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Estimation of Gompertz Mortality Parameter Models Based on the Indonesian Population Mortality Table 2023

Muhammad Rafael Andika Putra*[1], Nurjannah[1], and Mila Kurniawaty[2]

[1]*Department of Statistics, Universitas Brawijaya, Indonesia*
[2]*Department of Mathematics, Universitas Brawijaya, Indonesia*

Abstract

In actuarial practice, mortality modeling is essential, especially in life insurance and social protection planning. The purpose of this study is to estimate the parameters of the Gompertz Mortality Law using three distinct approaches: Poisson regression based on the Indonesian Population Mortality Table (TMPI) 2023, Weighted Least Squares (WLS), and Nonlinear Least Squares (NLLS) using the Gauss-Newton algorithm. The force of mortality is computed, and the Gompertz parameters are estimated using each method as part of the methodological framework. The analysis includes parameter estimation for each method, transformation of the Gompertz model, and computation of values in the mortality table. The WLS method is implemented by applying a natural-logarithm transformation to the force of mortality function using d_x as the weight, while Poisson regression optimizes the log-likelihood function based on the d_x function. The Root Mean Square Error (RMSE), which compares the expected and actual mortality rates, is used to evaluate each model's accuracy. The results show that the NLLS approach consistently yields the most accurate estimates. This study improves actuarial modeling procedures in Indonesia by providing a new comparative approach for parameter estimation using national mortality data.

Keywords: Gompertz mortality; Parameter estimation; RMSE; TMPI 2023

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1. Introduction

Life and death are inseparable, and the timing and causes of death often cannot be predicted with certainty [1]. Life insurance serves as a solution to provide financial protection to the heirs of the insured in facing unexpected mortality risk [2]. Actuarial calculations in life insurance strongly depend on mortality tables that reflect actual events; however, for more flexible applications, mortality law approaches such as the Gompertz and Makeham laws are needed.

Mortality laws such as Gompertz are used to model death rates that increase exponentially with age and are very useful in developing life insurance products and long-term financial planning [3]. In Indonesia, this law has begun to be used by insurance companies to refine mortality assumptions in actuarial calculations, such as in the development of life insurance products and the evaluation of technical reserves [4]. The Gompertz model helps project life expectancy more realistically and helps avoid the risk of funding shortfalls, both in the insurance sector and in public policy areas such as social security and elderly health financing [5].

Despite its usefulness, one of the main challenges in implementing accurate mortality models in Indonesia is the limited availability of high-quality mortality data [6]. This limitation highlights

*Corresponding author. E-mail: rafaelandkp@student.ub.ac.id

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the need for collaboration among industry practitioners, academics, and the government to develop models that are both statistically reliable and suitable for the Indonesian demographic context. Most existing research has used foreign datasets, and few studies have directly applied Indonesian mortality data in estimating mortality model parameters.

To address this gap, this study utilizes the 2023 Indonesian Population Mortality Table (TMPI) to estimate parameters of the Gompertz model. The parameters, α and β, which characterize the baseline mortality rate and its rate of exponential growth with age [6], are estimated using three methods: Nonlinear Least Squares (NLLS), Weighted Least Squares (WLS), and Poisson regression. Model accuracy is evaluated using Root Mean Square Error (RMSE) to assess the predictive performance [7].

This study contributes to the assessment of several estimation techniques and supports local actuarial calculations of premium reserves or mortality assumptions based on data-driven concepts, especially Indonesian mortality data. Prior studies, such as those by Tai and Noymer [7] and Putra et al. [8], suggest the advantages of WLS and Poisson regression in estimating Gompertz parameters [9]. However, comparative applications using Indonesian data remain limited. This study aims to fill that gap and demonstrate that the Gompertz model is especially suitable for adult age groups (35–60 years), where mortality risk tends to increase exponentially in line with the model's assumptions.

2. Methods

This section describes the research methods. The process begins with the preparation of mortality data, the construction of the Gompertz mortality model, and the use of the NLLS, WLS, and Poisson regression methods to estimate the parameters, calculate the value of q_x based on the estimated value from each method, and evaluate the parameter estimation results using RMSE.

2.1. Data Preparation (Force of Mortality Calculation (m_x))

Secondary data were obtained from the Indonesian Population Mortality Table (TMPI) 2023, published by the Indonesian Social Security Agency for Health. The table contains age-specific mortality probabilities for ages 0 to 111 years. Other mortality function values were derived from the table. In this step, the function value (m_x) was calculated based on other life-table functions. The calculation begins by initializing the value of (l_0) as 100,000, indicating that the mortality rate is assumed per 100,000 people [8]. Next, the value of (ℓ_x) can be calculated for $x = 1, 2, 3$, and so on, with $n = 1$, as follows

$$\begin{aligned} q_x &= \frac{\ell_x - \ell_{x+1}}{\ell_x} \\ q_x \ell_x &= \ell_x - \ell_{x+1} \\ \ell_{x+1} &= (1 - q_x)\ell_x. \end{aligned} \tag{1}$$

where ℓ_x denotes the population that remains alive exactly at age x, originating from the initial population (ℓ_0) [10]. Furthermore, the value of the d_x function, which represents the number of people who die exactly between age x and $x + n$ from the initial population l_0 [11], is given by

$$d_0 = \ell_0 - \ell_1 \tag{2}$$

The value of ${}_n\mathcal{L}_x$, which represents the number of person-years lived by individuals aged x to $x + n$, is given by:

$$\begin{aligned} {}_n\mathcal{L}_x &= n\left(\ell_{x+n} + {}_na_x \cdot {}_nd_x\right) \\ \mathcal{L}_0 &= \ell_1 + 0,5d_0 \end{aligned} \tag{3}$$

The value of 0.5 is based on the assumption that death occurs in the middle of the year, and this value is not calculated directly from the mortality table [12]. Furthermore, the value of the m_x function is calculated, which represents the force of mortality measured per unit time [13].

$${}_nm_x = \frac{{}_nd_x}{{}_n\mathcal{L}_x} \tag{4}$$

2.2. Gompertz Mortality Law

The Gompertz Mortality Law was first introduced by Benjamin Gompertz in 1825 [11]. This model has two parameters, namely α, which represents the initial value of the force of mortality function, and β, which represents the rate at which the force of mortality increases with age [14]. The Gompertz law models the death rate or force of mortality (μ_x), which increases exponentially with age [15].

The Gompertz mortality law is defined by the death rate as

$$\mu_x = \alpha\beta^x, \tag{5}$$

for $x > 0$, $\alpha > 0$, and $\beta > 1$. The Survival Density Function (SDF) is

$$S_X(x) = \exp\left[\int_0^x \mu_y \, dy\right] = \exp\left[\frac{\alpha}{\ln\ \beta}(1-\beta^x)\right]. \tag{6}$$

Furthermore, the Probability Density Function (PDF) of Gompertz mortality can be calculated by multiplying the death rate and the survival distribution function [16]. The resulting PDF for Gompertz mortality is

$$f_X(x) = \mu_x \cdot S_X(x) = \alpha\beta^x \cdot \exp\left[\frac{\alpha}{\ln\ \beta}(1-\beta^x)\right].$$

2.3. Estimation of parameters α and β on the Gompertz model

The parameters α and β of the Gompertz mortality law were estimated using three methods, namely NLLS through the Gauss-Newton algorithm, WLS, and Poisson regression.

Nonlinear Least Squares (NLLS). The parameters α and β are estimated using the NLLS approach by minimizing the sum of squared differences between empirical values and the prediction results of the Gompertz mortality model

$$S(\alpha,\beta) = \sum_{i=1}^{n} (m_i - \alpha\cdot\beta^{x_i})^2 \tag{7}$$

Then, the partial derivatives of the function in Eq. (7) with respect to the parameters α and β are

$$\frac{\partial S}{\partial \alpha} = -2\sum_{i=1}^{n} (m_i - \alpha\cdot\beta^{x_i})\,\beta^{x_i}$$
$$\frac{\partial S}{\partial \beta} = -2\alpha\sum_{i=1}^{n} (m_i - \alpha\cdot\beta^{x_i})\,x_i\beta^{x_i-1}$$

so that the NLLS system of equations is formed

$$\sum_{i=1}^{n} (m_i - \alpha\cdot\beta^{x_i})\,\beta^{x_i} = 0$$
$$\sum_{i=1}^{n} (m_i - \alpha\cdot\beta^{x_i})\,x_i\beta^{x_i-1} = 0$$

These equations do not have an exact solution, so a numerical approach is needed to solve the problem. The method used in this study is the Gauss-Newton algorithm. The Gauss-Newton method is an iterative algorithm used to estimate the parameters α and β in the Gompertz mortality model

$$m_i = \alpha\beta^{x_i}$$

by minimizing the squared error between the data and the model. The steps for Gauss-Newton iterations are [17]:

1. Determine the initial guess of the parameter α_0 and β_0.
2. Calculate the residual

$$r_i = m_i - \alpha\beta^{x_i}$$

which is the difference between the data and the model.

3. Calculate the Jacobian matrix containing the partial derivatives of the residuals with respect to α and β:

$$\frac{\partial r_i}{\partial \alpha} = -\beta^{x_i}, \quad \frac{\partial r_i}{\partial \beta} = -\alpha x_i \beta^{x_i - 1}$$

4. Update the parameter with the formula

$$\theta^{(k+1)} = \theta^{(k)} - (J^T J)^{-1} J^T \mathbf{r}$$

with Jacobian matrix $\theta = (\alpha, \beta)^T$, J, and residual vector $\mathbf{r}$.

5. Repeat steps 2–4 until the relative changes in parameters between iterations are very small (convergent), namely

$$\left| \frac{\hat{\theta}_{k+1} - \hat{\theta}_k}{\hat{\theta}_k} \right| < \delta$$

Through this process, the estimated parameters are iteratively improved until they reach the optimum value and minimum error.

Weighted Least Squares (WLS). According to Montgomery et al. [18], linear regression model parameters with nonconstant error variances can be estimated using the WLS method. In this method, the quadratic error function is assigned a weight w_i, so the function to be minimized is

$$S(\beta_0, \beta_1) = \sum_{i=1}^{n} w_i \left(y_i - \beta_0 - \beta_1 z_i\right)^2. \tag{8}$$

Based on Eq. (8), the WLS normal equations can be obtained

$$\hat{\beta}_0 \sum_{i=1}^{n} w_i + \hat{\beta}_1 \sum_{i=1}^{n} w_i z_i = \sum_{i=1}^{n} w_i y_i, \tag{9}$$

$$\hat{\beta}_0 \sum_{i=1}^{n} w_i z_i + \hat{\beta}_1 \sum_{i=1}^{n} w_i z_i^2 = \sum_{i=1}^{n} w_i y_i z_i. \tag{10}$$

The parameter estimators can be obtained by solving Eq. (9) and Eq. (10)

$$\hat{\beta}_1 = \frac{\left(\sum_{i=1}^{n} w_i y_i z_i\right) \sum_{i=1}^{n} w_i - \left(\sum_{i=1}^{n} w_i y_i\right) \left(\sum_{i=1}^{n} w_i z_i\right)}{\left(\sum_{i=1}^{n} w_i z_i^2\right) \sum_{i=1}^{n} w_i - \left(\sum_{i=1}^{n} w_i z_i\right)^2},$$

$$\hat{\beta}_0 = \frac{\sum_{i=1}^{n} w_i y_i - \hat{\beta}_1 \sum_{i=1}^{n} w_i z_i}{\sum_{i=1}^{n} w_i}.$$

In the Gompertz model, the weighted sum of squared errors for the linear regression of the natural-log-transformed data is defined as

$$S(\alpha^*, \beta^*) = \sum_{i=1}^{n} w_i \left(\ln \mu_{x_i} - \alpha^* - \beta^* x_i\right)^2, \tag{11}$$

where the weight w_i is usually taken from the number of deaths d_{x_i} at the age x_i.

By equating these variables to become

$$y_i = \ln \mu_{x_i}, \quad z_i = x_i, \quad \beta_0 = \alpha^*, \quad \beta_1 = \beta^*,$$

then the Gompertz parameter estimators are obtained by the WLS method

$$\widehat{\beta}^* = \frac{\left(\sum_{i=1}^n d_{x_i} y_i z_i\right) \sum_{i=1}^n d_{x_i} - \left(\sum_{i=1}^n d_{x_i} y_i\right) \left(\sum_{i=1}^n d_{x_i} z_i\right)}{\left(\sum_{i=1}^n d_{x_i} z_i^2\right) \sum_{i=1}^n d_{x_i} - \left(\sum_{i=1}^n d_{x_i} z_i\right)^2}, \tag{12}$$

$$\widehat{\alpha}^* = \frac{\sum_{i=1}^n d_{x_i} y_i - \widehat{\beta}^* \sum_{i=1}^n d_{x_i} z_i}{\sum_{i=1}^n d_{x_i}}. \tag{13}$$

The mortality-table data, μ_{x_i} and d_{x_i}, can be directly used to calculate the parameter estimates of the Gompertz model using the equation above.

Poisson Regression. According to Montgomery et al. [18], Poisson regression is one of the models used to explain the relationship between count data (the number of events) and predictor variables. In this model, it is assumed that the response variable y_i is in the form of count values, namely $y_i = 0, 1, 2, \ldots$. Count data are a statistical data type that describes the number of events that can be counted and have non-negative integer values [19]. The Gompertz model in this case can be written as

$$\mu_x = \alpha \beta^x = e^{\ln \alpha + x \ln \beta}. \tag{14}$$

Based on Eq. (14), we obtain

$$\begin{aligned} \frac{d_x}{l_x} &= e^{\ln \alpha + x \ln \beta} \\ d_x &= e^{\ln \alpha + x \ln \beta + \ln l_x} \end{aligned} \tag{15}$$

Eq. (15) can be considered a deterministic model. To incorporate stochastic aspects in accordance with the observed data, the model was developed into

$$d_x = e^{\ln \alpha + x \ln \beta + \ln l_x} + \varepsilon,$$

with ε as an error. Let $(\alpha^* = \ln \alpha)$ and $\beta^* = \ln \beta$, and assume that the variable d_{x_i} follows a Poisson distribution with parameter λ_i, so

$$\begin{aligned} d_{x_i} &= e^{\alpha^* + \beta^* x_i + \ln l_{x_i}} + \varepsilon_i \\ \lambda_i &= e^{\alpha^* + \beta^* x_i + \ln l_{x_i}}. \end{aligned}$$

Then, the log-likelihood function for the data d_{x_i} is

$$\begin{aligned} \ln L(\alpha^*, \beta^*) &= \sum_{i=1}^n \left(-\lambda_i + d_{x_i} \ln \lambda_i - \ln(d_{x_i}!)\right) \\ \ln L(\alpha^*, \beta^*) &= \sum_{i=1}^n \left(-e^{\alpha^* + \beta^* x_i + \ln l_{x_i}} + d_{x_i}(\alpha^* + \beta^* x_i + \ln l_{x_i}) - \ln(d_{x_i}!)\right) \end{aligned}$$

Finding the values of α^* and β^* that maximize $\ln L$ is done by solving the equations

$$\begin{aligned} \frac{\partial \ln L}{\partial \alpha^*} &= \sum_{i=1}^n \left(-e^{\alpha^* + \beta^* x_i + \ln l_{x_i}} + d_{x_i}\right) = 0, \\ \frac{\partial \ln L}{\partial \beta^*} &= \sum_{i=1}^n x_i \left(-e^{\alpha^* + \beta^* x_i + \ln l_{x_i}} + d_{x_i}\right) = 0. \end{aligned}$$

Because these equations are difficult to solve analytically, the estimation of the parameters α^* and β^* is generally carried out with the help of programming software. In this study, the computation was assisted by Python.

2.4. Calculating the Probability of Death (q_x)

After obtaining each parameter estimate, q_x is calculated for each resulting Gompertz model.

$$\begin{aligned} F_{T_x}(t) &= Pr(T_x \leq t) = Pr(X \leq x+t | X > x) \\ F_{T_x}(t) &= 1 - \frac{Pr(X > x+t)}{Pr(X > x)} \\ F_{T_x}(t) &= 1 - \frac{S_X(x+t)}{S_X(x)}. \end{aligned} \tag{16}$$

The calculation of q_x requires the value of $S_X(x)$, so it is calculated first using Eq. (6).

2.5. Evaluating the Estimating Results

Predictive accuracy is evaluated after the Gompertz parameters α and β have been obtained from each estimation method. The performance of the estimated model is measured using RMSE. As a model validation tool, RMSE can provide an overall picture of the error distribution [20].

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} {e_i}^2}, \tag{17}$$

where e_i is the error for the ith data point, which is the difference between the actual data and the prediction results.

3. Results and Discussion

The results and analysis of this study are presented in this section. The explanation of the results and analysis is divided into the construction of the mortality table, the values of the Gompertz model estimators for each method, the comparison of q_x values based on the parameter estimates from each method, and the evaluation of the parameter estimates.

3.1. Construction of Indonesian Population Mortality Table 2023

The following table presents the construction results of the function calculations in the TMPI 2023:

Table 1: Construction results of Indonesian Population Mortality Table 2023

x	q_x	p_x	l_x	d_x	$\mathcal{L}_x$	m_x
0	0,007880	0,992120	100000,00	788,00	99606,00	0,00791
1	0,002096	0,997904	99212,00	207,95	99108,03	0,00210
2	0,000900	0,999100	99004,05	89,10	98959,50	0,00090
⋮	⋮	⋮	⋮	⋮	⋮	⋮
109	0,518532	0,481468	187,24	97,09	138,70	0,70002
110	0,559684	0,440316	90,15	50,46	64,92	0,77717
111	1,000000	0,000000	39,70	39,70	19,85	2,00000

Table 1 presents the construction results and serves as a reference for determining the Gompertz mortality parameters, especially the value of m_x, which represents the force of mortality by age.

3.2. Estimation of Gompertz Parameters

Parameter estimation is carried out using four case-group models formed for each estimation method, with the TMPI 2023 data in Table 1 as the observed data.

1. Gompertz model for males with an age limit from 35 to 100 years,
2. Gompertz model for females with an age limit from 35 to 100 years,
3. Gompertz model for males with an age limit from 0 to 111 years, and
4. Gompertz model for females with an age limit from 0 to 111 years.

For each case-group, the Gompertz model parameters are estimated using the NLLS method through the Gauss-Newton algorithm with a convergence limit of 10^{-6}, WLS, and Poisson regression, yielding twelve estimated models in total. The following is a summary of the α and β values for each parameter in Table 2.

Table 2: Gompertz Model Parameter Estimation Results Based on Case Group

	$\hat{\alpha}$			$\hat{\beta}$		
Group	NLLS	WLS	Poisson Regression	NLLS	WLS	Poisson Regression
Male (35–100)	0,000025	0,000126	0,000200	1,099597	1,080947	1,073663
Female (35–100)	0,000007	0,000130	0,000151	1,110529	1,075340	1,072537
Male (0–111)	0,000037	0,000244	0,000247	1,095348	1,075315	1,070882
Female (0–111)	0,000003	0,000225	0,000170	1,119224	1,070744	1,071454

Table 2 shows that the $\widehat{\alpha}$ parameter, which represents the baseline hazard, is consistently small across all groups and methods. It tends to be lower in the elderly group (35–100 years), which aligns with the theoretical expectation that mortality accelerates at older ages, reducing the need for a large initial hazard rate. Meanwhile, the $\widehat{\beta}$ parameter shows more pronounced variation. NLLS generally estimates higher $\widehat{\beta}$ values compared to WLS, while Poisson regression, in contrast, produces more moderate and stable parameter values.

Notably, the estimates for female groups tend to have lower $\widehat{\alpha}$ and higher $\widehat{\beta}$ values, indicating greater variability and acceleration in mortality risk. For example, the NLLS method estimates the highest $\widehat{\beta}$ values (1.099597 for male and 1.110529 for female) in the age group 35–100 years, whereas Poisson regression provides the lowest values (1.073663 for male and 1.072537 for female). The results show that NLLS estimates faster mortality acceleration, especially in the elderly population. The extreme values produced by NLLS for both parameters in this age group indicate that the Gompertz model is more sensitive to the exponential pattern of mortality at older ages.

For the 0–111 years age group, the differences between methods become more moderate. For males, the estimated $\widehat{\beta}$ values range narrowly between 1.070882 and 1.095348 across methods. For females, the NLLS method estimates $\widehat{\beta}$ as 1.119224, considerably higher than WLS (1.070744) and Poisson (1.071454). This shows that the sensitivity of the parameter estimates to the estimation method is greater in the female population with broad age coverage.

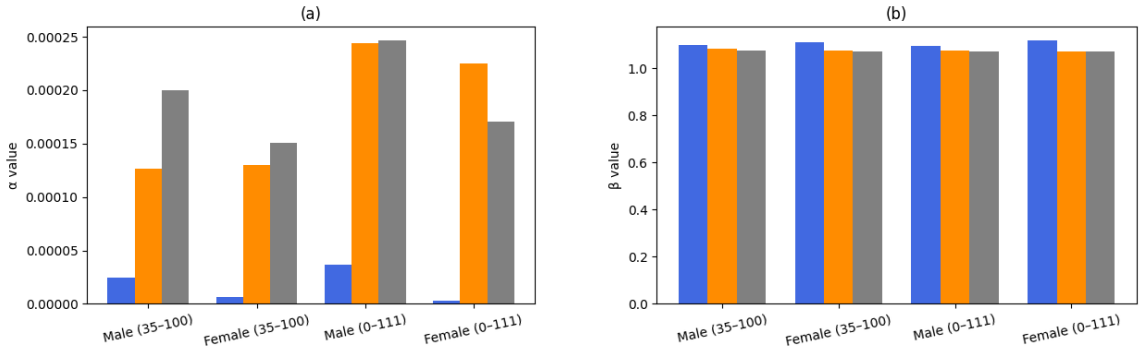


Fig. 1: Estimators $\hat{\alpha}$ and $\hat{\beta}$ based on Gompertz parameter estimation methods

Fig. 1 shows that Poisson regression generally produces the highest estimates, especially for the age group 0–111 years which includes the effects of infant mortality in subfigure (a). NLLS consistently reports the highest values for females aged 35–100 years in subfigure (b), showing its sensitivity to the exponential growth of mortality in adulthood. The figure reinforces the previous findings that NLLS captures the steep mortality curve in adulthood more effectively, while WLS and Poisson regression produce more conservative and stable estimates. Thus, the choice of estimation method affects not only the goodness of fit but also the interpretation of mortality behavior throughout life.

This is in line with theoretical expectations that, in old age, the probability of death increases sharply, so α as the baseline hazard is lower. On the other hand, the β parameter shows a more significant variation between methods and groups representing the level of mortality

acceleration. The estimation results show that the NLLS approach tends to provide a higher estimated acceleration of mortality than the WLS and Poisson regression methods, while Poisson regression gives relatively stable and moderate results. In addition, the greater differences in estimates for female groups than for male groups indicate the possibility of greater variability in the mortality distribution of female populations and deserve consideration in the selection of actuarial or demographic models.

3.3. Probability of Death (q_x) Based on Gompertz Estimators

$S_X(x)$ values are required in the calculation of q_x. Therefore, it is necessary to calculate $S_X(x)$ using Eq. (6) with the estimated parameter values of α and β for each Gompertz model. For the male Gompertz model with an age limit from 0 to 111 years using the NLLS method, the following is obtained

$$S_X(0) = \exp\left[\frac{0,0001002}{\ln(1,0826571)}(1-(1,0826571)^0)\right] = 1.$$

$S_X(1)$, $S_X(2)$, and so on are calculated in the same way. Next, q_x is calculated based on Eq. (16). For the male Gompertz model with an age limit from 0 to 111 years using the NLLS method, the following is obtained

$$q_0 = 1 - \frac{S_X(1)}{S_X(0)} = 1 - \frac{0,9998957}{1} \approx 0,0001043.$$

q_1, q_2, and so on are calculated in the same way and are also calculated for each of the other Gompertz models. An overview of the values of the function q_x for each model is displayed in Fig. 2.

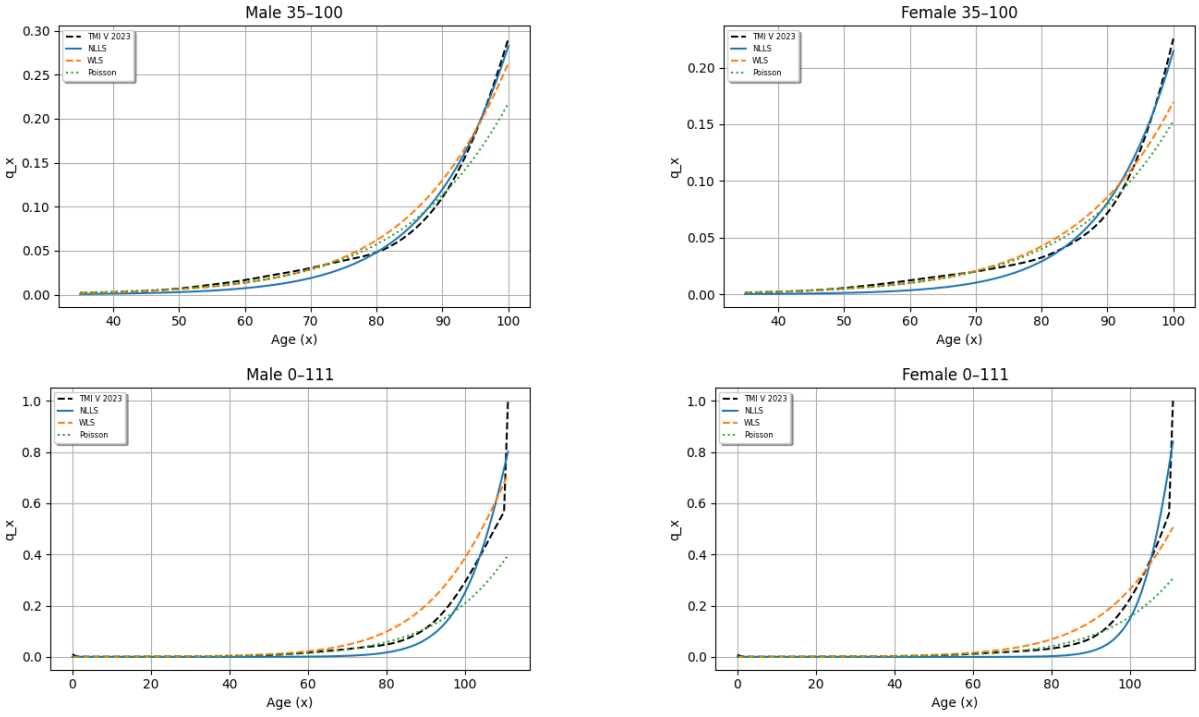


Fig. 2: Function graph of q_x for each estimated Gompertz model

Based on the results in Fig. 2, the NLLS curve closely follows the empirical pattern of the TMPI, especially in the adult and elderly age ranges, indicating consistency with the expected exponential growth in mortality. Meanwhile, Poisson regression tends to produce a flatter curve at older ages, especially in the mortality escalation after age 60. In addition, the WLS curve sometimes deviates in young adult age (20–40 years). This may be due to the linear transformation, which may reduce flexibility and sensitivity. The main finding is that the NLLS model not only gives numerical compatibility close to the TMPI values but also follows the visual

pattern of mortality development more accurately. The upward curvature that is consistent under NLLS for ages 35–100 years confirms that this method is more sensitive to the dynamics of real-life mortality in Indonesia's data.

3.4. Evaluation of Estimation Results Performance

The RMSE value for each model is calculated using Eq. (17). The error for the ith data point is denoted by e_i and defined as the difference between the value of the function q_x in TMPI 2023 and the value of the function q_x for the ith observation in the estimated model, namely

$$e_i = q_{x_i} \text{ TMPI } 2023 - q_{x_i} \text{ model.} \tag{18}$$

For the male Gompertz model with an age limit from 0 to 111 years using the NLLS method, the following is obtained

$$e_1 = q_0 \text{ TMPI } 2023 - q_0 \text{ model} = 0,00524 - 0,0001043 = 0,0051357.$$

e_2, e_3,..., e_{112} are calculated in the same way. Furthermore, the RMSE value is calculated using Eq. (17), namely

$$\text{RMSE} = \sqrt{\frac{1}{112}\sum_{i=1}^{112} {e_i}^2} = \sqrt{0,00034029} \approx 0,007461.$$

The comparison of RMSE values can be seen in Fig. 3.

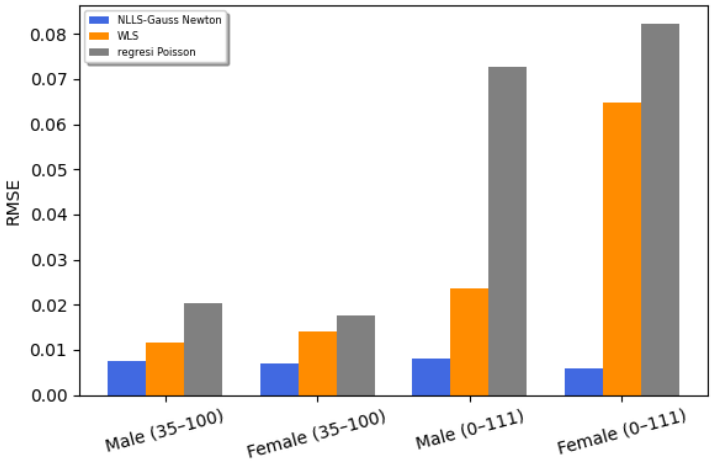


Fig. 3: Comparison of RMSE values for each estimated model

NLLS consistently produces the lowest RMSE values in Fig. 3 for all demographic scenarios based on gender and age and is confirmed as the most accurate approach. The RMSE differences among methods are more substantial in the age group of 35–100 years. This is in line with the assumptions of the Gompertz law, which show that model selection is very important when focusing on adult mortality. Poisson regression tends to produce the highest RMSE, perhaps due to overdispersion or underfitting in higher age groups.

4. Conclusion

Across all demographic groups and age ranges, the results consistently demonstrated that the NLLS method offered the best fit, both quantitatively and visually. However, this study has several limitations. First, it assumes the midpoint approximation for the central exposure, which may introduce bias in younger age groups. Second, the model does not incorporate confidence intervals or assess statistical uncertainty in the parameter estimates. Additionally, the use of a fixed weighting scheme in WLS may not be optimal across all data conditions. For future research, it is recommended to estimate parameters directly from the probability of death function q_x,

thereby avoiding assumptions related to the value of ${}_na_x$ in mortality table calculations. In addition, the value of d_x used as a weight in this study can be explored further using other weighting methods, such as Huber or Tukey. The results of this study are expected to be applied to actuarial modeling calculations, such as determining premium reserves and mortality assumptions in life insurance.

CRediT Authorship Contribution Statement

Muhammad Rafael Andika Putra: Conceptualization, Methodology, Formal Analysis, Software, Visualization, Writing–Original Draft Preparation, Project Administration. **Nurjannah:** Supervision, Validation, Funding Acquisition, Writing–Review & Editing. **Mila Kurniawaty:** Supervision, Data Curation, Writing–Review & Editing.

Declaration of Generative AI and AI-assisted technologies

The authors used Grammarly to assist in sentence construction so that the language structure was easier to understand and aligned with the intended meaning.

Declaration of Competing Interest

The authors declare no competing interests.

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Data Availability

The dataset analyzed during the current study is publicly available in the Society of Actuaries of Indonesia (PAI) repository[1].

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