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Comparison of Vector Autoregressive and Multiresponse Fourier Series for Cryptocurrency Forecasting after the 2024 Bitcoin Halving

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Abstract

The 2024 Bitcoin halving represents a major structural event in the cryptocurrency market, as the programmed reduction in block rewards directly alters supply dynamics and market expectations. This study examines cryptocurrency price forecasting in the immediate post-halving period, which is characterized by heightened volatility, increased institutional participation, macroeconomic tightening, and stronger cross-asset synchronization relative to previous halving episodes. The novelty of this study lies in applying a cosine-based multiresponse Fourier Series estimator as a multivariate forecasting benchmark for post-halving cryptocurrency markets, an approach that remains limited in the existing literature. The dataset consists of daily closing prices of Bitcoin, Ethereum, and Litecoin from April 2024 to August 2025 ($t = 480$), obtained from Investing.com, with 90% of observations used for training and 10% for testing. Bartlett's sphericity test confirms significant cross-asset correlations ($p < 0.001$), supporting the use of multivariate analysis. Forecasting performance is evaluated using the Mean Absolute Percentage Error (MAPE). The results indicate that the Fourier Series estimator with five oscillation parameters ($k = 5$) achieves superior accuracy, yielding a MAPE of 3.768%, compared to 8.503% for the VAR model. These findings suggest that periodic multiresponse regression more effectively captures cyclical and nonlinear dynamics in post-halving cryptocurrency prices.

Keywords: Cryptocurrency forecasting; Bitcoin halving; Multivariate time series; Vector autoregression; Fourier regression; Forecast evaluation

1. Introduction

Digital currencies or cryptocurrencies, especially Bitcoin, have attracted significant attention in the financial world due to their volatile nature and growing role in the global digital economy. In recent years, crypto assets have rapidly expanded as both investment instruments and cross-border payment alternatives. According to the TripleA report (2024), global crypto holders reached approximately 562 million people, or 6.8% of the world's population, representing a 34% increase from 2023 [1]. Their growing adoption in international transactions is driven by low transaction

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costs, fast settlement, and the absence of intermediaries. Countries such as El Salvador have even adopted Bitcoin as legal tender to promote financial inclusion and reduce remittance costs [2]. A defining characteristic of Bitcoin is the halving mechanism, a protocol-driven event that halves mining rewards approximately every four years. This mechanism regulates supply growth by reducing newly issued coins and ultimately constraining total supply to 21 million units [3]. To date, four halving events have occurred: 2012, 2016, 2020, and 2024. Although technical in nature, halving events have substantial implications for investor behavior, market expectations, and the broader digital economy. Supported by blockchain and decentralized finance (DeFi), cryptocurrencies contribute to financial inclusion, digital economic participation, and inclusive growth, aligning with Sustainable Development Goal (SDG) 8 on decent work and economic growth [4].

The 2024 Bitcoin halving represents a critical structural turning point in the cryptocurrency market. The reduction of block rewards from 6.25 to 3.125 Bitcoins constrained new supply, while demand continued to rise amid growing institutional participation and tighter global macroeconomic conditions. This imbalance contributed to price appreciation accompanied by persistent volatility. Moreover, altcoins exhibited stronger price co-movements with Bitcoin, trading activity intensified, and market sentiment became increasingly sensitive to external shocks and news announcements [5, 6]. These developments indicate a post-halving market environment characterized by heightened volatility, nonlinear dynamics, and increased cross-asset synchronization, posing challenges for conventional forecasting approaches. Despite the importance of this period, empirical studies focusing specifically on multivariate cryptocurrency forecasting in the immediate post-2024 halving phase remain limited [7].

Existing literature on cryptocurrency forecasting largely relies on either linear econometric models or machine learning techniques. While machine learning approaches such as Bi-LSTM and GRU have demonstrated improved predictive accuracy over standard LSTM models [8], and Dynamic Bayesian Networks have outperformed classical benchmarks including ARIMA, SVR, and SVM [9], these methods often sacrifice interpretability and are less transparent in capturing joint dynamics across multiple assets. On the other hand, classical econometric models remain essential for understanding multivariate interactions. Prior studies using VAR and VECM frameworks have shown that although Bitcoin exhibits long-term relationships with major fiat currencies, its short-term dynamics are primarily driven by its own lagged behavior [10]. However, most existing econometric studies emphasize linear dynamics and do not explicitly account for cyclical or nonlinear patterns that may dominate during structurally disruptive events such as halving episodes.

This study aims to bridge this gap by conducting a systematic comparison between two multivariate forecasting approaches in the immediate post-2024 Bitcoin halving period: the Vector Autoregressive (VAR) model and a multivariate Fourier series estimator. VAR serves as a well-established benchmark for capturing short-term linear interdependencies through lagged dynamics [11]. In contrast, the multivariate Fourier series is employed as a structured periodic benchmark designed to approximate synchronized cyclical and nonlinear patterns across multiple cryptocurrency price series, rather than as an explicit dynamic feedback system [12, 13]. By positioning the Fourier specification as a competing benchmark within a multivariate forecasting framework, this study provides an empirical evaluation of whether periodic regression can outperform or complement linear VAR models under post-halving market conditions. Accordingly, the research addresses the following question: *How does the forecasting performance of a multivariate VAR model compare with that of a multivariate Fourier series approach in the immediate aftermath of the 2024 Bitcoin halving?* Given the heightened volatility and nonlinear behavior observed during this period, this study hypothesizes that the Fourier-based multivariate model can achieve comparable or superior forecasting accuracy relative to the linear VAR benchmark.

2. Methods

To achieve the objectives of this study, a structured methodological framework was employed, focusing on Fourier series analysis and vector autoregressive analysis. The methodology consists of two main parts, namely the research data and the data analysis steps. The analysis includes data description, prediction using Fourier series and vector autoregressive models, and model performance evaluation. The following subsections explain these components in more detail.

2.1. Research Data

This study uses a quantitative approach by applying the Vector Autoregressive (VAR) method and Fourier Series. Based on the comparative method discussed earlier, the generated model is validated using actual data to assess its accuracy in predicting cryptocurrency price fluctuations. This study aims to analyze and evaluate the impact of the 2024 Bitcoin halving on the movement of several cryptocurrencies. This study uses secondary data obtained from the website *investing.com*, which consists of daily closing price data (in USD) collected at 00:00 UTC for Bitcoin (BTC), Ethereum (ETH), and Litecoin (LTC) covering the period from April 20, 2024, to August 12, 2025, with a total of 480 data points. Basically, the crypto market never stops ticking—24 hours a day.

The dataset is divided into two parts: training data (432 data points or 90%) used to build the analytical model, and testing data (48 data points or 10%) used to validate the model's predictive results against actual data.

2.2. Data Analysis Procedure

1. Collecting daily cryptocurrency price data.
2. Determining training and testing data with a ratio of 90% and 10%.
3. Identifying the characteristics of daily cryptocurrency price data based on descriptive statistics and time series plots.
4. Determine the Bartlett Sphericity Test to find out the correlation between variables. The hypothesis and test statistics used are.
   H_0 : $\mathbf{R} = \mathbf{I}$ (no correlation among variables)
   H_1 : $\mathbf{R} \neq \mathbf{I}$ (there is a correlation among variables)
   The test statistic is formulated as.
   $$\chi^2 = -\left[(n-1) - \frac{1}{6}(2p-5)\right] \ln |\mathbf{R}|$$
   where $\mathbf{I}$ is the identity matrix, n is the number of observations, p is the number of variables, and $|\mathbf{R}|$ is the determinant of the correlation matrix among variables. The decision criterion is as follows reject H_0 if $\chi^2 > \chi^2_{\alpha;p(p-1)}$ which indicates that there is a significant correlation among variables [14].
5. Make predictions with Vector Autoregressive using the following steps.
   a. Testing data stationarity. The stationarity of data can be formally tested using the Augmented Dickey-Fuller (ADF) test by examining the presence or absence of a unit root in the model. The test statistic for the stationarity test is determined using the ADF calculation with the following hypotheses [15].
   H_0: The data are non-stationary
   H_1: The data are stationary
   The ADF test statistic can be expressed as in the following equation.
   $$\tau = \frac{\hat{\delta}}{Se(\hat{\delta})}$$
   The critical region used is to reject H_0 when the p-value $< \alpha = 0.05$. If a variable is found to be non-stationary, data differencing is performed.

b. Determine the optimal lag by looking at the Akaike Information Criterion (AIC) value. The lag will be considered optimal if it has the minimum AIC value.

$$\text{AIC}(p) = log|\hat{\Sigma}_{\varepsilon}| + \frac{2}{T}pq^2$$

where $\hat{\Sigma}_{\varepsilon} = n^{-1}\sum_{t=1}^{n}\hat{\varepsilon}_t\hat{\varepsilon}_t'$ is the residual covariance estimator matrix for the VAR(p) model, n is the sample size, and q is the number of endogenous variables [16].

c. Testing data stability. This test is conducted using the Roots of Characteristic Polynomial method. A VAR model is considered stable if all of its characteristic roots are within the unit circle (have a modulus value of less than 1). However, if there are roots that are on or outside the unit circle, then the model is declared non-stationary or even explosive.

d. Perform a cointegration test using the Johansen Cointegration Test. This test is conducted when the data reaches stationarity at the first differentiation, with the aim of assessing the long-term equilibrium between variables and ensuring a linear relationship in the data. The test hypothesis is as follows.
H_0: No cointegration equation
H_1: There exists at least one cointegration equation
The test statistics are computed using the trace statistic and maximum eigenvalue statistic, as shown in Eq. (1) and Eq. (2) [17].

$$\lambda_{trace}(r) = -n\sum_{i=r+1}^{n}\ln(1-\hat{\lambda}_i) \tag{1}$$

$$\lambda_{max}(r, r+1) = -n\ln(1-\hat{\lambda}_{r+1}) \tag{2}$$

where,
r : cointegration rank
n : number of time periods or observations
$\hat{\lambda}_i$: estimated i-th largest eigenvalue
If $\lambda_{trace}(0) > \lambda_{trace,5\%}$ dan $\lambda_{max}(0) > \lambda_{trace,5\%}$ then reject H_0, which means there is one cointegration vector. This means that the Johansen test results indicate the existence of a long-term equilibrium relationship [16]. If the test results indicate no cointegration, the analysis can be continued using the VAR (Vector Autoregressive) method. Conversely, if cointegration is found, the more appropriate approach is the VECM (Vector Error Correction Model) [17].

e. Perform modeling and testing of the obtained VAR model. The equation of the VAR(1) model is as follows.

$$\begin{bmatrix} y_{1t} \\ y_{2t} \\ \vdots \\ y_{qt} \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_q \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1q} \\ a_{21} & a_{22} & \cdots & a_{2q} \\ \vdots & \vdots & \ddots & \vdots \\ a_{q1} & a_{q2} & \cdots & a_{qq} \end{bmatrix} \begin{bmatrix} y_{1,t-1} \\ y_{2,t-1} \\ \vdots \\ y_{q,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \vdots \\ \varepsilon_{qt} \end{bmatrix} \tag{3}$$

Using the VAR model equation, it is assumed that the general form of Eq. (3) is stable if

$$\mathbf{y}_t = \boldsymbol{\mu} + \sum_{i=0}^{\infty}\mathbf{A}_i\boldsymbol{\varepsilon}_{t-i} \tag{4}$$

where,

$$\mathbf{y}_t = \begin{bmatrix} y_{1t} \\ y_{2t} \\ \vdots \\ y_{qt} \end{bmatrix}, \boldsymbol{\mu} = \begin{bmatrix} \bar{y}_1 \\ \bar{y}_2 \\ \vdots \\ \bar{y}_q \end{bmatrix} \text{ and } \mathbf{A}_i = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1q} \\ a_{21} & a_{22} & \cdots & a_{2q} \\ \vdots & \vdots & \ddots & \vdots \\ a_{q1} & a_{q2} & \cdots & a_{qq} \end{bmatrix}, \boldsymbol{\varepsilon}_{t-i} = \mathbf{A}_i^{-1}\begin{bmatrix} \epsilon_{2,t-i} \\ \vdots \\ \epsilon_{q,t-i} \end{bmatrix}$$

f. Performing Impulse Response Function (IRF) analysis. The Impulse Response Function (IRF) describes how an endogenous variable reacts when there is a shock to another endogenous variable [18].
g. Performing Variance Decomposition (VD) analysis. Variance Decomposition (VD) analysis in the Vector Autoregressive (VAR) model is used to measure how much of the variation in an endogenous variable is explained by shocks from itself or from other variables in the system. This analysis shows the relative contribution of each variable to the fluctuations of the observed variable over time and complements the Impulse Response Function (IRF) by providing a quantitative measure of the effect of each shock across different time horizons [17].

6. Making predictions with Fourier series using the cosine function
   a. Fourier series are a combination of periodic functions used to approximate recurring data patterns. In cases where the pattern in the data is not known with certainty, Fourier series become an effective approach to represent data fluctuations, especially in time series data [19]. In this study, Fourier series approach was performed using only cosine functions because the observed data patterns were symmetric with respect to the time axis, so they could be adequately represented by cosine harmonic components [20]. To provide empirical support for this choice, an alternative Fourier specification including both cosine and sine terms was also considered. However, the inclusion of sine components does not yield a meaningful improvement in model performance. Therefore, the cosine-only specification is adopted in this study. The basic multiresponse nonparametric regression model is expressed in the following general form.
   $$y_{ij} = m_j(\lambda_i) + \varepsilon_{ij}, \quad \varepsilon_{ij} \sim IIDN(0, \sigma_j^2)$$
   In this context, $m_j(\lambda_i)$ represents the unknown regression function for the j-th response at the i-th observation, while ε_{ij} denotes the error component. The variance σ_j^2 is heterogeneous, indicating that each response exhibits a different level of error variation. As the next step, the function $m_j(t_i)$ is approximated using a combination of cosine functions based on the Fourier series, allowing the modeling process to better capture the fluctuating relationship patterns [21].
   b. The function $m_j(\lambda_i)$ in the above model is approximated using a cosine function derived from the Fourier series with a specified number of harmonics, λ. Accordingly, the regression function can be expressed as follows.
   $$m_j(t_i) = \frac{1}{2}\hat{\alpha}_{0j} + \hat{\gamma}_j t_i + \sum_{h=1}^{\lambda} \hat{\alpha}_{hj} \cos(ht_i)$$
   By substituting this function into the initial model, the formulation yields a multiresponse nonparametric Fourier series regression model with cosine functions, as follows.
   $$y_{ij} = \frac{1}{2}\alpha_{0j} + \gamma_j t_i + \sum_{h=1}^{\lambda} \alpha_{hj} \cos(ht_i) + \varepsilon_{ij}$$
   where,
   y_{ij} : predicted value of the j-th response at the i-th observation
   α_{0j} : constant coefficient for the j-th response
   γ_j : linear trend coefficient with respect to time for the j-th response
   t_{ij} : time corresponding to the i-th observation of the j-th response
   $\alpha_{\lambda j}$: cosine coefficient for the λ-th harmonic of the j-th response
   Through this approach, each response variable can exhibit a distinct periodic pattern while maintaining a similar functional relationship with time [22]. Once the regression

function has been formulated, the next step is to express it in matrix form to facilitate a more systematic parameter estimation process.

c. To simplify the estimation process, the model is represented in vector form. For each response j, the vector form is as follows.

$$\mathbf{y}_j = \mathbf{T}_j \boldsymbol{\beta}_j + \boldsymbol{\varepsilon}_j$$

where,

$$\mathbf{y}_j = \begin{bmatrix} y_{1j} \\ y_{2j} \\ \vdots \\ y_{nj} \end{bmatrix}, \quad \mathbf{T}_j = \begin{bmatrix} 1 & t_1 & \cos(t_1) & \cos(2t_1) & \cdots & \cos(\lambda t_1) \\ 1 & t_2 & \cos(t_2) & \cos(2t_2) & \cdots & \cos(\lambda t_2) \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & t_n & \cos(t_n) & \cos(2t_n) & \cdots & \cos(\lambda t_n) \end{bmatrix},$$

$$\boldsymbol{\beta}_j = \begin{bmatrix} \frac{1}{2}\alpha_{0j} \\ \gamma_j \\ \alpha_{1j} \\ \alpha_{2j} \\ \vdots \\ \alpha_{\lambda j} \end{bmatrix}, \quad \boldsymbol{\varepsilon}_j = \begin{bmatrix} \varepsilon_{1j} \\ \varepsilon_{2j} \\ \vdots \\ \varepsilon_{nj} \end{bmatrix}.$$

The entire multiresponse system can be written together in block-diagonal form as follows.

$$\mathbf{y} = \mathbf{T}_\lambda \boldsymbol{\beta}_\lambda + \boldsymbol{\varepsilon}$$

where,

$$\mathbf{T}_{\boldsymbol{\lambda}} = \begin{bmatrix} \mathbf{T_1} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{T_2} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{T_q} \end{bmatrix}, \quad \boldsymbol{\beta}_{\boldsymbol{\lambda}} = \begin{bmatrix} \boldsymbol{\beta_1} \\ \boldsymbol{\beta_2} \\ \vdots \\ \boldsymbol{\beta_q} \end{bmatrix}, \quad \boldsymbol{\varepsilon} = \begin{bmatrix} \boldsymbol{\varepsilon_1} \\ \boldsymbol{\varepsilon_2} \\ \vdots \\ \boldsymbol{\varepsilon_q} \end{bmatrix}$$

The index j denotes the j-th response, while q represents the total number of response variables being modeled. This block-diagonal representation is commonly used in multivariate nonparametric regression models [23]. The next step is to consider the correlation relationships among the responses, which are reflected in the structure of the error covariance matrix.

d. Since the response variables in a multiresponse model are generally correlated, the covariance structure among the responses needs to be incorporated into the model. The covariance structure can be expressed as follows:

$$\mathbf{E}(\varepsilon\varepsilon^T) = \boldsymbol{\Sigma} = \begin{bmatrix} \sigma_1^2\mathbf{I} & \sigma_{12}\mathbf{I} & \cdots & \sigma_{1q}\mathbf{I} \\ \sigma_{21}\mathbf{I} & \sigma_2^2\mathbf{I} & \cdots & \sigma_{2q}\mathbf{I} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{q1}\mathbf{I} & \sigma_{q2}\mathbf{I} & \cdots & \sigma_q^2\mathbf{I} \end{bmatrix}$$

Because the variance and covariance between responses differ, a Weighted Least Squares (WLS) approach is adopted. Since the true covariance matrix Σ is unknown, it is estimated from preliminary residuals, leading to a feasible GLS (WLS) implementation [24]. Under standard regularity conditions and with a consistent estimate of Σ, the resulting estimator is asymptotically efficient. The WLS weighting matrix is defined as:

$$\mathbf{W} = \boldsymbol{\Sigma} \otimes \mathbf{I}$$

With the estimated weight matrix, the estimation procedure accounts for heteroscedasticity and cross-response correlations. The efficiency claim holds under the assumption that the estimated Σ is consistent. In addition, the residuals are examined to ensure that any remaining autocorrelation does not invalidate the estimation results. Once the weight structure has been determined, the next step is to estimate the model parameters.

e. Parameter estimation is performed by minimizing the weighted sum of squares error function as follows:

$$S(\boldsymbol{\beta}_\lambda) = (\mathbf{y} - \mathbf{T}_\lambda \boldsymbol{\beta}_\lambda)^T \mathbf{W} (\mathbf{y} - \mathbf{T}_\lambda \boldsymbol{\beta}_\lambda)$$

By differentiating this function with respect to $\boldsymbol{\beta}_\lambda$ and setting it equal to zero, the following parameter estimates are obtained.

$$\hat{\boldsymbol{\beta}}_\lambda = (\mathbf{T}_\lambda^T \mathbf{W} \mathbf{T}_\lambda)^{-1} \mathbf{T}_\lambda^T \mathbf{W} \mathbf{y}$$

This estimator provides parameter solutions that simultaneously account for heteroscedasticity and correlations between responses [25]. Once the parameters are obtained, the model can be used to generate predicted values for each response variable.

f. After the parameter $\hat{\boldsymbol{\beta}}_\lambda$ is obtained, the result is substituted back into the model to generate a multiresponse Fourier series prediction based on the cosine function as follows.

$$\hat{y}_{ij} = \frac{1}{2}\hat{\alpha}_{0j} + \hat{\gamma}_j t_{ij} + \sum_{\lambda=1}^{k} \hat{\alpha}_{\lambda j} \cos(\lambda t_i) + \varepsilon_{ij}$$

This prediction model can capture oscillation patterns and periodic fluctuations in the data, while maintaining estimation stability across various variations. The next step is to determine the optimal number of harmonics λ so that the model is neither too smooth nor too complex.

g. The selection of the number of harmonics λ is an important factor in the Fourier series model because it affects the balance between bias and variance. A value of λ that is too large causes overfitting, while a value that is too small causes underfitting. Therefore, the optimal number of harmonics is determined using the Generalized Cross Validation (GCV) method with the following formulation.

$$GCV(\lambda) = \frac{MSE(\lambda)}{(nq)^{-1}[trace(\mathbf{I} - \mathbf{H}_\lambda)]^2}$$

Where,

$$\mathbf{H}_\lambda = \mathbf{T}_\lambda (\mathbf{T}_\lambda^T \mathbf{W} \mathbf{T}_\lambda)^{-1} \mathbf{T}_\lambda^T \mathbf{W}, \quad MSE(\lambda) = (nq)^{-1} \mathbf{y}'(\mathbf{I} - \mathbf{H}_\lambda)\mathbf{y}$$

The value of λ that gives the smallest GCV is considered the optimal harmonic sum because it shows the best balance between model complexity and accuracy of the data [26].

7. Method for comparing the effectiveness of the two models

a. Using 10% of the testing data to calculate the Mean Absolute Percentage Error (MAPE) of the Vector Autoregressive and Multiresponse Fourier Series models. The MAPE value represents the average percentage of absolute error between the predicted and actual values, which can be calculated using Eq. (5). The interpretation of MAPE values based on their range is presented in Table 1 as follows [27].

$$MAPE = \frac{1}{nq} \sum_{i=1}^{n} \sum_{j=1}^{q} \left| \frac{y_{ij} - \hat{y}_{ij}}{y_{ij}} \right| \times 100\% \tag{5}$$

Where:
y_{ij} : actual data response j in observation i
n : number of observations
q : number of responses
$\hat{y}_{ij}$: predicted data response j in observation i

Table 1: Interpretation of MAPE

MAPE Value	Interpretation
MAPE $< 10\%$	Very accurate prediction ability
$10\% \leq$ MAPE $\leq 20\%$	Good prediction ability
$20\% <$ MAPE $< 50\%$	Acceptable prediction ability
MAPE $\geq 50\%$	Poor prediction ability

b. Select the best method based on the minimum MAPE value.
c. Presenting a comparison of actual data and predictions in a time series plot.

8. Interpretation and Conclusion.

3. Results and Discussion

This section presents the empirical findings obtained from the implementation of the proposed methodology. To provide a clear and structured discussion, the results are organized into several subsections. The analysis begins with descriptive statistics to illustrate the general characteristics of the data, followed by correlation testing using the Bartlett test. Subsequently, the forecasting performance of the Vector Autoregressive (VAR) model and the Fourier Series estimator is evaluated and compared based on their predictive accuracy. Each subsection is designed to systematically support the overall findings of the study.

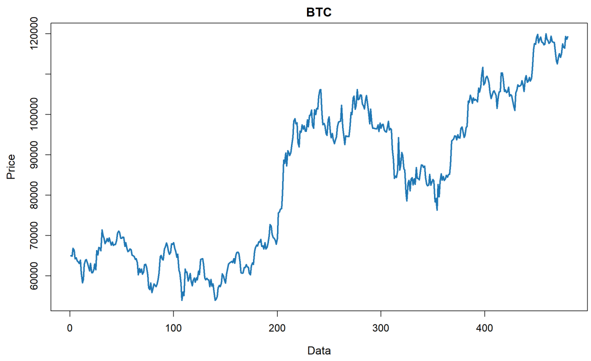
3.1. Descriptive Statistics

Descriptive statistics are used to provide a general overview of the daily extreme fluctuations in cryptocurrency prices. The general description of cryptocurrency prices from April 20, 2024, to August 12, 2025, is presented in Table 2 below.

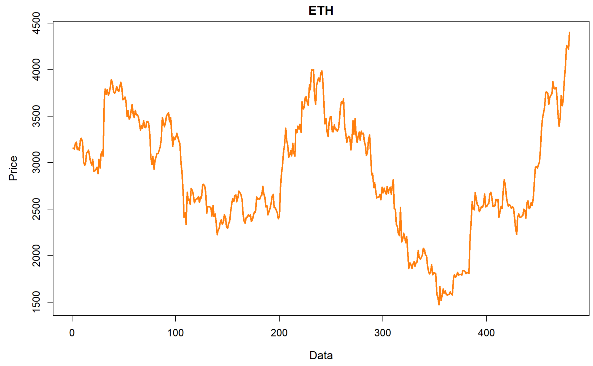
Table 2: Descriptive Statistics of Cryptocurrency Data

Variable	Mean	Variance	Minimum	Maximum
Bitcoin	84,069.92	377,466,300	53,966.8	119,965.5
Ethereum	2,860.45	388,233	1,473.4	4,400.61
Litecoin	88.329	375.989	55.94	136.99

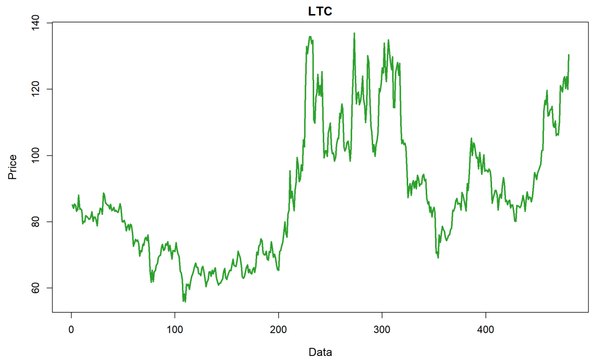
Based on Table 2, the three cryptocurrencies, namely Bitcoin (BTC), Ethereum (ETH), and Litecoin (LTC), show significant price fluctuations. Bitcoin has the highest average price (84,069.92 USD) and the largest variance (377,466,300), indicating very high volatility. Ethereum has an average of 2,860 USD with a variance of 388,233, indicating moderate stability, while Litecoin is the most stable with an average of 88.329 USD and a variance of 375.989. Overall, Bitcoin is the riskiest but dominates the market, Ethereum is relatively stable, and Litecoin is the calmest in terms of price movements. Furthermore, to see the trend patterns of cryptocurrency price data, refer to Fig. 1 below.



(a) Bitcoin



(b) Ethereum



(c) Litecoin

Fig. 1: Time Series Plot of Cryptocurrency Prices

Based on Fig. 1, Bitcoin's (BTC) lowest point on July 7, 2024, at 53,966.8 was influenced by the post-halving phase in April 2024, which historically triggered a short-term correction due to profit-taking, coupled with pressure from tight global monetary policy that reduced interest in risky assets. Conversely, BTC's highest point on July 22, 2025, at 119,965.5 was driven by increased institutional adoption through spot Bitcoin ETFs in the United States and looser macro conditions, which increased crypto market liquidity. Ethereum (ETH) reached its lowest point on April 8, 2025, at 1,473.40 amid doubts about the network's ability to address scalability issues and high gas fees, coupled with regulatory pressure on the DeFi sector. while its peak on August 12, 2025, at 4,400.61 occurred after a successful network upgrade that reduced transaction costs and increased capacity, thereby attracting investor interest once again. Litecoin (LTC) showed higher volatility, with a peak price of 136.99 on January 17, 2025, triggered by the continued effects of the August 2023 halving and the rally sentiment alongside BTC–ETH, while the low of 55.94 on April 6, 2025, was caused by profit-taking after the previous rally and a shift in investor focus to major assets like BTC and ETH. Overall, the prices of BTC, ETH, and LTC show that the three assets move relatively in the same direction, with periods of joint rallies at the end of May 2024 and mid-May to July 2025, as well as joint corrections in June to early July 2024 and early April 2025.

3.2. Barlett Test

Before conducting further analysis, Bartlett's test of sphericity was performed to examine the correlation among the variables. The decision rule is to reject H_0 if the p-value is less than 0.05. The test result produced a chi-square statistic of $\chi^2 = 8329.9$ with 3 degrees of freedom and a p-value of 0.001. Since the p-value is smaller than the significance level ($\alpha = 0.05$), H_0 is rejected, indicating that there is a significant correlation among the cryptocurrency variables. Therefore, simultaneous modeling using the Vector Autoregressive (VAR) and Fourier Series Estimator approaches is appropriate for the analyzed data.

3.3. Prediction using Vector Autoregressive (VAR)

Before making predictions using the Vector Autoregressive (VAR) method, the first step is to test the stationarity of the data. The results of the data stationarity test are presented in Table 3 below.

Table 3: Stationarity Test

Variable	DF	Prob.	Conclusion
Bitcoin (Y_1)	-0.590097	0.8698	Non-stationary
Ethereum (Y_2)	-0.974844	0.7634	Non-stationary
Litecoin (Y_3)	-1.735286	0.4128	Non-stationary

Based on Table 3, a probability value > 0.05 was obtained for each variable, which means that all variables are non-stationary. Therefore, first differencing is necessary for all variables. Table 4 below shows the test results after first differencing.

Table 4: Differencing One

Variable	DF	Prob.	Conclusion
Bitcoin (Y_1)	-22.99572	0.0000	Stationary
Ethereum (Y_2)	-21.22299	0.0000	Stationary
Litecoin (Y_3)	-21.27349	0.0000	Stationary

From the differencing results in Table 4, after the first differencing, the Augmented Dickey-Fuller (ADF) test probability values for each variable are less than 0.05. This indicates that all variables have become stationary at the first differencing level. Thus, the analysis can proceed to the optimal lag determination stage.

Table 5: Optimal Lag Determination

Lag	AIC
0	46.00840
1	34.41061*
2	34.43322
3	34.44301
4	34.45452
5	34.44989

Based on Table 5, the optimal lag selected is lag 1 because the AIC value for this lag is the smallest compared to other lags. Thus, lag 1 is set as the optimal lag to be used in the next stage of analysis. To ensure the stability of the estimation model, a stability test needs to be carried out, which can be seen in Table 6.

Table 6: Model Stability Test

Root	Modulus
0.991755 − 0.006335i	0.991775
0.991755 + 0.006335i	0.991775
0.967421	0.967421

The stability of the VAR model was evaluated using the characteristic root criterion. Based on Table 6, all modulus values are below 1, namely 0.991775 and 0.967421. Since all characteristic roots lie inside the unit circle, the VAR model satisfies the stability condition, so that the estimation results can be trusted for further analysis. Next, a Johansen Cointegration test was conducted to determine whether there was a long-term relationship between endogenous variables in the VAR system. The Johansen method was used by comparing the Trace Statistic value with the critical value. If the Trace Statistic is greater than the critical value, then there is a cointegration vector in the model.

Table 7: Johansen Cointegration Test

Cointegration	*Trace Statistics*	**t-critical value**	**P-Value**
None	20.10188	29.79707	0.4160
At Most 1	7.106536	15.49471	0.5653
At Most 2	1.722829	3.841465	0.1893

Based on Table 7, it can be seen that the probability values at all cointegration levels are greater than the significance level α (5%). This condition indicates that there is no cointegration relationship between the variables analyzed. Thus, there is no long-term equilibrium between the prices of Bitcoin, Ethereum, and Litecoin. Because the data are stationary at the first difference but no cointegration is found, further analysis focuses on the VAR model to examine short-term interactions between variables.

Since the Augmented Dickey–Fuller (ADF) test results indicate that all variables become stationary after the first differencing, the VAR model is estimated using the differenced series. Let ΔY_{1t}, ΔY_{2t}, and ΔY_{3t} denote the first differences of Bitcoin (BTC), Ethereum (ETH), and Litecoin (LTC) prices, respectively. Based on the optimal lag selection using the Akaike Information Criterion (AIC), the VAR model with lag order one is specified as follows.

$$\begin{aligned}
\Delta Y_{1t} &= 1114.007 + 1.006497\,\Delta Y_{1,t-1} - 0.192094\,\Delta Y_{2,t-1} - 11.30405\,\Delta Y_{3,t-1} \\
\Delta Y_{2t} &= 7.069506 + 0.0008031\,\Delta Y_{1,t-1} + 0.998288\,\Delta Y_{2,t-1} - 0.786693\,\Delta Y_{3,t-1} \\
\Delta Y_{3t} &= 0.197881 + 0.00004444\,\Delta Y_{1,t-1} + 0.000321\,\Delta Y_{2,t-1} + 0.946146\,\Delta Y_{3,t-1}
\end{aligned}$$

The estimated VAR model produces forecasts in terms of changes in cryptocurrency prices rather than their absolute levels. Therefore, the predicted differences are transformed back into price levels using

$$\hat{Y}_t = Y_{t-1} + \Delta\hat{Y}_t$$

where Y_{t-1} represents the last observed price and $\Delta\hat{Y}_t$ denotes the predicted change from the VAR model. The resulting level forecasts are then used to compute the Mean Absolute Percentage Error (MAPE) to ensure comparability with the Fourier model.After estimating the VAR model, the significance of the estimated coefficients was evaluated to determine whether the lagged variables have a statistically significant effect on the dependent variable. The significance test was conducted using the t-statistic at a 5% significance level. A coefficient is considered statistically significant if the absolute value of the t-statistic is greater than the critical value of 1.96. The results of the coefficient significance test are presented in Table 8.

Table 8: Significance Test of VAR Model Coefficients

Regressor	Coefficient	Std. Error	t-Statistic	Significance (5%)
BTC(-1)	1.006497	0.00741	135.843	Significant
ETH(-1)	-0.192094	0.16001	-1.20053	Not Significant
LTC(-1)	-11.30405	7.87554	-1.43534	Not Significant
C (Constant)	1114.007	548.976	2.02924	Significant

Based on Table 8, the lagged value of Bitcoin (BTC(-1)) has a statistically significant effect on the change in Bitcoin prices since the absolute value of the t-statistic (135.843) is greater than the critical value of 1.96. This result indicates that past movements in Bitcoin prices play an important role in explaining current changes in Bitcoin prices.

Meanwhile, the lagged values of Ethereum (ETH(-1)) and Litecoin (LTC(-1)) are not statistically significant because their absolute t-statistic values are smaller than 1.96. This suggests that the short-run influence of Ethereum and Litecoin prices on Bitcoin price changes is relatively weak within the VAR framework.In addition, the constant term is statistically significant, indicating that there exists a systematic component in the model that affects the dependent variable even when the lagged variables are held constant.

After evaluating the statistical significance of the VAR coefficients, residual diagnostic tests were conducted to assess whether the estimated VAR model satisfies the required assumptions for reliable time series analysis. In particular, the diagnostic test was performed to examine whether the residuals are free from serial correlation. Residual autocorrelation is an important assumption that must be satisfied in Vector Autoregression (VAR) models, as uncorrelated residuals indicate that the model has adequately captured the dynamic relationships among the variables. The serial correlation of the residuals was tested using the Lagrange Multiplier (LM) test, which is commonly applied to determine whether the residuals are serially uncorrelated so that the estimated VAR model can be considered valid [28–30]. In addition, portmanteau tests, such as the Ljung–Box and Box–Pierce tests, are also frequently used to evaluate model adequacy by examining the joint autocorrelations of residuals [28, 29, 31]. However, the LM test is widely considered sufficient as the primary diagnostic tool for detecting residual autocorrelation in VAR models, while portmanteau tests are often used as complementary tests to strengthen the diagnostic evaluation of the model specification. The results of the LM test are presented in Table 9.

Table 9: Residual Serial Correlation Test (LM Test)

Lag	LRE Statistic	df	p-value	Decision
1	11.94540	9	0.2164	No autocorrelation
2	13.46565	9	0.1426	No autocorrelation
3	10.65976	9	0.2998	No autocorrelation

Based on Table 9, the p-values for all tested lags are greater than the 5% significance level, indicating that the residuals are free from serial correlation. This result suggests that the estimated VAR model satisfies the assumption of no autocorrelation and is therefore adequate for further dynamic analysis.

After confirming the adequacy of the VAR model through the residual diagnostic test, the dynamic interactions among the variables are further examined using the Impulse Response Function (IRF) analysis. The IRF analysis illustrates how a shock in one variable affects the current and future values of other variables within the system. The results of the IRF analysis are presented in Fig. 2.

Fig. 2 presents the impulse response functions (IRFs) capturing the dynamic transmission of a one-standard-deviation shock among Bitcoin, Ethereum, and Litecoin prices during the post-2024 halving period. The results indicate that a positive shock to Bitcoin generates an immediate and pronounced response in Ethereum and Litecoin within the first one to two periods, reflecting strong short-run spillover effects from the dominant cryptocurrency to major altcoins. The response magnitudes decline progressively over subsequent periods and converge toward zero, suggesting that the impact of shocks is transitory rather than persistent.

In contrast, shocks originating from Ethereum and Litecoin induce comparatively weaker and less sustained responses in Bitcoin, as evidenced by the flatter and more rapidly stabilizing IRF trajectories. This asymmetric transmission mechanism underscores Bitcoin's role as the primary source of market-wide disturbances, whereas altcoins primarily absorb rather than propagate shocks. Such dynamics imply a hierarchical interdependence structure in which cross-asset

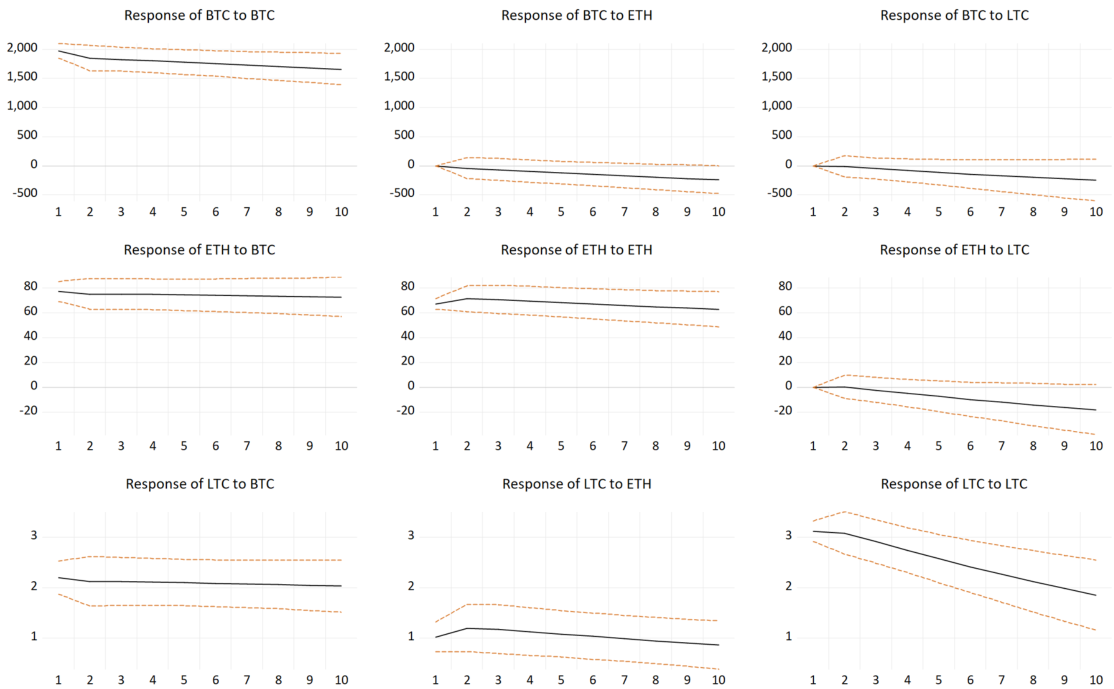


Fig. 2: IRF Analysis Result Chart

linkages intensify in the short horizon but dissipate over time. Overall, the convergence of all impulse responses toward equilibrium supports the stability of the estimated VAR system and indicates mean-reverting behavior in the post-halving cryptocurrency market.

Table 10: Result of Variance Decomposition

Period	**BTC**			**ETH**			**LTC**		
	BTC	ETH	LTC	BTC	ETH	LTC	BTC	ETH	LTC
1	100.000	0.000	0.000	56.649	43.351	0.000	30.867	6.957	62.176
2	99.976	0.008	0.016	56.903	43.067	0.029	31.921	6.979	61.100
3	99.921	0.027	0.052	57.133	42.773	0.094	32.977	6.996	60.028
4	99.839	0.056	0.106	57.338	42.469	0.193	34.032	7.006	58.962
5	99.729	0.095	0.175	57.521	42.157	0.322	35.085	7.012	57.903
6	99.596	0.146	0.259	57.682	41.839	0.479	36.134	7.011	56.854
7	99.439	0.207	0.355	57.824	41.514	0.661	37.178	7.005	55.817
8	99.261	0.278	0.461	57.948	41.185	0.867	38.214	6.994	54.792
9	99.063	0.360	0.577	58.054	40.853	1.094	39.241	6.978	53.781
10	98.846	0.452	0.701	58.144	40.518	1.338	40.257	6.957	52.786

The Variance Decomposition (VD) results presented in Table 10 indicate a clear dominance of Bitcoin in explaining forecast error variance across the system. At the beginning of the forecast horizon, Bitcoin price movements are entirely driven by its own innovations and remain overwhelmingly self-determined throughout the period, with a contribution of approximately 98.84%. In contrast, Ethereum and Litecoin display substantial exposure to cross-asset shocks. Ethereum's variance is initially explained largely by Bitcoin (56.64%) and itself (43.35%), with Bitcoin's contribution increasing over time, suggesting persistent dependence. Litecoin, while initially more influenced by its own shocks (62.17%), exhibits a declining self-contribution as the influence of Bitcoin and Ethereum gradually strengthens.

As the forecast horizon extends, the explanatory share of Bitcoin shocks decreases modestly, while the relative importance of each asset's own innovations becomes more pronounced. This transition implies that although Bitcoin serves as the primary information transmitter in the short run, market dynamics become relatively more distributed over longer horizons.

These VD findings are consistent with the IRF results and collectively reinforce the interpretation of a hierarchical interdependence structure in which Bitcoin functions as the central shock leader, whereas Ethereum and Litecoin primarily adjust to external disturbances rather than generating systemic impulses. This empirical pattern aligns with prior evidence documenting Bitcoin's dominant short-term influence within the cryptocurrency market [10].

3.4. Prediction using Fourier Series Estimator

Before performing forecasting, the model must first be constructed based on the in-sample data to obtain the appropriate functional form. In this study, the Fourier series estimator with a cosine basis was used to capture the fluctuation patterns in the time series data. The best model selection is determined by the optimal oscillation parameter (k), which is established based on the minimum Generalized Cross Validation (GCV) value. The smaller the GCV value, the better the model quality in representing the relationship between the response and predictor variables. Based on calculations using R software, the GCV, MSE, and R^2 values were obtained for various oscillation parameter (k) variations as shown in Table 11 below.

Table 11: Optimal k Selection

k	GCV	MSE	$\mathbf{R^2}$
1	0.0005489008	0.7015295	0.9522864
2	0.0005448030	0.6930499	0.9538582
3	0.0005461379	0.6915054	0.9540426
4	0.0005390503	0.6793381	0.9552702
5	**0.0005380751**	**0.6749293**	**0.9559479**

The results in Table 11 show that the smallest GCV value was obtained at $k = 5$ with a value of 0.0005380751, indicating that the cosine-based Fourier series model with five oscillations is the optimal choice. The MSE value of 0.6749293 is relatively low, while $R^2 = 0.9559479$ indicates that the model is able to explain more than 95% of the data variation. Thus, this model is considered the most accurate in predicting the research data.

The resulting simulation model meets the goodness-of-fit criteria based on test results. Parameter estimation was carried out using a nonparametric multivariate regression approach based on the cosine Fourier series, where the modeling process used the optimal number of oscillations $k = 5$. The selection of this k value was based on the minimum GCV result, ensuring that the resulting model can represent the data effectively. Therefore, an optimal regression model was obtained, suitable for the research dataset.

After obtaining the multivariate Fourier series model with a cosine basis and determining the optimal oscillation parameter, the next step is to perform parameter estimation to identify the contribution of each component in forming the model. This estimation includes the intercept value, time trend, and cosine coefficients at each oscillation level. The complete results of the parameter estimation are presented in the following Table 12.

Table 12: Estimation of Fourier Series Parameters

Parameter	BTC	ETH	LTC
$\frac{1}{2}\hat{\alpha}_{0j}$	-15,146.40	-14,740.27	-317.56
$\hat{\gamma}_j$	281.67	39.53	0.92
$\hat{\alpha}_{1j}$	276.49	23.27	0.13
$\hat{\alpha}_{2j}$	13.48	15.28	0.036
$\hat{\alpha}_{3j}$	-324.68	-4.35	0.22
$\hat{\alpha}_{4j}$	-426.19	-28.03	-0.72
$\hat{\alpha}_{5j}$	386.59	-30.21	0.46

The Fourier series estimator in this study is used to capture cyclical patterns in cryptocurrency price movements through sinusoidal components. The estimated parameters presented in Table 13 represent the amplitudes of the cosine terms that describe the periodic structure of the model. Since the Fourier model is applied as a deterministic approximation for forecasting purposes, the estimation focuses on the parameter values rather than statistical inference such as t-statistics or p-values. After obtaining the parameter estimates $\hat{\boldsymbol{\beta}}_\lambda$ with the optimal oscillation parameter $k = 5$, the estimated coefficients were substituted into the multiresponse Fourier regression model to obtain the specific prediction equations for each cryptocurrency. Let Y_{1t} denote Bitcoin (BTC), Y_{2t} denote Ethereum (ETH), and Y_{3t} denote Litecoin (LTC). The resulting estimated models are expressed as follows:

$$\begin{aligned}
\hat{Y}_{1t} &= \frac{1}{2}\hat{\alpha}_{01} + \hat{\gamma}_1 t_{i1} + \sum_{\lambda=1}^{5} \hat{\alpha}_{\lambda 1}\cos(\lambda t_i),\\
\hat{Y}_{2t} &= \frac{1}{2}\hat{\alpha}_{02} + \hat{\gamma}_2 t_{i2} + \sum_{\lambda=1}^{5} \hat{\alpha}_{\lambda 2}\cos(\lambda t_i),\\
\hat{Y}_{3t} &= \frac{1}{2}\hat{\alpha}_{03} + \hat{\gamma}_3 t_{i3} + \sum_{\lambda=1}^{5} \hat{\alpha}_{\lambda 3}\cos(\lambda t_i).
\end{aligned} \tag{6}$$

Based on the results in Table 12, each response variable shows different coefficient values, indicating the unique price movement characteristics of each cryptocurrency. The nonparametric multivariate regression using the cosine Fourier series estimator, whose general form is shown in Eq. (6), with $j = 1, 2, 3$ and $k = 5$.

Subsequently, this model is substituted with the estimated parameter values to obtain the specific equations for Bitcoin, Ethereum, and Litecoin where Y_{1t} represents Bitcoin, Y_{2t} represents Ethereum, and Y_{3t} represents Litecoin.

$$\begin{aligned}
\hat{y}_{1t} &= \tfrac{1}{2}(-15146.40) + 281.67t_i + 276.49\cos(t_i) + 13.48\cos(2t_i) - 324.68\cos(3t_i)\\
&\quad - 426.19\cos(4t_i) + 386.59\cos(5t_i),\\
\hat{y}_{2t} &= \tfrac{1}{2}(-14740.27) + 39.53t_i + 23.27\cos(t_i) + 15.28\cos(2t_i) - 4.35\cos(3t_i)\\
&\quad - 28.03\cos(4t_i) - 30.21\cos(5t_i),\\
\hat{y}_{3t} &= \tfrac{1}{2}(-317.56) + 0.92t_i + 0.13\cos(t_i) + 0.036\cos(2t_i) + 0.22\cos(3t_i)\\
&\quad - 0.72\cos(4t_i) + 0.46\cos(5t_i).
\end{aligned}$$

The prediction of cryptocurrency prices was carried out simultaneously on the testing data using a cosine-based Fourier series model. The prediction results indicate that the model is capable of providing a high level of accuracy, with an MSE value of 0.5709641, and an R^2 value of 0.9778967, demonstrating that the model can effectively capture the variation in cryptocurrency price movements and generate reliable forecasts for subsequent periods.

3.5. Comparison between VAR and Fourier Series Estimator Approach

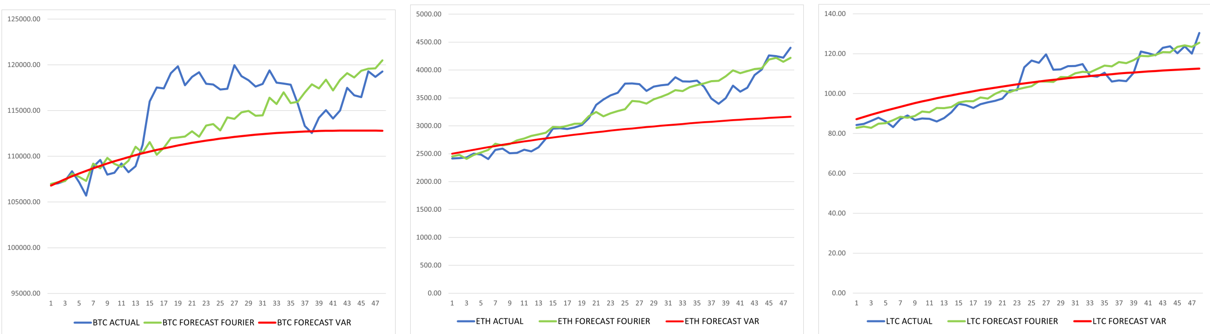
VAR models are powerful for multivariate time series analysis and forecasting, especially in economic contexts, but they can be limited by their linear assumptions and high dimensionality. On the other hand, nonparametric Fourier series offer flexibility and robustness for modeling periodic data, although they can be more complex to implement and interpret. Therefore, both approaches are considered in this study to compare their performance in modeling the observed time series data.

The evaluation of model prediction accuracy can be conducted using the Mean Absolute Percentage Error (MAPE). This measure represents the average relative error between the predicted values and the actual data. A smaller MAPE value indicates that the model has a higher level of accuracy in predicting the data. Therefore, comparing the MAPE values across different methods becomes an important step in determining the most optimal model. The comparison results of MAPE values between the Vector Autoregressive (VAR) method and the Fourier Series Estimator are presented in the following Table 13.

Table 13: Comparison of Testing MAPE

Method	**MAPE**
Vector Autoregressive (VAR)	8.503%
Fourier Series Estimator	3.768%

Based on the results in Table 13, it can be seen that the *Fourier Series Estimator* method has a *Mean Absolute Percentage Error* (MAPE) value of 3.768%, which is smaller than that of the *Vector Autoregressive (VAR)* method, which produces a MAPE value of 8.503%. The lower MAPE value indicates that the Fourier Series Estimator model provides more accurate predictions compared to the VAR model. In addition to the MAPE value comparison table, a visualization comparing the forecasting results of both methods for each variable using 10% testing data is also presented in the following Fig. 3.



(a) Bitcoin **(b)** Ethereum **(c)** Litecoin

Fig. 3: Comparison of Actual and Predicted Cryptocurrency Data Using the VAR and Fourier Series Methods

This graph shows the relationship between actual data and predictions generated by the VAR and Fourier methods. It is clear that the Fourier prediction line (green) more closely approximates the actual data pattern (blue) for the three variables BTC, ETH, and LTC, while the VAR prediction (red) tends to be more linear and less able to capture the dynamics of price fluctuations. This further confirms that for highly volatile cryptocurrency data, the Fourier series approach is superior in minimizing prediction errors compared to the VAR approach. Fourier is better at representing the ups and downs of the data, resulting in more accurate predictions.

4. Conclusion

This study compares the effectiveness of two multivariate time series analysis approaches, namely Vector Autoregressive (VAR) and Fourier Series Estimator based on cosine functions. The analysis results show that the Fourier Series Estimator method is superior to VAR in predicting time series data with complex fluctuation patterns. Validation using Bitcoin, Ethereum, and Litecoin price data after the 2024 Bitcoin halving event shows that Fourier produces a Mean Absolute Percentage Error (MAPE) value of 3.768%, which is lower than VAR. This finding confirms Fourier's ability to stably model nonlinear and periodic data dynamics. Methodologically, this research positions the Fourier Series Estimator as an efficient alternative for multivariate time series modeling, particularly in contexts with high volatility such as financial analysis. This study has several limitations. First, the analysis is restricted to a single post-halving period, which may not fully represent different market regimes. Second, forecast evaluation was conducted using a single data split and a single performance metric (MAPE). Third, structural identification in the VAR model depends on the selected variable ordering. Future research should incorporate longer time horizons, multiple forecast evaluation metrics, alternative model specifications, and formal forecast comparison tests such as the Diebold–Mariano test.

CRediT Authorship Contribution Statement

The contributions of each author are detailed based on the Contributor Roles Taxonomy (CRediT) as follows: **Dita Amelia**: Review of the supervision, guidance, and final manuscript. As the corresponding author and final contributor. **Rizky Dwi Kurnia Rahayu**: Data Curation, Formal Analysis, Writing-Review and Editing. **Bimo Okta Syahputra**: Methodology, formal analysis, data collection, and investigation.

Declaration of Generative AI and AI-assisted technologies

Generative AI tools were used during the preparation of this manuscript. Specifically, ChatGPT (version 4, OpenAI) was utilized to assist in language refinement, paraphrasing, and improving the clarity of certain sections. The final content was reviewed and approved by the authors to ensure accuracy and integrity.

Declaration of Competing Interest

The authors declare no conflict of interest.

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Data Availability

Data supporting the findings of this study were obtained from publicly accessible sources. Specifically, the historical cryptocurrency data used for the analysis were collected from the Investing.com website[1][2] [3]. This ensures transparency and facilitates the reproducibility of the study results.

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