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Applications of Reich-Perov α-Contractive Mapping in Vector Valued Metric Spaces

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Abstract

This paper presents two new applications of a fixed point theorem for Reich–Perov α-contractive mappings in vector-valued metric spaces. As a new application, we first demonstrate the existence and uniqueness of a solution to the vector valued Volterra-Fredholm integral equation system. By constructing suitable integral operators, we show that they satisfy the α-contractive Reich-Perov condition. The second application concerns the determination of the fixed point for discrete recursive systems with coupled components, for which the existence of a unique solution is established.

Keywords: α-contractive; fixed point; Reich-Perov contraction; vector valued metric space.

1. Introduction

A broad class of problems in nonlinear analysis, including those involving differential equations, integral equations, and dynamical systems, can be effectively studied using fixed point approaches. Following the Banach contraction principle, various generalizations of the fixed point theorem have been developed to expand the class of analyzable mappings. An important direction of development is the expansion of the structure of metric spaces and the forms of contraction used in them.

Perov [1] pioneered the use of vector valued metric spaces to analyze systems of simultaneous differential and integral equations. This approach was further developed by Varga [2] through a study of iterative matrix analysis, which provided a strong theoretical basis for using the spectral radius as a contraction criterion. Several studies on Perov type contractive mappings have been published in various spaces, including [3–10].

In addition to the expansion of spatial structures, the development of contractive conditions is also a major focus in fixed point theory. The Reich type contraction, which takes into account the distance of a point from its own image, is an important generalization of the Banach contraction [11]. Various studies have shown that the Reich contraction is more flexible and capable of handling nonlinear mappings that fail the classical (Banach) contraction. Several studies related to the Reich contraction and its various developments include those of [12–16].

The development of fixed-point theory extends beyond generalizations of vector-valued metric spaces to more flexible variations of contractive conditions. Almalki et al. and Alansari et al. [17,

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18] extended the Perov-type fixed-point theorem by incorporating ordered relation structures and equivalence relations, allowing for the analysis of coupled systems within a more general framework. Sarwar et al. [19] introduced Perov-type contractions formulated in terms of functions F and applied them to semilinear operator systems. Subsequently, Mirkov et al. [20] refined and clarified the form of these contractional mappings in vector-valued metric spaces. Furthermore, [21] studied this class of contractions in the context of generalized asymmetric metric spaces.

Later developments introduced the concept of α–admissible as a tool to introduce additional structure into metric spaces, such as partial ordering or certain relations between elements. This concept was first introduced by Samet et al. [22] and has since been widely used to expand the scope of the classical fixed point theorem. This approach is particularly useful for problems involving monotonicity or specific initial conditions. Several studies related to α–admissible in various spaces include [23–25].

Several studies, such as those by Altun et al. [26] and Mirkov et al. [27], have discussed Perov type contractions, α–contractive Perov contractions, and their applications to integral equations and differential systems. However, most of the results obtained are still limited to linear contractions. Further developments, including special formulations and variations of the contractivity condition, have been investigated by Sunarsini et al. in [28]. The results obtained ensure the presence of exactly one Reich–Perov type solution in vector-valued metric spaces. This serves as a theoretical basis for further research in this article.

On the other hand, vector valued Volterra-Fredholm integral-differential systems naturally require an analytical framework capable of handling the interrelationships between components and mixed memory contributions. This motivation drives this research, which applies the α-contractive Reich-Perov fixed point theorem, which has not been discussed in the previous literature. Specifically, this article provides a new application to Volterra-Fredholm integral equation systems. Thus, the results obtained not only extend the application of the Reich-Perov theorem in the context of coupled systems but also make new contributions to the development of fixed point theory in vector valued metric spaces. Furthermore, we establish new applications to coupled discrete recursive systems involving nonlinear functions, which demonstrate the effectiveness of the proposed approach. Therefore, the results obtained in this paper provide a meaningful extension of the existing theory and broaden its range of applications.

2. Preliminaries

The definitions, concepts, and auxiliary results required to establish the main results of this article are introduced below. Vector valued metric spaces were originally proposed by Perov [1] as a tool to analyze differential and integral equation systems simultaneously.

Definition 1. Assume that X is a set containing at least one element, and that $\mathbb{R}^n$ is equipped with the componentwise partial order, $\preceq$, defined by $\boldsymbol{k} \preceq \boldsymbol{l}$ whenever $k_i \leq l_i$ for all $i \in \{1, 2, \ldots, n\}$ Also, $\boldsymbol{k} \prec \boldsymbol{l} \iff k_i < l_i,$ for all $i \in \{1, 2, \ldots, n\}$ The function $d_v : X \times X \to \mathbb{R}^n$ is considered a vector valued metric if, the following assertions are fulfilled for every $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z} \in X$:

(i) $\boldsymbol{0} \preceq d_v(\boldsymbol{x}, \boldsymbol{y})$, and the equality $d_v(\boldsymbol{x}, \boldsymbol{y}) = \boldsymbol{0}$ occurs precisely when $\boldsymbol{x} = \boldsymbol{y}$, ($\boldsymbol{0}$ denotes the zero vector);

(ii) $d_v(\boldsymbol{x}, \boldsymbol{y})$ remains unchanged when $\boldsymbol{x}$ and $\boldsymbol{y}$ are interchanged;

(iii) $d_v(\boldsymbol{x}, \boldsymbol{y}) \preceq d_v(\boldsymbol{x}, \boldsymbol{z}) + d_v(\boldsymbol{z}, \boldsymbol{y})$.

Throughout this section, the notation X will be used to represent the vector-valued metric space (X, d_v).

Definition 2. The notions of Cauchy sequences and completeness corresponding to the metric d_v are taken as in [28].

The role of nonnegative matrices is crucial in Perov's theory, particularly as a replacement for the contraction constant in the Banach principle. The role of nonnegative matrices is crucial in Perov's theory, particularly as a replacement for the contraction constant in the Banach principle. Throughout this work, we use nonnegative matrices and the notion of matrices converging to zero, referring the reader to A. I. Perov for precise definitions [2].

A practical criterion for determining this property is given by the spectral radius.

Lemma 1. *[2] For a nonnegative matrix $\boldsymbol{A}$, convergence of $\boldsymbol{A}$ to the zero matrix occurs precisely when its spectral radius satisfies $\rho(\boldsymbol{A}) < 1$.*

The lemma plays an important role in the analysis of matrix-based contractive operators in vector metric spaces, as discussed in [2]. In later developments, Perov broadened Banach's fixed point framework by introducing a matrix oriented contraction approach.

Definition 3. [26] A matrix $\boldsymbol{A}$ satisfying the conditions of Lemma 1 is considered. The self-mapping $T : X \to X$ is Perov–contractive whenever $d_v(T\boldsymbol{s}, T\boldsymbol{t}) \preceq \boldsymbol{A}\, d_v(\boldsymbol{s}, \boldsymbol{t})$ holds for arbitrary $\boldsymbol{s}, \boldsymbol{t} \in X$.

Reich [11] introduced a more flexible form of contraction that involves the distance between a point and its own image. This concept was later combined with Perov's [28].

Definition 4. [28] A self-mapping T in X is called a Reich–Perov type contraction in X whenever there exist matrices $\boldsymbol{A}_1, \boldsymbol{A}_2, \boldsymbol{A}_3 \in M_{n\times n}(\mathbb{R}_0^+)$ fulfilling the condition $\rho(\boldsymbol{A}_1) + \rho(\boldsymbol{A}_2) + \rho(\boldsymbol{A}_3) < 1$, for which the inequality

$$d_v(T\boldsymbol{s}, T\boldsymbol{t}) \preceq \boldsymbol{A}_1\, d_v(\boldsymbol{s}, T\boldsymbol{s}) + \boldsymbol{A}_2\, d_v(\boldsymbol{t}, T\boldsymbol{t}) + \boldsymbol{A}_3\, d_v(\boldsymbol{s}, \boldsymbol{t}) \tag{1}$$

is satisfied for all $\boldsymbol{s}, \boldsymbol{t} \in X$.

The notion of α–admissibility was proposed by Samet et al. [22] to incorporate additional structures such as partial ordering or certain relations.

Definition 5. With $\alpha : X \times X \to \mathbb{R}^n$ given, the α–admissibility of $T : X \to X$ is characterized by the implication For every $\boldsymbol{s}, \boldsymbol{t} \in X$, condition $\alpha(\boldsymbol{s}, \boldsymbol{t}) \succeq \boldsymbol{I}$ ensures that $\alpha(T\boldsymbol{s}, T\boldsymbol{t}) \succeq \boldsymbol{I}$ is held for arbitrary $\boldsymbol{s}, \boldsymbol{t} \in X$, and $\boldsymbol{I}$ is an identity matrix of size n × n.

Remark 1. Analogously to Definition 1, a componentwise partial order, $\succeq$, on $\mathbb{R}^n$ is defined by $\boldsymbol{k} \succeq \boldsymbol{l}$ whenever $k_i \geq l_i$ for all $i \in \{1, 2, \ldots, n\}$ Moreover, $\boldsymbol{k} \succ \boldsymbol{l}$ whenever $k_i > l_i$ for all $i \in \{1, 2, \ldots, n\}$.

The main result that forms the basis of this article was developed by Sunarsini et al. [28].

Definition 6. Given a self mapping T on X. If there exist a function $\alpha : X \times X \to M_{n\times n}(\mathbb{R}_0^+)$ defined by $\alpha(\boldsymbol{s}, \boldsymbol{t}) \succeq \boldsymbol{I}$ and three matrices $\boldsymbol{A}_1, \boldsymbol{A}_2, \boldsymbol{A}_3 \in M_{n\times n}(\mathbb{R}_0^+)$ fulfilling the condition

$$\rho(\boldsymbol{A}_1) + \rho(\boldsymbol{A}_2) + \rho(\boldsymbol{A}_3) < 1,$$

such that for every $\boldsymbol{s}, \boldsymbol{t} \in X$ that satisfies (1), then T is called a Reich-Perov α-contractive mapping in X.

Theorem 1. *We assume that (X, d_v) is complete, together with a self-mapping T that satisfies Definition 6. Suppose that the following requirements are fulfilled:*

1. *the mapping T fulfills the α–admissibility condition;*
2. *an initial point $\boldsymbol{s}_0 \in X$ can be chosen such that $\alpha(\boldsymbol{s}_0, T\boldsymbol{s}_0) \succeq \boldsymbol{I}$ holds;*
3. *T is continuous.*

Under these assumptions, the operator T admits exactly one fixed point in X.

This theorem simultaneously extends the Banach, Reich, and Perov principles and serves as the main framework for discussing applications to Volterra–Fredholm integral equation systems and coupled discrete recursive systems in the following sections.

3. Application to Volterra–Fredholm Systems of Integral Equations

In this section, we discuss the application of the Reich–Perov α–contractive Fixed Point Theorem to Volterra–Fredholm systems of integral equations. As a first step, we first define the function space to be used and its vector-valued metric structure.

We denote by $C(I, \mathbb{R}^n)$, where $I := [0, 1]$, the space endowed with the supremum norm. For $\boldsymbol{x} = (x_1, \ldots, x_n) \in C(I, \mathbb{R}^n)$, the vector-valued metric d_v is defined by

$$d_v(\boldsymbol{x}, \boldsymbol{y}) = \begin{pmatrix} \|x_1 - y_1\|_\infty \\ \vdots \\ \|x_n - y_n\|_\infty \end{pmatrix}, \tag{2}$$

where $\|\cdot\|_\infty$ denotes the supremum norm in $C(I)$.

Lemma 2. *The vector-valued metric space $(C(I, \mathbb{R}^n), d_v)$ is complete.*

Proof. It can be verified without difficulty that d_v in Eq. (2) satisfies Definition 1. The completeness of the space $C([0, 1], \mathbb{R}^n)$ with respect to this metric is established as follows. Consider an arbitrary sequence $\{\boldsymbol{x}_k\} \subset C(I, \mathbb{R}^n)$ that is Cauchy in the sense of d_v. The Definition 2 guaranties that, given each $\boldsymbol{\varepsilon} \succ 0 \in \mathbb{R}^n$, one can find an integer $N > 0$ such that

$$d_v(\boldsymbol{x}_k, \boldsymbol{x}_m) \prec \boldsymbol{\varepsilon}$$

holds whenever $k, m \geq N$.
By Definition 2, given an arbitrary $\boldsymbol{\varepsilon} \succ 0 \in \mathbb{R}^n$, one can find an integer $N > 0$ such that for which

$$d_v(\boldsymbol{x}_k, \boldsymbol{x}_m) \prec \boldsymbol{\varepsilon} \quad \forall k, m \geq N.$$

Componently, this means

$$\|x_i^{(k)} - x_i^{(m)}\|_\infty < \varepsilon_i, \quad \forall i = 1, \ldots, n.$$

As a result, for every index i, the sequence $\{x_i^{(k)}\}$ is Cauchy in $C(I, \mathbb{R})$ with respect to the supremum norm. By completeness, one obtains $x_i \in C(I, \mathbb{R})$ satisfying

$$\|x_i^{(k)} - x_i\|_\infty \to 0 \quad \text{as } k \to \infty.$$

Now, define $\boldsymbol{x} = (x_1, \dots, x_n)$. It follows that $\boldsymbol{x} \in C(I, \mathbb{R}^n)$. Furthermore, for each i we have

$$\|x_i^{(k)} - x_i\|_\infty \to 0,$$

so that

$$d_v(\boldsymbol{x}^{(k)}, \boldsymbol{x}) \to 0 \quad \text{when } k \to \infty.$$

Hence, every Cauchy sequence in $C(I, \mathbb{R}^n)$ admits a limit in $C([I, \mathbb{R}^n)$, and therefore $C(I, \mathbb{R}^n)$ is complete. □

Consider the Volterra–Fredholm system of integral equations

$$x_i(t) = f_i(t) + \int_0^t K_i(t, s, \boldsymbol{x}(s))\, ds + \int_0^1 H_i(t, s, \boldsymbol{x}(s))\, ds, \qquad t, s \in I, \quad i = 1, \dots, n, \tag{3}$$

where

$$f_i : I \to \mathbb{R}, \quad K_i, H_i : I \times I \times \mathbb{R}^n \to \mathbb{R}$$

are continuous functions. Define a self-mapping T on $C(I, \mathbb{R}^n)$, by

$$T\boldsymbol{x}(t) = (T_1\boldsymbol{x}(t), T_2\boldsymbol{x}(t), \dots, T_n\boldsymbol{x}(t)),$$

where for each $i = 1, 2, \dots, n$,

$$T_i\boldsymbol{x}(t) = f_i(t) + \int_0^t K_i(t, s, \boldsymbol{x}(s))\, ds + \int_0^1 H_i(t, s, \boldsymbol{x}(s))\, ds, \quad t \in I. \tag{4}$$

Theorem 2. *Consider the Volterra–Fredholm system (3) in $(C(I, \mathbb{R}^n), d_v)$. Assume that*

1. *Matrices $\boldsymbol{A} = (a_{ij})$, $\boldsymbol{B} = (b_{ij})$, and $\boldsymbol{C} = (c_{ij}) \in (M_{n\times n}(\mathbb{R}_0^+)$ can be chosen in such a way that for every $t, s \in I$ and $\boldsymbol{x}, \boldsymbol{y} \in C(I, \mathbb{R}^n)$,*

$$|K_i(t, s, \boldsymbol{x}(s)) - K_i(t, s, \boldsymbol{y}(s))| \le \sum_{j=1}^n a_{ij}|x_j(s) - y_j(s)| + \sum_{j=1}^n b_{ij}|x_j(s) - T_j\boldsymbol{x}(s)| + \sum_{j=1}^n b_{ij}|y_j(s) - T_j\boldsymbol{y}(s)|, \tag{5}$$

$$|H_i(t, s, \boldsymbol{x}(s)) - H_i(t, s, \boldsymbol{y}(s))| \le \sum_{j=1}^n c_{ij}|x_j(s) - y_j(s)|.$$

2. *The kernels are monotone, that is*

$$\boldsymbol{x}(s) \preceq \boldsymbol{y}(s), \ s \in I, \quad \text{under which} \quad K_i(t, s, \boldsymbol{x}(s)) \le K_i(t, s, \boldsymbol{y}(s)), \quad H_i(t, s, \boldsymbol{x}(s)) \le H_i(t, s, \boldsymbol{y}(s)). \tag{6}$$

3. *$\rho(\boldsymbol{A}) + \rho(\boldsymbol{B}) + \rho(\boldsymbol{C}) < \frac{1}{4}$.*
4. *$f_i(t) \ge 0$, $K_i(t, s, 0) \ge 0$, $H_i(t, s, 0) \ge 0$.*

Let $\alpha : C(I, \mathbb{R}^n) \times C(I, \mathbb{R}^n) \to M_{n\times n}(\mathbb{R})$ be given by

$$\alpha(\boldsymbol{x}, \boldsymbol{y}) = \begin{cases} \boldsymbol{I}, & \text{if } \boldsymbol{x}(t) \preceq \boldsymbol{y}(t), \ t \in [0, 1], \\ \boldsymbol{O}, & \text{in any other case,} \end{cases}$$

where $\boldsymbol{I}$ and $\boldsymbol{O}$ denote the identity matrix and the zero matrix in $M_{n\times n}(\mathbb{R})$, respectively. Then the operator T satisfies the Reich–Perov α–contractive condition and satisfies Theorem 1, and hence, the system (3) admits a unique solution.

Proof. We proceed in several stages.

(i) Since f_i, K_i, and H_i are continuous, it follows that $T_i\boldsymbol{x}(t)$ is continuous for each $i = 1, 2, ..., n$. Hence T maps $C(I, \mathbb{R}^n)$ into itself.
Let $\boldsymbol{x}_n \to \boldsymbol{x}$ in $C(I, \mathbb{R}^n)$. Then $\boldsymbol{x}_n(s) \to \boldsymbol{x}(s)$ uniformly on I. By continuity of K_i and H_i, we have

$$K_i(t, s, \boldsymbol{x}_n(s)) \to K_i(t, s, \boldsymbol{x}(s)), \quad H_i(t, s, \boldsymbol{x}_n(s)) \to H_i(t, s, \boldsymbol{x}(s)).$$

Moreover, since $\boldsymbol{x}_n \to x$ uniformly and K_i, H_i are continuous on a compact domain, one may choose a constant $M > 0$ for which

$$|K_i(t, s, \boldsymbol{x}_n(s))| \le M, \quad |H_i(t, s, \boldsymbol{x}_n(s))| \le M,$$

for all $t, s \in I$ and all n.
Thus, by the Dominated Convergence Theorem,

$$\int_0^t K_i(t, s, \boldsymbol{x}_n(s))\, ds \to \int_0^t K_i(t, s, \boldsymbol{x}(s))\, ds,$$

and

$$\int_0^1 H_i(t, s, \boldsymbol{x}_n(s))\, ds \to \int_0^1 H_i(t, s, \boldsymbol{x}(s))\, ds.$$

Therefore,

$$T_i\boldsymbol{x}_n(t) \to T_i\boldsymbol{x}(t),$$

uniformly on I. Hence, T is continuous.

(ii) Let's take any $\boldsymbol{x}, \boldsymbol{y} \in C(I, \mathbb{R}^n)$. For every $i = 1, \dots, n$ and $t \in I$, the following holds:

$$\begin{aligned}|T_i\boldsymbol{x}(t) - T_i\boldsymbol{y}(t)| \le &\int_0^t |K_i(t, s, \boldsymbol{x}(s)) - K_i(t, s, \boldsymbol{y}(s))| ds \\ &+ \int_0^1 |H_i(t, s, \boldsymbol{x}(s)) - H_i(t, s, \boldsymbol{y}(s))| ds.\end{aligned}$$

Based on Eq. (5), the following holds:

$$\begin{aligned}|T_i\boldsymbol{x}(t) - T_i\boldsymbol{y}(t)| \le &\int_0^t \Big(\sum_{j=1}^n a_{ij}|x_j(s) - y_j(s)| + \sum_{j=1}^n b_{ij}|x_j(s) - T_j\boldsymbol{x}(s)| \\ &+ \sum_{j=1}^n b_{ij}|y_j(s) - T_j\boldsymbol{y}(s)|\Big) ds \\ &+ \int_0^1 \sum_{j=1}^n c_{ij}|x_j(s) - y_j(s)| ds.\end{aligned}$$

Since $|x_j(s) - y_j(s)| \le \|x_j - y_j\|_\infty$ for all $s \in I$, the integral can be bounded:

$$\int_0^t a_{ij}|x_j(s) - y_j(s)| ds \le a_{ij}\|x_j - y_j\|_\infty, \quad \int_0^t b_{ij}|x_j(s) - T_j\boldsymbol{x}(s)| ds \le b_{ij}\|x_j - T_j\boldsymbol{x}\|_\infty,$$

$$\int_0^1 c_{ij}|x_j(s) - y_j(s)| ds \le c_{ij}\|x_j - y_j\|_\infty.$$

Taking supremum norm and using $t \le 1$, we obtain,

$$\|T_i\boldsymbol{x} - T_i\boldsymbol{y}\|_\infty \le \sum_{j=1}^n (a_{ij} + c_{ij})\|x_j - y_j\|_\infty + \sum_{j=1}^n b_{ij}\|x_j - T_j\boldsymbol{x}\|_\infty + \sum_{j=1}^n b_{ij}\|y_j - T_j\boldsymbol{y}\|_\infty.$$

Based on 2, the above inequality can be written as

$$d_v(T\boldsymbol{x}, T\boldsymbol{y}) \preceq \boldsymbol{B}\, d_v(\boldsymbol{x}, T\boldsymbol{x}) + \boldsymbol{B}\, d_v(\boldsymbol{y}, T\boldsymbol{y}) + (\boldsymbol{A} + \boldsymbol{C})\, d_v(\boldsymbol{x}, \boldsymbol{y}).$$

By taking

$$\boldsymbol{A}_1 = \boldsymbol{B}, \quad \boldsymbol{A}_2 = \boldsymbol{B}, \quad \boldsymbol{A}_3 = 2\boldsymbol{A} + \boldsymbol{C},$$

then the contraction form becomes

$$d_v(T\boldsymbol{x}, T\boldsymbol{y}) \preceq \boldsymbol{A}_1 d_v(\boldsymbol{x}, T\boldsymbol{x}) + \boldsymbol{A}_2 d_v(\boldsymbol{y}, T\boldsymbol{y}) + \boldsymbol{A}_3 d_v(\boldsymbol{x}, \boldsymbol{y}).$$

We compute

$$\rho(\boldsymbol{A}_1) + \rho(\boldsymbol{A}_2) + \rho(\boldsymbol{A}_3) = 2\rho(\boldsymbol{B}) + \rho(\boldsymbol{A} + \boldsymbol{C}) \leq 2\rho(\boldsymbol{B}) + \rho(\boldsymbol{A}) + \rho(\boldsymbol{C}) < \frac{1}{4}$$

(by subadditivity of the spectral radius) . Thus,

$$\rho(\boldsymbol{A}_1) + \rho(\boldsymbol{A}_2) + \rho(\boldsymbol{A}_3) < 1,$$

and T is a Reich-Perov α-contractive mapping in $C(I, \mathbb{R}^n)$.

(iii) By the monotonicity assumption Eq. (6) and integrating both sides, we obtain

$$\begin{aligned} T_i\boldsymbol{x}(t) &= f_i(t) + \int_0^t K_i(t, s, \boldsymbol{x}(s))ds + \int_0^1 H_i(t, s, \boldsymbol{x}(s))ds \\ &\leq f_i(t) + \int_0^t K_i(t, s, \boldsymbol{y}(s))ds + \int_0^1 H_i(t, s, \boldsymbol{y}(s))ds \\ &= T_i\boldsymbol{y}(t). \end{aligned}$$

Thus,

$$T_i\boldsymbol{x}(t) \leq T_i\boldsymbol{y}(t), \quad \forall t \in I,$$

which implies

$$T\boldsymbol{x}(t) \preceq T\boldsymbol{y}(t), \quad \forall t \in I.$$

Therefore, by definition of α,

$$\alpha(T\boldsymbol{x}, T\boldsymbol{y}) = \boldsymbol{I}.$$

Hence, T is α–admissible. by Eq. (6),

$$T_i\boldsymbol{x}(t) \leq T_i\boldsymbol{y}(t).$$

Thus $\alpha(T\boldsymbol{x}, T\boldsymbol{y}) \succeq \boldsymbol{I}$.

(iii) Let $\boldsymbol{x}_0 \in C(I, \mathbb{R}^n)$ be defined by

$$\boldsymbol{x}_0(t) = (0, \ldots, 0), \quad \forall t \in I.$$

Then, for each $i = 1, \ldots, n$ and $t \in I$,

$$T_i\boldsymbol{x}_0(t) = f_i(t) + \int_0^t K_i(t, s, 0)\, ds + \int_0^1 H_i(t, s, 0)\, ds.$$

By assumption,

$$f_i(t) \geq 0, \quad K_i(t, s, 0) \geq 0, \quad H_i(t, s, 0) \geq 0,$$

for all $t, s \in I$, we have

$$T_i\boldsymbol{x}_0(t) \geq 0 = \boldsymbol{x}_{0,i}(t), \quad \forall t \in I.$$

Hence,

$$\boldsymbol{x}_0(t) \preceq T\boldsymbol{x}_0(t), \quad \forall t \in I,$$

which implies

$$\alpha(\boldsymbol{x}_0, T\boldsymbol{x}_0) = \boldsymbol{I}.$$

The existence of an initial point is verified. Consequently, Theorem 1 ensures that T admits exactly one fixed point, which corresponds to the sole solution of Eq. (3). □

4. Applications to Coupled Discrete Recursive Systems

The concept of coupled fixed points in ordered metric spaces has become a fundamental approach in the analysis of coupled nonlinear systems. The further development of this concept is evident in the results obtained, as seen in [29, 30]. This approach provides sufficient conditions to guaranty that the problem admits exactly one solution.

In this section, we concretely apply the Reich–Perov α–contractive fixed point theorem (Theorem 1) to both linearly and nonlinearly coupled discrete recursive systems with nonzero real constant coefficients. This approach yields fixed-point solutions in an exact form, making the results constructive and readily interpretable. This application explicitly demonstrates how the Reich–Perov α contractive theorem guaranties that the problem admits exactly one solution to such discrete systems.

Lemma 3. *Consider $\mathbb{R}^2$ and define $d_v : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}^2$ by*

$$d_v((x,y),(u,v)) = \begin{pmatrix} |x-u| \\ |y-v| \end{pmatrix}, \quad \forall (x,y),(u,v) \in \mathbb{R}^2,$$

then the pair $(\mathbb{R}^2, d_v)$ is a complete vector-valued metric space.

Proof. It is straightforward to verify that d_v meets Definition 1. Now, we show that $(\mathbb{R}^2, d_v)$ is complete.
Take a Cauchy sequence $\{(x_n, y_n)\}$ in $(\mathbb{R}^2, d_v)$. In other words, for any $\boldsymbol{\varepsilon} = (\varepsilon_1, \varepsilon_2) \succ (0,0)$, $\exists N \in \mathbb{N}$ such that

$$d_v((x_n,y_n),(x_m,y_m)) \prec \boldsymbol{\varepsilon}, \quad \forall n,m \geq N.$$

This is equivalent to

$$|x_n - x_m| < \varepsilon_1 \quad \text{and} \quad |y_n - y_m| < \varepsilon_2, \quad \forall n,m \geq N.$$

Thus, $\{x_n\}$ and $\{y_n\}$ are Cauchy in $\mathbb{R}$ and therefore converge to some $x^*, y^* \in \mathbb{R}$. Therefore,

$$d_v((x_n,y_n),(x^*,y^*)) = \begin{pmatrix} |x_n - x^*| \\ |y_n - y^*| \end{pmatrix} \to \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \text{when } n \to \infty.$$

Hence, $(\mathbb{R}^2, d_v)$ is complete. □

Theorem 3. *Given $\mathbb{R}^2$ as in Lemma 3 and a coupled discrete recursive system is considered*

$$\begin{cases} x_{n+1} = 0.2\,x_n + 0.2\,y_n + 1, \\ y_{n+1} = 0.1\,x_n + 0.1\,y_n + 2, \end{cases} \quad n \in \mathbb{N}. \tag{7}$$

We define a self-mapping $T : \mathbb{R}^2 \to \mathbb{R}^2$ associated with Eq. (7) by

$$T(x,y) = \begin{pmatrix} 0.2x + 0.2y + 1 \\ 0.1x + 0.1y + 2 \end{pmatrix}, \tag{8}$$

and $\alpha : \mathbb{R}^2 \times \mathbb{R}^2 \to M_{2\times 2}(\mathbb{R})$ by

$$\alpha(\boldsymbol{x}, \boldsymbol{y}) = \begin{cases} \boldsymbol{I}, & \text{if } x \leq u,\ y \leq v, \\ \boldsymbol{O}, & \text{otherwise}, \end{cases} \tag{9}$$

where $\boldsymbol{I}$ and $\boldsymbol{O}$ denote the identity matrix and the zero matrix in $M_{2\times 2}(\mathbb{R})$, respectively. Then T is a Reich–Perov α-contractive mapping. Consequently, the system (7) has exactly one fixed point $(x^*, y^*) \in \mathbb{R}^2$, and the iteration sequence $\{T^n(x_0, y_0)\}$ converges to (x^*, y^*) for every starting point $(x_0, y_0) \in \mathbb{R}^2$.

Proof. The proof is performed in several steps.

(i) For arbitrarily $(x, y), (u, v) \in \mathbb{R}^2$, direct computation Eq. (8) and applying the triangle inequality yield

$$\begin{aligned}|T_1(x,y) - T_1(u,v)| \le\ & 0.1\big(|x - T_1(x,y)| + |y - T_2(x,y)|\big)\\ &+ 0.1\big(|u - T_1(u,v)| + |v - T_2(u,v)|\big)\\ &+ 0.4\big(|x-u| + |y-v|\big).\end{aligned}$$

$$\begin{aligned}|T_2(x,y) - T_2(u,v)| \le\ & 0.2\big(|x - T_1(x,y)| + |y - T_2(x,y)|\big)\\ &+ 0.1\big(|u - T_1(u,v)| + |v - T_2(u,v)|\big)\\ &+ 0.4\big(|x-u| + |y-v|\big).\end{aligned}$$

Therefore,

$$d_v(T(x,y), T(u,v)) \preceq \boldsymbol{A}_1\, d_v((x,y), T(x,y)) + \boldsymbol{A}_2\, d_v((u,v), T(u,v)) + \boldsymbol{A}_3\, d_v((x,y),(u,v)),$$

where

$$\boldsymbol{A}_1 = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.2 \end{pmatrix}, \qquad \boldsymbol{A}_2 = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.1 \end{pmatrix}, \qquad \boldsymbol{A}_3 = \begin{pmatrix} 0.4 & 0 \\ 0 & 0.4 \end{pmatrix}.$$

The spectral radius of these three matrices is equal to the maximum value of their diagonal elements. Hence, $\rho(\boldsymbol{A}_1) = \max\{0.1, 0.2\} = 0.2, \rho(\boldsymbol{A}_2) = \max\{0.1, 0.1\} = 0.1, \rho(\boldsymbol{A}_3) = \max\{0.4, 0.4\} = 0.4$, and

$$\sum_{i=1}^{3} \rho(\boldsymbol{A}_i) < 1.$$

Consequently, the mapping T satisfies the Reich–Perov α contractive condition.

(ii) Since $\alpha((x,y),(u,v)) \succeq \boldsymbol{I}$ implies $(x,y) \preceq (u,v)$ and T is order-preserving, we have $T(x,y) \preceq T(u,v)$, hence $\alpha(T(x,y), T(u,v)) \succeq \boldsymbol{I}$.
Hence, T is α-admissible. Since T satisfies a Lipschitz condition, it follows that T is continuous on $\mathbb{R}^2$. For any initial point $(x_0, y_0) \in \mathbb{R}^2$, it follows that

$$\alpha((x_0,y_0), T(x_0,y_0)) = \boldsymbol{I} \succeq \boldsymbol{I},$$

so all assumptions of Theorem 1 are satisfied. As a consequence, T admits a unique fixed point $(x^*, y^*) \in \mathbb{R}^2$.
Solving the fixed point equation $T(x^*, y^*) = (x^*, y^*)$ leads to

$$\begin{cases} x^* = 0.2x^* + 0.2y^* + 1, \\ y^* = 0.1x^* + 0.1y^* + 2. \end{cases}$$

Equivalently,

$$\begin{pmatrix} 0.8 & -0.2 \\ -0.1 & 0.9 \end{pmatrix} \begin{pmatrix} x^* \\ y^* \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$$

Since the determinant of the coefficient matrix is equal to $0.7 \neq 0$, the system admits a unique solution given by

$$x^* = \frac{0.9 + 0.4}{0.7} \approx 1.857, \qquad y^* = \frac{1.6 + 0.1}{0.7} \approx 2.429.$$

Hence, the discrete system (7) possesses a unique equilibrium point (x^*, y^*).
Next, for each initial condition $(x_0, y_0) \in \mathbb{R}^2$, a recursive sequence is defined

$$(x_{n+1}, y_{n+1}) = T(x_n, y_n), \quad n \in \mathbb{N}.$$

This sequence represents the discrete evolution of system (7). Since the operator T satisfies the Reich–Perov α-contraction condition, then for every $n \in \mathbb{N}$ it holds

$$d_v(T^n(x_0, y_0), (x^*, y^*)) \longrightarrow 0 \quad \text{when } n \to \infty.$$

Thus,for every initial point $(x_0, y_0) \in \mathbb{R}^2$, the sequence generated by iteration of T converges to its fixed point. □

Theorem 4. *Given $\mathbb{R}^2$ as in Lemma 3. Consider the nonlinear coupled discrete system*

$$\begin{cases} x_{n+1} = 0.1\tanh(x_n) + 0.1\dfrac{y_n}{1+|y_n|} + 0.2x_n + 1, \\ y_{n+1} = 0.1\dfrac{x_n}{1+|x_n|} + 0.1\tanh(y_n) + 0.2y_n + 2. \end{cases} \tag{10}$$

We define $T : \mathbb{R}^2 \to \mathbb{R}^2$ by

$$T(x, y) = \begin{pmatrix} 0.1\tanh(x) + 0.1\dfrac{y}{1+|y|} + 0.2x + 1 \\ 0.1\dfrac{x}{1+|x|} + 0.1\tanh(y) + 0.2y + 2 \end{pmatrix},$$

and $\alpha : \mathbb{R}^2 \times \mathbb{R}^2 \to M_{2\times 2}(\mathbb{R})$ by

$$\alpha(\boldsymbol{x}, \boldsymbol{y}) = \begin{cases} \boldsymbol{I}, & \text{if } x \le u,\ y \le v, \\ \boldsymbol{O}, & \text{otherwise}, \end{cases} \tag{11}$$

where $\boldsymbol{I}$ and $\boldsymbol{O}$ denote the identity matrix and the zero matrix in $M_{2\times 2}(\mathbb{R})$, respectively. Consequently, system (10) admits exactly one fixed point $(x^*, y^*) \in \mathbb{R}^2$. Moreover, for every initial pair $(x_0, y_0) \in \mathbb{R}^2$, the iterates generated by the T approach (x^*, y^*).

Proof. The proof is performed in several steps.

(i) Take an arbitrary $(x, y), (u, v) \in \mathbb{R}^2$. We begin with the first component of the mapping T, that

$$T_1(x, y) = 0.1\tanh(x) + 0.1\frac{y}{1+|y|} + 0.2x + 1.$$

Applying the triangle inequality and using the Lipschitz property, we obtain

$$\begin{aligned} |T_1(x, y) - T_1(u, v)| &\le 0.1|x-u| + 0.1|y-v| + 0.2|x-u| \\ &= 0.3|x-u| + 0.1|y-v|. \end{aligned}$$

$$\begin{aligned} |T_2(x, y) - T_2(u, v)| &\le 0.1|x-u| + 0.1|y-v| + 0.2|y-v| \\ &= 0.1|x-u| + 0.3|y-v|, \end{aligned}$$

Next, observe that

$$x-u=(x-T_1(x,y))+(T_1(x,y)-T_1(u,v))+(T_1(u,v)-u),$$

$$y-v=(y-T_2(x,y))+(T_2(x,y)-T_2(u,v))+(T_2(u,v)-v).$$

By applying the triangle inequality to these decompositions, we derive the following estimates.

$$\begin{aligned}|T_1(x,y)-T_1(u,v)| \le\ & 0.1\big(|x-T_1(x,y)|+|y-T_2(x,y)|\big)\\ &+0.1\big(|u-T_1(u,v)|+|v-T_2(u,v)|\big)\\ &+0.3|x-u|+0.1|y-v|,\end{aligned}$$

$$\begin{aligned}|T_2(x,y)-T_2(u,v)| \le\ & 0.1\big(|x-T_1(x,y)|+|y-T_2(x,y)|\big)\\ &+0.1\big(|u-T_1(u,v)|+|v-T_2(u,v)|\big)\\ &+0.1|x-u|+0.3|y-v|.\end{aligned}$$

Hence,

$$d_v(T(x,y),T(u,v)) \preceq \boldsymbol{A}_1\, d_v((x,y),T(x,y))+\boldsymbol{A}_2\, d_v((u,v),T(u,v))+\boldsymbol{A}_3\, d_v((x,y),(u,v)),$$

where the matrices $\boldsymbol{A}_1$ and $\boldsymbol{A}_2$ are chosen as

$$\begin{pmatrix}0.1 & 0.1\\ 0.1 & 0.1\end{pmatrix},$$

whereas the matrix is defined by

$$\boldsymbol{A}_3 \text{ is defined by } \begin{pmatrix}0.3 & 0.1\\ 0.1 & 0.3\end{pmatrix}.$$

$$\boldsymbol{A}_1=\boldsymbol{A}_2=\begin{pmatrix}0.1 & 0.1\\ 0.1 & 0.1\end{pmatrix}, \quad \boldsymbol{A}_3=\begin{pmatrix}0.3 & 0.1\\ 0.1 & 0.3\end{pmatrix}.$$

A direct calculation yields

$$\sum_{i=1}^{3}\rho(\boldsymbol{A}_i)=0.8,$$

which is strictly less than 1. Therefore, T satisfies the α Reich–Perov contractive condition.

(ii) Since $\tanh x$, x and the function

$$\frac{x}{1+|x|}$$

are continuous on $\mathbb{R}$, so that both T_1 and T_2 are continuous. Hence, a self mapping T is continuous.

(iii) Recall that Eq. (11), assume $\alpha((x,y),(u,v))=\boldsymbol{I}$. We show that $T(x,y)\preceq T(u,v)$. Since the functions

$$\tanh x, \quad \frac{x}{1+|x|}, \quad x$$

are increasing on $\mathbb{R}$, we have

$$\begin{aligned}T_1(x,y)&=0.1\,\tanh(x)+0.1\,\frac{y}{1+|y|}+0.2x+1\\ &\le 0.1\,\tanh(u)+0.1\,\frac{v}{1+|v|}+0.2u+1=T_1(u,v),\end{aligned}$$

and similarly

$$T_2(x,y) \leq T_2(u,v).$$

Hence,

$$T(x,y) \preceq T(u,v),$$

which implies

$$\alpha((x,y),(u,v)) = \boldsymbol{I} \;\Rightarrow\; \alpha(T(x,y),T(u,v)) = \boldsymbol{I}.$$

Thus, T is an α-admissible.

(iv) We establish that one can find $\boldsymbol{s}_0 \in \mathbb{R}^2$ for which

$$\boldsymbol{s}_0 \preceq T(\boldsymbol{s}_0).$$

For instance, choosing $\boldsymbol{s}_0 = (0,0)$ yields $T(0,0) = (1,2)$, so the condition holds. Hence, by definition of α, we obtain

$$\alpha(\boldsymbol{s}_0, T\boldsymbol{s}_0) = \boldsymbol{I}.$$

Since $\boldsymbol{I} \succeq \boldsymbol{I}$ holds componentwise, it follows that $\alpha(\boldsymbol{s}_0, T\boldsymbol{s}_0) \succeq \boldsymbol{I}$.

Consequently, according to Theorem 1, the mapping T admits exactly one fixed point $(x^*, y^*) \in \mathbb{R}^2$. The fixed point satisfies the nonlinear system

$$\begin{cases} x^* = 0.1\,\tanh(x^*) + 0.1\,\dfrac{y^*}{1+|y^*|} + 0.2x^* + 1, \\ y^* = 0.1\,\dfrac{x^*}{1+|x^*|} + 0.1\,\tanh(y^*) + 0.2y^* + 2. \end{cases}$$

Finally, for any initial point $(x_0, y_0) \in \mathbb{R}^2$, define

$$(x_{n+1}, y_{n+1}) = T(x_n, y_n).$$

Then

$$d_v(T^n(x_0,y_0),(x^*,y^*)) \to 0 \quad \text{as } n \to \infty.$$

Hence, the limit of the sequence is a unique fixed point. □

5. Conclusion

This article presents an application of a Reich–Perov fixed point principle with α-contractive condition in vector-valued metric spaces to the Volterra–Fredholm system of integral equations.

By equipping the space $C([0,1],\mathbb{R}^n)$ using a vector valued metric and assuming matrix contraction that converges to zero, we show that the corresponding integral operator has a unique fixed point. This result guarantees that a solution to the Volterra–Fredholm system exists and is uniquely determined. An important addition to these results is that, when applied to coupled discrete recursive systems, the method not only ensures that a fixed point exists and is uniquely determined, but also enables the analytical determination of explicit solutions, thereby providing a clear interpretation of the system's stability and equilibrium states. Furthermore, the vector-valued metric-based formulation and Perov matrix contraction demonstrate advantages over classical scalar approaches, particularly in handling interactions between components in a straightforward and structured manner.

CRediT Authorship Contribution Statement

Sunarsini: Main research ideas, methodology, formal analysis, verification and validation of results, provision of sources and references, writing the initial draft of the article, and reviewing

and editing the final manuscript.
Mahmud Yunus: Development of concepts and theoretical framework, formulation of research methods, mathematical analysis and proof of theorems, and contribution to the writing and refinement of the initial draft.
Subiono: Conceptualization of the research, strengthening the methodological approach, formal analysis, provision of supporting sources, and writing and formulating the initial draft of the article.

Declaration of Generative AI and AI-assisted technologies

All scientific ideas used are a continuation of the paper (see [28]). The authors utilize generative AI technology, specifically *ChatGPT* (developed by OpenAI), as a tool to assist in sentence formulation, enhance text clarity, and explore the initial application possibilities of the theoretical framework proposed by the authors.

Declaration of Competing Interest

The authors declare no competing interests.

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Data and Code Availability

No datasets or source code were generated, analyzed, or used in this study.

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