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Box Fractal as an Iterated Function System in Fractal Interpolation for Determining the Approximate Value of Demand Data

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Abstract

A common problem in inventory planning is the uncertainty of demand. One technique for determining the demand approximation value is the fractal interpolation. The aim of this study is to develop a fractal interpolation technique using a Fractal Interpolated Function constructed by the affine function that forms the Box Fractal shape. The developed method is applied to interpolate rice demand data based on prices at a rice milling factory. Mean Absolute Percentage Error (MAPE) is used to measure the accuracy of the interpolation results. For the n^{th} iteration, the number of boxes formed is 5^n, and the number of pairs of points is 4×5^n. Based on the rice demand data from one of the factories, the best MAPE was obtained at the 6^{th} iteration, with a value of 16.319%, which falls into the good category. Based on the data used, the affine function forming the Box Fractal as a Fractal Interpolated Function can be applied in fractal interpolation techniques.

Keywords: Box Fractal; Demand; Fractal; Fractal Interpolation.

1. Introduction

Fractal interpolation is a technique for estimating values between two known values using the concept of fractals. Fractal interpolation and fractal concept has been applied in various fields. Among them is the application of fractal interpolation to seismic problems [1], time series data analysis [2–4], deep learning analyze [5], analyzing mass transfer in shrimp during convective drying [6], analysis of COVID-19 spread based [7], and in the financial sector [8]. The development of fractal interpolation is carried out by determining the variation of the vertical scaling factor [9–11]. In addition, development can be carried out on Fractal Interpolated Function (FIF) [12, 13]. FIF development can use nonlinear functions [10, 14] and affine function that forms a fractal shape. The affine function that forms the Sierpinski triangle is used as the FIF [15]. In the [16], the Sierpinski Carpet is used as the FIF. The results of developments [15] and [16] were applied to interpolate rice demand data. Based on the results of study [15], the MAPE value was obtained at 23.77% within the sufficient criteria and was carried out in 6 iterations. The results of study [16] obtained a MAPE value of 7.15% within the very good criteria. The results

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of this study illustrate that the selection of fractal shapes in formulating FIF greatly affects the interpolation results. In this study, an affine function is developed to form a box fractal structure as an FIF. The results of this development are applied to rice demand data at a rice milling factory. The novelty in this study is that the FIF used in fractal interpolation is an affine function that forms a box fractal structure to interpolate rice demand data.

2. Methods

In this study, a fractal interpolation scheme was developed with an affine FIF, that is a box fractal forming function. The FIF construct the Box Fractal is five affine functions defined as follows.

$$w_1\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}\frac{1}{3}&0\\0&\frac{1}{3}\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}+\begin{bmatrix}0\\0\end{bmatrix} \tag{1}$$

$$w_2\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}\frac{1}{3}&0\\0&\frac{1}{3}\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}+\begin{bmatrix}0\\\frac{2}{3}\end{bmatrix} \tag{2}$$

$$w_3\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}\frac{1}{3}&0\\0&\frac{1}{3}\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}+\begin{bmatrix}\frac{1}{3}\\\frac{1}{3}\end{bmatrix} \tag{3}$$

$$w_4\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}\frac{1}{3}&0\\0&\frac{1}{3}\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}+\begin{bmatrix}\frac{2}{3}\\0\end{bmatrix} \tag{4}$$

$$w_5\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}\frac{1}{3}&0\\0&\frac{1}{3}\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}+\begin{bmatrix}\frac{2}{3}\\\frac{2}{3}\end{bmatrix} \tag{5}$$

The function w_1, w_2, w_3, w_4, w_5 are mapped the initial box into a congruent box so that the size of the box is $\frac{1}{3}$ smaller than the shape produced in the previous iteration. Beside that, w_2 translates the shape along the vertical axis by a factor of $\frac{2}{3}$ units. Subsequently, w_3 is mapping the original shape to a new box and shifting it $\frac{1}{3}$ units in both the horizontal and vertical directions to the right. Then, w_4 shifts the box horizontally by $\frac{2}{3}$ units. The last, w_5 maps the original shape to a new box and shifts it $\frac{2}{3}$ units in both the horizontal and vertical directions to the right.

The fractal interpolation calculation algorithm is given as follows.

1. Given the initial data $\{(x_i, y_i)^T \in R^2 : i = 1, 2, 3, 4\}, x_1 < x_2 < x_3 < x_4$ and the number of iterations desired. The selection of initial conditions is based on the consideration that the box region described from the four initial conditions will include all values in the data or dummy. The four pairs of points selected must form a regular box shape.
2. Determine distance between x_1, x_2, x_3, x_4 and distance y_1, y_2, y_3, y_4

$$\begin{aligned} &d_1 = |x_1 - x_2|, && d_2 = |x_1 - x_3|, && d_3 = |x_1 - x_4|, && d_4 = |x_2 - x_3|,\\ &d_5 = |x_2 - x_4|, && d_6 = |x_3 - x_4|, && d_7 = |y_1 - y_2|, && d_8 = |y_1 - y_3|,\\ &d_9 = |y_1 - y_4|, && d_{10} = |y_2 - y_3|, && d_{11} = |y_2 - y_4|, && d_{12} = |y_3 - y_4| \end{aligned}$$

3. Determine $w_1\left(\begin{bmatrix}x^*\\y^*\end{bmatrix}\right)$ based on Eq. (1), $w_2\left(\begin{bmatrix}x^*\\y^*\end{bmatrix}\right)$ based on Eq. (2), $w_3\left(\begin{bmatrix}x^*\\y^*\end{bmatrix}\right)$ based on Eq. (3), $w_4\left(\begin{bmatrix}x^*\\y^*\end{bmatrix}\right)$ based on Eq. (4), and $w_5\left(\begin{bmatrix}x^*\\y^*\end{bmatrix}\right)$ based on Eq. (5).
4. We get a new box $B^* = \bigcup_{i=1}^{5} w_i(B^*) = w_1(B^*) \cup w_2(B^*) \cup w_3(B^*) \cup w_4(B^*) \cup w_5(B^*)$.
5. Proceed back to Step 2

6. B^* is now a new box involving the interpolated points. The iteration stops when a very good MAPE value is obtained.

3. Results and Discussion

This section reports the theoretical and empirical results of the proposed approach. We first formulate the Box Fractal mappings within the IFS framework and then apply the resulting FIF to the rice demand dataset to assess interpolation accuracy.

3.1. Fractal Interpolation Function

In fractal interpolation theory, an Iterated Function System (IFS) is defined as a set of contractive mappings used to construct a Fractal Interpolation Function (FIF) as its attractor. An attractor is a fixed set or curve obtained through infinite iterations of an IFS and remains invariant under the application of the system. In other words, the desired fractal curve, namely the FIF, is the attractor of the IFS. Given a complete metric space (X, d) and a finite family of contractions $(f_i)_{i\in I}$, where $f_i : X \to X$, there exists a unique nonempty compact subset A of X such that

$$A = \bigcup_{i\in I} f_i(A).$$

The set A is called the attractor of the IFS

$$S = ((X, d), (f_i)_{i\in I}) \text{ [12]}.$$

Given

$$Z = \{(x_i, y_i)^T \in I \times [\sigma, \varphi] : i = 0, 1, \ldots, N\},$$

where $I = [x_0, x_N] \subset \mathbb{R}$ is a closed interval and $\mathbb{R}$ denotes the set of all real numbers, consider a continuous function

$$f_i : I \to [\sigma, \varphi]$$

that interpolates the data points such that

$$f(x_i) = y_i, \quad \forall i = 0, 1, \ldots, N.$$

On the compact set

$$K = I \times [\sigma, \varphi],$$

the collection of continuous mappings

$$\{w_n : K \to K,\ n = 1, 2, \ldots, N\}$$

forms an IFS whose attractor G is the graph

$$G = \{(x, f(x))^T : x \in I\}.$$

Furthermore, the IFS is defined by

$$w_n(x, y) = (L_n(x),\ F_n(x, y)^T), \quad \forall n = 1, 2, \ldots, N,$$

which satisfies the following conditions:

$$L_n(x_0) = x_{n-1}, \quad L_n(x_N) = x_n,$$

$$|L_n(x_1) - L_n(x_2)| \le \lambda |x_1 - x_2|,$$

$$F_n(x_0) = y_{n-1}, \quad F_n(x_N, y_N) = y_n,$$

$$|F_n(x,\psi_1) - F_n(x,\psi_2)| \leq \nu|\psi_1 - \psi_2|,$$

for all $x_1, x_2 \in I$, all $\psi_1, \psi_2 \in [\sigma, \varphi]$, and some

$$0 < \lambda < 1, \quad 0 < \nu < 1 \text{ [17]}.$$

The existence and uniqueness of attractors from an FIF in fractal interpolation have been discussed in previous studies [18, 19]. Based on these results, a fractal interpolation technique was developed using a Fractal Interpolation Function (FIF) constructed by an affine function that forms a box fractal.

Proposition 1. *The affine mapping on a compact set*

$$B^* = \bigcup_{i=1}^{5} W_i(B^*)$$

is contractive.

Proof. Given the initial data

$$\{(x_i, y_i)^T \in \mathbb{R}^2 : i = 1,2,3,4\}, \quad x_1 < x_2 < x_3 < x_4,$$

and the collection of affine mappings

$$\{w_n : K \to K, \ n = 1,2,3,4,5\},$$

defined by

$$w_n \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{3} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} c \\ d \end{pmatrix}, \qquad 0 < c, d < 1.$$

Take two points on B^*, namely (x_1, y_1) and (x_2, y_2). Then, for simplicity of notation, the expression below is written in vector form.

$$w_n(x) - w_n(y) = \begin{pmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{3} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}.$$

Using Euclidean norm, we obtain

$$\|w_n(x) - w_n(y)\|^2 = \left(\frac{1}{3}\right)^2 (x_1 - y_1)^2 + \left(\frac{1}{3}\right)^2 (x_2 - y_2)^2.$$

For example, let

$$\nu = \max\left(\frac{1}{3}, \frac{1}{3}\right) = \frac{1}{3},$$

where $0 < \nu < 1$. This implies that

$$\|w_n(x) - w_n(y)\| < \nu\|x - y\|,$$

and therefore it can be concluded that w_n is contractive. □

3.2. Application of fractal interpolation

The results of the development were applied to interpolate rice demand data in the rice milling factory. Rice price and demand data are presented in Table 1.

Table 1: Rice Price and Demand Data

Period	Price (IDR)	Demand (Kg)
March 2020	8,900	46000
August 2020	9,000	42000
March 2021	9,300	48000
August 2021	10,300	41000
March 2022	10,500	63000
August 2022	10,700	52000
March 2023	11,000	51000
August 2023	10,600	47000
March 2024	11,700	57000
August 2024	12,000	40000
March 2025	12,500	40000
August 2025	12,700	37000

The data in Table 1 are the inventory data from the last five years, namely 2020 to 2025. The data period is March 2020 and August 2020 to March 2025 and August 2025. March and August are the main harvest periods. Based on the data in Table 1, the pairs of points selected as original conditions are (8900, 46000), (10700, 52000), (12700, 37000), (14500, 43000). The point pair (14500, 43000) is a dummy variable taken with the aim of forming a box. The dummy point is introduced solely to ensure that the initial conditions form a regular box required by the Box Fractal construction. This point is not included in the MAPE evaluation and does not represent observed demand data. Therefore, its role is limited to defining the geometric boundary of the interpolation domain rather than influencing the evaluation of interpolation accuracy. The selection of dummy point is based on the consideration that the iteration process can be carried out if the boxes formed are regular. In this case, the dummy is taken outside the known data interval with the aim that all data can be covered in the box area formed. The horizontal axis is price and the vertical axis is demand. The choice of original condition is not unique, alternative pairs of points can be chosen as original conditions. The visualization of the box fractal based on the original conditions is provided in Fig. 1 and Fig. 2.

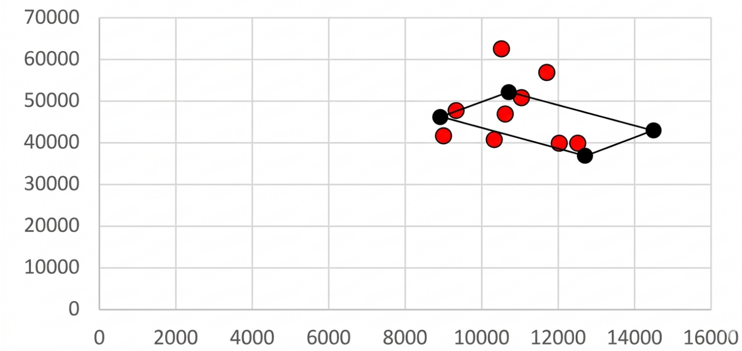

Fig. 1: Data Visualization based on Table 1

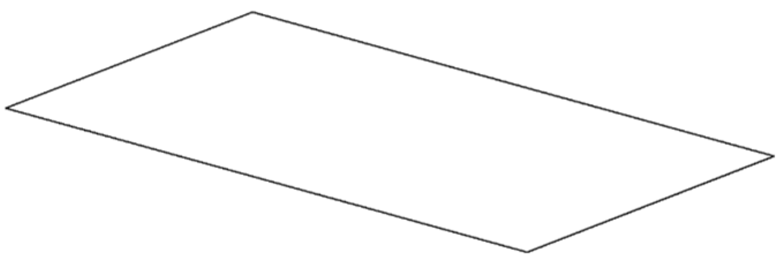

Fig. 2: Box Fractal Iteration 0

There are 4 pairs of points outside the box namely (9000,42000), (10300,41000), (10500, 63000) and (11700, 57000). These points will be used for the MAPE calculation. The following is the calculation of fractal interpolation iteration 1.

1. Let

$$x_1 = 8900,\ y_1 = 46000,$$
$$x_2 = 10700,\ y_2 = 52000,$$
$$x_3 = 12700,\ y_3 = 37000,$$
$$x_4 = 14500,\ y_4 = 43000$$

2. Determine the distance to the x and y axes

$$d_1 = |x_1 - x_2| = |8900 - 10700| = 1800$$
$$d_2 = |x_1 - x_3| = |8900 - 12700| = 3800$$
$$d_3 = |x_1 - x_4| = |8900 - 14500| = 5600$$
$$d_4 = |x_2 - x_3| = |10700 - 12700| = 2000$$
$$d_5 = |x_2 - x_4| = |10700 - 14500| = 3800$$
$$d_6 = |x_3 - x_4| = |12700 - 14500| = 1800$$
$$d_7 = |y_1 - y_2| = |46000 - 52000| = 6000$$
$$d_8 = |y_1 - y_3| = |46000 - 37000| = 9000$$
$$d_9 = |y_1 - y_4| = |46000 - 43000| = 3000$$
$$d_{10} = |y_2 - y_3| = |52000 - 37000| = 15000$$
$$d_{11} = |y_2 - y_4| = |52000 - 43000| = 9000$$
$$d_{12} = |y_3 - y_4| = |37000 - 43000| = 6000$$

   The distances $d_1, d_2, d_3, d_4, d_5, d_6$ are the distance calculation for the horizontal axis while $d_7, d_8, d_9, d_{10}, d_{11}, d_{12}$ are the vertical axis.

3. Determine fractal interpolation based on FIF Eq. (1) to Eq. (5). An illustration of the calculation process and application of FIF 1 to 5 based on Fig. 2 is given in Fig. 3 below.

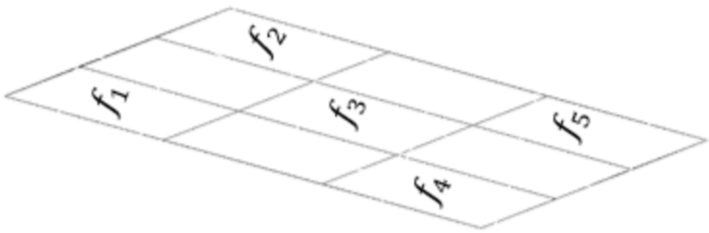

Fig. 3: An illustration Box Fractal Iteration 1

   All affine mappings $w_1, \ldots, w_5$ are applied independently to the same set of points in each iteration (parallel implementation). The equalities shown below indicate geometric equivalence (coincident coordinates) in the geometric construction and should not be interpreted as sequential function composition. Each iteration applies all mappings to the same current point set; no mapping output is recursively used as input within the same iteration step.

$$f_1(x_1^*, y_1^*) = (x_1, y_1) = (8900, 46000)$$
$$f_1(x_2^*, y_2^*) = (x_1 + \tfrac{1}{3}|x_1 - x_2|,\ y_1 + \tfrac{1}{3}|y_1 - y_2|)$$
$$= (8900 + \tfrac{1}{3}|8900 - 10700|,\ 46000 + \tfrac{1}{3}|46000 - 52000|)$$

$$
\begin{aligned}
&= \left(8900 + \tfrac{1}{3}(1800),\ 46000 + \tfrac{1}{3}(6000)\right) \\
&= (9500,\ 48000) \\
f_1(x_3^*, y_3^*) &= \Big(x_1 + \tfrac{1}{3}|x_1 - x_3|,\ y_1 + \tfrac{1}{3}|y_1 - y_3|\Big) \\
&= \Big(8900 + \tfrac{1}{3}|8900 - 12700|,\ 46000 + \tfrac{1}{3}|46000 - 37000|\Big) \\
&= \Big(8900 + \tfrac{1}{3}(3800),\ 46000 + \tfrac{1}{3}(9000)\Big) \\
&= (10167,\ 43000) \\
f_1(x_4^*, y_4^*) &= \Big(x_1 + \tfrac{1}{3}|x_1 - x_4|,\ y_1 - \tfrac{1}{3}|y_1 - y_4|\Big) \\
&= \Big(8900 + \tfrac{1}{3}|8900 - 14500|,\ 46000 - \tfrac{1}{3}|46000 - 43000|\Big) \\
&= \Big(8900 + \tfrac{1}{3}(5600),\ 46000 - \tfrac{1}{3}(3000)\Big) \\
&= (10767,\ 45000)
\end{aligned}
$$

$$
\begin{aligned}
f_2(x_1^*, y_1^*) &= \Big(x_1 + \tfrac{2}{3}|x_1 - x_2|,\ y_1 + \tfrac{2}{3}|y_1 - y_2|\Big) \\
&= \Big(8900 + \tfrac{2}{3}|8900 - 10700|,\ 46000 + \tfrac{2}{3}|46000 - 52000|\Big) \\
&= \Big(8900 + \tfrac{2}{3}(1800),\ 46000 + \tfrac{2}{3}(6000)\Big) \\
&= (10100, 50000) \\
f_2(x_2^*, y_2^*) &= (x_2, y_2) = (10700, 52000) \\
f_2(x_3^*, y_3^*) &= \Big(x_2 + \tfrac{1}{3}|x_2 - x_3|,\ y_2 - \tfrac{1}{3}|y_2 - y_3|\Big) \\
&= \Big(10700 + \tfrac{1}{3}|10700 - 12700|,\ 52000 - \tfrac{1}{3}|52000 - 37000|\Big) \\
&= \Big(10700 + \tfrac{1}{3}(2000),\ 52000 - \tfrac{1}{3}(15000)\Big) \\
&= (11367,\ 47000) \\
f_2(x_4^*, y_4^*) &= \Big(x_2 + \tfrac{1}{3}|x_2 - x_4|,\ y_2 - \tfrac{1}{3}|y_2 - y_4|\Big) \\
&= \Big(10700 + \tfrac{1}{3}|10700 - 14500|,\ 52000 - \tfrac{1}{3}|52000 - 43000|\Big) \\
&= \Big(10700 + \tfrac{1}{3}(3800),\ 52000 - \tfrac{1}{3}(9000)\Big) \\
&= (11967,\ 49000)
\end{aligned}
$$

$$
\begin{aligned}
f_3(x_1^*, y_1^*) &= f_1(x_4^*, y_4^*) = (10767,\ 45000) \\
f_3(x_2^*, y_2^*) &= f_2(x_3^*, y_3^*) = (11367,\ 47000) \\
f_3(x_3^*, y_3^*) &= \Big(x_3 - \tfrac{1}{3}|x_2 - x_3|,\ y_3 + \tfrac{1}{3}|y_2 - y_3|\Big) \\
&= \Big(12700 - \tfrac{1}{3}|10700 - 12700|,\ 37000 + \tfrac{1}{3}|52000 - 37000|\Big) \\
&= \Big(12700 - \tfrac{1}{3}(2000),\ 37000 + \tfrac{1}{3}(15000)\Big) \\
&= (12033,\ 42000) \\
f_3(x_4^*, y_4^*) &= \Big(x_4 - \tfrac{1}{3}|x_1 - x_4|,\ y_4 + \tfrac{1}{3}|y_1 - y_4|\Big) \\
&= \Big(14500 - \tfrac{1}{3}|8900 - 14500|,\ 43000 + \tfrac{1}{3}|46000 - 43000|\Big) \\
&= \Big(14500 - \tfrac{1}{3}(5600),\ 43000 + \tfrac{1}{3}(3000)\Big)
\end{aligned}
$$

$$= (12633,\ 44000)$$

$$\begin{aligned}
f_4(x_1^*, y_1^*) &= \left(x_1 + \tfrac{2}{3}|x_1 - x_3|,\ y_1 - \tfrac{2}{3}|y_1 - y_3|\right) \\
&= \left(8900 + \tfrac{2}{3}|8900 - 12700|,\ 46000 - \tfrac{2}{3}|46000 - 37000|\right) \\
&= \left(8900 + \tfrac{2}{3}(3800),\ 46000 - \tfrac{2}{3}(9000)\right) \\
&= (11433,\ 40000) \\
f_4(x_2^*, y_2^*) &= \left(x_3 - \tfrac{1}{3}|x_2 - x_3|,\ y_3 + \tfrac{1}{3}|y_2 - y_3|\right) \\
&= \left(12700 - \tfrac{1}{3}|10700 - 12700|,\ 37000 + \tfrac{1}{3}|52000 - 37000|\right) \\
&= \left(12700 - \tfrac{1}{3}(2000),\ 37000 + \tfrac{1}{3}(15000)\right) \\
&= (12033,\ 42000) \\
f_4(x_3^*, y_3^*) &= (x_3, y_3) = (12700,\ 37000) \\
f_4(x_4^*, y_4^*) &= \left(x_3 + \tfrac{1}{3}|x_4 - x_3|,\ y_3 + \tfrac{1}{3}|y_4 - y_3|\right) \\
&= \left(12700 + \tfrac{1}{3}|14500 - 12700|,\ 37000 + \tfrac{1}{3}|43000 - 37000|\right) \\
&= \left(12700 + \tfrac{1}{3}(1800),\ 37000 + \tfrac{1}{3}(6000)\right) \\
&= (13300,\ 39000)
\end{aligned}$$

$$\begin{aligned}
f_5(x_1^*, y_1^*) &= \left(x_4 - \tfrac{1}{3}|x_1 - x_4|,\ y_4 + \tfrac{1}{3}|y_1 - y_4|\right) \\
&= \left(14500 - \tfrac{1}{3}|8900 - 14500|,\ 43000 + \tfrac{1}{3}|46000 - 43000|\right) \\
&= \left(14500 - \tfrac{1}{3}(5600),\ 43000 + \tfrac{1}{3}(3000)\right) \\
&= (12633,\ 44000) \\
f_5(x_2^*, y_2^*) &= \left(x_4 - \tfrac{1}{3}|x_2 - x_4|,\ y_4 + \tfrac{1}{3}|y_2 - y_4|\right) \\
&= \left(14500 - \tfrac{1}{3}|10700 - 14500|,\ 43000 + \tfrac{1}{3}|52000 - 43000|\right) \\
&= \left(14500 - \tfrac{1}{3}(3800),\ 43000 + \tfrac{1}{3}(9000)\right) \\
&= (13233,\ 46000) \\
f_5(x_3^*, y_3^*) &= \left(x_3 + \tfrac{2}{3}|x_4 - x_3|,\ y_3 + \tfrac{2}{3}|y_4 - y_3|\right) \\
&= \left(12700 + \tfrac{2}{3}|14500 - 12700|,\ 37000 + \tfrac{2}{3}|43000 - 37000|\right) \\
&= \left(12700 + \tfrac{2}{3}(1800),\ 37000 + \tfrac{2}{3}(6000)\right) \\
&= (13900,\ 41000) \\
f_5(x_4^*, y_4^*) &= (x_4, y_4) = (14500,\ 43000)
\end{aligned}$$

The results of the first iteration calculation are given in Table 2.

Table 2: Fractal Interpolation Result 1st Iteration

Price (IDR)	Demand (Kg)	Demand Interpolation (Kg)
8900	46000	46000
		46000
9000	42000	
9300	48000	
10300	41000	
10500	63000	
10700	52000	52000
		52000
11000	51000	
10600	47000	
11700	57000	
12000	40000	
12500	40000	
12700	37000	37000
		37000
14500		43000
		43000
9500		48000
10767		45000
		45000
10167		43000
10100		50000
11367		47000
		47000
11967		49000
12633		44000
		44000
11433		40000
12033		42000
		42000
13233		46000
13300		39000
13900		41000

In Iteration 1, twenty pairs of points were obtained which will form five boxes. For the n^{th} iteration, the number of boxes formed is 5^n and 4×5^n pairs of points. The calculation continues to the next iteration using the results of the previous iteration. Iterations are carried out until the desired accuracy value is obtained. The visualization and the results of Iteration 5 and 6 are given in Fig. 4, Fig. 5 and Table 3.

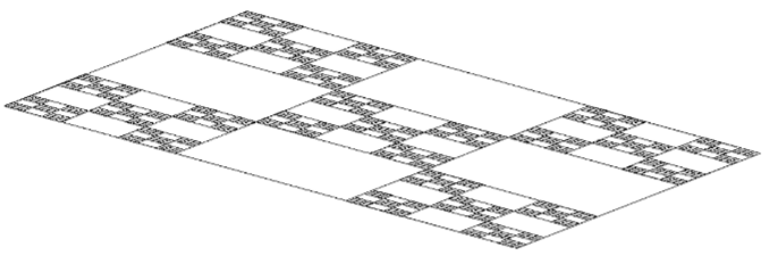

Fig. 4: Box Fractal Iteration 5

Fig. 4 is a visualization of the 5^{th} iteration. There are 5^5 boxes and 4×5^5 pairs of points are formed. While in Fig. 5, there are 5^6 boxes and 4×5^6 pairs of points. The resulting point pairs are the result of fractal interpolation.

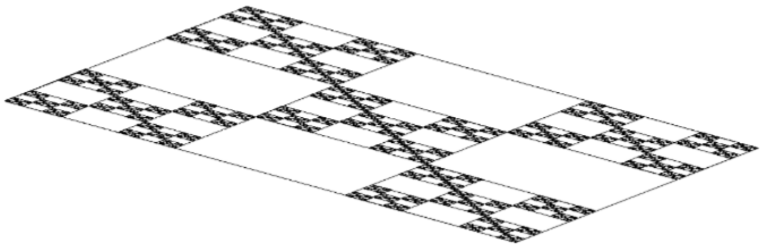

Fig. 5: Box Fractal Iteration 6

Table 3: Fractal Interpolation Result Iteration 5 and Iteration 6

Price (IDR)	Demand (Kg)	Demand Interpolation	
		Iteration 5	**Iteration 6**
8,900	46000	46000	46000
9,000	42000	45975	45778
9,300	48000	47333	47333
10,300	41000	50667	50667
10,500	63000	51333	51333
10,700	52000	52000	52000
11,000	51000	45420	51247
10,600	47000	51309	44593
11,700	57000	48914	48914
12,000	40000	42247	41741
12,500	40000	43556	43556
12,700	37000	37000	37000
MAPE		**16.437%**	**16.319%**

The following presents the MAPE calculation for the 5th iteration:

$$\begin{aligned}\text{MAPE} &= \frac{100\%}{4}\left(\left|\frac{42000-45975}{42000}\right| + \left|\frac{41000-50667}{41000}\right| + \left|\frac{63000-51333}{63000}\right| + \left|\frac{57000-48914}{57000}\right|\right)\\ &= 16.437\%.\end{aligned}$$

The MAPE obtained for the 5^{th} iteration is 16.437%. Using the same calculation, the MAPE obtained for the 6^{th} iteration is 16.319%. The best MAPE value, namely 16.319%, is achieved at Iteration 6, which falls into the good category. This indicates that the fractal interpolation technique using a Fractal Interpolation Function (FIF) based on the Box Fractal structure can effectively approximate demand values within the observed data range.

4. Conclusion

For the n^{th} iteration, the number of boxes formed is 5^n and 4×5^n pairs of points. Based on the rice demand data and demand interpolation results at one of the factories, the best MAPE was obtained at the 6^{th} Iteration, which was 16.319%. The MAPE value is in the good category. Based on the research data used, the development of fractal interpolation using box fractals can be used as a Fractal Interpolated Function and can provide good interpolation performance for the observed data.

CRediT Authorship Contribution Statement

Eka Susanti: Conceptualization, Methodology, Writing–Original Draft. **Oki Dwipurwani:** Data Curation, Formal Analysis. **Dian Cahyawati:**Validation, Investigation. **Novi Rustiana Dewi:** Supervision, Project Administration, Funding Acquisition. **Husnul Khotimah:** Writing–Review & Editing. **Wahyuni Apria Ningsih:** Software, Visualization.

Declaration of Generative AI and AI-assisted technologies

No artificial intelligence tools were used in the preparation of this research. All ideas, analyses, and conclusions presented in this paper are solely the authors' original work

Declaration of Competing Interest

The authors declare that they have no competing interests.

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Data and Code Availability

The data were derived from five years of inventory logs (2020-2025) from a rice milling factory. March and August serve as the main harvest periods. They include confidential business information and are available from the corresponding author upon reasonable request and subject to confidentiality constraints. Additional details concerning the dataset are provided in Section 3.

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