   H_1 : stock returns do not follow the specified distribution.
   The test criteria used are reject H_0 if the value of $A_{count} > A_{table}$ or Prob(A_{count})$< \theta$. In other words, if the test results show that the test statistical value is too large or the probability is very small, then the null hypothesis is considered not appropriate to the data
b) Significance Level: θ
c) Critical Value: The hypothesis regarding the decision is taken based on a comparison of the test statistical value A with the critical value at the predetermined significance level θ. If the test statistic value A is smaller than the critical value, then the null hypothesis H_0 is accepted, indicating that the data comes from a certain distribution. Conversely, if the test statistic value A is greater than the critical value, then H_0 is rejected, indicating that the data does not come from a certain distribution.

The Anderson-Darling test is used in this study because it provides a more robust assessment of the goodness-of-fit, particularly in evaluating the tails of the distribution. This is crucial in portfolio optimization, where extreme returns significantly influence risk measurement. The significance level ($\theta = 0.01$) is chosen to ensure a strict threshold for rejecting the null hypothesis, reducing the likelihood of incorrectly assuming that the data follows a specific distribution. This conservative approach aligns with the study's focus on accurately modeling financial risks

Entropic-VaR Model

This section explains an investment risk measure called Entropic Value-at-Risk (EVaR). By incorporating entropy, EVaR provides a more flexible and robust approach to measuring risk in chaotic environments where traditional methods, such as Mean-Variance, fall short [23]. EVaR is a new coherent risk measure introduced and studied by Ahmadi-Javid [12,13]. EVaR, at the confidence level $1-\alpha$ (or at-risk level α), is defined as shown in equation (10).

$$\mathrm{EVaR}_{1-\alpha}(X) := \inf_{\theta>0}\left\{\theta^{-1}\ln\left(\frac{M_x(\theta)}{\alpha}\right)\right\}, X \in \mathbf{L_M}, \alpha \in (0,1]\,. \tag{10}$$

EVaR is a risk measure that combines the concept of VaR with entropic information theory. Therefore, it can provide a new perspective on portfolio risk by considering uncertainty in the asset price distribution. For a normal distribution $X \sim N(\mu, \sigma^2)$ EVaR is given by equation (11).

$$\mathrm{EVaR}_{1-\alpha}(X) = \mu + \sqrt{-2\ln\alpha}\,\sigma. \tag{11}$$

Based on the normality assumption, it can be concluded that all risk measures, such as VaR, CVaR, and EVaR, are functions of the mean and variance

Determine Weight Vector, Mean Vector, Unit Vector, Covariance Matrix and Inverse Matrix

This step explains how to determine the weight vector, mean vector, unit vector, covariance matrix, and inverse matrix. In the context of this research, which focuses on the selected top N stocks, several basic elements in investment portfolio analysis, such as the weight vector, mean vector, unit vector, covariance matrix, and its inverse, can be explained as follows.

The weight vector, denoted as $\mathbf{w}$, in the context of an investment portfolio, represents the proportion of the total capital allocated to each stock within the portfolio. Each component of the weight vector corresponds to the specific portion of the total investment assigned to a particular stock [24,25]. It is essential for the weight vector to satisfy the condition that the sum of all its elements equals 1, ensuring that all available funds are fully invested in the portfolio without any remaining capital. Mathematically, this condition is expressed as follows (12).

$$\mathbf{w}^T = [w_1 \quad w_2 \quad \cdots \quad w_N], \tag{12}$$

with $\mathbf{w}^T$ is transpose of the weight vector $\mathbf{w}$, w_i is weight of the i-th stock, where $i = 1,2,\ldots,N$.

The vector $\mathbf{e}$ is a vector vector where all elements are equal to 1, with a length equal to the number of stocks. In this case, since there are N selected stocks, the unit vector e is given by equation (13).

$$\mathbf{e}^T = [1 \quad 1 \quad \dots \quad 1], \tag{13}$$

with $\mathbf{e}^T$ is tranpose of vector $\mathbf{e}$. [19]

The weight and unit vectors play a crucial role in determining the return of an investment portfolio. The mean vector, denoted as $\boldsymbol{\mu}$, consists of the expected or average returns for each stock in the portfolio. This vector represents the anticipated return for each stock over the research period. Each component of the mean vector corresponds to the expected return for a specific stock. The mean vector is expressed mathematically through the equation (14), which provides the foundation for calculating the overall return of the portfolio based on the individual stock returns. [19]

$$\boldsymbol{\mu}^T = [\mu_1 \quad \mu_2 \quad \cdots \ \mu_N], \tag{14}$$

with $\boldsymbol{\mu}^T$ is transpose of the mean vector $\boldsymbol{\mu}$, and μ_i mean return of the i-th stock, where $i = 1,2,\dots,N$.

The covariance matrix $\boldsymbol{\Sigma}$ represents the risk within a portfolio by representing the covariances between the returns of each pair of stocks. It provides insights into how stock returns interact with one another. The covariance matrix is of size N × N, where N is the number of stocks in the portfolio, and it computes both the variance of each individual stock and the covariance between each pair of stocks. This allows for a comprehensive assessment of the overall risk in the portfolio, as the interactions between the different stocks are considered. The covariance matrix $\boldsymbol{\Sigma}$ is mathematically expressed in the following equation (15).

$$\boldsymbol{\Sigma} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} & \cdots & \sigma_{1N} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} & \cdots & \sigma_{2N} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} & \cdots & \sigma_{3N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \sigma_{N1} & \sigma_{N2} & \sigma_{N3} & \cdots & \sigma_{NN} \end{bmatrix}, \tag{15}$$

with $\boldsymbol{\Sigma}$ is covariance matrix, and σ_{ij} covariance between the i-th and j-th stocks, where $i,j = 1,2,3,\dots,N$.

The inverse of the covariance matrix, denoted as $\boldsymbol{\Sigma^{-1}}$, is utilized to analyze how sensitive the portfolio is to fluctuations in the returns of individual stocks. It can be calculated using methods like Gauss-Jordan elimination. In this context, $\boldsymbol{\Sigma}^{-1}$ is represented by equation (16).

$$\boldsymbol{\Sigma^{-1}} = [\boldsymbol{\Sigma}|\boldsymbol{I}] = \left(\begin{array}{ccccc|ccccc} \sigma_{11} & \sigma_{12} & \sigma_{13} & \cdots & \sigma_{1N} & 1 & 0 & 0 & \dots & 0 \\ \sigma_{21} & \sigma_{22} & \sigma_{23} & \cdots & \sigma_{2N} & 0 & 1 & 0 & \dots & 0 \\ \sigma_{31} & \sigma_{32} & \sigma_{33} & \cdots & \sigma_{3N} & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \sigma_{N1} & \sigma_{N2} & \sigma_{N3} & \cdots & \sigma_{NN} & 0 & 0 & 0 & \cdots & 1 \end{array}\right), \tag{16}$$

with $\boldsymbol{I}$ is identity matrix. [19]

The inverse covariance matrix is a critical component in portfolio optimization, particularly when calculating the optimal portfolio weights that either minimize risk or enhance risk-adjusted returns. This matrix is integral to several optimization techniques, such as the Mean-VaR model [26], and the Mean-EVaR model. Portfolio variance, denoted as σ_p^2, can be expressed through a multiplication process as shown in equation (17) [27,28]. This formula is used to assess the total risk associated with a portfolio by considering the interactions between the assets within the portfolio.

$$\sigma_p^2 = \mathbf{w}^T \,\boldsymbol{\Sigma}\mathbf{w}. \tag{17}$$

Mean-EVaR Investment Portfolio Optimization

EVaR is considered a more comprehensive risk measure than VaR because it incorporates entropic information from the deeper tail of the distribution. Defined in Equation (17), EVaR evaluates risk by focusing on the tail distribution, offering more robust protection against extreme events when compared to VaR. It is because it accounts for extreme tail events and the deeper aspects of the distribution, which is crucial for accurately assessing risks in the presence of heavy tails and market volatility. According to Equation (17), the EVaR of a portfolio is derived based on this concept. The calculation of EVaR involves using the portfolio's mean, as specified in Equation (18), and the portfolio's variance, as presented in Equation (19), to determine the portfolio's EVaR value.

$$\mu_p = \sum_{i=1}^{N} w_i \mu_i = \boldsymbol{\mu}^T \mathbf{w} = \mathbf{w}^T \boldsymbol{\mu}, \tag{18}$$

$$\sigma_p^2 = \sum_{i=1}^{N} w_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \sigma_{ij} = \mathbf{w}^T \boldsymbol{\Sigma}\mathbf{w}. \tag{19}$$

Therefore, EVaR in equation (11), in the form of vector and matrix notation, EVaR_p can be calculated using equation (20).

$$\text{EVaR}_\text{p} = -W_0 \left(2\tau\boldsymbol{\mu}^T \mathbf{w} + \left(\boldsymbol{\mu}^T \mathbf{w} + \sqrt{-2 \ln \alpha}\, (\mathbf{w}^T \boldsymbol{\Sigma}\mathbf{w})^{\frac{1}{2}} \right) \right), \tag{20}$$

τ is the risk tolerance factor, which controls the trade-off between risk and return. A higher τ indicates higher risk tolerance, meaning the investor is willing to accept more risk for potentially higher returns. The problem to be addressed in this optimization is maximizing the return of the portfolio, while considering the risk, which is quantified using EVaR. When the risk is measured through EVaR, the optimization problem that needs to be solved is represented by equation (21).

$$\max\{2\tau\mu_p - \text{EVaR}_\text{p}\}, \tag{21}$$

$$st. \sum_{i=1}^{N} w_i = 1.$$

This objective function aims to balance the portfolio's expected return μ_p with its risk (as measured by EVaR), weighted by the risk tolerance factor τ. The optimization problem is subject to the constraint that the sum of the portfolio weights must equal 1. By utilizing the vectors in equation (18), the vector equation in (19), and equation (20), and assuming an initial capital of $W_0 = 1$ unit, the optimization problem presented in equation (21) can be restructured and represented as equation (22).

$$\max\left\{ 2\tau\boldsymbol{\mu}^T \mathbf{w} + \left(\boldsymbol{\mu}^\mathrm{T} \mathbf{w} + \sqrt{-2\, ln\, \alpha}\, (\mathbf{w}^T \boldsymbol{\Sigma}\mathbf{w})^{\frac{1}{2}} \right) \right\}, \tag{22}$$

$$st.\, \mathbf{e}^T\mathbf{w} = 1.$$

The solution to determine the optimal weight for a Mean-EVaR portfolio optimization problem, considering the risk tolerance factors as presented in equation (22), is as follows.:

1) When the risk tolerance factor $\tau \geq 0$, the solution to the Mean-EVaR optimization problem can be expressed as follows:

$$\mathbf{w} = \frac{(2\tau + 1)\mathbf{\Sigma}^{-1}\boldsymbol{\mu} + \lambda\mathbf{\Sigma}^{-1}\mathbf{e}}{(2\tau + 1)\mathbf{e}^T\mathbf{\Sigma}^{-1}\boldsymbol{\mu} + \lambda\mathbf{e}^T\mathbf{\Sigma}^{-1}\mathbf{e}}, \tag{23}$$

2) The Lagrangian multiplier λ used to enforce the constraint $e^T w = 1$, it can be determined using the following formula in equation (24).

$$\lambda_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}; \quad \lambda > 0, \tag{24}$$

where

$$a = \mathbf{e}^T\mathbf{\Sigma}^{-1}\mathbf{e},$$
$$b = (2\tau + 1)(\mathbf{e}^T\mathbf{\Sigma}^{-1}\boldsymbol{\mu} + \boldsymbol{\mu}^T\mathbf{\Sigma}^{-1}\mathbf{e}),$$
$$c = (2\tau + 1)^2\boldsymbol{\mu}^T\mathbf{\Sigma}^{-1}\boldsymbol{\mu} + \sqrt{-2\ln\alpha},$$

The Mean-EVaR portfolio optimization process incorporates the risk tolerance factor τ to balance risk and return. By determining the optimal portfolio weights through the use of matrix equations and Lagrange multipliers, the method provides a more nuanced approach to portfolio optimization compared to traditional models. The construction of the efficient frontier helps investors choose the best portfolio based on their risk preferences, ensuring that the portfolio is both optimized for return and appropriately aligned with the investor's risk tolerance

These steps are applied to the return data of the ten selected stocks, as described in the subsequent sections. By using the Mean-EVaR optimization model, the optimal portfolio weights are determined based on the selected stocks' return data, taking into account the risk tolerance factor and the covariance structure of the assets.

RESULTS AND DISCUSSION

Stock data and Returns

This section presents the stock price data for the top 10 performing stocks listed in Indonesia's LQ45 index, chosen based on their performance during the research period. Stock returns are computed using the log return formula, which provides an overview of each stock's profit or loss over a specific period. The data analyzed in this study consists of stock prices from the 10 best-performing stocks traded on the Indonesia Stock Exchange (IDX). This stock price analysis offers valuable insights into notable price fluctuations and patterns that serve as the foundation for calculating stock returns. The 10 selected stocks are part of the LQ45 index, which is evaluated every six months by the IDX, with this research period covering February 2024 to July 2024. For this analysis, stock data spanning three full years, from August 2, 2021, to August 1, 2024, is included. According to the LQ45 index, the top 10 stocks are ACES, BBRI, EXCL, ITMG, PTBA, ADRO, BBTN, GGRM, KLBF, and PTMP. However, since PTMP did not meet the three-year data requirement, it was replaced with the 11th best-performing stock, AKRA. The closing price data for these 10 stocks covers 724 trading days. This data is then used to calculate the stock returns during the research period, which is subsequently applied in the portfolio optimization model. Table 1 provides the names and abbreviations of the stocks used in this study, representing the top 10 performers selected based on the specified criteria.

Table 1. 10 best stocks selected based on the LQ45.

No.	Stocks	Best LQ45 Stocks
1	ACES	Aspirasi Hidup Indonesia Tbk PT
2	BBRI	Bank Rakyat Indonesia (Persero) Tbk PT
3	EXCL	XL Axiata Tbk PT
4	ITMG	Indo Tambangraya Megah Tbk PT
5	PTBA	Bukit Asam Tbk PT
6	ADRO	Adaro Energy Indonesia Tbk PT
7	BBTN	Bank Tabungan Negara (Persero) Tbk PT
8	GGRM	Gudang Garam Tbk PT
9	KLBF	Kalbe Farma Tbk PT
10	AKRA	AKR Corporindo Tbk PT

This study also includes charts illustrating the daily stock price movements from August 2, 2021, to August 1, 2024. In these charts, the horizontal axis represents the dates of trading, while the vertical axis shows the corresponding stock prices. For example, Figure 3 presents the daily stock price chart for ACES.

Figure 1. Daily Stock Price Movement of ACES.

In Figure 1, it can be observed that ACES stock prices exhibited a downward trend from the beginning of the period until the middle, followed by a slight increase and concluding with a period of consolidation at a lower level. Stock returns are calculated based on the changes in stock prices over time, representing the profit or loss generated from investing in these stocks. . This example illustrates the calculation of logarithmic returns for ACES stock between the two dates, focusing on a single stock out of the ten analyzed in the study. The same calculation method was applied to the subsequent periods. Figure 2 presents the return chart for ACES stock, which aids in understanding the volatility of ACES stock returns.

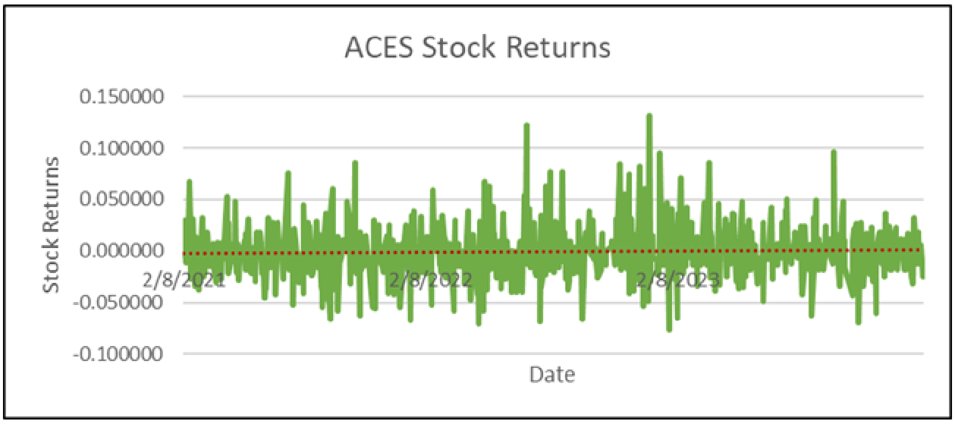

Figure 2. Daily Stock Returns of ACES.

Returns serve as a measure of investment performance and are a crucial component in portfolio analysis. By calculating stock returns, one can assess the potential profits and associated risks of each stock. In this study, return data for 10 stocks were calculated using equation (1) with the assistance of Microsoft Excel software. An example of the manual calculation of daily stock returns is provided using the closing price data of ACES stock from August 2, 2021, and August 3, 2021:

$$r_1 = ln\left(\frac{1325}{1305}\right).$$

Specifically, 1305 is the closing price on August 2, 2021, and 1325 is the closing price on August 3, 2021.

Descriptive statistics offer a numerical overview of the features of stock return data, including metrics like the mean return, variance, and standard deviation (which represents volatility). By analyzing these descriptive statistics, a better understanding of stock return patterns can be gained, which will inform decisions in portfolio optimization. The descriptive statistics for the returns are displayed in Table 2.

Table 2. Descriptive Statistics of the Returns of the Top 10 Stocks.

No.	Stocks	Mean Return (μ)	Variance (σ^2)	Standard Deviation (σ)
1	ACES	-0.000677	0.000698	0.026423
2	BBRI	0.000439	0.000257	0.016025
3	EXCL	-0.000276	0.000515	0.022701
4	ITMG	0.000608	0.000600	0.024496
5	PTBA	0.000248	0.000532	0.023067
6	ADRO	0.001182	0.000662	0.025772
7	BBTN	0.000196	0.000343	0.018511
8	GGRM	-0.001010	0.000395	0.019874
9	KLBF	0.000273	0.000322	0.017947
10	AKRA	0.001056	0.001056	0.023729

This descriptive statistic provides a brief overview of the average performance and the associated risk for each stock utilized in the portfolio optimization process.

Estimating and Testing the Distribution Fit for Top 10 Stock Returns

In estimating and testing the suitability of the return distribution model for the top 10 stocks, the most fitting statistical distribution is identified to accurately represent stock return behavior. This includes determining whether the returns follow a log-logistic, Burr, or other types of distribution. For instance, an analysis was conducted on the return distribution model of ACES stock. The stock returns for ACES were visually represented as a histogram, and after conducting tests using EasyFit software, it was concluded that the histogram in Figure 3 is best described by a log-logistic (3P) distribution.

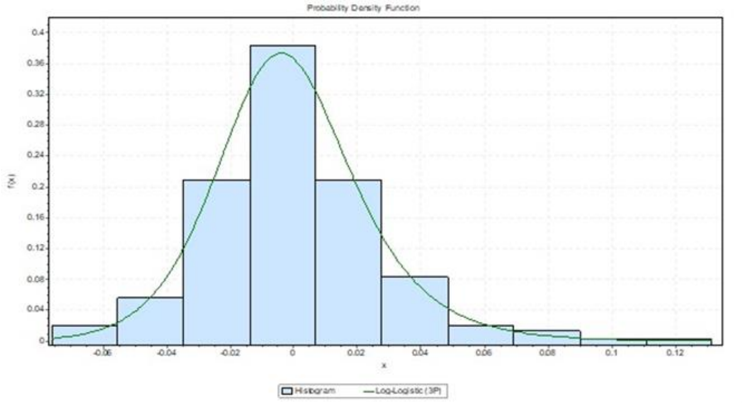

Figure 3. Histogram of ACES Stock Returns.

The parameter estimation for ACES stock was carried out using the Maximum Likelihood Estimation (MLE) method, as demonstrated by Mai et al. [29]. Once the parameter values were obtained, as shown in Table 3, a goodness-of-fit test was performed using the Anderson-Darling test at a significance level of $\theta = 0.01$. This step is crucial to verify that the portfolio optimization model is based on a suitable distribution.

For the distribution model testing on ACES stock, the hypotheses used are as follows:

H_0: The stock returns follow a Log-Logistic distribution (3P).

H_1: The stock returns do not follow a Log-Logistic distribution (3P).

The same testing method was applied to each stock. At the significance level of $\theta = 0.01$, the critical value of Anderson-Darling is $A^2_{table} = 3.9074$, which serves as a benchmark for determining whether the data distribution significantly deviates from the theoretical distribution being tested.

Table 3 reveals that several stocks, including ACES, ITMG, ADRO, GGRM, and KLBF, are better represented by the Log-Logistic distribution (3P). This distribution is known for effectively capturing more regular data patterns with lighter tails, suggesting that these stocks tend to have more stable return characteristics over the long term. In contrast, stocks such as BBRI, EXCL, PTBA, BBTN, and AKRA align more closely with the Burr distribution. The Burr distribution is capable of capturing patterns of higher volatility and heavier tails, indicating that these stocks are more prone to experiencing extreme fluctuations or outlier events in their returns. This suggests that risk analysis using the Burr distribution would be more appropriate for these stocks, as it can offer more realistic risk estimates, particularly in the presence of significant price fluctuations.

Table 3. Parameter Estimator Values and Fit of Return Distributions for 10 Stocks

No.	**Stocks**	**Distribution**	**Parameter Value**				**Anderson-Darling Goodness-of-Fit Test**	
			α	β	γ	k	A^2_{hitung}	H_0
1	ACES	Log-Logistic (3P)	16.21	0.23	-0.23	-	2.6958	Accepted
2	BBRI	Burr (4P)	2.14×10^4	178.49	-178.49	0.88	0.67056	Accepted
3	EXCL	Burr (4P)	610.05	6.30	-6.30	0.71	0.9668	Accepted
4	ITMG	Log-Logistic (3P)	73.55	0.94	-0.94	-	1.6247	Accepted
5	PTBA	Burr (4P)	1.84×10^8	2.13×10^6	-2.13×10^6	1.03	2.5731	Accepted
6	ADRO	Log-Logistic (3P)	91.07	1.25	-1.25	-	1.6213	Accepted
7	BBTN	Burr (4P)	298.76	2.29	-2.30	0.63	2.4064	Accepted
8	GGRM	Log-Logistic (3P)	45.41	0.44	-0.44	-	3.345	Accepted
9	KLBF	Log-Logistic (3P)	36.04	0.35	-0.35	-	1.0033	Accepted
10	AKRA	Burr (4P)	8.49×10^7	9.49×10^5	-9.49×10^5	0.76	2.6861	Accepted

Based on the parameter estimator values presented in Table 3, the subsequent step involves calculating the mean using equation (2) and the variance using equation (3). The outcomes of these mean and variance calculations are displayed in Table 4.

Table 4. Mean and Variance Estimator

No.	Stocks	Mean Return (μ)	Variance (σ^2)	Standard Deviation (σ)
1	ACES	-0.000783	0.000658	0.025660
2	BBRI	0.000426	0.000252	0.015870
3	EXCL	-0.000017	0.000469	0.021660
4	ITMG	0.000675	0.000534	0.023110
5	PTBA	0.000535	0.000502	0.022410
6	ADRO	0.001300	0.000620	0.024900
7	BBTN	0.000350	0.000297	0.017230
8	GGRM	-0.001240	0.000315	0.017740
9	KLBF	0.000182	0.000316	0.017770
10	AKRA	0.001260	0.000570	0.023880

Next, Table 5 presents the covariance between the returns of the stocks analyzed. The covariance matrix highlights the relationships between stock returns, indicating whether they move in the same or opposite directions. These covariance values have been computed using equation (4).

If the covariance between two stocks is positive, it indicates that the stocks tend to move in the same direction. For instance, the positive covariance of 0.000054 between ACES and BBRI suggests that when ACES' return increases, BBRI's return also tends to rise. On the other hand, a negative covariance, such as the one between ACES and ITMG at -0.000015, means that when ACES' return increases, ITMG's return tends to decrease, indicating an inverse relationship between the two stocks. The mean and variance estimators shown in Table 4, along with the covariance values presented in Table 5, are utilized in the portfolio optimization process within the Mean-EVaR model.

Determining the Mean Vector, Unit Vector, and Covariance Matrix of Returns for the 10 Best Stocks

The mean value, derived from Table 4, is rewritten in vector form following the structure of Equation (14) and is formulated as presented in Equation (25).

$$\boldsymbol{\mu}^T = [-0.0007830\ \ 0.000426\ -0.000017\ \ 0.000675\ \ 0.000535\ \ 0.001300\ \ 0.000350 - 0.001240\ \ 0.000182\ \ 0.001260] \tag{25}$$

Since there are 10 stocks being analyzed, and based on equation (13), the unit vector is formulated as shown in equation (26). The unit vector is written as below reflecting the presence of 10 stocks in the analysis.

$$\mathbf{e}^T = [1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1]. \tag{26}$$

Based on the volatility estimates presented in Table 4 and the covariance values among the 10 selected stocks listed in Table 5, as referenced in equation (15), the covariance matrix is structured according to equation (27). Following this, the matrix in equation (27) is processed by calculating its inverse covariance matrix as outlined in equation (16), with the result being shown in equation (28). The calculation of the inverse covariance matrix is crucial in the process of investment portfolio optimization, as it typically aims to determine asset weights that either minimize risk for a specified level of return or maximize return for a given level of risk. In essence, the inverse covariance matrix plays a key role in solving the optimization model by incorporating the variability and correlation between assets and the investor's risk tolerance, ultimately guiding the determination of the optimal portfolio composition in alignment with the investment goals.

CAUCHY: Jurnal Matematika Murni dan Aplikasi
Volume 10(1) (2025), Pages 238-261
p-ISSN: 2086-0382; e-ISSN: 2477-3344

Table 5. Covariance of Returns between 10 Stocks

Covariance of Returns

Stocks	ACES	BBRI	EXCL	ITMG	PTBA	ADRO	BBTN	GGRM	KLBF	AKRA
ACES	0.000658	0.000054	0.000080	-0.000015	0.000003	-0.000005	0.000083	0.000058	0.000039	0.000050
BBRI	0.000054	0.000252	0.000060	0.000031	0.000054	0.000052	0.000115	0.000013	0.000040	0.000030
EXCL	0.000080	0.000060	0.000469	0.000070	0.000060	0.000101	0.000078	0.000015	0.000050	0.000051
ITMG	-0.000015	0.000031	0.000070	0.000534	0.000303	0.000358	0.000035	0.000059	-0.000003	0.000118
PTBA	0.000003	0.000054	0.000060	0.000303	0.000502	0.000339	0.000049	0.000051	0.000023	0.000123
ADRO	-0.000005	0.000052	0.000101	0.000358	0.000339	0.000620	0.000049	0.000040	0.000010	0.000147
BBTN	0.000083	0.000115	0.000078	0.000035	0.000049	0.000049	0.000297	0.000027	0.000025	0.000057
GGRM	0.000058	0.000013	0.000015	0.000059	0.000051	0.000040	0.000027	0.000315	0.000044	0.000026
KLBF	0.000039	0.000040	0.000050	-0.000003	0.000023	0.000010	0.000025	0.000044	0.000316	0.000012
AKRA	0.000050	0.000030	0.000051	0.000118	0.000123	0.000147	0.000057	0.000026	0.000012	0.000570

No.	Stocks	Full Name of Best LQ45 Stocks	No.	Stocks	Full Name of Best LQ45 Stocks
1	ACES	Aspirasi Hidup Indonesia Tbk PT	6	ADRO	Adaro Energy Indonesia Tbk PT
2	BBRI	Bank Rakyat Indonesia (Persero) Tbk PT	7	BBTN	Bank Tabungan Negara (Persero) Tbk PT
3	EXCL	XL Axiata Tbk PT	8	GGRM	Gudang Garam Tbk PT
4	ITMG	Indo Tambangraya Megah Tbk PT	9	KLBF	Kalbe Farma Tbk PT
5	PTBA	Bukit Asam Tbk PT	10	AKRA	AKR Corporindo Tbk PT

$$\mathbf{\Sigma} = \begin{pmatrix} 0.000658 & 0.000054 & 0.000080 & -0.000015 & 0.000003 & -0.000005 & 0.000083 & 0.000058 & 0.000039 & 0.000050 \\ 0.000054 & 0.000252 & 0.000060 & 0.000031 & 0.000054 & 0.000052 & 0.000115 & 0.000013 & 0.000040 & 0.000030 \\ 0.000080 & 0.000060 & 0.000469 & 0.000070 & 0.000060 & 0.000101 & 0.000078 & 0.000015 & 0.000050 & 0.000051 \\ -0.000015 & 0.000031 & 0.000070 & 0.000534 & 0.000303 & 0.000358 & 0.000035 & 0.000059 & -0.000003 & 0.000118 \\ 0.000003 & 0.000054 & 0.000060 & 0.000303 & 0.000502 & 0.000339 & 0.000049 & 0.000051 & 0.000023 & 0.000123 \\ -0.000005 & 0.000052 & 0.000101 & 0.000358 & 0.000339 & 0.000620 & 0.000049 & 0.000040 & 0.000010 & 0.000147 \\ 0.000083 & 0.000115 & 0.000078 & 0.000035 & 0.000049 & 0.000049 & 0.000297 & 0.000027 & 0.000025 & 0.000057 \\ 0.000058 & 0.000013 & 0.000015 & 0.000059 & 0.000051 & 0.000040 & 0.000027 & 0.000315 & 0.000044 & 0.000026 \\ 0.000039 & 0.000040 & 0.000050 & -0.000003 & 0.000023 & 0.000010 & 0.000025 & 0.000044 & 0.000316 & 0.000012 \\ 0.000050 & 0.000030 & 0.000051 & 0.000118 & 0.000123 & 0.000147 & 0.000057 & 0.000026 & 0.000012 & 0.000570 \end{pmatrix} \tag{27}$$

$$\mathbf{\Sigma}^{-1} = \begin{pmatrix} 1636.919 & -139.840 & -207.916 & 107.285 & 13.024 & 61.787 & -323.216 & -268.326 & -87.780 & -113.009 \\ -139.840 & 4994.449 & -226.291 & 102.115 & -286.055 & -155.069 & -1750.503 & 81.229 & -426.806 & 28.826 \\ -207.916 & -226.291 & 2367.185 & -115.374 & 72.497 & -295.872 & -396.720 & 60.193 & -289.884 & -52.138 \\ 107.285 & 102.115 & -115.374 & 3491.007 & -1149.733 & -1327.131 & -4.510 & -337.390 & 196.490 & -123.903 \\ 13.024 & -286.055 & 72.497 & -1149.733 & 3622.755 & -1236.402 & -104.599 & -158.477 & 174.308 & -198.256 \\ 61.787 & -155.069 & -295.872 & -1327.131 & -1236.402 & 3170.436 & -4.551 & 70.432 & 42.120 & -248.825 \\ -323.216 & -1750.503 & -396.720 & -4.510 & -104.599 & -4.551 & 4323.113 & -180.392 & 24.192 & -244.149 \\ -268.326 & 81.229 & 60.193 & -337.390 & -158.477 & 70.432 & -180.392 & 3381.046 & -438.266 & -24.096 \\ -87.780 & -426.806 & -289.884 & 196.490 & 174.308 & 42.120 & 24.192 & -438.266 & 3349.412 & -12.638 \\ -113.009 & 28.826 & -52.138 & -123.903 & -198.256 & -248.825 & -244.149 & -24.096 & -12.638 & 1924.97 \end{pmatrix} \tag{28}$$

Table 6. Efficient Portfolio Weight Composition Based on the Mean-EVaR Optimization Model

τ	λ	w										μ_p	$EVaR_p$	σ	REVaR
		ACES	BBRI	EXCL	ITMG	PTBA	ADRO	BBTN	GGRM	KLBF	AKRA				
0	0.02248	0.05330	0.19277	0.07483	0.07209	0.03129	0.01428	0.11673	0.16882	0.18916	0.08673	0.00010	0.01507	0.00922	0.00652
0.1	0.02246	0.05243	0.19359	0.07424	0.07226	0.03075	0.01583	0.11734	0.16546	0.18993	0.08817	0.00011	0.01507	0.00923	0.00713
0.2	0.02245	0.05156	0.19441	0.07366	0.07243	0.03020	0.01739	0.11795	0.16209	0.19069	0.08961	0.00012	0.01507	0.00923	0.00775
0.3	0.02243	0.05069	0.19523	0.07307	0.07261	0.02966	0.01894	0.11857	0.15872	0.19146	0.09106	0.00013	0.01507	0.00924	0.00836
0.4	0.02241	0.04982	0.19605	0.07248	0.07278	0.02911	0.02050	0.11918	0.15534	0.19223	0.09250	0.00014	0.01507	0.00925	0.00897
0.5	0.02238	0.04894	0.19688	0.07189	0.07295	0.02856	0.02206	0.11980	0.15196	0.19301	0.09395	0.00014	0.01507	0.00925	0.00959
0.6	0.02236	0.04807	0.19770	0.07130	0.07313	0.02802	0.02362	0.12042	0.14857	0.19378	0.09540	0.00015	0.01508	0.00926	0.01020
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
4.1	0.02025	0.01419	0.22959	0.04850	0.07983	0.00686	0.08404	0.14426	0.01763	0.22367	0.15143	0.00051	0.01612	0.01011	0.03181
4.2	0.02015	0.01305	0.23066	0.04773	0.08006	0.00615	0.08607	0.14506	0.01324	0.22467	0.15331	0.00052	0.01618	0.01016	0.03243
4.3	0.02004	0.01190	0.23174	0.04696	0.08029	0.00543	0.08812	0.14587	0.00879	0.22569	0.15522	0.00054	0.01625	0.01021	0.03305
4.4	0.01994	0.01073	0.23284	0.04617	0.08052	0.00470	0.09021	0.14669	0.00427	0.22672	0.15715	0.00055	0.01632	0.01025	0.03367
4.5	0.01983	0.00955	0.23396	0.04537	0.08075	0.00396	0.09232	0.14753	-0.0003	0.22776	0.15911	0.00056	0.01639	0.01030	0.03429

No.	Stocks	Full Name of Best LQ45 Stocks
1	ACES	Aspirasi Hidup Indonesia Tbk PT
2	BBRI	Bank Rakyat Indonesia (Persero) Tbk PT
3	EXCL	XL Axiata Tbk PT
4	ITMG	Indo Tambangraya Megah Tbk PT
5	PTBA	Bukit Asam Tbk PT

No.	Stocks	Full Name of Best LQ45 Stocks
6	ADRO	Adaro Energy Indonesia Tbk PT
7	BBTN	Bank Tabungan Negara (Persero) Tbk PT
8	GGRM	Gudang Garam Tbk PT
9	KLBF	Kalbe Farma Tbk PT
10	AKRA	AKR Corporindo Tbk PT

Determining the Proportion of Optimal Investment Portfolio Weights Based on the Mean-EVaR Model

Portfolio optimization using the Mean-EVaR model also aims to determine the efficient portfolio composition by maximizing returns and minimizing risk measured using EVaR. The Mean-EVaR portfolio optimization process is based on Theorem 1. Using the vector $\boldsymbol{\mu}^T$ in equation (25), the vector $\mathbf{e}^T$ in equation (26), and the matrix $\boldsymbol{\Sigma}^{-1}$ in equation (28), the optimal weights of the investment portfolio are calculated using the equations of Theorem 1. Risk tolerance τ, with the condition $\tau \geq 0$, is determined through simulation with several values satisfying $\mathbf{e}^T\mathbf{w} = 1$ or $\Sigma_{i=1}^{10} w_i = 1$ for $i = 1,\ldots,10$. The simulation stops when substituting the risk tolerance into Theorem 1 produces weights w_i ($i = 1,\ldots,10$) that are not positive real numbers or do not satisfy $\Sigma_{i=1}^{10} = 1$. The results of the risk tolerance simulation and the calculation of the efficient portfolio weight composition based on the Mean-EVaR optimization model are presented in Table 6.

Based on Table 6, the risk tolerance values taken are in the range $0 \leq \tau \leq 4.4$ because risk tolerance values above 4.4 result in invalid negative weights. For a risk tolerance of $\tau = 0$, the portfolio yields an average return $\mu_p = 0.00010$ and $EVaR_p = 0.01507$, which are the minimum average return [15,17] and minimum EVaR values. Conversely, for a risk tolerance of $\tau = 4.4$, the average portfolio return is $\mu_p = 0.00055$ with an EVaR risk of $EVaR_p = 0.01632$, which are the maximum values.

An efficient portfolio can be obtained by balancing the average return and EVaR risk, where the risk tolerance must be optimally selected to maximize returns without producing negative weights. An efficient portfolio is achieved at the point where risk tolerance provides an optimal combination of return and risk, as seen at $\tau = 4.4$, which results in the highest average return without exceeding acceptable risk limits. The curve connecting pairs of average return and EVaR risk forms the efficient frontier, where each point on that line represents an optimal portfolio with a balanced combination of return and risk. Figure 4 illustrates how the relationship between average return and EVaR risk forms the surface of the efficient portfolio, with the valid range of risk tolerance being $0 \leq \tau \leq 4.4$.

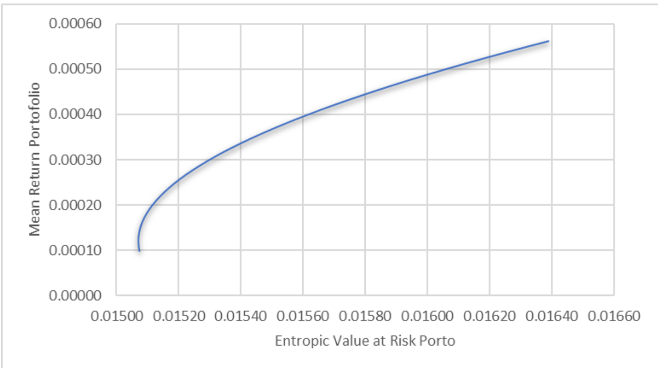

Figure 4. Mean-EVaR Efficient Portfolio Surface of 10 Stocks.

Among the efficient frontiers, there is one optimal portfolio that needs to be identified, which is the portfolio that offers the best combination of average return and EVaR risk. After obtaining a series of efficient portfolios, the next step is to determine the composition of the optimal portfolio. Investors generally desire a portfolio that generates high average returns with low risk [30, 31]. If the investor's preference is solely based on expected return and risk, the optimal portfolio can be determined from the efficient portfolio that has the highest ratio between average return and EVaR risk. The results of this ratio calculation can be seen in Table 6 and visualized in Figure 5.

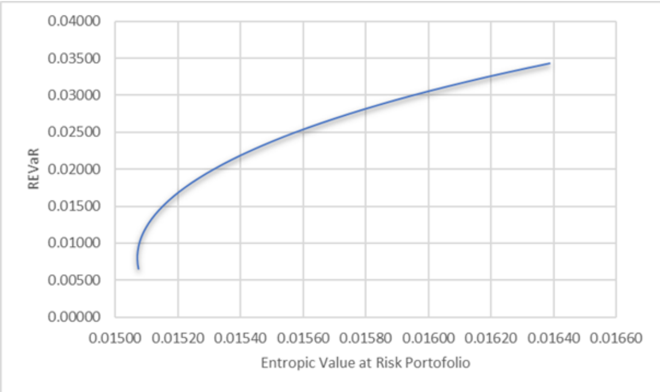

Figure 5. Plot of the Ratio and EVaR Risk of Portfolio Returns for 10 Stocks

Based on Figure 5 and Table 6, the highest ratio between the average portfolio return μ_p and $EVaR_p$ is recorded at 0.3367, achieved when the risk tolerance is at a value of $\tau = 4.4$. This ratio continues to increase throughout the risk tolerance interval of $0 \leq \tau \leq 4.4$. According to the results of portfolio optimization using the Mean-EVaR model, the optimal portfolio formed consists of the 10 best stocks with the following stock weight vector. The stock weights are rewritten from Table 6 and expressed in vector form, as stated in equation (12).

$$\mathbf{w}^T = [0.01073 \;\; 0.23284 \;\; 0.04617 \;\; 0.08052 \;\; 0.00470 \;\; 0.09021 \;\; 0.14669 \;\; 0.00427 \;\; 0.22672 \;\; 0.15715].$$

This optimal portfolio is capable of generating an average return of 0.00055 with a risk, measured using EVaR, of 0.01632.

Discussion

Based on the results of the optimization, the portfolio weights that are considered optimal, as determined by the Mean-EVaR model with a risk tolerance of $\tau = 4.4$, are summarized in Table 6. The specific details of these weights can be found in Table 7. The distribution of weights shows consistency in the formation of the portfolio across the range of risk tolerance applied, highlighting the efficiency of the Mean-EVaR model in portfolio optimization.

Table 7. Optimal Portfolio Weight Composition, Average, and Risk

	Stocks	Variance	Standard Deviation
$\mathbf{w}^T$	ACES	0.01058	0.01073
	BBRI	0.23299	0.23284
	EXCL	0.04607	0.04617
	ITMG	0.08055	0.08052
	PTBA	0.00460	0.00470
	ADRO	0.09048	0.09021
	BBTN	0.14680	0.14669
	GGRM	0.00367	0.00427
	KLBF	0.22685	0.22672
	AKRA	0.15741	0.15715
μ_p		0.00055	
$EVaR_p$		0.01632	

In the Mean-EVaR model, at a risk tolerance of $\tau = 4.4$, the optimal portfolio yields an average portfolio return of $\mu_p = 0.00055$ with a slightly lower of $EVaR_p = 0.01632$.

These results indicate that the Mean-EVaR model effectively reduces potential extreme losses, as EVaR is more sensitive to tail risk. This makes the Mean-EVaR model particularly appealing to investors who prioritize tail risk management.

Furthermore, to evaluate the performance of the optimal portfolio, a modified Sharpe Ratio can be used by measuring risk with EVaR. The REVaR values are summarized in Table 6 and presented in Table 8. Based on Table 8, the highest Sharpe ratio, or REVaR value, in the Mean-EVaR model is 0.3367 at $\tau = 4.4$.

Table 8. REVaR Values

No.	τ	REVaR
1	0	0.00652
2	0.1	0.00713
3	0.2	0.00775
4	0.3	0.00836
5	0.4	0.00897
6	0.5	0.00959
7	0.6	0.01020
8	0.7	0.01081
9	0.8	0.01143
10	0.9	0.01204
11	1.0	0.01266
12	1.1	0.01327
13	1.2	0.01389
14	1.3	0.01450
15	1.4	0.01512
16	1.5	0.01573
17	1.6	0.01635
18	1.7	0.01696
19	1.8	0.01758
20	1.9	0.01820
21	2.0	0.01881
22	2.1	0.01943
23	2.2	0.02005
24	2.3	0.02066
25	2.4	0.02128
26	2.5	0.21898
27	2.6	0.02252
28	2.7	0.02313
29	2.8	0.02375
30	2.9	0.02437
31	3.0	0.02499
32	3.1	0.02561
33	3.2	0.02623
34	3.3	0.02685
35	3.4	0.02746
36	3.5	0.02808
37	3.6	0.02870
38	3.7	0.02932
39	3.8	0.02994
40	3.9	0.03057
41	4.0	0.03119
42	4.1	0.03181
No.	τ	**REVaR**
43	4.2	0.03243
44	4.3	0.03305
45	4.4	0.03367

The claim that the EVaR method is simpler compared to other approaches is supported by the fact that the optimal solution can be calculated more efficiently through vector and matrix approaches, reducing computational complexity. Nevertheless, this study acknowledges that the approximation approach in EVaR requires adjustments to minimize potential errors, as explained in detail in the methodology and results sections.

In the Mean-EVaR model, diversification is directed at reducing risk measured by EVaR. The larger EVaR value offers better protection against extreme loss scenarios, making it important to focus on reducing tail risk. An EVaR-optimized portfolio seeks not only to reduce average risk but also to address infrequent yet significant loss risks.

Diversification also plays a key role in enhancing the Sharpe Ratio, which measures portfolio efficiency in combining return with risk. By achieving a well-diversified weight allocation, the portfolio can reach a higher Sharpe Ratio. This indicates that proper diversification can yield optimal average returns with lower risk.

In forming the efficient portfolio frontier, diversification is evident in how the optimal combination between average return and portfolio risk is linked. The efficient portfolio frontier formed shows how diversification helps achieve a balance between risk and return, allowing investors to choose the optimal portfolio.

However, it is important to acknowledge some limitations of the study. The findings may not be fully generalizable to other markets or asset classes due to the focus on the Indonesian capital market and the specific characteristics of the dataset. The impact of dataset characteristics, such as market conditions or the selection of stocks, could affect the robustness of the results. Additionally, the choice of hyperparameters, particularly the risk tolerance factor (τ), can significantly influence the results. The influence of these choices should be carefully considered in future research.

Furthermore, potential sources of bias or confounding factors that may have affected the model's performance were not fully addressed. For example, market anomalies or external shocks could impact the reliability of the optimization process. Future studies should incorporate sensitivity analyses or robustness checks to ensure the validity of the results under different conditions.

In terms of interpreting the results, the analysis primarily focuses on optimizing the portfolio's risk-return profile through diversification and EVaR optimization. The identified portfolio weights, while providing a balanced risk-return trade-off, also highlight the importance of managing tail risk and adjusting the portfolio according to different risk tolerance levels. The findings of this study contribute to the broader literature on portfolio optimization, especially in emerging markets, by demonstrating how the Mean-EVaR model can be applied effectively to mitigate extreme risk and enhance portfolio efficiency.

In conclusion, the results of this study show that the Mean-EVaR model, when applied with careful diversification and risk tolerance adjustments, can offer a more efficient and robust portfolio optimization strategy, particularly in the context of the Indonesian capital market. Future research could expand on these findings by considering alternative risk measures, additional market conditions, and further refinements to the optimization process.

CONCLUSIONS

Based on the results and discussions presented in the previous sections and in reference to the research objectives, the conclusions are as follows: (1) A formulation and solution for optimizing investment portfolio weights using the Mean-EVaR model with vector and matrix equation approaches has been conducted. This study successfully formulates and solves the optimization of investment portfolio weights using the Mean-EVaR model with vector and matrix equation approaches. The model maximizes portfolio returns while accounting for risk through entropic VaR. The optimal portfolio weights are derived by considering stock return averages and associated risks, as detailed in the weight formulation. The model ensures that the total portfolio weight equals 1, representing full allocation of invested funds. Additionally, the portfolio weights obtained maximize the objective function while minimizing risk, with a minimum weight indicating the lowest-risk portfolio distribution. These results serve as a foundation for determining efficient asset allocation. (2) In the Mean-EVaR model, valid risk tolerance falls within the range of 0 to 4.4. At the highest risk tolerance of 4.4, the portfolio produces an average return of 0.00055 with an EVaR risk of 0.01632. If risk tolerance exceeds this limit, the portfolio weights become negative. The optimal portfolio is achieved at a risk tolerance of 4.4, where the ratio between return and EVaR risk reaches its highest value, thus forming an efficient portfolio with an optimal return without exceeding the acceptable risk limit. (3) Diversification in the Mean-EVaR model successfully reduces portfolio risk, ensuring that no single stock dominates the allocation. Diversification plays a crucial role in reducing rare extreme risks. With optimal diversification, the portfolio not only achieves competitive returns but also maintains controlled risk and provides better protection against significant loss scenarios.

Additionally, The findings of this study offer valuable insights for investors and portfolio managers, especially in managing extreme risks. The Mean-EVaR model proves effective in reducing such risks while maintaining an optimal return-to-risk ratio. Future studies could explore multi-period portfolio models or assess the approach in dynamic market conditions, providing deeper insights into its effectiveness across various market scenarios. Additionally, extending the model to other emerging markets could enhance its global relevance and applicability. Besides, this research contributes by integrating portfolio weight optimization using the Mean-EVaR model with vector and matrix equation approaches, offering more accurate and efficient solutions in asset allocation and risk management, particularly in uncertain market conditions. However, the study has some limitations, such as being restricted to a single-period analysis and the use of data from the Indonesian stock market, which may limit the generalizability of the findings. Future research should consider applying the model under dynamic market conditions and across multiple periods, as well as exploring its use in other emerging markets to assess its broader applicability.

ACKNOWLEDGMENTS

Thanks, are conveyed to Universitas Padjadjaran, who provided the Padjadjaran Overseas Student Scholarship for Master Program Number: 2818/UN6.RKT/Kep/HK/2023, and the grant of "Academic Leadership Grant (ALG)" under the Chairperson of Prof. Dr. Sukono, M.M., M.Si.

REFERENCES

[1] Singh, A. 2012. An overview of the optimization modelling applications. *Journal of Hydrology*, 466, 167-182. https://doi.org/10.1016/j.jhydrol.2012.08.004.

[2] Caunhye, A. M., Nie, X. and Pokharel, S. 2012. Optimization models in emergency logistics: A literature review. *Socio-economic planning sciences*, 46(1), 4-13. https://doi.org/10.1080/00014788.1998.9728901 .

[3] Zhang, G., Lu, J., & Gao, Y. (2015). Multi-level decision making. *Models, Methods and Applications*.

[4] Cheng, S.R., 2007. A study on the factors affecting stock liquidity. International *Journal of Services and Standards*, 3(4), pp.453-475. https://doi.org/10.1504/IJSS.2007.015227.

[5] Dempsey, M. 1998. Capital gains tax: Implications for the firm's cost of capital, share valuation and investment decision-making. *Accounting and Business Research*, 28(2). https://doi.org/10.1080/00014788.1998.9728901.

[6] Hubbard, R.G., 1997. Capital-market imperfections and investment. https://doi.org/10.1504/IJSS.2007.01522710.3386/w5996.

[7] Abuselidze, G. and Slobodianyk, A. 2019. Investment of the financial instruments and their influence on the exchange stock market development. In Economic Science for Rural Development Conference Proceedings. No 52, Jelgava, LLU ESAF, 9-10 May 2019. 211-221. https://doi.org/10.22616/ESRD.2019.124.

[8] Ahmadi-Javid, A. Entropic Value-at-Risk: A New Coherent Risk Measure. *Journal of Optimization Theory and Applications* 2012, 155(3), 1124–1128. https://doi.org/10.1007/s10957-011-9968-2.

[9] Ahmed, D.; Soleymani, F.; Ullah, M. Z.; Hasan, H. Managing the risk based on entropic value-at-risk under a normal-Rayleigh distribution. *Applied Mathematics and Computation* 2021, 402, 126129. https://doi.org/10.1016/j.amc. 2021.126129.

[10] Rockafellar, R.T. Coherent approaches to risk optimization under uncertainty. In: Klastorin, Tutorials in Operations Research: OR Tools and Applications−Glimpses of Future Technologies, INFORMS, Catonsville, MD 2007, pp. 38-61. https://doi.org/10.1287/educ.1073.0032.

[11] Rockafellar, R.T.; Uryasev, S. Conditional value-at-risk for general loss distributions. *Journal of Banking and Finance* 2002, 26(7), 1443–1471. https://doi.org/10.1016/S0378-4266(02)00271-6.

[12] Ahmadi-Javid, A. An information–theoretic approach to constructing coherent risk measures. In: Proceedings of IEEE Interna-tional Symposium on Information Theory, August 2011, St. Petersburg, Russia, pp. 2125–2127. https://doi.org/10.1109/ISIT.2011.6033932.

[13] Ahmadi-Javid, A. Stochastic optimization via entropic value-at-risk: A new coherent risk measure. In: International Conference on Operations Research and Optimization, January 2011, Tehran, Iran. file:///C:\Users\asus\Downloads\s10957-011-9968-2-2.pdf.

[14] Ahmadi-Javid, A.; Fallah-Tafti, M. Portfolio optimization with entropic value-at-risk. *European Journal of Operational Research* 2019, 279(1), 225-241. https://doi.org/10.1016/j.ejor.2019.02.007.

[15] Giunta, N., Orlando, G., Carleo, A., and Ricci, J.M. 2024. Exploring entropy-based portfolio strategies: empirical analysis and cryptocurrency impact. Risks 12: 78. https://doi.org/10.3390/risks12050078.

[16] Mishur, Y., Ralchenko, K., Zelenko, P., and Zubchenko, V. 2024. Properties of the entropic risk measure EVaR in relation to se-lected distributions. Modern

Stochastics: Theory and Applications 11 (4), 373–394. https://doi.org/10.15559/24-VMSTA255.

[17] Bodie, Z.; Kane, A.; Marcus, A. J. Investments (10th ed.). McGraw-Hill Education, 2014.

[18] Cajas, D. Portfolio Optimization of Relativistic Value at Risk 2023. https://ssrn.com/ abstract=4378498.

[19] Suhaimi, N.B.A., and Rusyn, V. 2024. Risk measurement of investment portfolio using VaR and CVaR from the top 10 traded stocks on the IDX. *International Journal of Quantitative Research and Modeling,* Vol. 5, No. 1, 12-19. DOI: 10.46336/ijqrm.v5i1.600.

[20] Burr, I. W. (1942). Cumulative Frequency Functions. *The Annals of Mathematical Statistics*, 13(2), 215–232. http://www.jstor.org/stable/2235756.

[21] Christian, K., & Samuel, K. (2023). *Statistical size distributions in economics and actuarial sciences*. Wiley.

[22] Bello, J.F., Taiwo, E.S., and Adinya, I. 2024. Modified models for constrained mean absolute deviation portfolio optimization. *International Journal of Mathematical Sciences and Optimization: Theory and Applications,* 10(1), 12 – 24. https://doi.org/10.5281/zenodo.10449126.

[23] Ricci, J.M. 2024. Stability of entropic risk measures. Roma Tre University - Department of Business Studies, Via S. D'Amico 77, 00145 Rome, Italy. 1-30. http://romatrepress.uniroma3.it.

[24] Bello, J.F., Taiwo, E.S., and Adinya, I. 2024. Modified models for constrained mean absolute deviation portfolio optimization. *International Journal of Mathematical Sciences and Optimization: Theory and Applications,* 10(1), 12 – 24. https://doi.org/10.5281/zenodo.10449126.

[25] Gubu, L. and Hilmi, M.R. 2024. Beyond mean-variance Markowitz portfolio selection: a comparison of mean-variance-skewness-kurtosis model and robust mean-variance model*. Economic Computation and Economic Cybernetics Studies and Research*, Vol. 58, Issue 1/2024, 298-313. DOI: 10.24818/18423264/58.1.24.19.

[26] Sukono; Sidi; P. Bon, A. T; Bin; Supian, S. Modeling of Mean-VaR portfolio optimization by risk tolerance when the utility func-tion is quadratic. AIP Conference Proceedings 2017, 1827. https://doi.org/10.1063/1.4979451.

[27] Gaivoronski, A. A.; Pflug, G. Value-at-risk in portfolio optimization: properties and computational approach. *Journal of risk* 2005, 7(2), 1-31. https://doi.org/ 10.21314/JOR.2005.106.

[28] Artzner, P.; Delbaen, F.; Eber, J. M.; Heath, D. Coherent measures of risk. *Mathematical finance* 1999, 9(3), 203-228. https://doi.org/10.1111/1467-9965.00068.

[29] Mai, A.T.; Bastin, F.; Toulouse, M. On Optimization Algorithms for Maximum Likelihood Estimation, Interuniversity Research Center on Enterprise Networks, Logistics and Transportation 2014.

[30] Sahu, S., Vázquez, J.H.O., Ramírez, A.F. and Kim, J-M. 2024. Analyzing portfolio optimization in cryptocurrency markets: A comparative study of short-term investment strategies using high-frequency data. *Preprints.org* (www.preprints.org). 1-16. doi:10.20944/preprints202403.0048.v1.

[31] Zlatniczki, A. and Telcs, A. 2024. Application of portfolio optimization to achieve persistent time series. *Journal of Optimization Theory and Applications*, 201: 932–954. https://doi.org/10.1007/s10957-024-02426-1.