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ARIMA–GGJR–GARCH Modeling of Asymmetric Conditional Volatility in Wind Speed Time Series

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Abstract

Volatility modeling plays a crucial role in time-series forecasting, particularly for wind speed, where variability and asymmetric responses to shocks are commonly observed. Accurate wind speed forecasting can help mitigate potential risks associated with extreme or uncontrolled wind events. While the Autoregressive Integrated Moving Average (ARIMA) model is widely used to model the conditional mean of time series, it does not capture time-varying volatility or asymmetric effects. To address this limitation, we combine ARIMA with Generalized Autoregressive Conditional Heteroskedasticity (GARCH) and asymmetric extensions, including the Glosten-Jagannathan-Runkle GARCH (GJR-GARCH) and its generalized form (GGJR-GARCH). This framework allows simultaneous modeling of the conditional mean and conditional variance, accommodating heteroscedasticity and leverage effects in wind speed data. The empirical results indicate that negative shocks exert a stronger impact on conditional volatility than positive shocks, confirming the presence of asymmetry. Based on forecasting performance evaluation, the ARIMA(2,0,1)-GGJR-GARCH(1,1) specification provides the most accurate predictions among the competing models.

Keywords: ARIMA-GGJR-GARCH model; Asymmetric Conditional Volatility; Conditional Heteroscedasticity; Wind Speed Time Series

1. Introduction

Wind refers to the movement of air from a region of higher air pressure to a region of lower air pressure. Wind is a renewable energy source that is environmentally friendly and can support sustainable development [1]. In wind power plants, wind energy is primarily used to drive turbines that generate clean electrical energy. In addition, wind plays an important role in transportation and aviation and contributes to natural processes such as plant pollination. Thus, wind energy is not only an alternative energy source but also supports sustainability across various sectors [2–4]. Compared with fossil fuels such as petroleum and coal, wind energy produces significantly lower emissions and has a smaller environmental impact. However, wind energy is inherently variable and fluctuating, which may create operational risks if not properly managed [5]. Therefore, accurate estimation and prediction of wind characteristics are essential to optimize energy utilization while minimizing potential adverse impacts. Wind movement and speed exhibit substantial variability over time and space, requiring stochastic modeling approaches to capture

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their dynamic behavior. This complexity arises from the highly irregular and uncertain nature of wind speed, which is often difficult to predict accurately. Consequently, reliable predictive models are needed to improve wind speed forecasting performance [6]. Meteorological variables such as humidity, temperature, rainfall, and atmospheric pressure gradients are known to influence wind speed dynamics [7, 8]. Hence, predictive models should consider these influencing factors and their potential interactions.

Prediction refers to the process of estimating future values using historical observations and currently available information. In this context, time series analysis plays a central role in modeling temporal dependence and projecting future behavior based on past data patterns [9]. Weather related prediction has become an important research area due to its substantial influence on various aspects of daily life and economic activities. Because weather systems are inherently uncertain, predictive models should incorporate measures of uncertainty to support risk aware decision making and improve preparedness for possible outcomes [10]. Volatility forecasting represents a key component of such analyses, as it captures the dynamic variability and uncertainty present in many real world processes. In the energy sector, volatility prediction is relevant for understanding fluctuations in oil prices, energy consumption, electricity pricing, and power generation from both conventional and renewable sources. In meteorological applications, volatility modeling is particularly important for wind speed analysis because variability in wind directly affects energy production efficiency [1]. Numerous wind speed forecasting approaches have been proposed to improve predictive accuracy, including statistical time series models, machine learning methods, and hybrid modeling strategies that combine multiple analytical frameworks [11, 12].

In measuring wind speed volatility, it is essential to consider the intrinsic characteristics of wind data. One important feature is time varying heteroscedasticity, reflecting changes in variance over time, a phenomenon also observed in other complex stochastic systems [13]. Previous studies have reported that wind speed series often exhibit persistence, volatility clustering, and asymmetric responses to shocks, in addition to heteroscedastic behavior [14, 15]. Fluctuations in wind speed are commonly modeled using mean processes such as the ARIMA (Autoregressive Integrated Moving Average) framework. However, relying solely on a mean model is insufficient to capture the full stochastic structure of wind speed variability. Empirical evidence suggests that higher wind speeds tend to be associated with increased variability, indicating the presence of conditional heterogeneity in the variance process [16]. In time series analysis, volatility is typically defined as the conditional standard deviation, which is not directly observable but can be modeled through appropriate variance equations [17–19]. Modeling volatility provides insight into how the dispersion of observations evolves over time and how fluctuations respond to past shocks. Therefore, an appropriate wind speed modeling framework should integrate both the conditional mean structure and the dynamic volatility component to accurately represent changes in wind speed behavior.

The Autoregressive Conditional Heteroskedasticity (ARCH) model was first introduced by [20] to capture time-varying conditional variance in financial time series. Later, [21] extended this framework to the Generalized ARCH (GARCH) model, which incorporates lagged conditional variances and has been widely applied in empirical studies. In standard ARCH and GARCH specifications, volatility depends only on the magnitude of past innovations. Consequently, these models imply a symmetric response, whereby positive and negative shocks of equal size generate identical effects on conditional variance. Empirical evidence, however, suggests that volatility often reacts asymmetrically to shocks. To accommodate this feature, [22] proposed the GJR-GARCH model, which introduces an indicator function to capture leverage effects. Subsequently, [23] proposed a generalized extension, referred to as the GGJR-GARCH model, to enhance flexibility in modeling complex asymmetric volatility dynamics.

Several previous studies have employed both homoscedastic and heteroscedastic models in wind speed time series analysis. For instance, [24] used an ARIMA model to capture the conditional mean and ARCH and GARCH models to represent volatility, reporting that the ARIMA-GARCH specification provided superior forecasting performance. Similarly, [25] modeled wind speed using an ARMA Component GARCH-M framework. In another study, [26] compared MSARIMA, MSARIMA-GARCH, and MSARIMA-EGARCH models for wind speed forecasting and found that MSARIMA-GARCH delivered the most accurate results in supporting operational management and wind energy integration in India. To account for asymmetric volatility, [2] employed the Benchmark Symmetric Curve (BSC) and Asymmetric Curve Index (ACI) to analyze volatility dynamics in wind power forecasting. Their findings indicated that negative shocks exerted a weaker impact on volatility than positive shocks. Additionally, [27] applied ARIMA and GARCH models to meteorological variables, including temperature and humidity, and concluded that the ARIMA-GARCH model achieved higher predictive accuracy. More recently, [1] implemented a Finite Mixture GARCH model estimated using the Expectation Maximization (EM) algorithm for wind speed forecasting, demonstrating improved volatility modeling compared with ARMA and ARMA-GARCH models. Likewise, [9] applied the ARCH model to analyze and forecast wind speed fluctuations in Ireland and reported satisfactory predictive performance.

This study proposes a novel two component approach to forecast the mean and variance of wind speed time series data. This approach combines the Autoregressive Moving Average (ARMA) model with an asymmetric GARCH model. Unlike previous studies that primarily focused on wind speed prediction, this study emphasizes volatility forecasting and examines the performance of volatility estimation within the GARCH family, particularly the GGJR-GARCH model. Evaluating volatility forecasts is challenging because true volatility is latent and cannot be directly observed. Furthermore, this study highlights the asymmetric characteristics of wind speed volatility, which remain relatively underexplored in the existing literature. The GGJR-GARCH model is employed to capture asymmetric volatility dynamics with greater flexibility.

The remainder of this paper is organized as follows. Section 2 describes the methodology used in the empirical analysis and introduces the ARMA model together with the asymmetric GARCH type model applied in this study. Section 3 presents the descriptive statistics of the sample and reports the results of the empirical analysis, including a comparison of forecasts from different estimation models. Finally, Section 4 presents the conclusions of this paper.

2. Methods

This study employs a time series modeling framework to jointly analyze conditional mean and conditional variance. The procedure consists of (i) stationarity assessment and preliminary analysis, (ii) specification of an ARMA model for the mean process, (iii) diagnostic testing for conditional heteroskedasticity, and (iv) estimation of symmetric and asymmetric GARCH-type models when volatility clustering is detected. The final model integrates the selected ARMA structure with an asymmetric GARCH specification. Parameter estimation is conducted using maximum likelihood estimation (MLE), followed by model selection using information criteria and evaluation of forecasting performance.

2.1. ARMA Model for Conditional Mean

For notational simplicity, the theoretical development in this section is presented for the ARMA(1,1)-GGJR-GARCH(1,1) specification. This formulation serves as a representative case that captures the essential interaction between the conditional mean and conditional variance. The results can be extended in a straightforward manner to more general ARMA(p, q) structures without altering the underlying modeling framework. The conditional mean of the process is modeled using an ARMA$(1, 1)$ specification. Let $\{\varepsilon_t\}$ denote the innovation process [28, 29]. The model is defined as

$$Y_t = c + \phi_1 Y_{t-1} + \theta_1 \varepsilon_{t-1} + \varepsilon_t, \tag{1}$$

where $\{\varepsilon_t\}$ is a sequence of innovations satisfying

$$\mathbb{E}[\varepsilon_t \mid \mathcal{F}_{t-1}] = 0, \quad \mathrm{Var}(\varepsilon_t \mid \mathcal{F}_{t-1}) = \sigma_t^2.$$

Thus, the conditional mean is given by

$$\mu_t = \mathbb{E}[Y_t \mid \mathcal{F}_{t-1}] = c + \phi_1 Y_{t-1} + \theta_1 \varepsilon_{t-1}. \tag{2}$$

Under the Gaussian assumption,

$$Y_t \mid \mathcal{F}_{t-1} \sim N(\mu_t, \sigma_t^2). \tag{3}$$

The stationarity condition requires $|\phi_1| < 1$ [30].

2.2. GARCH Model for Conditional Variance

To model time-varying volatility, the innovation process is expressed as

$$\varepsilon_t = \sigma_t \mathcal{Z}_t, \quad \mathcal{Z}_t \sim \mathcal{N}(0, 1), \tag{4}$$

where $\{\mathcal{Z}_t\}$ is an i.i.d. sequence. The conditional variance σ_t^2 follows a GARCH-type process. For example, the GARCH$(1, 1)$ model is defined as

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \tag{5}$$

with constraints $\alpha_0 > 0$, $\alpha_1, \beta_1 \geq 0$, and $\alpha_1 + \beta_1 < 1$ [21, 31].

2.3. Asymmetric GARCH Extensions

The ARCH and GARCH models assume that volatility depends only on the magnitude of shocks, implying symmetric effects between positive and negative shocks [32, 33]. However, wind speed data often exhibit nonlinear and asymmetric behavior, where positive and negative shocks may have different impacts on volatility, leading to volatility clustering that cannot be captured by linear models such as ARMA [34]. To address this, asymmetric GARCH models are used.

The GJR-GARCH(1,1) model introduces an indicator function to allow different volatility responses to negative shocks [22, 35]:

$$\sigma_t^2 = \alpha_0 + (\alpha_1 + \gamma_1 I_{\{\varepsilon_{t-1}<0\}})\varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \tag{6}$$

where $I_{\{\varepsilon_{t-1}<0\}} = 1$ if $\varepsilon_{t-1} < 0$ and 0 otherwise. The parameters satisfy $\alpha_0 > 0,\ \alpha_1, \beta_1 \geq 0$, while no sign restriction is imposed on γ_1, provided that the conditional variance remains positive, i.e., $\alpha_1 + \gamma_1 I_{\{\varepsilon_{t-1}<0\}} \geq 0$. The model is weakly stationary if $\alpha_1 + \frac{1}{2}\gamma_1 + \beta_1 < 1$. The GGJR-GARCH(1,1) model extends this by incorporating asymmetry in both the shock and volatility components through an additional parameter ξ_1 [23]:

$$\sigma_t^2 = \alpha_0 + (\alpha_1 + \gamma_1 I_{\{\varepsilon_{t-1}<0\}})\varepsilon_{t-1}^2 + (\beta_1 + \xi_1 I_{\{\varepsilon_{t-1}<0\}})\sigma_{t-1}^2, \tag{7}$$

where ξ_1 captures asymmetry in the volatility persistence. Similar to γ_1, no sign restriction is imposed on ξ_1, provided that $\beta_1 + \xi_1 I_{\{\varepsilon_{t-1}<0\}} \geq 0$. The stationarity condition is given by $\alpha_1 + \frac{1}{2}\gamma_1 + \beta_1 + \frac{1}{2}\xi_1 < 1$.

2.4. ARMA-Asymmetric GARCH Models

Volatility models can be classified into homoscedastic and heteroscedastic specifications. Homoscedastic models, such as ARMA with white-noise errors, assume constant variance, while heteroscedastic models like GARCH allow time-varying conditional variance driven by past information. However, standard GARCH models do not capture asymmetric volatility effects [32, 33].

Empirical data, including wind speed, often exhibit nonlinear and asymmetric volatility patterns, where negative shocks have stronger and more persistent impacts, leading to volatility clustering [34]. To address this, ARMA models are combined with asymmetric GARCH specifications, allowing separate modeling of the conditional mean and asymmetric conditional variance. In particular, the GJR-GARCH model captures asymmetry through an indicator function [22], while the GGJR-GARCH model further extends this by incorporating asymmetry in both the ARCH and GARCH components [23].

Definition 1. Let $\{\mathcal{Z}_t\}$ be an i.i.d. sequence with $\mathcal{Z}_t \sim N(0,1)$. The process $\{Y_t\}$ follows an ARMA(1,1)-GGJR-GARCH(1,1) model if

$$Y_t = \mu_t + \varepsilon_t, \tag{8}$$

$$\mu_t = c + \phi_1 Y_{t-1} + \theta_1 \varepsilon_{t-1}, \tag{9}$$

$$\varepsilon_t = \sigma_t \mathcal{Z}_t, \tag{10}$$

$$\sigma_t^2 = \alpha_0 + (\alpha_1 + \gamma_1 I_{\{\varepsilon_{t-1}<0\}})\varepsilon_{t-1}^2 + (\beta_1 + \xi_1 I_{\{\varepsilon_{t-1}<0\}})\sigma_{t-1}^2, \tag{11}$$

where $\mu_t = \mathbb{E}[Y_t \mid \mathcal{F}_{t-1}]$, $\alpha_0 > 0$, $\alpha_1, \beta_1 \geq 0$, and the parameters γ_1 and ξ_1 are unrestricted in sign, provided that the conditional variance remains positive.

This framework nests several models: ARMA when $\alpha_1 = \gamma_1 = \beta_1 = \xi_1 = 0$, GARCH when $\gamma_1 = \xi_1 = 0$, and GJR-GARCH when $\xi_1 = 0$. The parameters γ_1 and ξ_1 capture asymmetry in the shock and volatility components, respectively. For weak stationarity, the process must have constant mean and variance. Under this condition, the ARMA(1,1)-GGJR-GARCH(1,1) model satisfies:

Proposition 1. *If $\{Y_t\}$ is weakly stationary, then:*
i. The unconditional mean is given by

$$E[Y_t] = \frac{c}{1-\phi_1}, \quad \textit{for } |\phi_1| < 1.$$

ii. The process admits a finite unconditional variance provided that

$$\alpha_1 + \tfrac{1}{2}\gamma_1 + \beta_1 + \tfrac{1}{2}\xi_1 < 1.$$

The expression for the unconditional mean is obtained by taking expectations on both sides of the ARMA(1,1) mean equation and using the property $\mathbb{E}[\varepsilon_t] = 0$, which yields $\mathbb{E}[Y_t] = c + \phi_1 \mathbb{E}[Y_{t-1}]$, and hence $\mathbb{E}[Y_t] = c/(1-\phi_1)$ under $|\phi_1| < 1$. The condition for the existence of a finite unconditional variance follows from the GGJR-GARCH(1,1) specification, which ensures that the second-order moment of the innovation process $\{\varepsilon_t\}$ exists. In particular, under the condition $\alpha_1 + \frac{1}{2}\gamma_1 + \beta_1 + \frac{1}{2}\xi_1 < 1$, the unconditional variance of ε_t is finite, which in turn implies that $\{Y_t\}$ admits finite unconditional moments. These conditions are standard in asymmetric GARCH-type models and are sufficient to ensure that the process has well-defined unconditional moments consistent with the weak stationarity assumption [22, 23].

2.5. Model Estimation and Evaluation

Model parameters are estimated using maximum likelihood estimation (MLE) [36]. Model selection for the conditional mean and variance equations is conducted using the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC), defined as

$$\mathrm{AIC} = -2LL + 2K, \quad \mathrm{BIC} = -2LL + K \log N, \tag{12}$$

where LL denotes the log-likelihood value, K is the number of estimated parameters, and N is the sample size.

To evaluate the out-of-sample forecasting performance, a one-step-ahead forecasting scheme is employed. The dataset is partitioned into an in-sample (estimation) period and an out-of-sample (evaluation) period. Specifically, the first T_0 observations (approximately 80% of the total sample) are used for model estimation, while the remaining $T - T_0$ observations (approximately 20%) are reserved for forecast evaluation. Forecasts are generated using a recursive (expanding window) procedure. The model is initially estimated using observations $t = 1, \ldots, T_0$. A one-step-ahead forecast is then produced for time $t = T_0 + 1$. Subsequently, the sample is expanded to include the observation at time $t = T_0 + 1$, the model is re-estimated, and a new one-step-ahead forecast is generated for time $t = T_0 + 2$. This process continues recursively until forecasts are obtained for all observations in the evaluation sample, i.e., for $t = T_0 + 1, \ldots, T$.

This recursive scheme allows the model parameters to be updated as new information becomes available, which is particularly suitable for capturing evolving volatility dynamics in time series data. Forecast accuracy is evaluated by comparing the predicted values $\hat{Y}_t$ with the realized observations Y_t over the out-of-sample period $t = T_0 + 1, \ldots, T$. The performance is measured using the Root Mean Squared Error (RMSE) and Mean Absolute Percentage Error (MAPE), defined as

$$\text{RMSE} = \left[\frac{1}{T - T_0} \sum_{t=T_0+1}^{T} (Y_t - \hat{Y}_t)^2\right]^{1/2}, \quad \text{MAPE} = \frac{1}{T - T_0} \sum_{t=T_0+1}^{T} \left|\frac{Y_t - \hat{Y}_t}{Y_t}\right| \times 100\%. \tag{13}$$

It should be noted that MAPE is interpreted with caution when observed values are close to zero, as this may lead to inflated percentage errors. Since true volatility is unobservable, it is proxied by squared residuals [21, 37]. This forecasting and evaluation procedure corresponds to the final stage of the modeling framework illustrated in Fig. 1, where the selected model is used for prediction and performance assessment.

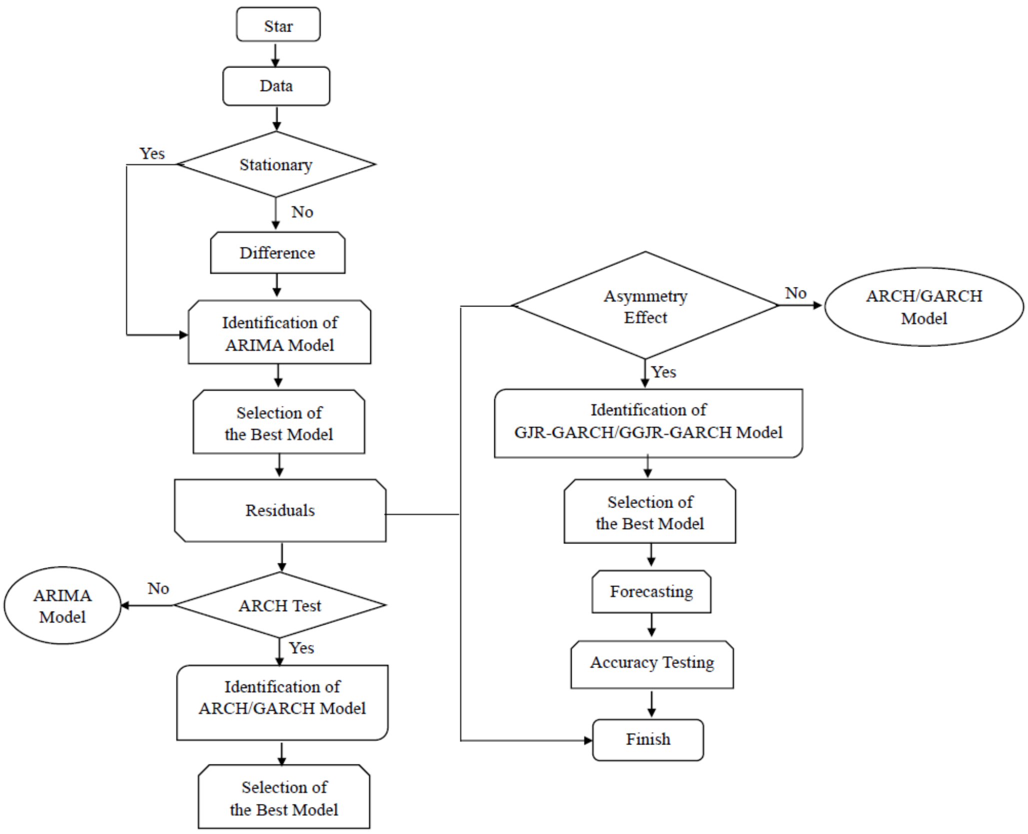

Fig. 1: Procedure for Fitting the ARIMA-Asymmetric GARCH Model

3. Results and Discussion

This section presents the empirical findings derived from the proposed modeling framework and provides a comprehensive discussion of the results. The analysis begins with a description of the data and preliminary statistical examination, including stationarity testing and volatility inspection. Subsequently, the conditional mean structure is identified and estimated using ARMA specifications, followed by diagnostic checking to ensure adequacy of the mean model. The presence of conditional heteroskedasticity is then examined, leading to the estimation of symmetric GARCH and asymmetric GARCH-type models to capture volatility dynamics. Finally, competing models are compared based on information criteria and forecasting accuracy measures to determine the most appropriate specification for modeling and predicting wind speed dynamics in Jakarta and Bandung.

3.1. Data and Preliminary Analysis

The data used in this study are daily wind speed observations from Jakarta and Bandung. Jakarta is located on the island of Java at an altitude of approximately 8 meters above sea level and at coordinates of about 6° south latitude and 106° east longitude. Bandung is surrounded by mountainous terrain and has a cooler climate than Jakarta. It is situated at an altitude of approximately 768 meters above sea level, with coordinates around 6° south latitude and 107° east longitude. Wind speed data for Jakarta were obtained from the Halim Perdanakusuma Meteorological Station, while data for Bandung were collected from the Husein Sastranegara Meteorological Station. The observation period spans from January 1, 2021, to July 31, 2024. These data provide an overview of wind speed patterns in both cities, including seasonal variations influenced by local topography and atmospheric conditions.

Table 1 presents the descriptive statistics of the selected series. Each series consists of 1.306 daily observations. Bandung exhibits a higher mean wind speed than Jakarta; however, this pattern is not consistently observed for the minimum and maximum values. Both series display positive skewness and leptokurtic distributions, as indicated by skewness values greater than zero and kurtosis values exceeding three. This suggests that the wind speed distributions are right-skewed with relatively heavier tails and a sharper peak around the mean.

Table 1: Summary Statistics of Daily Wind Speed

Location	N	Mean	Std. Dev.	Skewness	Kurtosis
Jakarta	1306	7.09	2.62	1.10	4.47
Bandung	1306	7.64	2.46	1.04	4.07

Time plots are used to visually assess the temporal behavior of the series, as shown in Fig. 2. The plots suggest the presence of non-constant variance in both Jakarta and Bandung wind speed series. This pattern may indicate heteroscedasticity and possible volatility clustering, which should be formally examined using appropriate statistical tests.

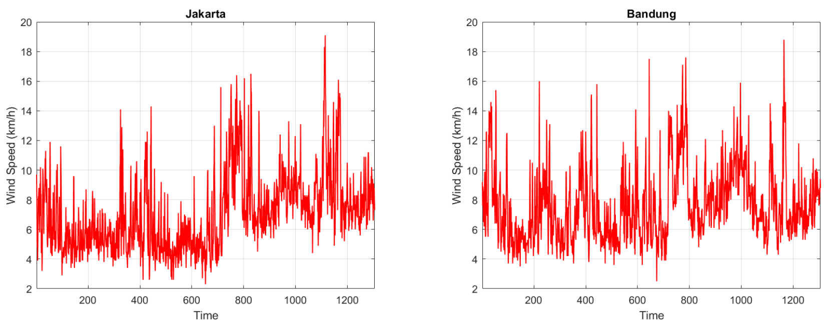

Fig. 2: The Daily Wind Speed Ranges from 1 January 2021 to 31 July 2024

To assess whether the data are stationary, the Augmented Dickey-Fuller (ADF) test is employed. In addition, the Phillips-Perron (PP) test is conducted to complement and validate the results obtained from the ADF test. Both tests examine the presence of a unit root in a stochastic process [38]. The ADF test extends the Dickey-Fuller procedure by including lagged differences of the dependent variable to control for higher-order serial correlation in the error term, thereby improving the reliability of the stationarity assessment [39]. The hypotheses of the ADF test are formulated as follows: H_0 : The series contains a unit root (non-stationary) and H_1 :The series does not contain a unit root (stationary).

Based on the ADF test results presented in Table 2, the p-value is less than 0.05. Therefore, the null hypothesis of a unit root is rejected, indicating that the series can be considered stationary. To further validate the findings, the Phillips-Perron (PP) test is also performed. According to [39], the PP test is based on a non-parametric approach to account for autocorrelation and heteroscedasticity in the residuals. The hypotheses of the PP test are identical to those of the ADF test. Hence, if the p-value is less than 0.05, the null hypothesis of a unit root is rejected, suggesting that the series is stationary.

Table 2: Testing for Stationarity

Test	Statistic	Jakarta	Bandung	Stationary
ADF Test	Dickey–Fuller	-3.575	-3.173	TRUE
	p-value	0.001	0.002	
PP Test	Phillips–Perron	-3.975	-3.544	TRUE
	p-value	0.001	0.001	

3.2. Fitting of Models

After establishing stationarity, the next step is to identify the appropriate ARIMA(p, d, q) specification based on the sample Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF), as presented in Fig. 3. The orders p and q are initially inferred from the behavior of the ACF and PACF plots. If the ACF decays gradually while the PACF exhibits a sharp cutoff after lag p, the process may be identified as AR(p). Conversely, if the ACF shows a sharp cutoff after lag q and the PACF decays gradually, the process may be identified as MA(q). If both the ACF and PACF decay gradually without a clear cutoff, the process may be modeled as ARIMA(p, d, q).

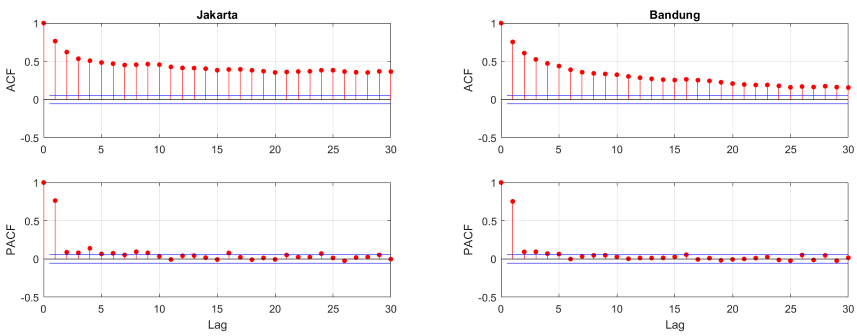

Fig. 3: ACF and PACF of Daily Wind Speed

Furthermore, Table 3 presents the AIC and BIC values used to select the conditional mean model. These information criteria are employed for in-sample model selection, where competing specifications are compared based on their goodness-of-fit to the observed data while penalizing model complexity. In particular, the Akaike Information Criterion (AIC) emphasizes predictive accuracy, whereas the Bayesian Information Criterion (BIC) imposes a stronger penalty for overparameterization, thereby favoring more parsimonious models. The results indicate that the ARIMA(2,0,1) model yields the lowest AIC and BIC values and is therefore selected as the appropriate conditional mean specification.

Although the theoretical framework in Section 2 is developed for the ARMA(1,1) specification, the empirical model selected here corresponds to an ARIMA(2,0,1) process based on information criteria. This choice is entirely data-driven. The theoretical formulation readily extends to this more general ARMA(p,q) structure by incorporating additional autoregressive terms in the conditional mean equation. In particular, the empirical model preserves the same hierarchical structure as the theoretical specification, where the conditional mean is modeled by an ARMA process and the innovation term follows a GGJR-GARCH-type conditional variance. Therefore, the ARMA(1,1) case presented in Section 2 should be viewed as an illustrative and tractable representation of the more general modeling framework implemented in this study. This finding is consistent with the fact that the time series is already stationary, so no differencing is required. Accordingly, although no differencing is required, the model is consistently denoted as ARIMA(2,0,1) to maintain standard notation in the ARIMA framework.

Table 3: The Best-Fitted Mean Conditional Models

Data	ARIMA	AIC	BIC
Jakarta	(0,0,0)	6220.308	6225.482
	(0,0,1)	5554.172	5640.521
	(0,0,2)	5291.852	5307.377
	(1,0,0)	5077.400	5087.749
	(1,0,1)	5065.447	5080.971
	(1,0,2)	5038.982	5059.681
	(2,0,0)	5069.066	5084.590
	(2,0,1)	4998.172	5018.871
	(2,0,2)	4999.827	5025.701
Bandung	(0,0,0)	6063.378	6068.553
	(0,0,1)	5401.011	5411.360
	(0,0,2)	5192.207	5207.731
	(1,0,0)	4972.521	4982.871
	(1,0,1)	4959.141	4974.665
	(1,0,2)	4944.715	4965.413
	(2,0,0)	4963.219	4978.743
	(2,0,1)	4935.430	4956.129
	(2,0,2)	4937.396	4963.269

After obtaining the best conditional mean model, namely ARIMA(2,0,1), the residuals are further examined, as shown in Fig. 4 and Fig. 5. According to [38], it is necessary to verify that no significant autocorrelation remains in the residuals. This step ensures that the mean equation is adequately specified. Any remaining dependence in the squared residuals can then be attributed to conditional heteroscedasticity rather than unmodeled linear dynamics.

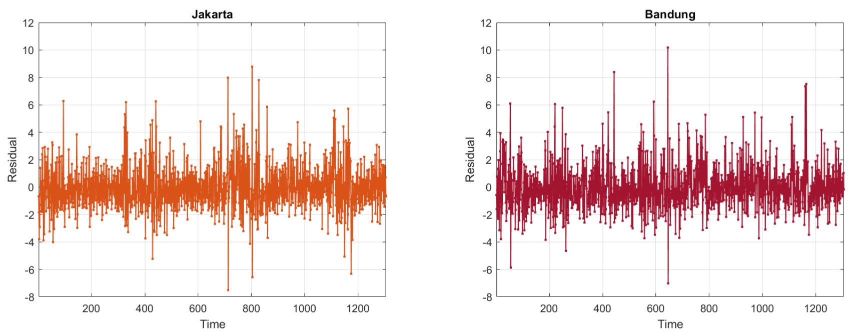

Fig. 4: Residual from the ARIMA(2,0,1) for Jakarta and Bandung Data

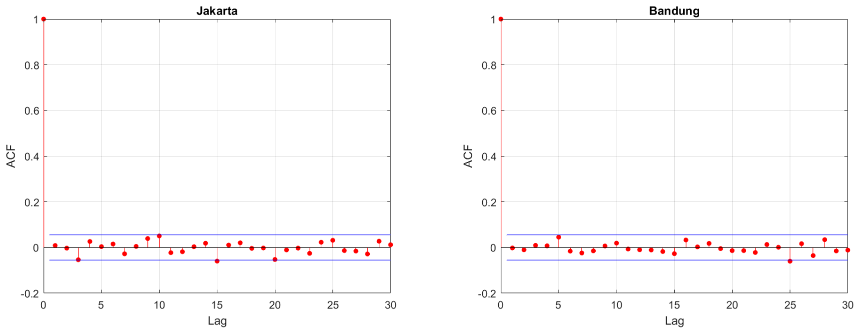

Fig. 5: ACF of Residuals for Jakarta and Bandung Data

To determine the appropriate stochastic model for the residuals, the presence of heteroscedasticity is tested using the ARCH-LM test proposed by [20]. The hypotheses are defined as H_0: no ARCH effect, and H_1: presence of ARCH effect. The results in Table 4 strongly reject the null hypothesis, indicating the presence of conditional heteroscedasticity. This finding justifies the use of GARCH-type models for modeling volatility.

Table 4: ARCH-LM Test

Data	χ^2	df	P-value	ARCH
Jakarta	767.3179	10	0.000	TRUE
Bandung	709.8880	10	0.000	TRUE

Furthermore, Table 5 presents the AIC and BIC values used to select the conditional variance model. Similar to the mean equation, these criteria are used for in-sample model selection. The results indicate that the GARCH(1,1) specification provides the best symmetric variance model based on AIC and BIC values.

Table 5: The Best-Fitted Variance Conditional Models

Data	GARCH	AIC	BIC
Jakarta	(0,1)	4941.604	4946.778
	(1,1)	4856.827	4872.351
Bandung	(0,1)	4890.930	4896.105
	(1,1)	4884.798	4900.322

However, to account for potential asymmetric effects in volatility, asymmetric GARCH-type models such as GJR-GARCH and GGJR-GARCH are further considered and evaluated in the forecasting stage.

3.3. Asymmetric Effects

The ARCH and GARCH models described above assume a symmetric response of volatility to shocks. In these models, the conditional variance depends solely on the magnitude of past shocks and does not distinguish between their signs. Consequently, positive shocks (increases in wind speed) and negative shocks (decreases in wind speed) of the same magnitude are assumed to have identical effects on volatility. In other words, the standard GARCH framework does not account for potential differences in the impact of positive and negative shocks on volatility dynamics. To examine the presence of asymmetric effects, the relationship between past shocks and future volatility can be investigated through the correlation between residuals and squared residuals. According to [40], the asymmetric effect in a time series is defined as the correlation between the residual at time t and the squared residual at time $t + \tau$, which is expressed as $L(\tau) = \mathrm{Corr}(\varepsilon_t, \varepsilon^2_{t+\tau})$, where τ denotes the lag. A non zero value of $L(\tau)$ indicates that the sign of past shocks influences future volatility, suggesting the presence of asymmetry in the conditional variance dynamics. Since volatility is not directly observable, it is commonly proxied by the squared residuals. Under the assumption that $\varepsilon_t = \sigma_t Z_t$, with Z_t being independently and identically distributed with zero mean and unit variance, it follows that $E(\varepsilon_t^2 \mid \mathcal{F}_{t-1}) = \sigma_t^2$.

Thus, the squared residuals ε_t^2 serve as an observable approximation of the latent conditional variance σ_t^2, allowing the leverage correlation $L(\tau)$ to be used as an empirical measure of asymmetric volatility effects.

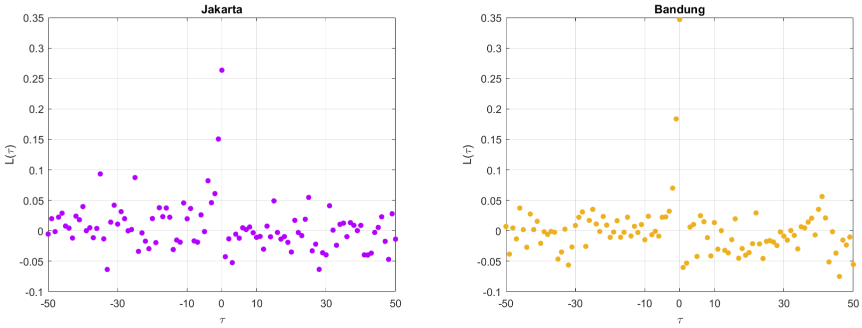

Fig. 6: Asymmetric Effect in Wind Speed Data

Based on Fig. 6, differences can be observed in the behavior of $L(\tau)$ for positive and negative values of τ. The empirical results indicate that $L(\tau) \neq L(-\tau)$, suggesting asymmetric dependence between past shocks and future volatility. Moreover, the correlation values differ significantly from zero, providing evidence of asymmetric effects. In particular, negative residuals appear to generate a larger increase in volatility compared to positive residuals of the same magnitude.

To accommodate this asymmetric volatility response, extensions of the standard GARCH model are considered. In the literature, asymmetry is often interpreted as a stronger volatility reaction to negative shocks than to positive shocks. Based on the ARIMA(2,0,2)–GARCH(1,1) specification, the correlation between the residuals and their squared values, shown in Fig. 6, indicates the presence of asymmetry. Given this evidence, asymmetric GARCH-type models such as GJR-GARCH and GGJR-GARCH are employed to capture the leverage effect. Table 6 reports the AIC and BIC values for selecting the appropriate asymmetric conditional variance model. Among the competing specifications, the GGJR-GARCH(1,1) model yields the lowest AIC and BIC values and is therefore selected as the preferred model.

Table 6: The Best-Fitted Variance Conditional Asymmetric Models

Data	Model	AIC	BIC
Jakarta	GJR(1,1)	4843.102	4858.626
	GGJR(1,1)	4834.521	4850.045
Bandung	GJR(1,1)	4838.470	4853.994
	GGJR(1,1)	4799.063	4814.587

3.4. Forecasting Comparison

Comparing predictive performance across competing models is essential for identifying the most appropriate specification for the data. Since forecasting accuracy depends on reliable parameter estimation, the adequacy of the estimated coefficients directly influences predictive results. In this study, ARIMA(2,0,1) is employed as the conditional mean equation, while GARCH(1,1), GJR-GARCH(1,1), and GGJR-GARCH(1,1) are considered for the conditional variance equations. The parameter estimation results for the ARIMA(2,0,1)-GARCH(1,1), ARIMA(2,0,1)-GJR-GARCH(1,1), and ARIMA(2,0,1)-GGJR-GARCH(1,1) models are presented in Fig. 7 and Table 7.

Although Fig. 7 provides a visual comparison of the fitted and forecasted values, the differences among the competing models are relatively subtle due to the strong overlap of the series over the full sample period. As a result, it may be difficult to distinguish the relative performance of the models based solely on the graphical representation. Therefore, the evaluation of forecasting performance is primarily based on quantitative accuracy measures, namely RMSE and MAPE, which provide a more precise and objective comparison of model performance, as reported in Table 8. This approach is consistent with standard practices in time series forecasting, where numerical evaluation metrics are preferred over visual inspection when model outputs exhibit strong overlap. For visualization purposes, Fig. 7 presents the full time series, including both in-sample fitted values and out-of-sample forecasts. However, the evaluation of forecasting performance is conducted exclusively on the out-of-sample period.

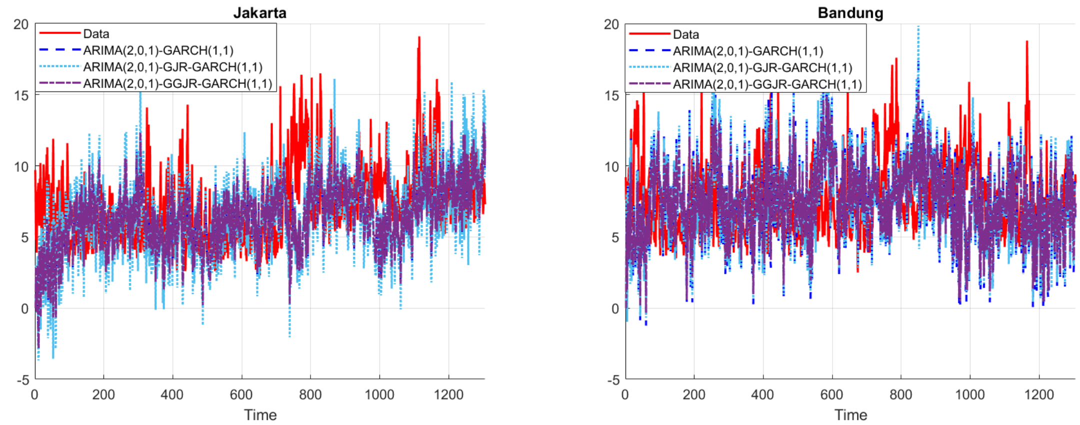

Fig. 7: Comparison of Observed and Forecasted Values

Table 7: Comparison of Parameter Estimation Results

Model	Parameter	Jakarta	Bandung
ARIMA(2,0,1)	c	0.029 (0.010)**	0.121 (0.045)**
	ϕ_1	1.560 (0.120)**	1.531 (0.110)**
	ϕ_2	-0.564 (0.090)**	-0.547 (0.085)**
	θ_1	-0.932 (0.070)**	-0.872 (0.065)**
GARCH(1,1)	α_0	0.067 (0.020)**	0.537 (0.150)**
	α_1	0.071 (0.025)**	0.099 (0.030)**
	β_1	0.903 (0.040)**	0.690 (0.080)**
LL: ARIMA(2,0,1)-GARCH(1,1)		-2425.413	-2439.399
GJR-GARCH(1,1)	α_0	0.065 (0.018)**	1.228 (0.300)**
	α_1	0.087 (0.030)**	0.393 (0.120)**
	γ_1	-0.081 (0.040)*	-0.393 (0.180)*
	β_1	0.922 (0.035)**	0.322 (0.090)**
LL: ARIMA(2,0,1)-GJR-GARCH(1,1)		-2418.551	-2416.235
GGJR-GARCH(1,1)	α_0	0.022 (0.010)**	0.298 (0.120)**
	α_1	0.102 (0.030)**	0.060 (0.020)**
	γ_1	0.021 (0.009)**	0.185 (0.060)**
	β_1	0.847 (0.045)**	0.696 (0.070)**
	ξ_1	0.072 (0.025)**	0.202 (0.080)**
LL: ARIMA(2,0,1)-GGJR-GARCH(1,1)		-2414.260	-2396.531

**, * denote significance at the 5% and 10% levels.

Based on the parameter estimation results presented in Table 7, the ARIMA(2,0,1)-GGJR-

GARCH(1,1) model provides the best specification for both Jakarta and Bandung. This model yields the highest log-likelihood values compared to the ARIMA(2,0,1)-GARCH(1,1) and ARIMA(2,0,1)-GJR-GARCH(1,1) alternatives. Most of the estimated parameters are statistically significant at conventional levels, indicating that the models are well specified. In particular, the asymmetry parameters γ_1 and ξ_1 in the GGJR-GARCH(1,1) model are statistically significant for both datasets. This result confirms the presence of asymmetric volatility effects in wind speed dynamics. It implies that positive and negative shocks have different impacts on conditional variance. In comparison, the asymmetry parameter in the GJR-GARCH(1,1) model is only weakly significant. This suggests that the GGJR-GARCH specification captures asymmetric effects more effectively. In addition, the GGJR-GARCH(1,1) model allows for greater flexibility by incorporating asymmetry in both the ARCH and GARCH components. This leads to a more comprehensive representation of volatility dynamics. Overall, the superiority of the ARIMA(2,0,1)-GGJR-GARCH(1,1) model is supported by both its higher log-likelihood values and the statistical significance of its asymmetric parameters. Furthermore, wind speed forecasts are generated using a one-step-ahead forecasting scheme based on the estimated parameters of the selected model.

It is important to clarify the relationship between the theoretical specification and the empirical estimates of the asymmetry parameters. In the theoretical formulation, no sign restriction is imposed on γ_1 and ξ_1, as long as the conditional variance remains positive. While many studies assume $\gamma_1 \geq 0$ to capture the leverage effect, this assumption is not required for model validity. In the empirical results, the estimated values of γ_1 in the GJR-GARCH(1,1) model are negative. This indicates that negative shocks have a smaller impact on conditional volatility than positive shocks in the wind speed data, reflecting an alternative form of asymmetry. Such behavior differs from the conventional leverage effect commonly observed in financial time series but is consistent with the more flexible specification adopted in this study. Therefore, the theoretical conditions and empirical findings are fully consistent, as the model allows for both positive and negative asymmetry depending on the underlying data characteristics.

Table 8: Accuracy Testing of the Best-Fitted Model

Data	Model	RMSE	MAPE
Jakarta	ARIMA(2,0,1)-GARCH(1,1)	5.207	63.618
	ARIMA(2,0,1)-GJR-GARCH(1,1)	5.111	61.900
	ARIMA(2,0,1)-GGJR-GARCH(1,1)	4.860	59.270
Bandung	ARIMA(2,0,1)-GARCH(1,1)	6.139	66.982
	ARIMA(2,0,1)-GJR-GARCH(1,1)	5.251	57.333
	ARIMA(2,0,1)-GGJR-GARCH(1,1)	5.219	56.828

The results of the one-step-ahead forecast evaluation are reported in Table 8. These results support the previous findings. The ARIMA(2,0,1)-GGJR-GARCH(1,1) model shows the best predictive performance. This is indicated by the lowest RMSE and MAPE values at both locations. For Jakarta, the RMSE is 4.860, and the MAPE is 59.270. For Bandung, the RMSE is 5.219, and the MAPE is 56.828. These results indicate that the model performs better than the competing specifications. In addition, Fig. 7 shows that the fitted and forecasted values follow the observed data more closely. However, the MAPE values remain relatively high. The values range from about 56% to 59%. This indicates that the forecasting errors are still large in absolute terms. This result reflects the high variability of wind speed data. Wind speed is difficult to predict, especially in tropical regions where fluctuations are frequent. Similar levels of error are also reported in previous studies. From a practical point of view, the model has limitations for precise prediction. It may not be suitable for exact short-term forecasting. However, the model is still useful for capturing volatility patterns. It can describe asymmetric responses to shocks.

This information is important for risk assessment and uncertainty analysis. It is also useful for supporting wind energy planning. In practical applications, this information can support decision-making in wind energy systems. For example, a better understanding of volatility can help operators anticipate periods of unstable wind conditions. This is important for scheduling power generation and managing supply fluctuations. It can also support risk mitigation by identifying periods with higher uncertainty. In addition, the model can be used as an input for energy forecasting systems that require information on variability rather than only point predictions. Overall, the ARIMA(2,0,1)-GGJR-GARCH(1,1) model is selected as the most suitable model. This conclusion is based on RMSE, MAPE, and the ability to model asymmetric volatility.

4. Conclusion

This study aimed to identify an appropriate stochastic model for capturing the volatility dynamics of wind speed data in Jakarta and Bandung. The analysis focused on heteroscedasticity and asymmetric effects. The empirical results show that the wind speed series exhibits conditional heteroscedasticity. The results also indicate asymmetric responses in volatility. Negative shocks have a stronger impact on future volatility than positive shocks of the same magnitude. The statistical significance of the asymmetry parameter in the GGJR-GARCH model supports this result. This finding justifies the use of an asymmetric specification instead of a symmetric GARCH model. In addition, the asymmetric models consistently produce lower RMSE and MAPE values than the symmetric GARCH model. This indicates better predictive performance. Among the competing models, the ARIMA(2,0,1)-GGJR-GARCH(1,1) model provides the best overall performance. It captures both volatility clustering and asymmetric effects. It also achieves the lowest forecasting errors. Therefore, this model offers a more adequate representation of wind speed dynamics in Jakarta and Bandung.

The results highlight the importance of accounting for asymmetric volatility behavior in meteorological time series. Ignoring this feature may reduce model accuracy and lead to less reliable forecasts. From a practical perspective, the model is useful for understanding variability and uncertainty in wind speed. In practice, it can assist energy operators in managing variability and supporting scheduling decisions. It can also help identify periods with higher uncertainty. This information is important for risk assessment in wind energy systems. Nevertheless, this study is limited to two locations and a one-step-ahead forecasting framework. Future research may extend the analysis to longer forecast horizons. It may also include additional explanatory variables or apply multivariate volatility models to capture spatial dependence across regions.

CRediT Authorship Contribution Statement

Nurhayati: Conceptualization, Methodology, Writing–Original Draft, Data Curation, Resources, Investigation, Formal Analysis, Software, Visualization, Writing–Review & Editing. **Andi Fitriawati:** Validation, Visualization, Supervision, Writing – original draft, Writing – review and editing, Project Administration,

Declaration of Generative AI and AI-assisted technologies

Generative AI tools, specifically ChatGPT, were used solely to assist with language clarity and grammar refinement during the preparation of this manuscript. All AI-generated suggestions were carefully reviewed, edited, and approved by the authors. The analysis, interpretation, data, figures, references, and main content of the manuscript were entirely developed by the authors without AI involvement.

Declaration of Competing Interest

The authors declare no competing interests.

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Data and Code Availability

The wind speed dataset and forecasting code analyzed in this study are available from the corresponding author upon reasonable request.

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