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Estimation of Gompertz Mortality Parameter Models on Indonesian Population Mortality Table 2023

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Abstract

In actuarial practice, mortality modeling is essential, especially in life insurance and social protection planning. The purpose of this study is to estimate the parameters of the Gompertz Mortality Law using three distinct approaches: Poisson Regression on the Indonesian Population Mortality Table (TMPI) 2023, Weighted Least Squares (WLS), and Nonlinear Least Squares (NLLS) using the Gauss-Newton algorithm. The force of mortality is computed and Gompertz parameters are estimated using each method as part of the methodological framework. The analysis comprises the estimation of parameters for each method, the transformation of the Gompertz model, and the computation of values in the mortality table. The WLS method work in a way by converting natural logarithms from the force of mortality function with d_x as weight, creating the d_x function, and optimizing the log ordered function on Poisson Regression, the WLS method reduces the number of squares of error. The Root Mean Square Error (RMSE), which compares the expected and actual mortality rates, is used to evaluate each models accuracy. The NLLS approach consistently yields the most accurate estimates, according to the results. This study improves actuarial modeling procedures in Indonesia by providing a new comparative method for parameter estimation on national mortality data.

Keywords: Gompertz mortality; Parameter estimation; RMSE; TMPI 2023

1 Introduction

Life and death are two inseparable things, where death is often unpredictable with certainty both time and caused [1]. Life insurance is present as a solution to provide financial protection to heirs from the insured in the face of unexpected risk of death [2]. Actuarial calculations in life insurance are very dependent on the mortality table that reflects the actual events, but for more flexible applications a mortality law approach such as Gompertz and Makeham law is needed.

Mortality law such as Gompertz is used to model the death rate that increases exponentially as we get older and are very useful in the development of life insurance products and long-term financial planning [3]. In Indonesia, the application of this law began to be used by insurance companies to perfect mortality assumptions in actuarial calculations, such as in the development of life insurance products and evaluating technical reserves [4]. The Gompertz model helps project life expectations more realistically in avoiding the risk of lack of funds, both in the insurance sector and public policy such as social security and elderly health financing [5].

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Despite its usefulness, one of the main challenges in implementing accurate mortality models in Indonesia is the limited availability of high-quality mortality data [6]. This limitation highlights the need for collaboration among industry practitioners, academics, and the government to develop models that are both statistically reliable and suitable for the Indonesian demographic context. Most existing research has used foreign datasets, and few studies have directly applied Indonesian mortality data in estimating mortality model parameters.

To address this gap, this study utilizes the 2023 Indonesian Population Mortality Table (TMPI) to estimate parameters of the Gompertz model. The parameters, α and β, which characterize the baseline mortality rate and its rate of exponential growth with age [6], are estimated using three methods such as Nonlinear Least Squares (NLLS), Weighted Least Squares (WLS), and Poisson Regression. Model accuracy is evaluated using Root Mean Square Error (RMSE) to assess the predictive performance [7].

The study contributes into the effectiveness of several estimation techniques and contributes to the local actuarial calculations of premium reserves or mortality assumption based from data driven concept, especially Indonesian mortality data. Prior studies, such as those by Tai and Noymer [7] and Putra et al. [8], suggest the advantage of WLS and Poisson Regression in estimating Gompertz parameters [9]. However, comparative applications using Indonesian data remain limited. This study aims to fill that gap and demonstrate that the Gompertz model is especially suitable for adult age groups (35–60 years) where mortality risk tends to increase exponentially in line with the models assumptions.

2 Methods

This section will be conveyed regarding this research method. The process begins with the preparation of mortality data, the preparation of the Gompertz mortality model and the OLS, WLS, and Poisson regression methods in estimating the parameters, calculating the value of q_x based on the estimated value of each method, and evaluating the results of the estimation of parameters using RMSE.

2.1 Data Preparation (Force of Mortality Calculations (m_x))

Secondary data obtained from the Indonesian Population Mortality Table (TMPI) 2023, published by the Indonesian Social Security Agency for Health. The table contains age-specific mortality probabilities for age 0 to 111 years. Other death function values will be obtained through the table. In this step, function value (m_x) calculated based on the value of the function (m_x). Calculation begins by initializing the value of (l_0) is 100,000 which indicates that it is assumed to cover the mortality rate per 100,000 people [8]. Next, the value can be calculated $(\updownarrow_x)$ for $x = 1,2,3,$ and etc, with $n = 1$, then obtained

$$\begin{aligned} q_x &= \frac{\ell_x - \ell_{x+1}}{\ell_x} \\ q_x \ell_x &= \ell_x - \ell_{x+1} \\ \ell_{x+1} &= (1 - q_x)\ell_x. \end{aligned} \tag{1}$$

where ℓ_x states that the population that remains alive right at the age of x originating from the initial population (ℓ_0) [10]. Furthermore, the value of the d_x function which states the number of people who died exactly between the age of x and $x+n$ from the initial population l_0 [11]), that is

$$d_0 = \ell_0 - \ell_1 \tag{2}$$

and ℓ_x which states the number of years of life that someone survive x between x to $x+n$.

$$\begin{aligned} {}_n\mathscr{L}_x &= n\left(\ell_{x+n} + {}_na_x \cdot {}_nd_x\right) \\ \mathscr{L}_0 &= \ell_1 + 0,5d_0 \end{aligned} \tag{3}$$

The value of 0.5 is the assumption that death occurs in the middle of the year and the value is not calculated by the value in the mortality table [12]. Furthermore, the value of the m_x function will be calculated which states the force of mortality measured per unit time [13].

$$ {}_n m_x = \frac{{}_n d_x}{{}_n \mathscr{L}_x} \tag{4} $$

2.2 Gompertz Mortality Law

Gompertz Mortality Law was first introduced by Benjamin Gompertz in 1825 [11]. This model has two parameters, namely the parameter α as the initial value of the function of the force of mortality and parameter β as a speed of the speed of increasing the force of mortality to age [14]. Gompertz law modeling that the rate of death or force of mortality (μ_x) increases exponentially with age [15].

The law of Gompertz mortality is defined by the rate of death as

$$ \mu_x = \alpha\beta^x, \tag{5} $$

for $x > 0$, $\alpha > 0$, and $\beta > 1$. The Survival Densitiy Function (SDF) is

$$ S_X(x) = \exp\left[\int_0^x \mu_y\, dy\right] = \exp\left[\frac{\alpha}{\ln\,\beta}(1-\beta^x)\right]. \tag{6} $$

Furthermore, it can be calculated Probability Density Function (PDF) Gompertz mortality by multiplying the rate of death and the survival distribution function [16]. Obtained PDF for Gompertz mortality is

$$ f_X(x) = \mu_x \cdot S_X(x) = \alpha\beta^x \cdot \exp\left[\frac{\alpha}{\ln\,\beta}(1-\beta^x)\right]. $$

2.3 Estimation of parameters α and β on the Gompertz model

Parameters α and β on the Gompertz mortality law were conducted using three methods, namely NLLS through the Gauss-Newton, WLS, and Poisson regression.

1. Non Linear Least Square (NLLS)
   Estimated parameters of α and β carried out with an NLLS approach, namely by minimizing the number of quadratic differences between empirical values and the prediction results of the Gompertz mortality model

$$ S(\alpha,\beta) = \sum_{i=1}^{n} (m_i - \alpha \cdot \beta^{x_i})^2 \tag{7} $$

   Then, the partial derivative of the Eq. 7 function of the parameters α and β becomes

$$ \frac{\partial S}{\partial \alpha} = -2\sum_{i=1}^{n} (m_i - \alpha \cdot \beta^{x_i})\,\beta^{x_i} $$

$$ \frac{\partial S}{\partial \beta} = -2\alpha\sum_{i=1}^{n} (m_i - \alpha \cdot \beta^{x_i})\,x_i\beta^{x_i-1} $$

   so that the NLLS equation system is formed

$$ \sum_{i=1}^{n} (m_i - \alpha \cdot \beta^{x_i})\,\beta^{x_i} = 0 $$

$$ \sum_{i=1}^{n} (m_i - \alpha \cdot \beta^{x_i})\,x_i\beta^{x_i-1} = 0 $$

   The equation has no exact solution, so a numerical approach is needed to solve the problem. The method used in this study is the Gauss-Newton. The Gauss-Newton method is an iterative algorithm to estimate the parameters α dan β in the Gompertz mortality model

$$ m_i = \alpha\beta^{x_i} $$

   by minimizing the square error between data and models. The steps for Gauss-Newton iterations are [17]:

(a) Determine the initial guess of the parameter α_0 and β_0.

(b) Calculate the residual

$$r_i = m_i - \alpha\beta^{x_i}$$

which is the difference between data and models.

(c) Calculate the Jacobian matrix containing a partial residual derivative to α and β:

$$\frac{\partial r_i}{\partial \alpha} = -\beta^{x_i}, \quad \frac{\partial r_i}{\partial \beta} = -\alpha x_i \beta^{x_i - 1}$$

(d) Update the parameter with the formula

$$\theta^{(k+1)} = \theta^{(k)} - (J^T J)^{-1} J^T \mathbf{r}$$

with Jacobian matrix $\theta = (\alpha, \beta)^T$, J, and residual vector $\mathbf{r}$.

(e) Repeat steps 2-4 until the relative changes of parameters between iterations are very small (convergent), namely

$$\left| \frac{\hat{\theta}_{k+1} - \hat{\theta}_k}{\hat{\theta}_k} \right| < \delta$$

Through this process, the estimated parameters are improved in stage until they reach the optimum value and the minimum error.

2. Weighted Least Square (WLS)
According to Montgomery et al. [18], linear regression model parameters with non-constant variants of errors can be estimated using the WLS method. In this method, the quadratic function of the error is given weight w_i so that the function is minimized to be

$$S(\beta_0, \beta_1) = \sum_{i=1}^{n} w_i (y_i - \beta_0 - \beta_1 z_i)^2 . \tag{8}$$

Based on the Eq. 8, WLS normal equation can be obtained

$$\hat{\beta}_0 \sum_{i=1}^{n} w_i + \hat{\beta}_1 \sum_{i=1}^{n} w_i z_i = \sum_{i=1}^{n} w_i y_i, \tag{9}$$

$$\hat{\beta}_0 \sum_{i=1}^{n} w_i z_i + \hat{\beta}_1 \sum_{i=1}^{n} w_i z_i^2 = \sum_{i=1}^{n} w_i y_i z_i. \tag{10}$$

Parameter estimator can obtain with completing the Eq. 9 and Eq. 10

$$\hat{\beta}_1 = \frac{\left(\sum_{i=1}^{n} w_i y_i z_i\right) \sum_{i=1}^{n} w_i - \left(\sum_{i=1}^{n} w_i y_i\right)\left(\sum_{i=1}^{n} w_i z_i\right)}{\left(\sum_{i=1}^{n} w_i z_i^2\right) \sum_{i=1}^{n} w_i - \left(\sum_{i=1}^{n} w_i z_i\right)^2},$$

$$\hat{\beta}_0 = \frac{\sum_{i=1}^{n} w_i y_i - \hat{\beta}_1 \sum_{i=1}^{n} w_i z_i}{\sum_{i=1}^{n} w_i}.$$

In the Gompertz model, the quadratic function of the error weighted for linear regression in the natural logarithm transformation data is defined as

$$S(\alpha^*, \beta^*) = \sum_{i=1}^{n} w_i \left(\ln \mu_{x_i} - \alpha^* - \beta^* x_i\right)^2 , \tag{11}$$

where the weight w_i is usually taken from the number of deaths d_{x_i} at the age x_i.

By equating these variables to become

$$y_i = \ln \mu_{x_i}, \quad z_i = x_i, \quad \beta_0 = \alpha^*, \quad \beta_1 = \beta^*,$$

then the Gompertz parameter estimator is obtained by the WLS method

$$\widehat{\beta}^* = \frac{\left(\sum_{i=1}^n d_{x_i} y_i z_i\right) \sum_{i=1}^n d_{x_i} - \left(\sum_{i=1}^n d_{x_i} y_i\right)\left(\sum_{i=1}^n d_{x_i} z_i\right)}{\left(\sum_{i=1}^n d_{x_i} z_i^2\right) \sum_{i=1}^n d_{x_i} - \left(\sum_{i=1}^n d_{x_i} z_i\right)^2}, \tag{12}$$

$$\widehat{\alpha}^* = \frac{\sum_{i=1}^n d_{x_i} y_i - \widehat{\beta}^* \sum_{i=1}^n d_{x_i} z_i}{\sum_{i=1}^n d_{x_i}}. \tag{13}$$

The data in the mortality table is μ_{x_i} and d_{x_i} can be directly used to calculate the parameter estimation of the Gompertz model with the above equation.

3. Poisson Regression
According to Montgomery et al. [18], Poisson regression is one of the models used to explain the relationship between observational data in the form of a count (number of events) with predictor variables. In this model, it is assumed that the variable response y_i in the form of chopped numbers, that are $y_i = 0, 1, 2, \ldots$. Count is a statistical data type that describes the number of events that can be calculated and have a non-negative integer value [19]. The Gompertz model in this case can be written as

$$\mu_x = \alpha \beta^x = e^{\ln \alpha + x \ln \beta}. \tag{14}$$

Based on the Eq. 14, obtained

$$\begin{aligned} \frac{d_x}{l_x} &= e^{\ln \alpha + x \ln \beta} \\ d_x &= e^{\ln \alpha + x \ln \beta + \ln l_x} \end{aligned} \tag{15}$$

Eq. 15 can be considered a deterministic model. To enter stochastic aspects in accordance with observation data, the model was developed into

$$d_x = e^{\ln \alpha + x \ln \beta + \ln l_x} + \varepsilon,$$

with ε as an error. For example $(\alpha^* = \ln \alpha)$ and $\beta^* = \ln \beta$ and variable d_{x_i} is assumed to distribute poisson with parameters λ_i, so

$$\begin{aligned} d_{x_i} &= e^{\alpha^* + \beta^* x_i + \ln l_{x_i}} + \varepsilon_i \\ \lambda_i &= e^{\alpha^* + \beta^* x_i + \ln l_{x_i}}. \end{aligned}$$

Then, the Logarithmic Function for the data d_{x_i} is

$$\begin{aligned} \ln L(\alpha^*, \beta^*) &= \sum_{i=1}^n \left(-\lambda_i + d_{x_i} \ln \lambda_i - \ln(d_{x_i}!)\right) \\ \ln L(\alpha^*, \beta^*) &= \sum_{i=1}^n \left(-e^{\alpha^* + \beta^* x_i + \ln l_{x_i}} + d_{x_i}(\alpha^* + \beta^* x_i + \ln l_{x_i}) - \ln(d_{x_i}!)\right) \end{aligned}$$

To find the values of α^* and β^* which maximizes $\ln l$, done by completing the equation

$$\begin{aligned} \frac{\partial \ln L}{\partial \alpha^*} &= \sum_{i=1}^n \left(-e^{\alpha^* + \beta^* x_i + \ln l_{x_i}} + d_{x_i}\right) = 0, \\ \frac{\partial \ln L}{\partial \beta^*} &= \sum_{i=1}^n x_i \left(-e^{\alpha^* + \beta^* x_i + \ln l_{x_i}} + d_{x_i}\right) = 0. \end{aligned}$$

Because the equation is difficult to resolve analytically, the completion of the parameter estimation α^* and β^* are generally carried out using the help of programming software. In this study will be assisted using Python.

2.4 Calculating the Probability of Death (q_x)

After obtaining the value of each parameter estimator, q_x will be calculated for each of the resulting Gompertz models.

$$F_{T_x}(t) = Pr(T_x \leq t) = Pr(X \leq x+t | X > x)$$
$$F_{T_x}(t) = 1 - \frac{Pr(X > x+t)}{Pr(X > x)}$$
$$F_{T_x}(t) = 1 - \frac{S_X(x+t)}{S_X(x)}. \quad (16)$$

Calculation of q_x requires the value of $S_x(x)$, so that it will be calculated first using the Eq. 6.

2.5 Evaluating the Estimating Results

The process of evaluating predictive accuracy measurements is carried out for Gompertz parameters α and β are obtained from each of the estimation methods. For the performance of the estimated model, it will be measured using RMSE. As a model validation tool, RMSE can provide an overall error distribution picture [20].

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} {e_i}^2}, \quad (17)$$

with e_i is error for i data, which is the difference between actual data and prediction results.

3 Results and Discussion

The results and analysis of this research will be explained in this section. Explanation of the results and analysis will be divided into the results of the construction of the mortality table, the value of the Gompertz Model estimator with each method, comparison of the value of q_x based on the results of the parameter estimates in each method, and the evaluation of the parameter estimates.

3.1 Construction of Indonesian Population Mortality Table 2023

The following are the results of the construction of the calculation of the functions in the TMPI 2023

Table 1: Construction results of Indonesian Population Mortality Table 2023

x	q_x	p_x	l_x	d_x	$\mathscr{L}_x$	m_x
0	0,007880	0,992120	100000,00	788,00	99606,00	0,00791
1	0,002096	0,997904	99212,00	207,95	99108,03	0,00210
2	0,000900	0,999100	99004,05	89,10	98959,50	0,00090
⋮	⋮	⋮	⋮	⋮	⋮	⋮
109	0,518532	0,481468	187,24	97,09	138,70	0,70002
110	0,559684	0,440316	90,15	50,46	64,92	0,77717
111	1,000000	0,000000	39,70	39,70	19,85	2,00000

Table 1 as the construction results will be a reference in determining the value of the Gompertz mortality parameter, especially the value of m_x which will review the force of mortality based on age.

3.2 Estimation of Gompertz Parameters

Estimation of the parameters is done using four case group model that has been formed for each estimation method using TMPI 2023 data in Table 1 as a calculated observation data.

1. Gompertz Model for male with age limitation from 35 to 100 years,

2. Gompertz Model for female with age limitation from 35 to 100 years,
3. Gompertz Model for male with age limitation from 0 to 111 years, and
4. Gompertz Model for female with age limitation from 0 to 111 years.

For each case group of Gompertz model will be an estimation of parameters by the NLLS method through the Gauss-Newton algorithm where the convergence limit is 10^{-6}, WLS, and Poisson regression so that as a whole produces twelve models of the estimated process. The following is a summary of values for the α and β from each parameter in Table 2

Table 2: Gompertz Model Parameter Estimation Results based on case group

Group	$\hat{\alpha}$			$\hat{\beta}$		
	NLLS	WLS	Poisson Regression	NLLS	WLS	Poisson Regression
Male (35–100)	0,000025	0,000126	0,000200	1,099597	1,080947	1,073663
Female (35–100)	0,000007	0,000130	0,000151	1,110529	1,075340	1,072537
Male (0–111)	0,000037	0,000244	0,000247	1,095348	1,075315	1,070882
Female (0–111)	0,000003	0,000225	0,000170	1,119224	1,070744	1,071454

Table 2 displays the $\widehat{\alpha}$ parameter represent the baseline hazard is consistently small across all groups and methods. It tends to be lower in the elderly group (35–100 years), which aligns with theoretical expectations that mortality accelerates at older age, reducing the need for a large initial hazard rate. Meanwhile, the $\widehat{\beta}$ parameter shows more pronounced variation. NLLS generally estimates higher $\widehat{\beta}$ values compared to WLS and Poisson regression are contrast produces more moderate and stable parameter values.

Notably, the estimates for female groups tend to have lower $\widehat{\alpha}$ and higher $\widehat{\beta}$ indicating greater variability and acceleration in mortality risk. For example, the NLLS method estimates highest $\widehat{\beta}$ values (1.099597 for male and 1.110529 for female) in the age group 35–100 years, where the Poisson regression provides lowest values (1.073663 for male and 1.072537 for female). The results show that NLLS estimates faster acceleration in death, especially in the elderly population. The extreme values produced by NLLS for both parameters in this age group showed that Gompertz was more sensitive to the exponential nature of the death of people at the age.

For the 0–111 years age group, the differences between methods become more moderate. For male, the estimated $\widehat{\beta}$ range narrowly between 1.070882 and 1.095348 across methods. For female, the NLLS method estimates $\widehat{\beta}$ are 1.119224, considerably higher than WLS (1.070744) and Poisson (1.071454). This shows that the sensitivity of the parameter of the estimation method is greater in the female population with broad age coverage.

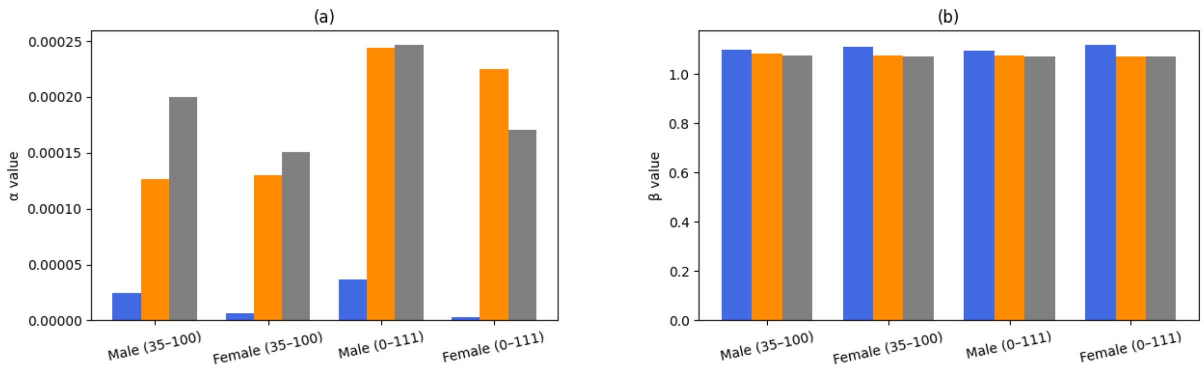

Figure 1: Estimator $\hat{\alpha}$ and $\hat{\beta}$ based on the method of estimating gompertz parameters

Figure 1 shows that Poisson regression generally produces the highest estimates, especially for the age group 0–111 years which includes the effects of infant death in subfigure (a). NLLS consistently reports the highest values for female 35–100 years in subfigure (b) which shows their sensitivity to exponential growth of death in adulthood. The figures reinforces the findings previously that NLLS captured the steep death curve in adulthood more effectively, while WLS and Poisson regression produced a more conservative and stable estimation. Thus, the choice of estimation methods affects not only the goodness of compatibility but also the interpretation of death behavior throughout life.

This is in line with theoretical expectations that in old age where the probability of death increases sharply, so that α as baseline hazard is lower. On the other hand, the β parameter shows a more significant variation between methods and groups where representing the level of mortality acceleration. The estimated results show that NLLS approach tends to provide an estimated acceleration of death that is higher than the WLS and Poisson regression methods while Poisson regression gives relatively stable and moderate results. In addition, differences in the results of greater estimates in female groups than male show the possibility of greater mortality distribution variability in female populations and deserve a concern in the selection of actuarial or demographic models.

3.3 Probability of Death (q_x) based on Gompertz Estimator

$S_x(x)$ value are required in the calculation of q_x. Therefore, it is necessary to calculate $S_x(x)$ using the Eq. 6 with the parameter values of α and β estimated results for each Gompertz model. For the male Gompertz model with age limits from 0 to 111 years in the NLLS method, obtained

$$S_X(0) = \exp\left[\frac{0,0001002}{\ln(1,0826571)}(1-(1,0826571)^0)\right] = 1.$$

$S_x(1)$, $S_x(2)$, and so on is calculated in the same way. Next, the q_x is calculated based on the Eq. 16. For the male Gompertz model with age limits from 0 to 111 years in the NLLS method, obtained

$$q_0 = 1 - \frac{S_X(1)}{S_X(0)} = 1 - \frac{0,9998957}{1} \approx 0,0001043.$$

q_1, q_2, and so on is calculated in the same way and also calculated for each other Gompertz model. Overview of the value of the function q_x each model is displayed with a graph in the Figure 2.

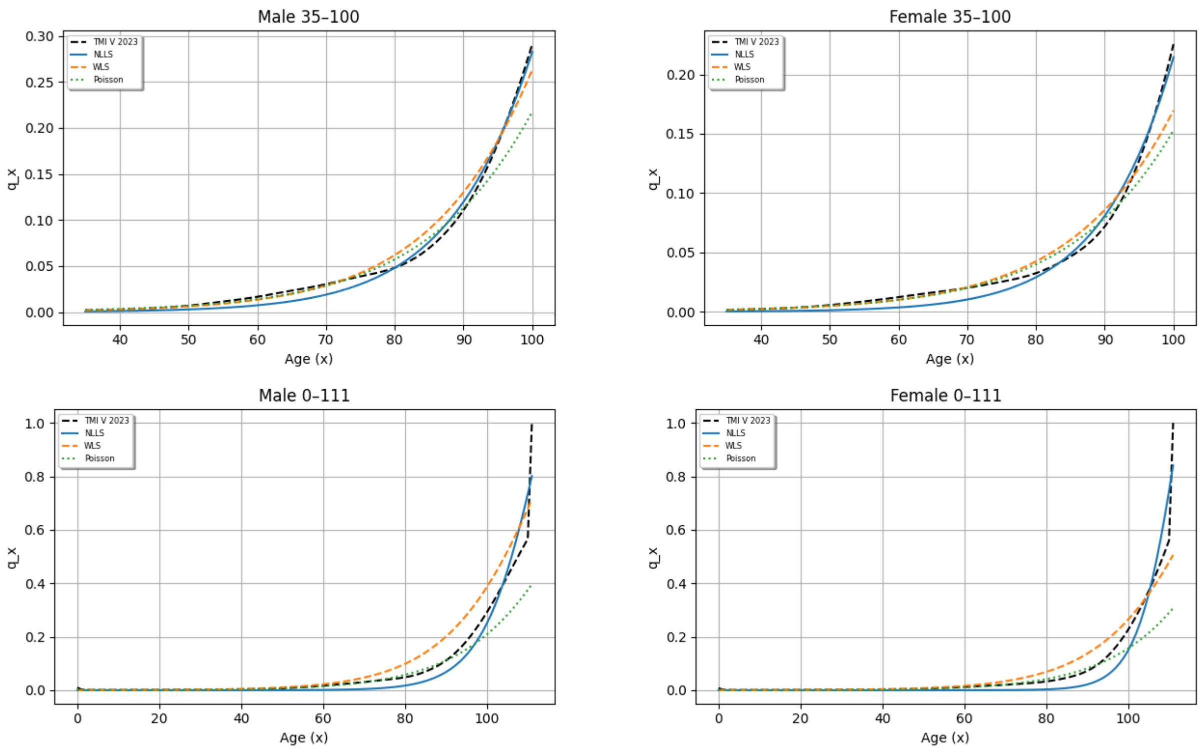

Figure 2: Function graph q_x every Gompertz Model Estimated

Based from results of Figure 2, the NLLS curve is parallel to the empirical pattern of the TMPI, especially in the age range and the elderly prove that in line with the expected exponential growth in death.

Meanwhile, Poisson regression tends to produce a more flat curve at an older age, especially the escalation of death after the age of 60. In addition, the WLS curve sometimes deviates in young adult age (20–40 years). This may be due to linear transformation which has an impact on decreased flexibility and sensitivity. The main highlights that NLLS model not only gives a numerical compatibility that is close to the value of the TMPI, but also follows the visual form of the development of death more accurately. The upward curvature that is consistent under NLLS for the age of 35–100 years confirms that this method is more sensitive to the dynamics of real life deaths in Indonesia's data.

3.4 Evaluation of Estimation Results Performance

The RMSE value for each model is calculated by the Eq. 17. Defined the error for i data denoted with e_i, as a difference between the value of the function q_x on TMPI 2023 and the function value of q_x to i in the estimated model, namely

$$e_i = q_{x_i} \text{ TMPI 2023} - q_{x_i} \text{ model.} \tag{18}$$

For the male Gompertz model with age limits from 0 to 111 years in the NLLS method, obtained

$$e_1 = q_0 \text{ TMPI 2023} - q_0 \text{ model} = 0,00524 - 0,0001043 = 0,0051357.$$

For $e_2, e_3, \ldots, e_{112}$ is calculated in the same way. Furthermore, the RMSE value is calculated using the Eq. 18, namely

$$\text{RMSE} = \sqrt{\frac{1}{112}\sum_{i=1}^{112} {e_i}^2} = \sqrt{0,00034029} \approx 0,007461.$$

The ratio of RMSE values can be seen in the picture 3

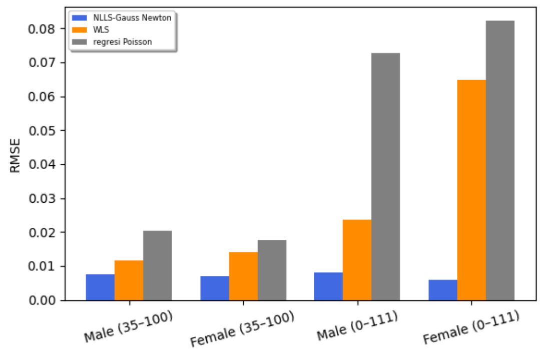

Figure 3: Comparison of RMSE values for each estimated model

NLLS consistently produces the lowest RMSE in Figure 3 for all demographic scenarios based on gender and age and confirmed as the most accurate approach methods. The difference in RMSE between the more substantial methods in the age group 35–100 years. There is in line with the Gompertz law assumptions where shows the model selection is very important when focusing on adult death. Poisson regression tends to produce the highest RMSE perhaps due to overdispersion or underfitting in higher age groups.

4 Conclusion

Across all demographic groups and age ranges, the results consistently demonstrated that the NLLS method offered the best fit, both quantitatively and visually. First, it assumes the midpoint approximation for the central exposure, which may introduce bias in younger age groups. Second, the model does not incorporate confidence intervals or assess statistical uncertainty in the parameter estimates. Additionally, the use of a fixed weighting scheme in WLS may not be optimal across all data conditions. For future research, it is recommended to estimate parameters directly from the probability of death function q_x, thereby avoiding assumptions related to the value of ${}_na_x$ in mortality table calculations. In addition, the value of d_x used as weighted in this study can be explored further using other weighting methods, such as Huber or Tukey. The results of this study hope can be applied to the calculation of actuarial modeling, such as determining premium reserves and mortality assumption in life insurance.

CRediT Authorship Contribution Statement

This research has involved many roles carried out by each writer. For details of workload and job descriptions is **Muhammad Rafael Andika Putra:** Conceptualization, Methodology, Formal Analysis, Software, Visualization, Writing–Original Draft Preparation, Project Administration. **Nurjannah:** Supervision, Validation, Funding Acquisition, Writing–Review & Editing. **Mila Kurniawaty:** Supervision, Data Curation, Writing–Review & Editing.

Declaration of Generative AI and AI-assisted technologies

The authors using Grammarly for assembling sentences in writing, so that the structure of language used is easily understood and following the conditions that occur.

Declaration of Competing Interest

The authors declare no competing interests

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Data Availability

The dataset analyzed during the current study is publicly available in the The Society of Actuaries of Indonesia (PAI) repository[1].

References

[1] S. Haberman and A. Renshaw, "A comparative study of parametric mortality projection models," *Insurance: Mathematics and Economics*, vol. 48, no. 1, pp. 35–55, 2011. DOI: 10.1016/j.insmatheco.2010.09.003.

[2] N. L. Bowers, *Actuarial Mathematics*. Society of Actuaries, 1986. Available online.

[3] A. S. Macdonald, S. J. Richards, and I. D. Currie, *Modelling Mortality With Actuarial Applications*. Cambridge University Press, 2018. DOI: 10.1017/9781107051386.

[4] M. Dahl, "Stochastic mortality in life insurance: Market reserves and mortality-linked insurance contracts," *Insurance: Mathematics and Economics*, vol. 35, no. 1, pp. 113–136, 2004. DOI: 10.1016/j.insmatheco.2004.05.003.

[1] https://www.aktuaris.or.id/page/news_detail/251/tabel-mortalitas-dan-morbiditas-penduduk-indonesia-tahun-2023-dari-bpjs-kesehatan

[5] e. a. Ramadhan M. A., "Joint-life insurance premium model using archimedean copula: The study of mortality in indonesia," *Journal of the Indonesian Mathematical Society*, vol. 31, no. 1, p. 1783, 2025. DOI: 10.22342/jims.v31i1.1783.

[6] Indonesian Social Security Agency for Health, *Tabel Mortalitas Penduduk Indonesia 2023*. Indonesia, 2023. Available online.

[7] T. H. Tai and A. Noymer, "Models for estimating empirical gompertz mortality: With an application to evolution of the gompertzian slope," *The Society of Population Ecology*, vol. 60, 2018. DOI: 10.1007/s10144-018-0609-6.

[8] D. A. Putra, N. Fitriyati, and Mahmudi, "Fit of the 2011 indonesian mortality table to gompertz law and makeham law using maximum likelihood estimation," *InPrime: Indonesian Journal of Pure and Applied Mathematics*, vol. 1, no. 2, pp. 68–76, 2019. DOI: 10.15408/inprime.v1i2.13276.

[9] J. R. Goldstein, M. Osborne, S. Atherwood, and C. F. Breen, "Mortality modeling of partially observed cohorts using administrative death records," *Population Research and Policy Review*, vol. 42, no. 3, 2023. DOI: 10.1007/s11113-023-09785-z.

[10] R. J. Cunningham, T. N. Herzog, and R. L. London, *Models for Quantifying Risk*, 2nd ed. Actex Publications, 2006. Available online.

[11] A. A. Al-Shomrani, "An improvement in maximum likelihood estimation of the gompertz distribution parameters," *Journal of Statistical Theory and Applications*, vol. 22, no. 1–2, pp. 98–115, 2023. DOI: 10.1007/s44199-023-00057-5.

[12] K. W. Watcher, *Essential Demographic Methods*. London: Harvard University Press, 2014. Available online.

[13] D. C. M. Dickson, M. R. Hardy, and H. R. Waters, *Actuarial Mathematics for Life Contingent Risks*, 2nd ed. Cambridge University Press, 2013. Available online.

[14] A. Lenart, *The gompertz distribution and maximum likelihood estimation of its parameters - a revision*, http://www.demogr.mpg.de, 2012.

[15] S. D. Miasary, "Annual premium determination for joint life insurance with de moivre and gompertz's mortality laws," *Square: Journal of Mathematics and Mathematics Education*, vol. 4, no. 2, pp. 117–123, 2022. DOI: 10.21580/square.2022.4.2.13212.

[16] M. A. Hegazy, R. E. A. El-Kader, A. A. El-Helbawy, and G. R. Al-Dayian, "Bayesian estimation and prediction of discrete gompertz distribution," *Journal of Advances in Mathematics and Computer Science*, vol. 36, no. 2, pp. 1–21, 2021. DOI: 10.9734/jamcs/2021/v36i230335.

[17] H. G. Bock, J. Gutekunst, A. Potschka, and M. E. S. Garcés, "A flow perspective on nonlinear least-squares problems," *Vietnam Journal of Mathematics*, vol. 48, no. 4, pp. 987–1003, 2020. DOI: 10.1007/s10013-020-00441-z.

[18] D. C. Montgomery, E. A. Peck, and G. G. Vining, *Introduction to Linear Regression Analysis*, 5th ed. New Jersey: John Wiley & Sons, Inc, 2012. Available online.

[19] M. K. Bhusal and S. P. Khanal, "Statistical models for predicting the number of under-five mortality in nepal," *PLoS ONE*, vol. 20, no. 5, e0324321, 2025. DOI: 10.1371/journal.pone.0324321.

[20] T. Chai and R. R. Draxler, "Root mean square error (rmse) or mean absolute error (mae) – arguments against avoiding rmse in the literature," *Geoscientific Model Development*, vol. 7, pp. 1247–1250, 2014. DOI: 10.5194/gmd-7-1247-2014.