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Dynamic Analysis of a Modified $SETPR$ Terrorism Model Incorporating Recidivism of Imprisoned Terrorists

Achla Fauziyah and Zulaikha*

Department of Mathematics, Faculty of Science and Technology, Universitas Islam Negeri Walisongo, Semarang, Indonesia

Abstract

Terrorism poses a serious threat to national security, particularly when former terrorist inmates return to terrorist activities because deradicalization programs have failed. This study modifies the $SETPR$ model by adding a direct recidivism pathway from the imprisoned compartment (P) to the active terrorist compartment (T), represented by the parameter k. The model consists of five subpopulations: susceptible individuals (S), exposed individuals (E), active terrorists (T), imprisoned terrorists (P), and individuals who have ceased terrorist activities (R). The basic reproduction number, $\mathcal{R}_0$, is derived using the next-generation matrix method and serves as a threshold parameter for the spread of terrorism. The local stability analysis shows that the terrorism-free equilibrium is locally asymptotically stable when $\mathcal{R}_0 < 1$, whereas the endemic equilibrium is locally asymptotically stable when $\mathcal{R}_0 > 1$. Numerical simulations using MATLAB R2013a and Maple 11 show that, when $\mathcal{R}_0 = 0.0039 < 1$, the system converges to the terrorism-free equilibrium, indicating that terrorism eventually becomes extinct. Conversely, when $\mathcal{R}_0 = 5.8535 > 1$, the system converges to the endemic equilibrium, indicating that terrorism persists at a positive level. These results highlight the importance of controlling key parameters through integrated intervention strategies to reduce $\mathcal{R}_0$ below unity.

Keywords: terrorism; $SETPR$ model; recidivism; imprisonment; basic reproduction number

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1. Introduction

Terrorist acts are considered among the greatest threats to humanity, both now and in the future [1]. In Indonesia, terrorism has posed a serious challenge to the national ideology of Pancasila [2]. Since the early 2000s, several major attacks have occurred, including the 2002 Bali bombings, which killed more than 200 people. These events prompted the government to enact Law No. 5 of 2018 as a comprehensive legal framework for counterterrorism [3]. However, severe punishments, such as the death penalty or life imprisonment, are not always effective in reducing terrorist crimes. Scholars have argued that eradication efforts should focus on addressing causal factors such as economic inequality and political injustice [4]. In this context, the sustainable rehabilitation of terrorist inmates is crucial to prevent them from re-engaging in terrorist activities after release [5].

Between 2000 and 2018, 1,494 perpetrators of terrorist crimes were recorded, 906 of whom had been released, including 52 recidivists [6]. Data collected from 2016 to 2020 show that six out

*Corresponding author. E-mail: zulaikha@walisongo.ac.id

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of ten convicted terrorists were recidivists, while the remainder had previously been convicted of other crimes [7]. A total of 658 suspected terrorists were arrested by Densus 88 between 2020 and March 2022 [8]. Cases of recidivism, such as those involving Sunakim and Yayat Cahdiat in the 2016 Thamrin bombing and the 2017 Kampung Melayu attack, indicate weaknesses in deradicalization programs because extremist beliefs are difficult to eliminate and not all inmates participate in rehabilitation.

Mathematical models have been developed for various social phenomena [9–11]. In the context of terrorism, several models have been proposed. Santoprete and Xu [12] divided the population into susceptible individuals, extremists, recruiters, and individuals undergoing treatment, with a focus on radicalization and deradicalization. Okoye, Collins, and Mbah [13] modeled susceptible, terrorist, and former-terrorist populations to understand and reduce terrorist activity, particularly in the context of Boko Haram in Nigeria. In addition, Gambo and Olarewaju [14] considered six subpopulations—susceptible individuals, moderates, terrorists, leaders, foot soldiers, and detained individuals—and highlighted the possibility that imprisoned terrorists may leave detention and return to terrorist groups. In 2023, Isa, Hwere, et al. [15] developed a model comprising susceptible, exposed, terrorist, imprisoned, and recovered subpopulations to address the possibility that individuals who had been punished and had repented might later become involved in terrorist activities again.

In the model of Isa, Hwere, et al. [15], individuals in the imprisoned compartment (P) can move only to the recovered compartment (R), without the possibility of returning directly to the active terrorist compartment (T). This assumption does not account for the possibility that some imprisoned terrorists may return to terrorist activities after release because of factors such as inadequate deradicalization programs, gaps in detention supervision, pressure from active terrorist networks, persistent extremist beliefs, and limited participation in rehabilitation programs. Consequently, some former terrorist inmates may immediately re-engage in terrorist activities after release, even after receiving rehabilitation in correctional institutions. To address this gap, the present study modifies the $SETPR$ model by adding a transition pathway from the imprisoned compartment (P) to the active terrorist compartment (T), represented by the parameter k. This parameter denotes the rate at which imprisoned terrorists return to terrorist activities.

Structurally, the proposed model retains all inter-compartment transition rates from the model of Isa, Hwere, et al. [15]. It incorporates recruitment by terrorist groups, periods of detention, and the possibility of relapse after individuals have served their sentences and have been declared rehabilitated or have ceased terrorist activities. Individuals who have ceased terrorist activities may lose their fear of the authorities and become re-exposed through contact with active terrorists. This relapse process is represented by the term $(1-\delta)\omega TR$. The main difference from the model of Isa, Hwere, et al. [15] is that, in the previous model, imprisoned terrorists (P) could transition only to the compartment of individuals who had ceased terrorist activities (R). The present model adds a direct recidivism pathway $P \to T$, represented by the parameter k. We derive the basic reproduction number $\mathcal{R}_0$ using the next-generation matrix method and analyze the local asymptotic stability of both the terrorism-free and endemic equilibria using the Routh–Hurwitz criterion. Numerical simulations are performed using the fourth-order Runge–Kutta method in MATLAB R2013a and Maple 11.

2. Methods

This theoretical and analytical study develops and analyzes a mathematical model of terrorist activity by modifying the model of Isa, Hwere, et al. [15]. The modification introduces the parameter k, which represents the rate at which imprisoned terrorists return to the active terrorist subpopulation. The analysis proceeds through several steps. First, a mathematical model of terrorism is formulated with five subpopulations: susceptible individuals (S), exposed individuals (E), active terrorists (T), imprisoned terrorists (P), and individuals who have ceased terrorist activities (R).

The model is formulated under the following assumptions. Individuals in the active terrorist subpopulation (T) may die from natural causes at rate μ or from terrorist activities at rate d. Natural mortality applies to all subpopulations, whereas deaths associated with terrorist activities, such as police raids, suicide bombings, or internal violence, are restricted to active terrorists. Imprisoned terrorists (P) and individuals who have ceased terrorist activities (R) are not subject to the latter type of mortality because they are no longer actively engaged in violent acts.

The susceptible population increases at a constant recruitment rate Λ, representing the entry of individuals aged 17–24 years, an age group identified by the Indonesian National Intelligence Agency (BIN) as particularly vulnerable to radicalization [8]. Susceptible individuals become exposed to terrorist ideology through direct contact with active terrorists. This transmission process is modeled by the bilinear incidence term βST, where β is the contact rate.

Exposed individuals are not yet involved in terrorist acts and progress to the active terrorist compartment at rate ξ. Active terrorists may be arrested and moved to the imprisoned compartment at rate σ, or they may voluntarily cease their activities because of repentance or awareness of the wrongfulness of their actions and move to the compartment of individuals who have ceased terrorist activities at rate γ. Imprisoned terrorists (P) may return to terrorist activities at rate k because of factors such as inadequate deradicalization programs, gaps in detention supervision, pressure from active terrorist networks, persistent extremist beliefs, and limited participation in rehabilitation programs. Alternatively, imprisoned terrorists may move to the compartment of individuals who have ceased terrorist activities (R) at rate θ.

Furthermore, individuals who have ceased terrorist activities may lose their fear of the authorities and become re-exposed through interaction with active terrorists. This relapse process is modeled by the term $(1-\delta)\omega TR$, where ω is the contact rate between individuals who have ceased terrorist activities and active terrorists, and $0 < \delta < 1$. Specifically, a proportion δ of individuals who have ceased terrorist activities maintain their ideological resilience and remain resistant to re-exposure, whereas the remaining proportion $(1-\delta)$ returns to the exposed compartment E. The transitions between the model compartments are illustrated in Fig. 1, while the variables and parameters are defined in Table 1 and Table 2, respectively.

Table 1: Variables of the Mathematical Model of Terrorism

Variable	Description	Unit	Condition
$S(t)$	Susceptible individuals	Individuals	$S(t) \geq 0$
$E(t)$	Individuals exposed to terrorist ideology	Individuals	$E(t) \geq 0$
$T(t)$	Active terrorists	Individuals	$T(t) \geq 0$
$P(t)$	Imprisoned terrorists	Individuals	$P(t) \geq 0$
$R(t)$	Individuals who have ceased terrorist activities	Individuals	$R(t) \geq 0$

Based on the assumptions and the relationships among the variables and parameters illustrated in Fig. 1, the nonlinear system governing the terrorism model is given by

$$\begin{cases} \frac{dS}{dt} &= \Lambda - \beta ST - \mu S \\ \frac{dE}{dt} &= \beta ST + (1-\delta)\omega TR - (\mu + \xi)E \\ \frac{dT}{dt} &= \xi E + kP - (\mu + \gamma + \sigma + d)T \\ \frac{dP}{dt} &= \sigma T - (\mu + \theta + k)P \\ \frac{dR}{dt} &= \gamma T + \theta P - (\mu + (1-\delta)\omega T)R \end{cases}, \tag{1}$$

where $N = S + E + T + P + R$.

Second, the positivity and boundedness of the solutions are established for all $t \geq 0$ under nonnegative initial conditions. Third, the terrorism-free and endemic equilibria are determined,

Table 2: Parameters of the Mathematical Model of Terrorism

Parameter	Description	Unit	Condition
Λ	Recruitment rate of individuals aged 17–24 years into S	$\frac{\text{Individuals}}{\text{Month}}$	$\Lambda > 0$
μ	Natural death rate (deaths not caused by terrorist activities)	$\frac{1}{\text{Month}}$	$\mu > 0$
d	Death rate of active terrorists (T) due to terrorist activities	$\frac{1}{\text{Month}}$	$d > 0$
k	Rate at which imprisoned terrorists return to terrorist activities	$\frac{1}{\text{Month}}$	$k > 0$
β	Contact rate between susceptible individuals and active terrorists	$\frac{1}{\text{Month}\cdot\text{Individual}}$	$\beta > 0$
ω	Contact rate between individuals who have ceased terrorist activities and active terrorists	$\frac{1}{\text{Month}\cdot\text{Individual}}$	$\omega > 0$
θ	Rate at which imprisoned terrorists cease terrorist activities	$\frac{1}{\text{Month}}$	$\theta > 0$
σ	Imprisonment rate of active terrorists	$\frac{1}{\text{Month}}$	$\sigma > 0$
δ	Proportion of individuals in R who remain resistant to re-exposure	–	$0 < \delta < 1$
ξ	Progression rate from exposure to active terrorist involvement	$\frac{1}{\text{Month}}$	$\xi > 0$
γ	Rate at which active terrorists voluntarily cease terrorist activities	$\frac{1}{\text{Month}}$	$\gamma > 0$

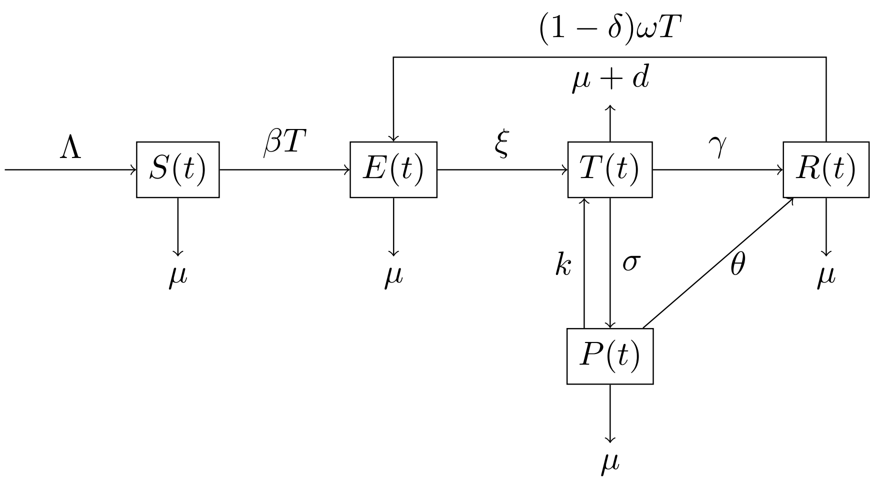


Fig. 1: Compartment diagram of the $SETPR$ model.

and the basic reproduction number $\mathcal{R}_0$ is calculated using the next-generation matrix method. The system is then linearized to obtain the Jacobian matrix, and the Routh–Hurwitz criterion is used to determine the local stability conditions around each equilibrium. Fourth, numerical simulations are conducted using MATLAB R2013a and Maple 11 to illustrate the effects of different values of $\mathcal{R}_0$ on the dynamics of terrorism. The parameter values are obtained from the literature or specified as assumptions. Finally, conclusions are drawn regarding the conditions under which terrorism becomes extinct ($\mathcal{R}_0 < 1$) or persists at a positive level ($\mathcal{R}_0 > 1$).

3. Results and Discussion

This section analyzes the model in Eq. (1) with respect to several key dynamical properties, including positivity and boundedness, the existence of equilibrium points, the basic reproduction number, and the local stability of the equilibria. The results are obtained analytically and illustrated numerically.

3.1. Positivity and Boundedness

The dynamical system governed by Eq. (1), subject to appropriate initial conditions, has nonnegative solutions within the feasible region Ω. This closed set is positively invariant under the system dynamics and is defined by

$$\Omega = \left\{(S, E, T, P, R) \in \mathbb{R}^5_+ : 0 \le N \le \frac{\Lambda}{\mu}\right\}. \tag{2}$$

Theorem 1. *Suppose that $N(t) = (S(t), E(t), T(t), P(t), R(t))$ is a solution of the system in Eq. (1) with initial condition $N(0) \in \mathbb{R}^5_+$. Then, for all $t \ge 0$, the solution remains nonnegative and bounded. In particular, there exists a feasible region $\Omega \subset \mathbb{R}^5_+$ that is positively invariant under the system dynamics.*

Proof. To establish positivity, observe that each equation in Eq. (1) has a nonnegative derivative whenever the corresponding state variable reaches zero. For the first equation, when $S = 0$, we obtain

$$\left.\frac{dS}{dt}\right|_{S=0} = \Lambda \ge 0.$$

so that $S(t) \ge 0$ for all $t \ge 0$.
Similarly, when $E = 0$, we obtain

$$\left.\frac{dE}{dt}\right|_{E=0} = \beta ST + (1-\delta)\omega TR \ge 0.$$

so that $E(t) \ge 0$ for all $t \ge 0$.
For the third equation, when $T = 0$, we obtain

$$\left.\frac{dT}{dt}\right|_{T=0} = \xi E + kP \ge 0.$$

so that $T(t) \ge 0$ for all $t \ge 0$.
For the fourth equation, when $P = 0$, we obtain

$$\left.\frac{dP}{dt}\right|_{P=0} = \sigma T \ge 0.$$

so that $P(t) \ge 0$ for all $t \ge 0$.
Finally, when $R = 0$, we obtain

$$\left.\frac{dR}{dt}\right|_{R=0} = \gamma T + \theta P \ge 0.$$

so that $R(t) \ge 0$ for all $t \ge 0$.
Therefore, all variables remain nonnegative for $t \ge 0$. Consequently, the solution trajectory remains in $\mathbb{R}^5_+$ for all $t \ge 0$.
To establish boundedness of the feasible region Ω, we sum all equations in Eq. (1) to obtain the dynamics of the total population:

$$\frac{dN}{dt} = \Lambda - \mu N - dT,$$

where $N(t) = S(t) + E(t) + T(t) + P(t) + R(t)$. Since $d > 0$ and $T(t) \geq 0$ for all $t \geq 0$, we have $-dT \leq 0$. Therefore,

$$\frac{dN}{dt} \leq \Lambda - \mu N.$$

Solving this differential inequality yields

$$\begin{aligned} N(t) &\leq \frac{\Lambda}{\mu} + \left(N(0) - \frac{\Lambda}{\mu}\right) e^{-\mu t}, \\ \lim_{t\to\infty} N(t) &\leq \frac{\Lambda}{\mu}. \end{aligned} \tag{3}$$

Hence, all solutions remain in the bounded region Ω, and the model admits feasible solutions that are nonnegative and bounded. $\square$

3.2. Equilibrium Points

An equilibrium point is a state in which the size of each subpopulation remains constant over time; equivalently, all rates of change are zero. The model in Eq. (1) admits a terrorism-free equilibrium, denoted by X^0, given by

$$X^0 = (S^0, E^0, T^0, P^0, R^0) = \left(\frac{\Lambda}{\mu}, 0, 0, 0, 0\right). \tag{4}$$

The endemic equilibrium, denoted by $X^* = (S^*, E^*, T^*, P^*, R^*)$, represents a state in which terrorism persists in the population, that is, $T^* > 0$. From the first equation in Eq. (1),

$$\frac{dS}{dt} = \Lambda - \beta ST - \mu S \quad \Rightarrow \quad S^* = \frac{\Lambda}{\beta T^* + \mu}.$$

From the fourth equation in Eq. (1),

$$\frac{dP}{dt} = \sigma T - (\mu + \theta + k)P \quad \Rightarrow \quad P^* = \frac{\sigma T^*}{\mu + \theta + k}.$$

From the third equation in Eq. (1),

$$\frac{dT}{dt} = \xi E + kP - (\mu + \gamma + \sigma + d)T \quad \Rightarrow \quad E^* = \frac{\big[(\mu + \gamma + \sigma + d)(\mu + \theta + k) - k\sigma\big]T^*}{\xi(\mu + \theta + k)}.$$

From the fifth equation in Eq. (1),

$$\frac{dR}{dt} = \gamma T + \theta P - (\mu + (1 - \delta)\omega T)R \quad \Rightarrow \quad R^* = \frac{\gamma T^*(\mu + \theta + k) + \theta\sigma T^*}{(\mu + \theta + k)(\mu + (1 - \delta)\omega T^*)}.$$

To determine T^*, these expressions are substituted into the equation obtained by combining $dE/dt = 0$ and $dT/dt = 0$, namely,

$$\xi\left(\frac{\beta ST + (1 - \delta)\omega TR}{\mu + \xi}\right) + kP - (\mu + \gamma + \sigma + d)T = 0.$$

Substituting S^*, P^*, and R^* yields

$$\xi\left(\frac{\beta\frac{\Lambda}{\beta T^* + \mu}T^* + (1 - \delta)\omega T^* \frac{\gamma T^*(\mu+\theta+k) + \theta\sigma T^*}{(\mu+\theta+k)(\mu+(1-\delta)\omega T^*)}}{\mu + \xi}\right) + k\frac{\sigma T^*}{\mu + \theta + k} - (\mu + \gamma + \sigma + d)T^* = 0.$$

Multiplying both sides by the common denominator and simplifying yields the following quadratic equation for T^*:

$$c_1 T^{*2} + c_2 T^* + c_3 = 0, \tag{5}$$

with

$$c_1 = a_4\beta\left(\xi\left(\gamma a_1 + \theta\sigma\right) - a_2\left(a_1a_3 - k\sigma\right)\right),$$
$$c_2 = \xi a_4\left(\beta\Lambda a_1 + \gamma\mu a_1 + \theta\sigma\mu\right) - \mu a_2(a_1a_3 - k\sigma)(\beta + a_4),$$
$$c_3 = \mu^2 a_2(a_1a_3 - k\sigma)(\mathcal{R}_0 - 1),$$

where

$$a_1 = (\mu + \theta + k), \quad a_2 = \mu + \xi, \quad a_3 = \mu + \gamma + \sigma + d, \quad a_4 = (1 - \delta)\omega.$$

The endemic equilibrium is determined by solving Eq. (5). For a biologically meaningful endemic equilibrium to exist, T^* must be real, positive, and unique. Eq. (5) has a unique positive root when its discriminant satisfies $D = c_2^2 - 4c_1c_3 > 0$ and $c_3/c_1 < 0$. The constant term c_3 is positive when $\mathcal{R}_0 > 1$ and $a_1a_3 - k\sigma > 0$. Moreover, $c_1 < 0$ when $\xi(\gamma a_1 + \theta\sigma) < a_2(a_1a_3 - k\sigma)$, a condition satisfied by the parameter values in Table 3. Because $c_1 < 0$ and $c_3 > 0$, the product of the roots satisfies $c_3/c_1 < 0$, implying that the roots have opposite signs and that exactly one root is positive. Furthermore, $-4c_1c_3 > 0$, so the discriminant satisfies

$$D = c_2^2 - 4c_1c_3 > c_2^2 \geq 0.$$

Thus, $D > 0$, and the two roots are real and distinct. The unique positive root is

$$T^* = \frac{-c_2 - \sqrt{D}}{2c_1}.$$

Once T^* is determined, the remaining equilibrium components are positive:

$$S^* = \frac{\Lambda}{\beta T^* + \mu} > 0, E^* = \frac{[(\mu + \gamma + \sigma + d)(\mu + \theta + k) - k\sigma]T^*}{\xi(\mu + \theta + k)} > 0, P^* = \frac{\sigma T^*}{\mu + \theta + k} > 0,$$
$$R^* = \frac{\gamma T^*(\mu + \theta + k) + \theta\sigma T^*}{(\mu + \theta + k)(\mu + (1 - \delta)\omega T^*)} > 0.$$

Thus, under the conditions $\mathcal{R}_0 > 1$, $a_1a_3 - k\sigma > 0$, and $\xi(\gamma a_1 + \theta\sigma) < a_2(a_1a_3 - k\sigma)$, the endemic equilibrium $X^* = (S^*, E^*, T^*, P^*, R^*)$ exists, is unique, and has strictly positive components.

3.3. Basic Reproduction Number

The potential spread of terrorist influence is characterized by the basic reproduction number $\mathcal{R}_0$. Following the next-generation matrix method of Driessche and Watmough [16], only the compartments directly involved in the transmission and progression of terrorist ideology are considered, namely, E (exposed individuals), T (active terrorists), and P (imprisoned terrorists). Let $\mathcal{F}$ denote the vector of new ideological transmissions and $\mathcal{V}$ the vector of transitions among and out of the affected compartments. From Eq. (1),

$$\mathcal{F} = \begin{bmatrix} \beta ST + (1 - \delta)\omega TR \\ 0 \\ 0 \end{bmatrix}, \qquad \mathcal{V} = \begin{bmatrix} (\mu + \xi)E \\ -\xi E - kP + (\mu + \gamma + \sigma + d)T \\ -\sigma T + (\mu + \theta + k)P \end{bmatrix}.$$

The matrices F and V are obtained by differentiating $\mathcal{F}$ and $\mathcal{V}$ with respect to (E, T, P) and evaluating the derivatives at the terrorism-free equilibrium $X^0 = (\Lambda/\mu, 0, 0, 0, 0)$:

$$F = \begin{bmatrix} 0 & \frac{\beta\Lambda}{\mu} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad V = \begin{bmatrix} \mu + \xi & 0 & 0 \\ -\xi & \mu + \gamma + \sigma + d & -k \\ 0 & -\sigma & \mu + \theta + k \end{bmatrix}.$$

The basic reproduction number $\mathcal{R}_0$ is the spectral radius of the next-generation matrix $K = FV^{-1}$.

$$\det(K - \lambda I) = 0$$

$$\begin{vmatrix} \frac{\beta\Lambda\xi(\mu+\theta+k)}{\mu(\mu+\xi)((\mu+\gamma+\sigma+d)(\mu+\theta+k)-k\sigma)} - \lambda & \frac{\beta\Lambda(\mu+\theta+k)}{\mu((\mu+\gamma+\sigma+d)(\mu+\theta+k)-k\sigma)} & \frac{\beta\Lambda k}{\mu((\mu+\gamma+\sigma+d)(\mu+\theta+k)-k\sigma)} \\ 0 & -\lambda & 0 \\ 0 & 0 & -\lambda \end{vmatrix}$$

which gives $\lambda_1 = \lambda_2 = 0$ and $\lambda_3 = \frac{\beta\Lambda\xi(\mu+\theta+k)}{\mu(\mu+\xi)[(\mu+\gamma+\sigma+d)(\mu+\theta+k)-k\sigma]}$. Therefore, the basic reproduction number for the model in Eq. (1) is

$$\mathcal{R}_0 = \frac{\beta\Lambda\xi(\mu+\theta+k)}{\mu(\mu+\xi)\left[(\mu+\gamma+\sigma+d)(\mu+\theta+k)-k\sigma\right]}. \tag{6}$$

The basic reproduction number $\mathcal{R}_0$ represents the average number of secondary terrorist recruits generated by a single active terrorist in a fully susceptible population under the modified $SETPR$ dynamics. A value of $\mathcal{R}_0 > 1$ indicates that terrorism can invade and persist at a positive level in the community, whereas $\mathcal{R}_0 < 1$ indicates that terrorist activity eventually becomes extinct. The effects of key parameters, including the recidivism rate k, imprisonment rate σ, rehabilitation rate θ, voluntary cessation rate γ, and mortality rate d, on $\mathcal{R}_0$ are examined through sensitivity analysis in a subsequent subsection.

3.4. Local Stability Analysis

Equilibrium stability analysis is an important component of dynamical modeling because it determines whether terrorist activity becomes extinct or persists. This subsection investigates the local stability of the equilibrium points by linearizing the system in Eq. (1) around each equilibrium. The resulting Jacobian matrix is

$$J = \begin{bmatrix} -\beta T - \mu & 0 & -\beta S & 0 & 0 \\ \beta T & -(\mu+\xi) & \beta S + (1-\delta)\omega R & 0 & (1-\delta)\omega T \\ 0 & \xi & -(\mu+\gamma+\sigma+d) & k & 0 \\ 0 & 0 & \sigma & -(\mu+\theta+k) & 0 \\ 0 & 0 & \gamma - (1-\delta)\omega R & \theta & -\mu - (1-\delta)\omega T \end{bmatrix} \tag{7}$$

Theorem 2. *The terrorism-free equilibrium of the system in Eq. (1) is locally asymptotically stable if $\mathcal{R}_0 < 1$ and unstable if $\mathcal{R}_0 > 1$.*

Proof. Substituting the terrorism-free equilibrium into the Jacobian matrix in Eq. (7) yields

$$J(X^0) = \begin{bmatrix} -\mu & 0 & \frac{-\beta\Lambda}{\mu} & 0 & 0 \\ 0 & -(\mu+\xi) & \frac{\beta\Lambda}{\mu} & 0 & 0 \\ 0 & \xi & -(\mu+\gamma+\sigma+d) & k & 0 \\ 0 & 0 & \sigma & -(\mu+\theta+k) & 0 \\ 0 & 0 & \gamma & \theta & -\mu \end{bmatrix} \tag{8}$$

The characteristic equation is obtained from $\det(J(X^0) - \lambda I) = 0$, which yields

$$\det\Big(J(X^0) - \lambda I\Big) = 0$$
$$\iff \begin{vmatrix} -\mu-\lambda & 0 & \frac{-\beta\Lambda}{\mu} & 0 & 0 \\ 0 & -(\mu+\xi)-\lambda & \frac{\beta\Lambda}{\mu} & 0 & 0 \\ 0 & \xi & -(\mu+\gamma+\sigma+d)-\lambda & k & 0 \\ 0 & 0 & \sigma & -(\mu+\theta+k)-\lambda & 0 \\ 0 & 0 & \gamma & \theta & -\mu-\lambda \end{vmatrix} = 0 \qquad (9)$$

Applying cofactor expansion [17] to Eq. (9) gives

$$(-\mu-\lambda)^2\Big(\mu(-(\mu+\xi)-\lambda)(-(\mu+\gamma+\sigma+d)-\lambda)(-(\mu+\theta+k)-\lambda) - \beta\Lambda\xi(-(\mu+\theta+k)-\lambda) - k\sigma\mu(-(\mu+\xi)-\lambda)\Big) = 0$$

Define

$$a_1 = (\mu+\theta+k), \quad a_2 = \mu+\xi, \quad a_3 = \mu+\gamma+\sigma+d, \quad a_4 = (1-\delta)\omega.$$

Then

$$(-\mu-\lambda)^2\Big(\mu\lambda^3 + \mu(a_1+a_2+a_3)\lambda^2 + (\mu(a_2a_3+a_1a_2+a_1a_3-k\sigma) - \beta\Lambda\xi)\lambda + (a_1a_2a_3\mu - a_1\beta\Lambda\xi - a_2k\sigma\mu)\Big) = 0 \qquad (10)$$

From Eq. (10), two eigenvalues of the Jacobian matrix $J(X^0)$ are $\lambda_1 = \lambda_2 = -\mu < 0$. The remaining eigenvalues λ_3, λ_4, and λ_5 satisfy

$$\mu\lambda^3 + b_1\lambda^2 + b_2\lambda + b_3 = 0, \qquad (11)$$

where

$$b_0 = \mu > 0, \quad b_1 = \mu(a_1+a_2+a_3) > 0, \quad b_2 = \mu(a_2a_3+a_1a_2+a_1a_3-k\sigma) - \beta\Lambda\xi,$$

and

$$b_3 = a_1a_2a_3\mu - a_1\beta\Lambda\xi - a_2k\sigma\mu.$$

We can express b_2 in terms of the basic reproduction number $\mathcal{R}_0$ as

$$b_2 = \mu a_2a_1a_3(1-\mathcal{R}_0) + \mu a_1^2a_2 + \mu a_1(a_1a_3-k\sigma) + \mu a_2k\sigma\mathcal{R}_0.$$

Since $a_1a_3 > k\sigma$, it follows that $b_2 > 0$ whenever $\mathcal{R}_0 < 1$. Similarly, b_3 can be rewritten as

$$b_3 = \mu a_2(a_1a_3-k\sigma)(1-\mathcal{R}_0),$$

and the same inequality $a_1a_3 > k\sigma$ implies $b_3 > 0$ when $\mathcal{R}_0 < 1$. To show that all roots of the polynomial in Eq. (11) have negative real parts, we apply the Routh–Hurwitz criterion, which requires

$$\Delta_1 = b_1 > 0, \quad \Delta_2 = b_1b_2 - b_0b_3 > 0.$$

From the above, it is clear that $\Delta_1 = b_1 > 0$. For $\Delta_2 = b_1b_2 - b_0b_3 > 0$, we obtain

$$\begin{aligned} & b_1b_2 - b_0b_3 > 0 \\ \iff\ & \mu(a_1+a_2+a_3)\big(\mu(a_2a_3+a_1a_2+a_1a_3-k\sigma) - \beta\Lambda\xi\big) - \mu(a_1a_2a_3\mu - a_1\beta\Lambda\xi - a_2k\sigma\mu) > 0 \\ \iff\ & 2a_1^2a_2a_3\mu^2 + a_1^3a_2\mu^2 + \mu^2a_1^2(a_1a_3-k\sigma) + a_2^2a_3a_1\mu^2(1-\mathcal{R}_0) + a_2^2a_1^2\mu^2 + a_1a_3^2a_2\mu^2(1-\mathcal{R}_0) \\ & \quad + a_1a_3\mu^2(a_1a_3-k\sigma) + \mu^2a_2^2k\sigma\mathcal{R}_0 + \mu^2a_2a_3k\sigma\mathcal{R}_0 > 0. \end{aligned}$$

Given that $a_1a_3 > k\sigma$, it follows that $b_1b_2 - b_0b_3 > 0$ if $\mathcal{R}_0 < 1$.
These conditions imply that all eigenvalues have negative real parts when $\mathcal{R}_0 < 1$. Therefore, the terrorism-free equilibrium is locally asymptotically stable, which proves Theorem 2. □

Theorem 3. *The endemic equilibrium of the system in Eq. (1) is locally asymptotically stable if $\mathcal{R}_0 > 1$ and unstable if $\mathcal{R}_0 < 1$.*

Proof. Substituting the endemic equilibrium into the Jacobian matrix in Eq. (7) yields

$$J(X^*) = \begin{bmatrix} -\beta T^* - \mu & 0 & -\beta S^* & 0 & 0 \\ \beta T^* & -(\mu+\xi) & \beta S^* + (1-\delta)\omega R^* & 0 & (1-\delta)\omega T^* \\ 0 & \xi & -(\mu+\gamma+\sigma+d) & k & 0 \\ 0 & 0 & \sigma & -(\mu+\theta+k) & 0 \\ 0 & 0 & \gamma-(1-\delta)\omega R^* & \theta & -\mu-(1-\delta)\omega T^* \end{bmatrix} \tag{12}$$

Similarly, the characteristic equation is obtained from $\det(J(X^*) - \lambda I) = 0$, which yields

$$\begin{aligned} &\det\Big(J(X^*) - \lambda I\Big) = 0 \\ \iff &\begin{vmatrix} -\beta T^* - \mu - \lambda & 0 & -\beta S^* & 0 & 0 \\ \beta T^* & -a_2 - \lambda & \beta S^* + a_4 R^* & 0 & a_4 T^* \\ 0 & \xi & -a_3 - \lambda & k & 0 \\ 0 & 0 & \sigma & -a_1 - \lambda & 0 \\ 0 & 0 & \gamma - a_4 R^* & \theta & -\mu - a_4 T^* - \lambda \end{vmatrix} = 0 \end{aligned} \tag{13}$$

Applying cofactor expansion [17] to Eq. (13) gives the following characteristic polynomial:

$$b_0\lambda^5 + b_1\lambda^4 + b_2\lambda^3 + b_3\lambda^2 + b_4\lambda + b_5 = 0, \tag{14}$$

where

$$\begin{aligned}
b_0 &= 1 > 0, \\
b_1 &= a_2 + a_3 + a_1 + a_4T^* + 2\mu + \beta T^* > 0, \\
b_2 &= \mu\mathcal{R}_0(\mu + a_4T^*) + \frac{\xi a_4(\gamma a_1 + \theta\sigma)}{a_2(a_1a_3 - k\sigma)} T^*(\beta T^* + \mu) + (a_3a_1 - \sigma k) + 2\mu(a_1 + a_3 + a_2) + \\
&\quad a_2a_1 + \beta T^*(a_1 + a_3 + a_2) + T^*(a_1a_4 + a_3a_4 + a_2a_4) + \left(\frac{a_2k\sigma}{a_1}\right) > 0, \\
b_3 &= (a_1 + a_2 + a_3)\Big(\mu\mathcal{R}_0(\mu + a_4T^*) + \frac{\xi a_4(\gamma a_1 + \theta\sigma)}{a_2(a_1a_3 - k\sigma)} T^*(\beta T^* + \mu)\Big) + \\
&\quad (a_1a_2 + a_1a_3 + a_2a_3 - \sigma k)\Big(a_4T^* + \beta T^* + 2\mu\Big) + 2\mu a_2\left(\frac{k\sigma}{a_1} - a_3\right) - \\
&\quad \xi a_4T^*(\beta S^* + \gamma + \beta R^*) \\
b_4 &= (a_1a_3 - k\sigma)\Big(\mu a_4T^* + a_2a_4T^* + \beta T^*a_2 + \beta T^{*2}a_4 + \beta T^*\mu + \mu^2\Big) + \mu^2\left(\frac{a_2k\sigma}{a_1}\right) + \mu^2a_2a_3 \\
&\quad (\mathcal{R}_0 - 1) + \mu^2\mathcal{R}_0a_1a_2 + a_2(a_1 + a_3)\Big(\mu\mathcal{R}_0a_4T^* + \frac{\xi a_4(\gamma a_1 + \theta\sigma)}{a_2(a_1a_3 - k\sigma)} T^*(\beta T^* + \mu)\Big) - \xi a_4T^* \\
&\quad \Big(\sigma\theta + \gamma(\mu + \beta T^* + a_1)\Big) - \xi\beta T^*a_4(a_1 + \mu)(S^* + R^*), \\
b_5 &= a_2(a_1a_3 - k\sigma)\Big(\mu\mathcal{R}_0a_4T^* + \frac{\xi a_4(\gamma a_1 + \theta\sigma)}{a_2(a_1a_3 - k\sigma)} T^*(\beta T^* + \mu)\Big) + \mu^2a_2(a_1a_3 - k\sigma)(\mathcal{R}_0 - 1) - \\
&\quad \mu\xi\beta a_1a_4T^*\Big(S^* + R^*\Big) - \xi a_4T^*(\mu + \beta T^*)(\gamma a_1 + \sigma\theta).
\end{aligned}$$

For all roots of the characteristic polynomial in Eq. (14) to have negative real parts, the Routh–Hurwitz conditions are

$$\begin{aligned}
\Delta_1 &= b_1 > 0,\\
\Delta_2 &= b_1 b_2 - b_0 b_3 > 0,\\
\Delta_3 &= b_1 b_2 b_3 - b_0 b_3^2 - b_1^2 b_4 > 0,\\
\Delta_4 &= (b_1 b_4 - b_0 b_5)(b_1 b_2 b_3 - b_0 b_3^2 - b_1^2 b_4) - b_5 (b_1 b_2 - b_0 b_3)^2 - b_0 b_1 b_5^2 > 0.
\end{aligned}$$

Evaluating these conditions shows that all eigenvalues have negative real parts when $\mathcal{R}_0 > 1$. Therefore, the endemic equilibrium is locally asymptotically stable, which proves Theorem 3. □

3.5. Parameter Sensitivity Analysis

Sensitivity analysis is used to determine how each parameter influences the potential spread of terrorism. The normalized sensitivity index of each parameter with respect to the basic reproduction number is computed to quantify its effect on $\mathcal{R}_0$. Parameters with large absolute sensitivity indices have a stronger theoretical influence on the model threshold. The normalized sensitivity index of a parameter y is defined as

$$G_y^{\mathcal{R}_0} = \frac{\partial \mathcal{R}_0}{\partial y} \times \frac{y}{\mathcal{R}_0}, \tag{15}$$

where $y \in \{\beta, \Lambda, \xi, \mu, \theta, k, \gamma, \sigma, d\}$.

Table 3: Parameter values for the conditions $\mathcal{R}_0 < 1$ and $\mathcal{R}_0 > 1$.

Parameters	$\mathcal{R}_0 < 1$		$\mathcal{R}_0 > 1$	
	Value	Source	Value	Source
Λ	12	[13]	12	[13]
μ	0.013077	[15]	0.013077	[15]
d	0.07	[15]	0.07	[15]
k	0.0016	[14]	0.0016	[14]
β	0.000001	[18]	0.0015	Assumption
ω	0.02	Assumption	0.02	Assumption
θ	0.56	[15]	0.56	[15]
σ	0.07	[15]	0.07	[15]
δ	0.5	[15]	0.5	[15]
ξ	0.12502	[15]	0.12502	[15]
γ	0.06	[15]	0.06	[15]

The contact rate β is selected to illustrate the two principal regimes of the model: $\mathcal{R}_0 < 1$, in which terrorism becomes extinct, and $\mathcal{R}_0 > 1$, in which terrorism persists. The value $\beta = 0.000001$ is adopted from the terrorism-modeling literature [18] and yields $\mathcal{R}_0 = 0.0039 < 1$. Conversely, $\beta = 0.0015$ is selected to represent the endemic scenario, yielding $\mathcal{R}_0 = 5.8535 > 1$. The sensitivity analysis shows that $\mathcal{R}_0$ is highly sensitive to β, with a sensitivity index of $+1$, so a proportional change in β produces the same proportional change in $\mathcal{R}_0$. Thus, the two selected values of β capture the qualitative behavior of the model in the two threshold regimes.

The parameter ω does not appear in the expression for $\mathcal{R}_0$. Consequently, ω does not affect the threshold value and influences only the transient dynamics toward equilibrium. In the absence of empirical data, ω is set to 0.02 per month, reflecting the assumption that interactions between former terrorists and active networks are relatively infrequent. Because this value is illustrative, no conversion to a smaller time unit is performed. Varying ω over a reasonable range, such as 0.01–0.1, does not change the value of $\mathcal{R}_0$ or the corresponding stability conclusions.

Using the derived expression for $\mathcal{R}_0$ and the parameter values in Table 3 for the case $\mathcal{R}_0 > 1$, the normalized sensitivity index of each parameter is computed. The resulting values are presented in Table 4.

Table 4: Sensitivity indices of the model parameters with respect to $\mathcal{R}_0$.

Parameters	Value	Sensitivity Index	Impact on $\mathcal{R}_0$
Λ	12	1.0000	Positive (Increasing $\mathcal{R}_0$)
β	0.0015	1.0000	Positive (Increasing $\mathcal{R}_0$)
ξ	0.12502	0.0938	Positive (Increasing $\mathcal{R}_0$)
k	0.0016	0.0009	Positive (Increasing $\mathcal{R}_0$)
μ	0.013077	-1.1445	Negative (Decreasing $\mathcal{R}_0$)
d	0.07	-0.3277	Negative (Decreasing $\mathcal{R}_0$)
σ	0.07	-0.3268	Negative (Decreasing $\mathcal{R}_0$)
γ	0.06	-0.2811	Negative (Decreasing $\mathcal{R}_0$)
θ	0.56	-0.0009	Negative (Decreasing $\mathcal{R}_0$)

The sensitivity indices of β, Λ, ξ, and k are positive. Thus, increasing any of these parameters increases $\mathcal{R}_0$ and, consequently, the potential spread of terrorism. Conversely, the sensitivity indices of μ, θ, γ, σ, and d are negative, indicating that increases in these parameters reduce $\mathcal{R}_0$.

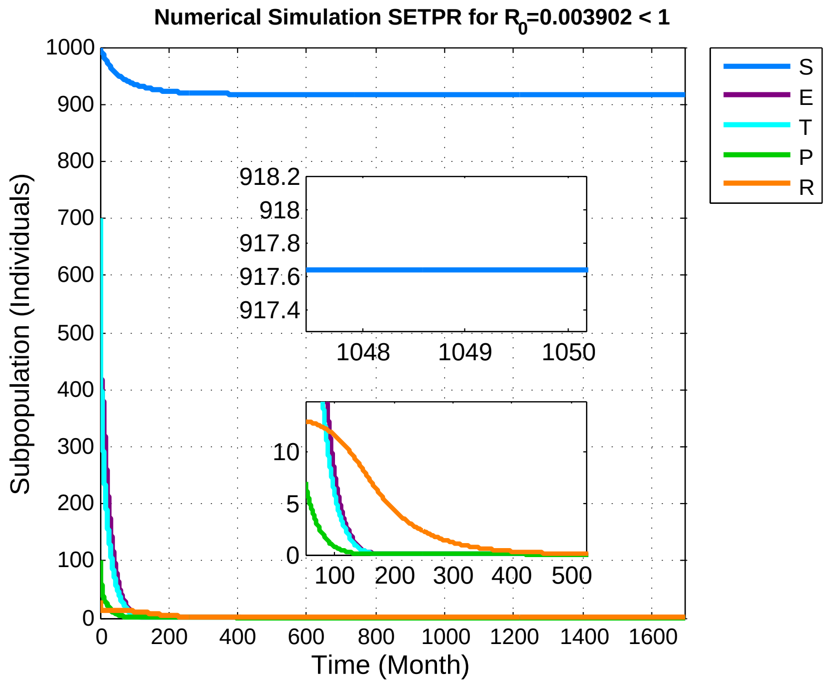
The parameter values used to obtain the sensitivity indices in Table 4 yield $\mathcal{R}_0 \approx 5.85$. The table reports the magnitude and direction of each parameter's sensitivity index. The contact rate β and recruitment rate Λ have the largest positive indices, both equal to 1, and therefore exert the strongest proportional influence on $\mathcal{R}_0$ within the model. If the model is subsequently calibrated using empirical data, parameters with large absolute sensitivity indices would have the greatest theoretical influence on the threshold and on the dynamics of terrorist activity.

3.6. Numerical Simulations

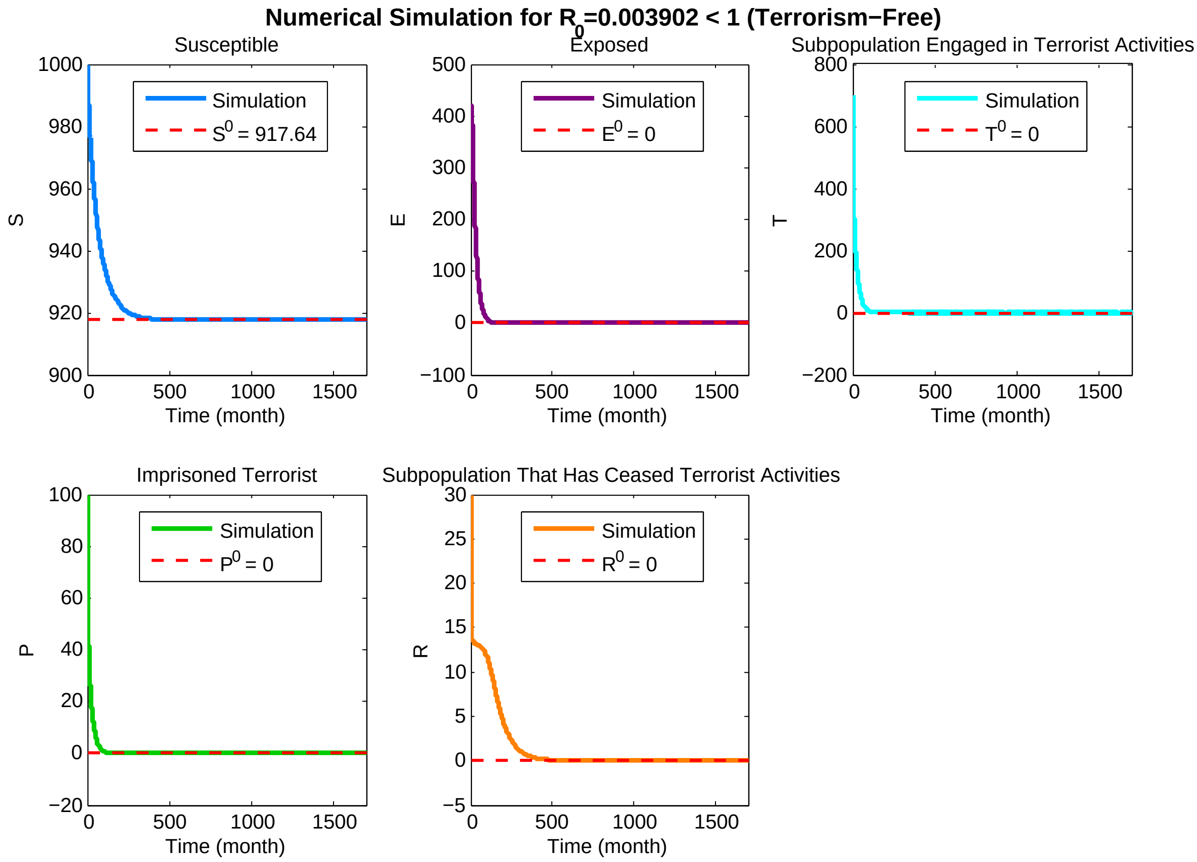
Numerical simulations are conducted to compare the analytical calculations with the numerical behavior of the system in Eq. (1). The simulations are performed using MATLAB R2013a and Maple 11. The initial values for the subpopulations S, E, T, and P are based on Gambo and Olarewaju [14], while the initial value for R is assumed. Specifically, $S(0) = 1000$, $E(0) = 300$, $T(0) = 700$, $P(0) = 100$, and $R(0) = 30$. The simulation horizon is 1700 months for the terrorism-free scenario and 450 months for the endemic scenario. These long horizons are used to display convergence to the respective equilibria clearly and to ensure consistency between the numerical solutions generated by MATLAB and Maple.

The parameter values used in the first simulation are listed in Table 3. For $\beta = 0.000001$, the basic reproduction number is $\mathcal{R}_0 = 0.0039023769 < 1$, indicating that terrorist activity eventually becomes extinct. This behavior is shown in Fig. 2. Under this condition, the equilibrium is $X^0 = (S^0, E^0, T^0, P^0, R^0) = (\Lambda/\mu, 0, 0, 0, 0)$. Over time, the active terrorist subpopulation T approaches zero. The imprisoned subpopulation P and the subpopulation that has ceased terrorist activities R initially decline, subsequently increase, and then approach zero. The exposed subpopulation E initially increases because some exposed individuals remain in the population, but it eventually approaches zero once active terrorism disappears. Meanwhile, the susceptible subpopulation S converges to $\Lambda/\mu = 917.6417$. The numerical values $S \approx 917.6417$, $E \approx 0$, $T \approx 0$, $P \approx 0$, and $R \approx 0$ agree with the analytical equilibrium, confirming convergence to the terrorism-free equilibrium when $\mathcal{R}_0 < 1$.

For $\beta = 0.0015$, the basic reproduction number is $\mathcal{R}_0 = 5.8535653326 > 1$, yielding the endemic equilibrium $X^* = (S^*, E^*, T^*, P^*, R^*) = (72.3753, 173.3722, 101.8169, 12.4021, 12.6586)$. The results for this scenario are shown in Fig. 3. When $\mathcal{R}_0 > 1$, the exposed subpopulation E initially increases sharply and then stabilizes near 173.3722, indicating persistent terrorist influence. The active terrorist subpopulation T exhibits similar behavior and stabilizes near 101.8169. The susceptible subpopulation S decreases and eventually approaches 72.3753, while

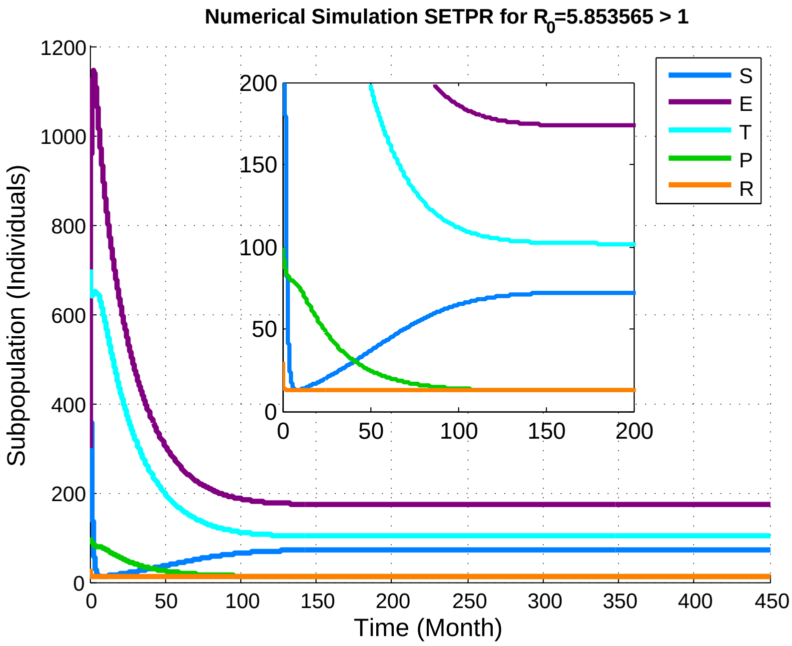


(a)

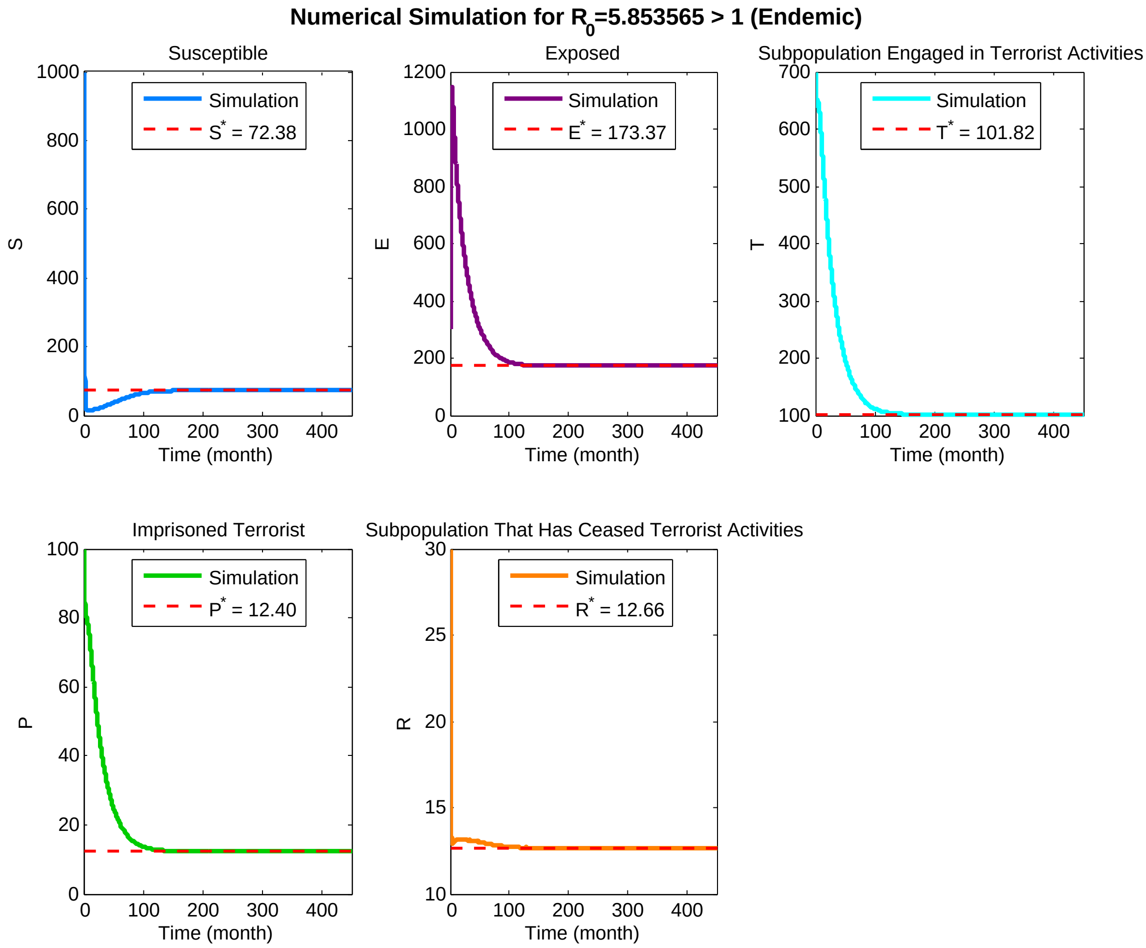


(b)

Fig. 2: Numerical simulations of the model in Eq. (1) for $\mathcal{R}_0 = 0.003902 < 1$.



(a)



(b)

Fig. 3: Numerical simulations of the model in Eq. (1) for $\mathcal{R}_0 = 5.853565 > 1$.

the imprisoned subpopulation P stabilizes near 12.4021. The subpopulation that has ceased terrorist activities R increases and approaches 12.6586. The numerical values agree with the analytical equilibrium, confirming that the system converges to the endemic equilibrium and that terrorism persists at a positive level when $\mathcal{R}_0 > 1$.

4. Conclusion

This study develops a modified $SETPR$ model of terrorism by adding a direct recidivism pathway from the imprisoned compartment (P) to the active terrorist compartment (T), represented by the parameter k. The model comprises susceptible individuals (S), exposed individuals (E), active terrorists (T), imprisoned terrorists (P), and individuals who have ceased terrorist activities (R). Two equilibrium points are obtained: the terrorism-free equilibrium X^0 and the endemic equilibrium X^*. The basic reproduction number $\mathcal{R}_0$ is derived using the next-generation matrix method. The local stability analysis shows that X^0 is locally asymptotically stable when $\mathcal{R}_0 < 1$, whereas X^* is locally asymptotically stable when $\mathcal{R}_0 > 1$. Numerical simulations using the fourth-order Runge–Kutta method in MATLAB R2013a and Maple 11 support the analytical results and show convergence to the corresponding equilibrium. The theoretical sensitivity analysis indicates that β, Λ, μ, γ, and σ have substantial effects on $\mathcal{R}_0$. The contact rate β and recruitment rate Λ have the strongest positive influence, each with a sensitivity index of $+1$, whereas the recidivism rate k has a small positive effect at the baseline parameter values. The imprisonment rate σ and voluntary cessation rate γ have negative effects on $\mathcal{R}_0$.

However, because the model is not calibrated using empirical terrorism data, the findings are illustrative and should not be translated directly into policy recommendations. The interpretation remains conceptual because several parameter values are assumed and the model has not been empirically validated. Future research could extend the framework by estimating parameters from historical terrorism data, applying optimal-control methods to intervention strategies, and validating the model using empirical observations.

CRediT Authorship Contribution Statement

Achla Fauziyah: Conceptualization; Data Curation; Formal Analysis; Funding Acquisition; Investigation; Project Administration; Resources; Software; Validation; Visualization; Writing–Original Draft; Writing–Review and Editing. Zulaikha: Supervision; Validation; Writing–Review and Editing.

Declaration of Generative AI and AI-Assisted Technologies

The authors acknowledge the use of generative AI and AI-assisted technologies in preparing this manuscript. DeepSeek AI was used to organize ideas, refine the content structure, and support exploratory drafting; Humanize AI was used to assist with paraphrasing, grammar, and fluency; and DeepL Translator was used to assist with translation into English.

All original intellectual contributions, interpretations, and final decisions regarding the content were made solely by the authors. The use of AI technologies complied with institutional, ethical, and publication standards. Furthermore, all AI-generated content underwent critical review and editing to ensure accuracy, originality, and scholarly integrity.

Declaration of Competing Interests

The authors declare that they have no competing interests.

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Data and Code Availability

All parameter values and initial conditions required to reproduce the numerical simulations are provided in Table 3 and in the text. The MATLAB R2013a and Maple 11 source code is available from the corresponding author upon reasonable request. Requests should be sent to `zulaikha@walisongo.ac.id` with a brief description of the intended use.

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