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Bayesian Estimation of ECM-NARDL: Inflation and Growth in Indonesia

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Abstract

This study develops a Bayesian estimation framework for the Error Correction Model–Nonlinear Autoregressive Distributed Lag (ECM–NARDL), applied to examine the dynamic and asymmetric relationship between inflation and economic growth in Indonesia. Using annual data from 1990–2024 (35 observations), the model is estimated via Gibbs sampling within a Markov Chain Monte Carlo (MCMC) framework, allowing parameter uncertainty to be quantified through posterior distributions rather than asymptotic approximations. Unit root tests confirm that the variables are integrated of order one, and the Bounds test supports a long-run equilibrium relationship. The error-correction coefficient is negative and statistically significant, indicating that roughly 73% of disequilibrium is corrected within one period. The results further indicate that the long-run effects of inflation are asymmetric, as a sustained decline in inflation is associated with lower economic growth, whereas the long-run effect of a sustained increase in inflation is not statistically distinguishable from zero. Most posterior estimates show satisfactory convergence, although the asymmetric long-run inflation parameters exhibit comparatively higher R-hat values and lower effective sample size, likely reflecting near-collinearity between the decomposed inflation components. These limitations are discussed alongside the study's small annual sample size.

Keywords: Bayesian Inference; ECM–NARDL; Economic Growth Gibbs Sampling; Inflation.

1. Introduction

The Nonlinear Autoregressive Distributed Lag (NARDL) model is a time-series econometric framework designed to capture nonlinear and asymmetric relationships between economic variables in both the short run and the long run [1–3]. Unlike conventional linear models, NARDL allows the response of a dependent variable to positive and negative changes in an explanatory variable to differ in magnitude and direction, thereby providing a more realistic representation of economic dynamics that are often driven by asymmetric shocks [4, 5]. The NARDL model was developed as an extension of the Autoregressive Distributed Lag (ARDL) framework, which has been widely applied in empirical economics due to its flexibility in handling variables integrated of different orders, provided that none is integrated of order two [6, 7]. The ARDL approach enables the simultaneous estimation of short-run dynamics and long-run equilibrium relationships through the bounds testing procedure for cointegration. However, the standard ARDL model relies on the

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assumption of linear and symmetric effects, which may be restrictive in many economic contexts where adjustment processes differ across expansionary and contractionary phases [8, 9].

To address this limitation, Shin et al. (2014) introduced the NARDL model by decomposing explanatory variables into their positive and negative partial sums. This decomposition allows for the identification of asymmetric effects in both the short run and the long run. Furthermore, the NARDL model can be reparameterized into an Error Correction Model (ECM), commonly referred to as the ECM-NARDL specification. Within this framework, the Error Correction Term (ECT) plays a central role in capturing the speed and direction of adjustment toward the long-run equilibrium following a disturbance, offering a richer economic interpretation of dynamic adjustment mechanisms.

Despite its conceptual appeal and modeling flexibility, the ECM-NARDL framework entails an increasingly complex parameter structure as it moves from linear to nonlinear and asymmetric specifications [10, 11]. Most existing empirical studies employing ECM-NARDL continue to rely on classical estimation methods, such as Ordinary Least Squares (OLS) or Maximum Likelihood Estimation (MLE). These approaches typically yield point estimates and inferential results that are highly dependent on a set of restrictive assumptions, including residual normality, homoskedasticity, and independence. In the context of nonlinear dynamic models with interdependent parameters, violations of these assumptions may undermine the reliability and interpretability of statistical inference, particularly in samples of limited size. As an alternative, the Bayesian approach provides a flexible and informative inferential framework for estimating ECM-NARDL models. In the Bayesian paradigm, model parameters are treated as random variables and summarized through posterior distributions, allowing uncertainty to be explicitly quantified [12, 13]. This framework enables the systematic incorporation of prior information alongside observed data and produces inference that does not rely solely on large-sample asymptotic properties [14, 15]. Consequently, Bayesian estimation tends to yield more stable and interpretable results in complex dynamic models, especially when the available sample size is relatively small.

Within the ECM-NARDL setting, Bayesian inference offers distinct advantages beyond conventional significance testing. Rather than focusing exclusively on whether the ECT is statistically significant and negative, the Bayesian framework allows for the direct evaluation of the posterior probability that the ECT lies within the region consistent with long-run stability. Moreover, long-run asymmetries can be examined through comparisons of posterior distributions associated with positive and negative changes in explanatory variables, an inferential perspective that cannot be obtained from OLS-based ECM-NARDL models relying solely on point estimates. Accordingly, the primary contribution of this study lies in extending the inferential scope of ECM-NARDL analysis from traditional parameter significance toward a probabilistic assessment of long-run stability and adjustment mechanisms. Specifically, the contribution of this study lies in its methodological framework, which formulates Bayesian Gibbs sampling for the ECM–NARDL likelihood and employs posterior credible intervals, rather than classical significance tests, to evaluate both the error-correction mechanism and the long-run asymmetry between θ^+ and θ^-. Bayesian estimation is implemented using Markov Chain Monte Carlo (MCMC) algorithms, which enable efficient exploration of posterior distributions in models characterized by nonlinear dynamics and high-dimensional parameter spaces [16–18]. In the context of ECM-NARDL modeling, MCMC techniques facilitate the simultaneous capture of short-run dynamics, long-run relationships, and parameter interactions, thereby providing a comprehensive inferential representation of the underlying economic system.

As an empirical illustration, the proposed Bayesian ECM-NARDL framework is applied to examine the relationship between inflation and economic growth[19–21]. This relationship is widely recognized as dynamic and potentially asymmetric, making it well suited for nonlinear modeling. An asymmetric response is particularly plausible in the Indonesian context, where monetary and fiscal policy responses to inflationary episodes, such as the 1997–1998 Asian

Financial Crisis and the post-2008 stabilization period, have historically differed in intensity from policy responses during periods of disinflation, suggesting that the growth effects of rising and falling inflation need not be symmetric. Nevertheless, the primary emphasis of this study remains on the methodological and inferential aspects of Bayesian ECM-NARDL modeling, while the empirical application serves mainly to demonstrate the practical relevance and interpretative advantages of the proposed approach.

2. Methods

This section describes the methodological framework employed in the study, beginning with the data and empirical workflow and followed by the econometric and Bayesian procedures used for model specification, estimation, inference, and evaluation.

2.1. Data Description and Study Framework

This study employs secondary data obtained from the official database of the International Monetary Fund (IMF). The variables under investigation consist of the inflation rate and economic growth, where economic growth is proxied by the growth rate of Indonesia's real Gross Domestic Product (GDP). The sample period spans from 1990 to 2024, comprising 35 annual observations. This time horizon is selected to capture the structural and cyclical dynamics of the Indonesian economy, including the pre-crisis period, the 1997–1998 Asian Financial Crisis, the subsequent economic recovery phase, the post-2008 global financial crisis stabilization period, and recent structural economic adjustments [22, 23].

The empirical analysis follows an integrated econometric and Bayesian workflow. First, the stationarity properties of the time series are tested using the Augmented Dickey-Fuller (ADF) test to ensure that no variable is integrated of order two I(2), as required by the NARDL framework [24]. Second, the optimal lag length for the NARDL specification is selected using the Akaike Information Criterion (AIC). Third, the Bounds testing procedure is applied to examine the existence of a long-run equilibrium relationship between inflation and economic growth [25, 26]. Fourth, if cointegration is confirmed, the NARDL model is reparameterized into the Error Correction Model-Nonlinear Autoregressive Distributed Lag (ECM-NARDL) form [26]. Fifth, parameter estimation is conducted within a Bayesian framework using the Gibbs sampling algorithm [27]. Sixth, convergence diagnostics are evaluated through trace plots, density plots, Monte Carlo (MC) error, and the Gelman-Rubin statistic [27, 28]. Finally, model adequacy is validated using Posterior Predictive Checks (PPC) [27, 29]. All analyses are performed using RStudio software.

2.2. Unit Root Test

The stationarity of each time series is examined using the Augmented Dickey-Fuller (ADF) test [24]. The ADF test is employed to examine whether a variable contains a unit root, which would indicate non-stationarity. The test is applied to both the level and the first difference of each variable. The general form of the ADF test equation in Eq. (1) is derived from an autoregressive model of order one.

$$Y_t = \phi Y_{t-1} + \varepsilon_t, \quad t = 1, 2, \dots, T \tag{1}$$

where ε_t is a white noise error term [1]. If $|\phi| = 1$, the data are non-stationary (random walk). To test this, the model is transformed into the first-difference form shown in Eq. (2) [30].

$$\Delta Y_t = \beta Y_{t-1} + \varepsilon_t, \quad \text{where } \beta = (\phi - 1) \tag{2}$$

In practice, however, the test equation actually estimated extends Eq. (2) by including an intercept, a linear time trend, and lagged difference terms to correct for higher-order serial correlation in the residuals, as shown in Eq. (3).

$$\Delta Y_t = \alpha + \delta t + \beta Y_{t-1} + \sum_{i=1}^{k} \gamma_i \Delta Y_{t-i} + \varepsilon_t \tag{3}$$

where α is an intercept, δ is the coefficient on a linear time trend t, and k is the augmentation lag order, determined for each series using a standard rule of thumb based on sample size.

The null hypothesis $H_0 : \beta = 0$ states that the series contains a unit root, i.e., it is non-stationary. This is tested against the alternative hypothesis $H_1 : \beta < 0$, which indicates that the series is stationary. The test statistic is given in Eq. (4).

$$DF = \frac{\hat{\beta}}{SE(\hat{\beta})} \tag{4}$$

Under H_0, DF is compared against the non-standard Dickey-Fuller critical values tabulated for a test equation with an intercept and a linear trend, these critical values are approximately -4.15, -3.50, and -3.18 at the 1%, 5%, and 10% significance levels, respectively, for the sample sizes considered in this study. If the p-value is less than α or the calculated DF is less than (more negative than) the critical value, the null hypothesis is rejected, indicating that the series is stationary. If a variable is non-stationary at the level but becomes stationary after taking the first difference, it is classified as integrated of order one, denoted $I(1)$. If a variable is already stationary at the level, it is classified as $I(0)$. The NARDL framework requires that no variable be integrated of order two $I(2)$; thus, the ADF results are used to verify this condition before proceeding with model estimation [25, 26].

2.3. Estimation of the NARDL Model

To accommodate potential nonlinear dynamics and asymmetric responses among the variables, this study employs the Nonlinear Autoregressive Distributed Lag (NARDL) approach [26]. Within this framework, the inflation variable is decomposed into two components—positive and negative changes—using the partial sum decomposition method. This approach allows the model to distinguish the effects of positive and negative movements in inflation on the dependent variable and enables the identification of potential asymmetric responses in the relationship between the variables [5, 31].

Shin et al. [26] developed the NARDL model of order (p, q), as shown in Eq. (5).

$$Y_t = \alpha + \sum_{j=1}^{p} \phi_j Y_{t-j} + \sum_{j=0}^{q} \left(\theta_j^+ X_{t-j}^+ + \theta_j^- X_{t-j}^- \right) + \varepsilon_t \tag{5}$$

where Y_t denotes economic growth (GDP), X_t denotes inflation, and $X_t = X_0 + X_t^+ + X_t^-$. The parameter ϕ_j represents the autoregressive coefficients, while θ_j^+ and θ_j^- denote the asymmetric distributed lag parameters for positive and negative changes in inflation, respectively. The error term ε_t is assumed to be independently and identically distributed with zero mean and constant variance σ^2 [32]. The positive and negative partial sums based on a zero threshold are defined in Eq. (6) [26].

$$X_t^+ = \sum_{j=1}^{t} \max(\Delta X_j, 0), \quad X_t^- = \sum_{j=1}^{t} \min(\Delta X_j, 0) \tag{6}$$

This decomposition allows the model to differentiate between positive and negative changes in the growth rate of inflation, thereby enabling the capture of asymmetric responses of economic growth to upward and downward movements in inflation [4].

In this study, the lag structure (p and q) is determined using the Akaike Information Criterion (AIC) to ensure an appropriate balance between model fit and parsimony. The estimated NARDL model serves as the basis for further analysis of the dynamic properties of the relationship between inflation and economic growth, and for conducting subsequent tests, including the Bounds test for cointegration and the Error Correction Model representation [10].

2.4. Cointegration and ECM-NARDL Specification

The presence of a long-run relationship among the variables is examined using the Bounds test for cointegration within the NARDL framework [25, 26]. This test determines whether the variables included in the model share a stable long-run equilibrium relationship. The Bounds test is an extension of the cointegration approach proposed by Pesaran et al. [25], adapted by Shin et al. [26] to the nonlinear framework.

The null hypothesis $H_0 : \rho = \theta^+ = \theta^- = 0$ states that there is no cointegration. This is tested against the alternative hypothesis $H_1 : \rho \neq 0$ or $\theta^+ \neq 0$ or $\theta^- \neq 0$, which indicates that cointegration exists. The F-statistic proposed by Pesaran et al. [25] is computed as shown in Eq. (7).

$$F = \frac{(JK_{GR} - JK_{GUR})/q}{JK_{GUR}/(n - k_{UR})} \tag{7}$$

where JK_{GR} is the sum of squared residuals from the restricted model (without level terms), JK_{GUR} is the sum of squared residuals from the unrestricted model (with level terms), q is the number of restrictions (coefficients of level terms being tested), n is the number of effective observations, and k_{UR} is the number of parameters estimated in the unrestricted model. The computed F-statistic is compared against the asymptotic critical value bounds tabulated by Pesaran et al. [25] for Case III (unrestricted intercept, no trend) with $k = 2$ level regressors, corresponding to the two decomposed components of inflation. If the calculated F-statistic exceeds the upper bound critical value I(1) at conventional significance levels, the null hypothesis of no cointegration is rejected, implying that cointegration exists. If the F-statistic is below the lower bound critical value I(0), the null hypothesis cannot be rejected, indicating no cointegration. If the F-statistic falls between the I(0) and I(1) critical values, the result is inconclusive [25].

If the Bounds test indicates the existence of cointegration, the NARDL model can be reparameterized into the Error Correction Model (ECM) form proposed by Shin et al. [26], as presented in Eq. (8).

$$\Delta Y_t = \rho Y_{t-1} + \theta^+ X^+_{t-1} + \theta^- X^-_{t-1} + \sum_{j=1}^{p-1} \gamma_j \Delta Y_{t-j} + \sum_{j=0}^{q-1} \left(\phi^+_j \Delta X^+_{t-j} + \phi^-_j \Delta X^-_{t-j} \right) + \varepsilon_t \tag{8}$$

The equivalent representation using the nonlinear error correction term, following Shin et al. [26], is shown in Eq. (9).

$$\Delta Y_t = \rho \xi_{t-1} + \sum_{j=1}^{p-1} \gamma_j \Delta Y_{t-j} + \sum_{j=0}^{q-1} \left(\phi^+_j \Delta X^+_{t-j} + \phi^-_j \Delta X^-_{t-j} \right) + \varepsilon_t \tag{9}$$

where:

- $\rho = \sum_{j=1}^{p} \phi_j - 1$ is the error-correction coefficient, representing the speed of adjustment toward long-run equilibrium. Theoretically, ρ is expected to be negative and statistically significant, implying that the system converges back to equilibrium following a deviation [1, 10].
- $\gamma_j = -\sum_{i=j+1}^{p} \phi_i$ for $j = 1, \ldots, p-1$ are the short-run dynamic coefficients of the dependent variable [26].
- $\theta^+ = \sum_{j=0}^{q} \theta^+_j$ is the long-run coefficient associated with positive changes in inflation.
- $\theta^- = \sum_{j=0}^{q} \theta^-_j$ is the long-run coefficient associated with negative changes in inflation.
- $\phi^+_0 = \theta^+_0$ and $\phi^+_j = -\sum_{i=j+1}^{q} \theta^+_i$ for $j = 1, \ldots, q-1$ are short-run coefficients for positive changes.
- $\phi^-_0 = \theta^-_0$ and $\phi^-_j = -\sum_{i=j+1}^{q} \theta^-_i$ for $j = 1, \ldots, q-1$ are short-run coefficients for negative changes.

The nonlinear error correction term, as defined by Shin et al. [26], is shown in Eq. (10).

$$\xi_t = Y_t - \beta^+ X_t^+ - \beta^- X_t^- \tag{10}$$

where $\beta^+ = -\theta^+/\rho$ and $\beta^- = -\theta^-/\rho$ represent the long-run multipliers for positive and negative changes in inflation, respectively. If the Bounds test indicates the absence of cointegration, the NARDL model cannot be transformed into the ECM representation. In this case, the analysis focuses on short-run dynamics using the short-run NARDL specification proposed by Shin et al. [26], as presented in Eq. (11).

$$\Delta Y_t = \sum_{j=1}^{p-1} \gamma_j \Delta Y_{t-j} + \sum_{j=0}^{q-1} \left(\phi_j^+ \Delta X_{t-j}^+ + \phi_j^- \Delta X_{t-j}^- \right) + \varepsilon_t \tag{11}$$

To address potential contemporaneous correlation between explanatory variables and residuals, the data-generating process for ΔX_t proposed by Shin et al. [26] is presented in Eq. (12).

$$\Delta X_t = \sum_{j=1}^{q-1} \Lambda_j \Delta X_{t-j} + v_t \tag{12}$$

where $v_t \sim iid(0, \Sigma_v)$ and Σ_v is a positive definite covariance matrix of dimension $k \times k$. The residual term ε_t is expressed in Eq. (13).

$$\varepsilon_t = \omega v_t + e_t = \omega \left(\Delta X_t - \sum_{j=1}^{q-1} \Lambda_j \Delta X_{t-j} \right) + e_t \tag{13}$$

Substituting this into the ECM equation yields the nonlinear conditional ECM specification which accounts for potential weak endogeneity arising from non-stationary explanatory variables [26].

2.5. Hypothesis Testing in ECM-NARDL (Classical Framework)

Before conducting Bayesian estimation, the following hypotheses are tested using the classical Ordinary Least Squares (OLS) approach to establish baseline inferences and model validity, with the parameter estimates from OLS serving as a reference for comparison with Bayesian estimates [10]. For partial significance tests (t-test), the t-statistic for each parameter proposed by Gujarati and Porter [32] is computed as presented in Eq. (14).

$$t_{stat} = \frac{\hat{\theta}}{SE(\hat{\theta})} \tag{14}$$

This test statistic follows a t-distribution with $n-K$ degrees of freedom, where K is the number of parameters. For the error correction term (ECT) coefficient ρ, the null hypothesis $H_0 : \rho = 0$ (no long-run adjustment) is tested against $H_1 : \rho < 0$ (there is long-run equilibrium adjustment); rejection of H_0 (with a negative $\hat{\rho}$) indicates that deviations from long-run equilibrium are corrected over time [1, 10]. For the short-run coefficients γ_j (lagged GDP changes), the null $H_0 : \gamma_j = 0$ is tested against $H_1 : \gamma_j \neq 0$; rejection indicates that past GDP values (at lag j) significantly influence current GDP growth [32]. For the short-run positive inflation coefficients ϕ_j^+, the null $H_0 : \phi_j^+ = 0$ is tested against $H_1 : \phi_j^+ \neq 0$; rejection indicates that positive inflation changes at lag j have a statistically significant effect on economic growth [26]. For the short-run negative inflation coefficients ϕ_j^-, the null $H_0 : \phi_j^- = 0$ is tested against $H_1 : \phi_j^- \neq 0$; rejection indicates that negative inflation changes at lag j have a statistically significant effect on economic growth [26]. The decision rule for each test is: if $|t_{stat}| > t_{table}$ (or if p-value $< \alpha$), then reject H_0 at the chosen significance level (typically 5%) [32].

Regarding asymmetry tests (Wald test) within the NARDL framework, one of the key advantages is the ability to capture asymmetric responses of the dependent variable to positive and negative changes in the independent variable [26]. To test whether the relationship between inflation and economic growth is symmetric or asymmetric, the Wald test is employed [26], evaluating the equality of coefficients between positive and negative components, both in the long run and in the short run [5, 31]. For long-run asymmetry, the null hypothesis $H_0 : \theta^+ = \theta^-$ (no long-run asymmetry) is tested against $H_1 : \theta^+ \neq \theta^-$ (long-run asymmetry exists). For short-run asymmetry, the null $H_0 : \sum_{j=0}^{q-1} \phi_j^+ = \sum_{j=0}^{q-1} \phi_j^-$ (no short-run asymmetry) is tested against $H_1 : \sum_{j=0}^{q-1} \phi_j^+ \neq \sum_{j=0}^{q-1} \phi_j^-$ (short-run asymmetry exists). The Wald statistic is defined in Eq. (15) [26].

$$W = (R\hat{\beta} - r)^T [R\widehat{Var}(\hat{\beta})R^T]^{-1}(R\hat{\beta} - r) \tag{15}$$

where $\hat{\beta}$ is the vector of estimated parameters $(\theta^+, \theta^-, \phi_j^+, \phi_j^-)$, R is the restriction matrix representing the hypothesis being tested, and r is the vector of restriction values (typically zero for equality restrictions). The Wald statistic asymptotically follows a chi-square distribution with k degrees of freedom, where k is the number of restrictions [32]. If the computed Wald statistic exceeds the critical value from the χ^2 distribution (or if the p-value $< \alpha$), then H_0 is rejected, indicating the presence of asymmetry in the relationship between inflation and economic growth [4, 26].

2.6. Bayesian Approach to ECM-NARDL Modeling

The Bayesian approach is a statistical inference framework that treats model parameters as random variables characterized by probability distributions [27]. Unlike the frequentist approach, which considers parameters as fixed and relies on large-sample theory for inference, the Bayesian framework explicitly models parameter uncertainty using probabilistic distributions that are updated as new information from the data becomes available [14]. This approach is grounded in Bayes' Theorem, which states that the posterior distribution is obtained by updating the prior distribution with information contained in the likelihood function.

The prior distribution reflects initial beliefs about the parameters before observing the data. In this study, conjugate weakly informative priors are employed to ensure analytical tractability and computational stability, and the term *weakly informative* is used consistently throughout this study to describe these priors, as distinct from a *strongly informative* prior that would impose a sharp belief about the parameter values before the data are observed. Specifically, all model coefficients are assumed to follow a normal distribution, while the error variance is assumed to follow an inverse-gamma distribution. For the ECM-NARDL model with parameter vector $\theta = (\alpha, \rho, \gamma_j, \phi_j^+, \phi_j^-, \sigma^2)$ each coefficient is independently specified to follow a normal distribution $N(\mu_0, \sigma_0^2)$ centered at $\mu_0 = 0$, with $\sigma_0^2 = 1$ for ρ, γ_j, ϕ_j^+, and ϕ_j^-, and a wider $\sigma_0^2 = 10$ for the intercept α to allow more flexibility around the average level of GDP growth. These prior variances are deliberately set well above the posterior standard deviations ultimately obtained for each coefficient, so that the prior contributes comparatively little information once the 35 annual observations are incorporated through the likelihood. The error variance σ^2 is specified to follow an inverse-gamma distribution $\sigma^2 \sim IG(a_0, b_0)$ with $a_0 = 5$ and $b_0 = 5$, a diffuse choice that keeps the prior for σ^2 weakly informative while still guaranteeing a proper posterior distribution. These hyperparameter values are held fixed across all coefficients of the corresponding type and across the three parallel chains used in estimation. To update these prior beliefs with empirical evidence, we construct the likelihood function, which measures the compatibility of the parameters with the observed data.

For the ECM-NARDL model, the likelihood function formulated by Gelman et al. [27] is presented in Eq. (16).

$$f(\Delta Y \mid \theta) = \prod_{t=T_0}^{T} N\left(\Delta Y_t \,\middle|\, \rho\xi_{t-1} + \sum_{j=1}^{p-1} \gamma_j \Delta Y_{t-j} + \sum_{j=0}^{q-1} \left(\phi_j^+ \Delta X_{t-1}^+ + \phi_j^- \Delta X_{t-1}^-\right), \sigma^2\right) \tag{16}$$

The exponential form of the likelihood function is shown in Eq. (17).

$$\begin{aligned} f(\Delta Y \mid \theta) = \prod_{t=T_0}^{T} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\Bigg\{ -\frac{1}{2\sigma^2}\Bigg[\Delta Y_t - \rho\xi_{t-1} \\ - \sum_{j=1}^{p-1}\gamma_j \Delta Y_{t-j} - \sum_{j=0}^{q-1}\left(\phi_j^+ \Delta X_{t-j}^+ + \phi_j^- \Delta X_{t-j}^-\right)\Bigg]^2\Bigg\}. \end{aligned} \tag{17}$$

For computational convenience, the likelihood function is transformed into the log-likelihood form, as presented in Eq. (18).

$$\begin{aligned} \ln f(\Delta Y \mid \theta) = \sum_{t=T_0}^{T} \Bigg[-\frac{1}{2}\ln(2\pi\sigma^2) - \frac{1}{2\sigma^2}\Bigg(\Delta Y_t - \rho\xi_{t-1} \\ - \sum_{j=1}^{p-1}\gamma_j \Delta Y_{t-j} - \sum_{j=0}^{q-1}\left(\phi_j^+ \Delta X_{t-j}^+ + \phi_j^- \Delta X_{t-j}^-\right)\Bigg)^2\Bigg]. \end{aligned} \tag{18}$$

Having specified both the prior distribution and the likelihood function, the next step is to combine them to form the posterior distribution, which reflects the updated beliefs about the parameters after observing the data. The posterior distribution is obtained by combining the prior and likelihood according to Bayes' Theorem [27]. However, due to the complexity of the ECM-NARDL model, the analytical form of the posterior distribution is often difficult to derive. Therefore, this study employs a numerical approximation using the Markov Chain Monte Carlo (MCMC) method, specifically the Gibbs Sampling algorithm [17, 18].

The Gibbs sampling algorithm iteratively samples from the full conditional posterior distributions of each parameter [27, 28]. Starting from initial values for all parameters, each iteration updates $\rho^{(t+1)} \sim p(\rho \mid \gamma^{(t)}, \phi^{+(t)}, \phi^{-(t)}, \sigma^{2(t)}, D)$, then $\gamma_j^{(t+1)} \sim p(\gamma_j \mid \rho^{(t+1)}, \gamma_{-j}^{(t)}, \phi^{+(t)}, \phi^{-(t)}, \sigma^{2(t)}, D)$, followed analogously by ϕ_j^+, ϕ_j^-, and finally $\sigma^{2(t+1)} \sim p(\sigma^2 \mid \rho^{(t+1)}, \gamma^{(t+1)}, \phi^{+(t+1)}, \phi^{-(t+1)}, D)$. Because the priors specified above are conjugate to the normal likelihood in Eq. (16), each full conditional posterior distribution used in the Gibbs sampler has a closed form rather than requiring a separate approximation step. Writing the conditional ECM specification in Eq. (9) as a linear regression of ΔY_t on the error-correction term ξ_{t-1}, the lagged differences ΔY_{t-j}, and the differenced inflation terms ΔX_{t-j}^+ and ΔX_{t-j}^-, let β denote any one of the regression coefficients α, ρ, γ_j, ϕ_j^+, or ϕ_j^-, with z_t denoting its corresponding regressor and r_t denoting the partial residual obtained after removing the contribution of every other coefficient at its current sampled value. Given the normal prior $N(\mu_0, \sigma_0^2)$ for β and the current draw of σ^2, the full conditional posterior of β is also normal, as shown in Eq. (19).

$$\beta \mid \text{rest}, D \sim N(M_\beta, V_\beta) \tag{19}$$

with posterior variance and mean given by

$$V_\beta = \left(\frac{1}{\sigma_0^2} + \frac{1}{\sigma^2}\sum_t z_t^2\right)^{-1} \qquad \text{and} \qquad M_\beta = V_\beta\left(\frac{\mu_0}{\sigma_0^2} + \frac{1}{\sigma^2}\sum_t z_t r_t\right) \tag{20}$$

This update is applied in turn to α, to ρ, to each γ_j, and to each ϕ_j^+ and ϕ_j^-, with the partial residual r_t recomputed at every step using the most recently sampled values of the other coefficients. For the error variance, conjugacy between the inverse-gamma prior $IG(a_0, b_0)$ and the normal likelihood gives a full conditional posterior that is also inverse-gamma, as shown in Eq. (21).

$$\sigma^2 \mid \rho, \gamma, \phi^+, \phi^-, D \sim IG(a_n, b_n) \tag{21}$$

with updated shape and scale parameters

$$a_n = a_0 + \frac{n}{2} \qquad \text{and} \qquad b_n = b_0 + \frac{1}{2}\sum_t \varepsilon_t^2 \tag{22}$$

where n is the number of effective observations used in estimation and ε_t is the model residual evaluated at the current draw of all regression coefficients. These closed-form full conditionals are the distributions actually sampled at every iteration of the Gibbs sampler described above. This process is repeated until convergence diagnostics indicate that the Markov chains have reached a stationary distribution, after which a burn-in period (5,000 iterations) is discarded, and thinning (interval of 10) is applied to reduce autocorrelation [28]. In this study, the MCMC procedure is run for 25,000 iterations using three parallel chains, retaining 2,000 posterior samples per chain (6,000 total) for statistical inference [27].

To assess whether the retained samples adequately represent the target distribution, convergence diagnostics are essential to ensure that MCMC samples reliably represent the target posterior distribution [27]. This study employs both visual and numerical methods. Trace plots assess chain stability and mixing quality, where a converged chain exhibits stable fluctuations around a constant mean [28]. Density plots from multiple chains should overlap substantially, indicating sampling from the same stationary distribution [27]. Monte Carlo Error measures numerical approximation error, defined as $MC\ Error(\bar{\theta}) = \sqrt{\widehat{Var}(\theta)/ESS}$; a small MC Error (e.g., <5% of posterior SD) indicates sufficient precision. Finally, the Gelman-Rubin statistic $\hat{R}$ compares within-chain and between-chain variances; values $\hat{R} < 1.1$ indicate convergence [27].

2.7. Parameter Significance in Bayesian Framework (Credible Interval)

In the Bayesian framework, parameter inference is conducted using credible intervals rather than p-values. Unlike classical confidence intervals, a 95% credible interval has a direct probabilistic interpretation: there is a 95% probability that the true parameter value lies within the interval given the observed data and prior information [15]. The credible interval obtained from the posterior samples generated through the MCMC procedure is shown in Eq. (23) [27].

$$(CI_{95\%}(\theta) = [Q_{0.025}(\theta), Q_{0.975}(\theta)] \tag{23}$$

A parameter is considered to have a meaningful effect if the 95% credible interval excludes zero, indicating high posterior probability (at least 95%) that the parameter is truly different from zero [27]. This approach provides a direct probabilistic assessment of parameter significance, which is more intuitive than the classical p-value interpretation.

2.8. Model Adequacy and Evaluation

Posterior Predictive Check (PPC) is a Bayesian model evaluation method that compares observed data with replicated data generated from the posterior predictive distribution [27]. The underlying principle is that if the model is correctly specified, the simulated data should resemble the observed data. Systematic discrepancies indicate model misspecification [29]. The posterior predictive distribution defined by Gelman et al. [27] is presented in Eq. (24).

$$p(y^{rep} \mid y) = \int p(y^{rep} \mid \theta)p(\theta \mid y)d\theta \tag{24}$$

where y^{rep} denotes replicated data generated from the model, y is the observed data, and θ is the parameter vector. This distribution combines parameter uncertainty (reflected in the posterior distribution) with the data-generating structure (defined by the likelihood).

In practice, PPC is implemented as follows [27, 29]:

1. After MCMC convergence, draw a sample of parameters $\theta^{(s)}$ from the posterior distribution
2. Generate replicated data $y^{rep(s)}$ from the likelihood $p(y^{rep} \mid \theta^{(s)})$

3. Repeat for many MCMC iterations to obtain a distribution of y^{rep}
4. Compare summary statistics (e.g., mean, standard deviation) or graphical displays between observed data and replicated data

Consistency between observed and replicated statistics indicates that the model adequately captures the underlying data-generating process [27]. Common summary statistics for comparison include the sample mean, sample variance, autocorrelation functions, and extreme values. If the observed data fall well within the distribution of replicated statistics, the model is considered adequate [29].

3. Results and Discussion

This section presents and discusses the empirical findings obtained from the proposed analytical framework. The results are reported sequentially from the characteristics of the study data and model specification to the stationarity, cointegration, and Bayesian estimation outcomes.

3.1. Data Description and Study Framework

This study employs annual data obtained from the IMF World Economic Outlook (WEO) database, accessed in March 2026. The dependent variable, denoted GDP_t, is economic growth, measured as the annual percentage change in Indonesia's real Gross Domestic Product. The independent variable, denoted $INFLASI_t$, is the inflation rate, measured as the annual percentage change in the average Consumer Price Index (CPI). The sample period spans from 1990 to 2024, comprising 35 annual observations. The selected time horizon is intended to capture the structural and cyclical dynamics of the Indonesian economy, including the pre-crisis period, the 1997–1998 Asian Financial Crisis, the subsequent economic recovery phase, the post-2008 global financial crisis stabilization period, and more recent structural economic adjustments [23]. This extended timeframe provides a comprehensive representation of macroeconomic fluctuations and policy regimes, thereby ensuring robust empirical analysis of the long-run and short-run interactions between inflation and economic growth [24].

Stationarity is tested using the ADF test, specified with an intercept and a linear trend (lag order set by a standard rule of thumb). A robustness check without the trend term, with lag selected via AIC, yields unchanged conclusions for the differenced series; the level-series results, however, are sensitive to the trend term, likely reflecting the short annual sample ($T = 35$), noted as a limitation. Following the stationarity assessment, the lag order (p, q^+, q^-) of the NARDL specification was determined via a grid search over lag combinations up to a maximum lag of three for each of p, q^+, and q^-, with model fit compared using the Akaike Information Criterion (AIC). Of the combinations evaluated within this bound, ten were successfully estimated; the three with the lowest AIC were NARDL $(1, 2, 1)$, NARDL $(2, 1, 1)$, and NARDL $(2, 3, 3)$, with AIC values of 135.2023, 135.4381, and 135.8081, respectively. The NARDL $(1, 2, 1)$ specification produced the lowest AIC value and was therefore selected as the optimal model, corresponding to one lag of the dependent variable, two lags of the positive component of inflation changes, and one lag of the negative component.

With the data and lag structure established, the remainder of this section proceeds through the same sequence introduced in Section 2, testing stationarity, evaluating cointegration, and then estimating the Bayesian ECM-NARDL model, reporting at each step the results specific to the inflation-growth relationship in Indonesia rather than repeating the underlying procedures already described in the Methods section.

3.2. Unit Root Test

In this study, the stationarity of the variables is tested using the Augmented Dickey–Fuller (ADF) test. The ADF test is employed to examine the null hypothesis that a variable contains a unit root (i.e., is non-stationary) against the alternative hypothesis that the variable is stationary

[24]. The test is conducted at both the level and first difference. If a variable is non-stationary at the level but becomes stationary after taking the first difference, the variable is said to be integrated of order one, denoted as I(1). Conversely, if a variable is already stationary at the level, it is considered to be integrated of order zero, denoted as I(0). The results of the ADF test for each variable are presented in Table 1.

Table 1: Unit root tests

Unit Root	ADF Level	ADF 1st Diff.
GDP	-2.7296 [0.2898]	-3.8836 [0.0264]*
INFLASI	-3.0530 [0.1642]	-4.1001 [0.0177]*

Notes: [ ] denotes the p-value while * represents the rejection of the null hypothesis at the 5% significance level.

The findings indicate that GDP and inflation are non-stationary at the level, as the associated p-values exceed the conventional 5 percent significance level, implying that the null hypothesis of a unit root cannot be rejected. However, after transforming the variables into their first differences, the ADF test statistics become statistically significant, indicating that both variables become stationary after first differencing. These results suggest that GDP and inflation are integrated of order one, I(1). Since none of the variables is integrated of order two I(2), the integration properties satisfy the necessary conditions for applying the Nonlinear Autoregressive Distributed Lag (NARDL) model and its corresponding Error Correction representation (ECM–NARDL) in the subsequent empirical analysis.

3.3. Estimation of the NARDL Model

To accommodate potential nonlinear dynamics and asymmetric responses among the variables, this study employs the *Nonlinear Autoregressive Distributed Lag* (NARDL) approach. Within this framework, the inflation variable is decomposed into two components, namely positive and negative changes, using the *partial sum decomposition* method [5, 26]. This approach allows the model to distinguish the effects of positive and negative movements in inflation on the dependent variable and enables the identification of potential asymmetric responses in the relationship between the variables [31]. In addition, the model incorporates lagged values of both the dependent and explanatory variables to represent the dynamic adjustment process commonly observed in time series data. The inclusion of lagged terms allows the model to capture temporal dependence and delayed responses that may occur across different periods.

In this study, the estimated specification follows a *NARDL(1,2,1)* structure, indicating that the model includes one lag of the dependent variable, two lags of the positive component of inflation changes, and one lag of the negative component of inflation changes. Accordingly, the estimated model is specified in Eq. (25).

$$\begin{aligned} GDP_t = \alpha + \phi_1 GDP_{t-1} + \theta_0^+ INFLASI_t^+ + \theta_1^+ INFLASI_{t-1}^+ + \theta_2^+ INFLASI_{t-2}^+ \\ + \theta_0^- INFLASI_t^- + \theta_1^- INFLASI_{t-1}^- + \varepsilon_t \end{aligned} \tag{25}$$

where GDP_t denotes the dependent variable, while $INFLASI_t^+$ and $INFLASI_t^-$ represent the positive and negative partial sum components of inflation obtained from the decomposition process. The lag structure in the model allows the analysis to capture the dynamic interaction between variables while also accommodating the possibility of different responses to positive and negative changes in inflation. The estimated model serves as the basis for further analysis of the dynamic properties of the relationship between the variables and for conducting subsequent tests within the NARDL framework, including the *Bounds test for cointegration* and the *Error Correction Model* (ECM) representation to examine the short-run adjustment dynamics.

3.4. Cointegration

The presence of a long-run relationship among the variables is examined using the Bounds test for cointegration within the NARDL framework. This test is used to determine whether the variables included in the model share a stable long-run equilibrium relationship. The results of the Bounds test are presented in Table 2.

Table 2: Bounds Test Results

Test	F-statistic	p-value
Bounds Test for Cointegration	7.2141	0.0036

Note: The null hypothesis of the Bounds test states that there is no cointegration among the variables.

The test results indicate an F-statistic value of 7.214 with a p-value of 0.0036. Since the F-statistic (7.214) exceeds the upper bound critical value $I(1)$ even at the 1% significance level (6.36), the null hypothesis of no cointegration is rejected decisively, indicating the existence of a long-run equilibrium relationship between GDP and the decomposed components of inflation. The evidence of cointegration suggests that although short-run fluctuations may occur, the variables in the model tend to move together toward a stable long-run equilibrium over time. Therefore, the NARDL model can be further expressed in its Error Correction Model (ECM) form to analyze both the short-run dynamics and the speed of adjustment toward the long-run equilibrium. The ECM formulation reparameterizes the original NARDL model by incorporating the lagged level variables, which represent the long-run relationship, together with the differenced variables that capture short-run adjustments. In this framework, the coefficient of the lagged dependent variable reflects the error-correction mechanism, indicating how quickly deviations from the long-run equilibrium are corrected over time. Accordingly, the ECM representation of the NARDL model is presented in Eq. (26).

$$\begin{aligned}\Delta GDP_t = {} & \rho GDP_{t-1} + \theta^{+} INFLASI^{+}_{t-1} + \theta^{-} INFLASI^{-}_{t-1} \\ & + \phi_0^{+} \Delta INFLASI^{+}_{t} + \phi_1^{+} \Delta INFLASI^{+}_{t-1} + \phi_0^{-} \Delta INFLASI^{-}_{t} + \epsilon_t\end{aligned} \tag{26}$$

The coefficients in the ECM–NARDL model describe both the long-run relationship and the short-run dynamics between inflation and economic growth. The parameter ρ represents the error-correction coefficient associated with the lagged dependent variable GDP_{t-1}, indicating the speed at which deviations from the long-run equilibrium are corrected over time. The coefficients θ^{+} and θ^{-} are the level coefficients of the partial-sum inflation terms in the conditional ECM, from which the long-run multipliers are subsequently obtained following Equation (10) (see Section 3.5). Equivalently, substituting $\rho\xi_{t-1} = \rho\,\mathrm{GDP}_{t-1} + \theta^{+}\mathrm{INFLASI}^{+}_{t-1} + \theta^{-}\mathrm{INFLASI}^{-}_{t-1}$ shows that Eq. (26) is the conditional-ECM counterpart of the nonlinear error-correction specification in Eq. (9). Estimating Eq. (26) and recovering $\beta^{+} = -\theta^{+}/\rho$ and $\beta^{-} = -\theta^{-}/\rho$ is therefore algebraically equivalent to estimating the model directly in terms of the nonlinear error-correction term ξ_t defined in Eq. (10). Meanwhile, the coefficients ϕ_0^{+}, ϕ_1^{+}, and ϕ_0^{-} represent the short-run effects of positive and negative inflation changes through their differenced terms. Finally, ε_t denotes the stochastic error term that captures other random shocks affecting GDP outside the model.

3.5. Bayesian Estimation Results

The ECM–NARDL model parameters were estimated using a Bayesian approach with the Gibbs Sampling algorithm, a *Markov Chain Monte Carlo* (MCMC) technique for generating samples from posterior distributions through iterative conditional sampling. Weakly informative priors were applied, consisting of normal distributions for regression coefficients and inverse-gamma distributions for error variance. The MCMC process was run for 25,000 iterations with a burn-in of 5,000 iterations and a thinning interval of 10. Three parallel Markov chains were employed, yielding a total of 6,000 posterior samples for parameter inference. The Bayesian ECM–NARDL estimation results, including MCMC diagnostics and posterior predictive checks, are presented in Table 3, while the corresponding posterior standard deviations and 95% credible intervals for every parameter, which support the interpretation of statistical significance discussed below, are reported alongside the same point estimates in Table 4.

Table 3: Bayesian ECM–NARDL Estimation and Posterior Predictive Check Results

Parameter (Variable)	Bayesian Estimate	MC Error	R-hat	ESS
α (Intercept)	5.1542* [1.4768]	0.0410	1.0015	1297.56
ρ (GDP_{t-1})	-0.7312* [0.1663]	0.0043	1.0035	1469.12
θ^+ ($INFLASI^+_{t-1}$)	0.2574 [0.1401]	0.0231	1.1283	36.80
θ^- ($INFLASI^-_{t-1}$)	0.2618* [0.1344]	0.0212	1.1276	40.06
ϕ_0^+ ($\Delta INFLASI^+_t$)	-0.3526* [0.0360]	0.0006	1.0003	3713.35
ϕ_1^+ ($\Delta INFLASI^+_{t-1}$)	-0.1070 [0.0986]	0.0078	1.0427	160.36
ϕ_0^- ($\Delta INFLASI^-_t$)	0.2241 [0.1511]	0.0154	1.0715	95.76
σ^2 (Error variance)	2.7087* [0.6954]	0.0101	1.0014	4695.92
Model Diagnostics				
MCMC iterations	25,000			
Burn-in	5,000			
Thinning interval	10			
Number of chains	3			
Posterior samples (per chain)	2,000			
Total posterior samples	6,000			
Posterior Predictive Check (PPC)				
Statistic	Observed Data	Posterior Predictive Mean	Difference	
Mean	-0.0467	-0.0969	0.0502	
Standard Deviation	4.5568	4.5345	0.0223	

Notes, * indicates 95% credible interval excludes zero, with the corresponding interval bounds for each parameter reported in Table 4, [ ] = posterior SD, MC Error, R-hat, ESS = MCMC diagnostics, posterior predictive based on y_{rep}.

The estimation results of the Bayesian ECM–NARDL model presented in Table 3 show that inflation and economic growth have a dynamic and asymmetric relationship. The intercept coefficient (α) is positive and statistically significant, indicating that GDP tends to increase when other variables remain constant. The error correction term represented by the lagged GDP coefficient ($\rho = -0.7312$) is also statistically significant. The negative value indicates that deviations from the long-run equilibrium are corrected over time. Economically, this means that around 73% of the imbalance from the previous period is adjusted within one period, suggesting a relatively fast return to long-run equilibrium.

In the long run, the results in Table 3 show asymmetric effects of inflation. The coefficient of positive inflation (θ^+) is positive but not statistically significant, while the coefficient of negative inflation (θ^-) is positive and statistically significant. Following the definition in Eq. (10), these conditional-ECM coefficients are converted into long-run multipliers by transforming each retained MCMC draw as $\beta^+ = -\theta^+/\rho$ and $\beta^- = -\theta^-/\rho$. The posterior mean of β^+ is 0.3646, with a 95% credible interval of -0.019 to 0.845, and the posterior mean of β^- is 0.3695, with a 95% credible interval of 0.009 to 0.822 (Table 4). Because X_t^+ and X_t^- are constructed as partial

Table 4: Long-Run Multipliers and Underlying ECM–NARDL Coefficients

Parameter	Definition	Posterior Mean	Posterior SD	95% Credible Interval
α	Intercept	5.1542*	1.4768	[2.1760, 7.9465]
ρ	GDP_{t-1} coefficient	−0.7312*	0.1663	[−1.0504, −0.3953]
θ^+	$INFLASI^+_{t-1}$ coefficient	0.2574	0.1401	[−0.0117, 0.5277]
θ^-	$INFLASI^-_{t-1}$ coefficient	0.2618*	0.1344	[0.0063, 0.5200]
ϕ^+_0	$\Delta INFLASI^+_t$ coefficient	−0.3526*	0.0360	[−0.4230, −0.2809]
ϕ^+_1	$\Delta INFLASI^+_{t-1}$ coefficient	−0.1070	0.0986	[−0.2964, 0.0919]
ϕ^-_0	$\Delta INFLASI^-_t$ coefficient	0.2241	0.1511	[−0.0783, 0.5184]
σ^2	Error variance	2.7087*	0.6954	[1.6723, 4.3454]
β^+	$-\theta^+/\rho$	0.3646	0.2256	[−0.0188, 0.8455]
β^-	$-\theta^-/\rho$	0.3695	0.2134	[0.0089, 0.8217]

Notes: * indicates 95% credible interval excludes zero. α, ρ, θ^+, θ^-, ϕ^+_0, ϕ^+_1, ϕ^-_0, and σ^2 are the conditional-ECM coefficients reported in Table 3. β^+ and β^- are the long-run multipliers defined in Eq. (10), obtained by transforming every retained MCMC draw, $\beta^{(s)} = -\theta^{(s)}/\rho^{(s)}$, and summarizing the resulting posterior sample directly.

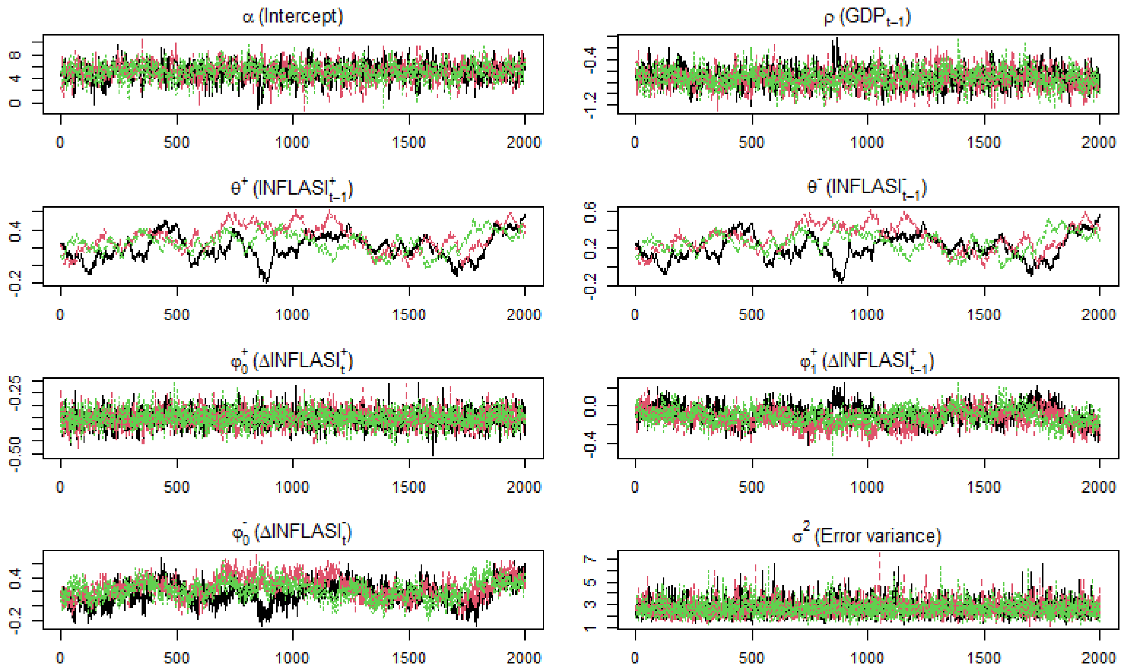

Fig. 1: Trace plots of posterior draws for the Bayesian ECM–NARDL parameters.

sums of, respectively, the positive and negative changes in inflation, a one-percentage-point permanent increase in inflation raises X^+_t by one unit while leaving X^-_t unchanged, so the implied long-run change in GDP growth is $+\beta^+$, or about 0.36 percentage points. Conversely, a one-percentage-point permanent decrease in inflation lowers X^-_t by one unit while leaving X^+_t unchanged, so the implied long-run change in GDP growth is $-\beta^-$, or about -0.37 percentage points. Because the 95% credible interval of β^+ includes zero while that of β^- excludes zero, albeit narrowly, only the long-run effect attached to declining inflation, written as $-\beta^-$, can be considered reliably different from zero. The effect attached to rising inflation, written as $+\beta^+$, should still be interpreted with caution given the lack of statistical support for θ^+. This asymmetry suggests that movements in the negative, or declining, component of inflation, rather than the positive, or rising, component, are the channel through which the long-run growth effect operates. A sustained decline in inflation is therefore associated with a reduction in long-run economic growth of about 0.37 percentage points for each percentage point of decline. This

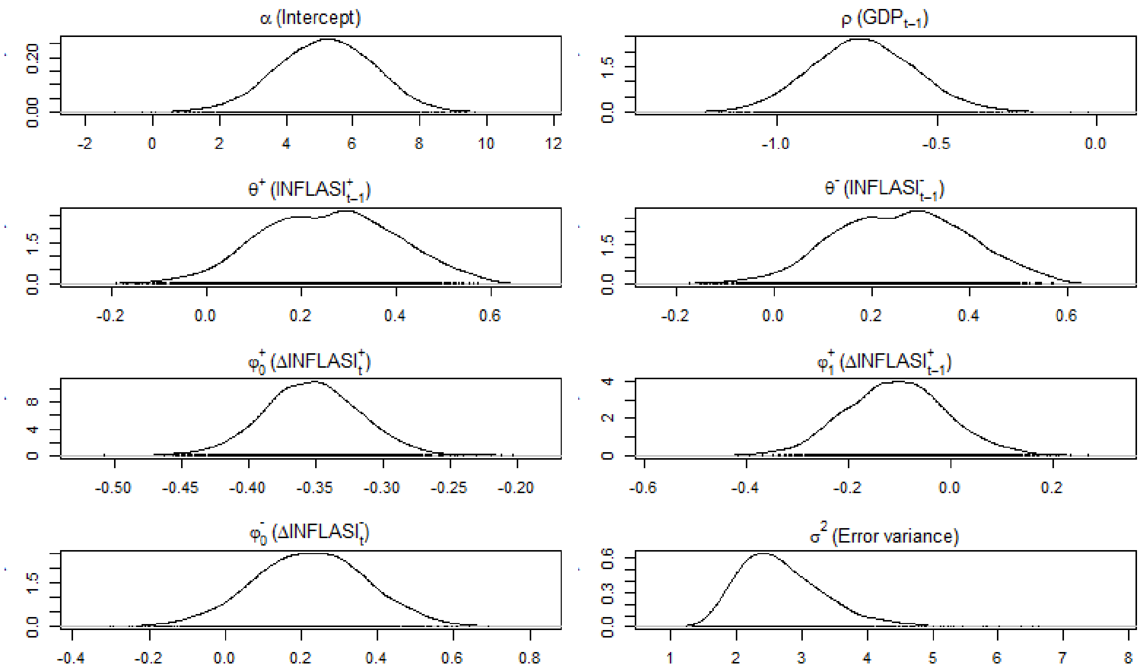

Fig. 2: Posterior density plots of the Bayesian ECM–NARDL parameters.

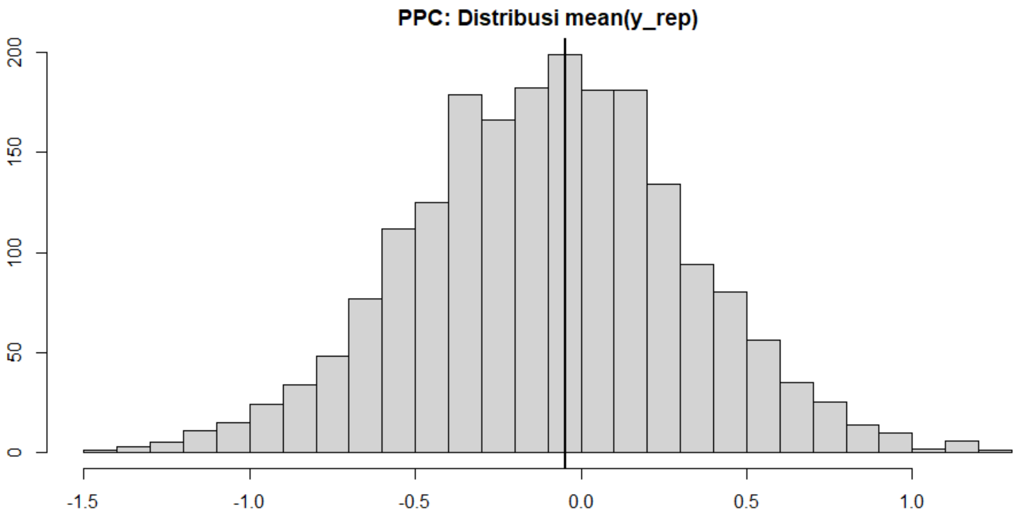

Fig. 3: Posterior predictive distribution plot for the Bayesian ECM–NARDL model.

pattern is broadly consistent with the joint contraction of inflation and GDP growth observed around the 2020 pandemic shock in the sample. As with θ^+ and θ^- themselves, the posterior estimates of β^+ and β^- inherit the comparatively wider uncertainty associated with the near-collinearity between the decomposed inflation components, noted as a limitation of this study. In the short run, increases in inflation have a negative and significant effect on GDP, as shown by the coefficient of (ϕ_0^+), while the other short-run inflation effects are not statistically significant.

Taken together, the empirical findings point to a fairly clear overall picture of how inflation and economic growth interact in Indonesia over the sample period. The economy returns to long-run equilibrium quickly once disturbed, the short-run response to inflation is negative, and the long-run response is asymmetric, with the growth-dampening channel running mainly through declining rather than rising inflation. Economically, this pattern is consistent with several mechanisms commonly discussed in the inflation-growth literature. On the short-run side, the immediate negative effect of rising inflation on GDP growth is consistent with standard menu-cost and price-rigidity arguments, where firms facing higher and more volatile inflation incur greater costs in repricing goods and inputs, which temporarily weighs on output. On the long-run side, the asymmetry toward the declining-inflation channel is consistent with the idea that disinflation episodes in Indonesia have often coincided with tight monetary policy or weak aggregate demand rather than occurring on their own, so that periods of falling inflation tend to be periods of subdued growth for reasons connected to, but not limited to, the price level itself. This interpretation is in line with the asymmetric monetary and fiscal policy responses to inflationary and disinflationary episodes already noted in the Introduction, and it is also visible directly in the sample, since the sharp disinflation observed in 2020 coincided with one of the deepest growth contractions in the data. These findings are broadly in line with other applications of the NARDL framework to the inflation-growth relationship in different country settings, including the asymmetric effects reported for Iran by Khalili et al. [4], the asymmetric cointegration evidence summarized by Hossain et al. [5], and the asymmetric inflation-growth relationship documented for Mexico by Trejo-García et al. [31], all of which support the general conclusion that inflation does not affect growth symmetrically. Within the Indonesian context specifically, this result also complements the discussion in Saefulloh et al. [23] on the relationship between inflation and economic growth in Indonesia, extending it through an explicitly asymmetric and Bayesian lens rather than a single linear coefficient.

The Bayesian diagnostic results presented in Figure 1 and Figure 2 indicate that the Markov Chain Monte Carlo (MCMC) estimation performs adequately for most parameters, although convergence is weaker for the asymmetric long-run inflation coefficients. The Gelman–Rubin statistic (R-hat) reported in Table 3 remains below the 1.1 threshold stated in Section 2.6 for α, ρ, ϕ_0^+, ϕ_1^+, ϕ_0^-, and σ^2, supporting convergence for these six parameters. For θ^+ and θ^-, however, R-hat reaches 1.1283 and 1.1276 respectively, both above this threshold, so convergence for these two coefficients should not be claimed outright. The effective sample size shows a related pattern, with θ^+ and θ^- retaining only 36.80 and 40.06 effective samples out of the 6,000 retained posterior draws, and with ϕ_1^+ and ϕ_0^- also showing comparatively low effective sample sizes of 160.36 and 95.76, while the remaining four parameters retain effective sample sizes in the thousands. The correspondingly wide 95% credible intervals reported in Table 4 for θ^+, and to a lesser extent θ^-, further reflect this elevated posterior uncertainty. This pattern is consistent with the near-collinearity between the decomposed inflation components noted earlier in this study, and means that the posterior summaries for θ^+ and θ^- in particular should be interpreted with more caution than the other parameters. The trace plots in Figure 1 and the density plots in Figure 2 are broadly consistent with this picture, showing stable mixing for most parameters while the chains for θ^+ and θ^- display visibly slower mixing. Taken together, these diagnostics support convergence and reliability for most of the estimated parameters, while the asymmetric long-run inflation coefficients remain a qualified exception that warrants caution rather than a claim of full convergence. The Posterior Predictive Check (PPC) results in Table 3 and the posterior predictive distribution shown in Figure 3 nonetheless support the adequacy of the model as a whole. The mean and standard deviation of the observed data are very close to those generated from the posterior predictive distribution, indicating that the model can reproduce the main characteristics of the data. Overall, these results suggest that the Bayesian ECM–NARDL model provides a reasonable representation of the relationship between inflation and economic growth, while the weaker convergence of the asymmetric long-run inflation parameters should be kept in mind when interpreting the corresponding long-run multipliers discussed above.

4. Conclusion

This study applies a Bayesian framework to estimate the Error Correction Model–Nonlinear Autoregressive Distributed Lag (ECM–NARDL) model in order to analyze the relationship between economic growth and inflation in Indonesia. The Bayesian estimation is implemented using the Gibbs sampling algorithm within a Markov Chain Monte Carlo (MCMC) framework, which allows the posterior distributions of the model parameters to be obtained through iterative sampling. Compared to conventional estimation approaches, the Bayesian method provides more comprehensive information about parameter uncertainty through posterior distributions and credible intervals. The empirical results indicate the presence of a long-run equilibrium relationship between economic growth and inflation, as confirmed by the Bounds test for cointegration, with an F-statistic of 7.2141 against an upper bound critical value of 6.36 at the 1 percent level. The error correction coefficient is negative and statistically significant at -0.7312, indicating that around 73 percent of any deviation from the long-run equilibrium is corrected within a single year. Furthermore, the ECM–NARDL specification allows the model to capture asymmetric effects of inflation, with a sustained decline in inflation associated with a reduction in long-run GDP growth of about 0.37 percentage points per percentage point of decline, while the corresponding effect of a sustained increase in inflation is not statistically distinguishable from zero. In the short run, a rise in inflation is associated with a contemporaneous reduction in GDP growth of about 0.35 percentage points.

From the Bayesian perspective, the estimation results demonstrate satisfactory convergence and reliability for most parameters, although the asymmetric long-run inflation coefficients converge more slowly and should be interpreted with additional caution. The MCMC diagnostics, including the Gelman–Rubin statistic (R-hat), Effective Sample Size (ESS), trace plots, and posterior density plots, indicate that the Markov chains converge properly for the majority of parameters and produce stable posterior estimates, while R-hat values above the conventional 1.1 threshold and comparatively low effective sample sizes for θ^+ and θ^- point to weaker mixing for these two coefficients, likely driven by near-collinearity between the decomposed inflation components. In addition, the Posterior Predictive Check (PPC) indicates that the model reproduces the mean and standard deviation of the observed data reasonably well, although this check covers only these two summary statistics and should not be read as a complete validation of the model. Overall, this study suggests that integrating the Bayesian approach with the ECM–NARDL model offers a flexible framework for analyzing dynamic and asymmetric relationships in macroeconomic data, with the Bayesian estimation providing a fuller account of parameter uncertainty than a single point estimate, while the limitations noted below indicate that these conclusions should be treated as suggestive rather than definitive.

Several limitations of this study should be kept in mind when interpreting these findings. First, the annual sample used here spans only 35 observations from 1990 to 2024, a sample size that is small by the standards of standard asymptotic theory and that limits the precision with which some parameters, particularly the asymmetric long-run inflation coefficients, can be estimated, as already noted in Section 3.1 and Section 3.5. Second, the sample period contains several major economic episodes, including the 1997 to 1998 Asian Financial Crisis, the 2008 global financial crisis, and the 2020 pandemic shock, and the model does not explicitly test for or accommodate possible structural breaks in the underlying relationship across these episodes, so the estimated coefficients should be interpreted as an average relationship over the full period rather than as necessarily stable throughout. Third, the model relates GDP growth to inflation alone, without controlling for other macroeconomic variables such as exchange rates, interest rates, or fiscal policy stance that could also influence growth and that may be correlated with inflation, so the estimated relationship should not be interpreted as fully isolating the causal effect of inflation on growth. Fourth, as discussed in Section 3.5, the Markov chains for the asymmetric long-run inflation coefficients θ^+ and θ^- show R-hat values above the conventional 1.1 threshold and comparatively low effective sample sizes, so the corresponding posterior summaries, and the long-run multipliers derived from them, carry more sampling uncertainty than the other parameters in the model and should be interpreted with corresponding caution. Taken together, these limitations suggest that the results of this study are best viewed as an initial Bayesian characterization of an asymmetric inflation-growth relationship in Indonesia, rather than a definitive or fully robust account, and they point to a small annual sample, the possibility of structural breaks, the limited set of variables considered, and the convergence behavior of the asymmetric long-run coefficients as natural directions for further work.

Credit Authorship Contribution Statement

Meidy Indhira Putri: Conceptualization, Methodology, Software, Formal Analysis, Data Curation, Visualization, Writing–Original Draft. **Nurjannah:** Supervision, Methodology, Validation, Writing–Review & Editing. **Efendi, A:** Supervision, Validation, Writing–Review & Editing. **Astuti, A. B:** Supervision, Writing–Review & Editing.

Declaration of Generative AI and AI-assisted technologies

No generative AI or AI-assisted technologies were used during the preparation of this manuscript.

Declaration of Competing Interest

The authors declare no competing interests

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Data and Code Availability

The dataset analyzed during the current study is publicly available from the official database of the International Monetary Fund (IMF)[1].

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