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Modified Multigroup Ramsey RESET for Detecting Linear and Truncated Spline Specifications in Semiparametric Path Models

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Abstract

This study develops a modified multigroup Ramsey RESET (Regression Equation Specification Error Test) for detecting linear and truncated-spline specifications in multigroup semiparametric path models. The proposed procedure extends the classical Ramsey RESET by incorporating dummy-variable interactions, allowing alternative linear and truncated-spline specifications across groups to be evaluated simultaneously within a single integrated model. A simulation study was conducted under 16 combinations of linear and truncated-spline data-generating mechanisms representing different group-specific relationship structures. The performance of the proposed method was evaluated based on its ability to correctly identify the true data-generating specification, using p-values, the Correct-to-Incorrect p-value Ratio, Dominance Ratio, and Accuracy. The results indicate that specifications corresponding to the true data-generating mechanism generally produce larger p-values and stronger performance metrics than competing alternatives, demonstrating the ability of the proposed procedure to correctly identify the underlying relationship structure. An empirical application using students' AI literacy data shows that several relationships initially evaluated under a linear specification are better represented by truncated-spline forms.

Keywords: AI Literacy; Multigroup Analysis; Ramsey RESET; Semiparametric Path Model; Truncated Spline.

1. Introduction

The development of semiparametric methods has advanced statistical research and data science in recent decades. Semiparametric approaches provide an important alternative when classical linear parametric models cannot optimally describe relationships between variables [1]. In many applications, relationships exhibit nonlinear patterns, making strict linear assumptions insufficient for accurately representing data [2, 3]. Truncated spline regression offers a flexible approach for modeling such patterns through knot points while maintaining interpretability [4, 5]. At the same time, multigroup path models are widely used to examine differences in structural relationships across groups in social, educational, and behavior research [6].

Multigroup analyses are commonly performed separately for each group, which may reduce efficiency and complicate the interpretation of structural differences across groups [7, 8]. These

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challenges become more pronounced when relationship structures vary across groups and involve both linear and nonparametric forms. Consequently, identifying appropriate specifications for structural relationships simultaneously across groups remains an important issue in semiparametric modeling [9]. Dummy-variable interactions provide a promising framework because they allow multiple groups to be represented within a single integrated model [10].

One widely used method for detecting model specification errors is the Ramsey Regression Equation Specification Error Test (RESET) [11]. The Ramsey RESET evaluates whether a fitted model adequately represents the relationship between predictors and responses by introducing augmentation terms to detect potential misspecification [12, 13]. However, most developments of the Ramsey RESET have focused on classical linear models [14]. Its application to semiparametric models, particularly truncated spline regression, remains limited, and specification testing for multigroup semiparametric path models has not been fully developed, especially when linear and nonparametric relationships coexist [15].

Several studies have extended the Ramsey RESET to nonparametric and semiparametric settings. Nurdin et al. applied a modified Ramsey RESET to identify smoothing spline polynomial orders in nonparametric path models [16]. Hidayatulloh et al. extended the approach to hybrid models combining truncated splines and Fourier series [17]. Hidayat et al. further developed a modified Ramsey RESET for evaluating nonlinear specifications in semiparametric path models [18]. Although these studies demonstrate the potential of the Ramsey RESET in increasingly complex settings, they remain largely restricted to single-group analyses and do not provide a unified framework for evaluating alternative linear and truncated-spline specifications simultaneously across groups.

To address these limitations, this study develops a modified multigroup Ramsey RESET based on dummy-variable interactions within a multigroup semiparametric path model. The proposed procedure evaluates hypothesized linear and truncated-spline specifications for structural relationships across groups within a single integrated system. The method is primarily formulated as a specification test. In simulation studies, where the true data-generating mechanism is known, the procedure is used to assess whether the corresponding specification can be correctly identified among competing alternatives. In empirical applications, where the true relationship structure is unknown, the procedure is used to detect whether structural relationships are adequately represented by a baseline linear specification or whether alternative truncated-spline forms should be considered. Larger p-values are interpreted as indicating weaker evidence against a candidate specification rather than evidence that the specification is true.

The objectives of this study are to: (1) develop a modified multigroup Ramsey RESET for semiparametric path models; (2) evaluate its ability to distinguish linear and truncated-spline specifications; (3) evaluate its ability to identify simultaneous relationship structures across groups; and (4) assess its performance through simulation studies and an empirical application based on students' AI literacy data. The results are expected to contribute to the development of specification-testing procedures for multigroup semiparametric models and provide an alternative approach for identifying linear and nonparametric relationship structures across groups.

2. Methods

This study employed both empirical and simulation data. The empirical data were collected using a Likert-scale questionnaire designed to measure students' perceptions of the role of Artificial Intelligence (AI) in the learning process. The population consisted of students in the Department of Statistics, while the sample included students from the Statistics and Data Science Program enrolled in the third to eighth semesters. Using quota sampling, a total of 83 respondents were obtained. The empirical dataset was used to illustrate the practical application of the proposed method and to provide realistic parameter references for the simulation settings. Because the primary objective of the empirical application was to illustrate the proposed specification-testing procedure rather than draw population-level inferences, the dataset was considered adequate for methodological demonstration. The simulation study was conducted to evaluate the performance of the modified multigroup Ramsey RESET under various semiparametric multigroup relationship scenarios involving linear and truncated-spline specifications.

For the multigroup analysis, respondents were classified into two groups based on their AI readiness scores. AI readiness scores were calculated as the average of all questionnaire items related to AI preparedness, producing a composite score for each respondent. AI readiness refers to students' preparedness to understand and utilize AI technologies in learning activities. The mean score was selected as the grouping threshold because no established theoretical cut-off AI readiness is currently available, and the mean provided a relatively balanced partition of respondents across groups. Respondents with scores below the overall mean were assigned to Group 1, representing lower AI readiness, while respondents with scores above the overall mean were assigned to Group 2, representing higher AI readiness. This classification resulted in 42 respondents in Group 1 and 41 respondents in Group 2. The grouping structure was subsequently used to investigate differences in relationship pattern across levels of AI readiness.

Four conceptual constructs were considered in this study: (1)Attitude towards AI usage (X_1), measured by 6 indicators and 13 items; (2) Availability of AI Learning Resources (X_2), measured by 3 indicators and 9 items; (3) Academic Adaptation (Y_1) measured by 3 indicators and 12 items and treated as a mediating construct; and (4) AI Usage Literacy (Y_2) measured by 4 indicators and 15 items and treated as the final endogenous construct. The mean score of each construct was used as a composite manifest variable in the subsequent path analysis, yielding four observed variables: X_1, X_2, Y_1, and Y_2, consistent with the conceptual framework shown in the path diagram Fig. 1

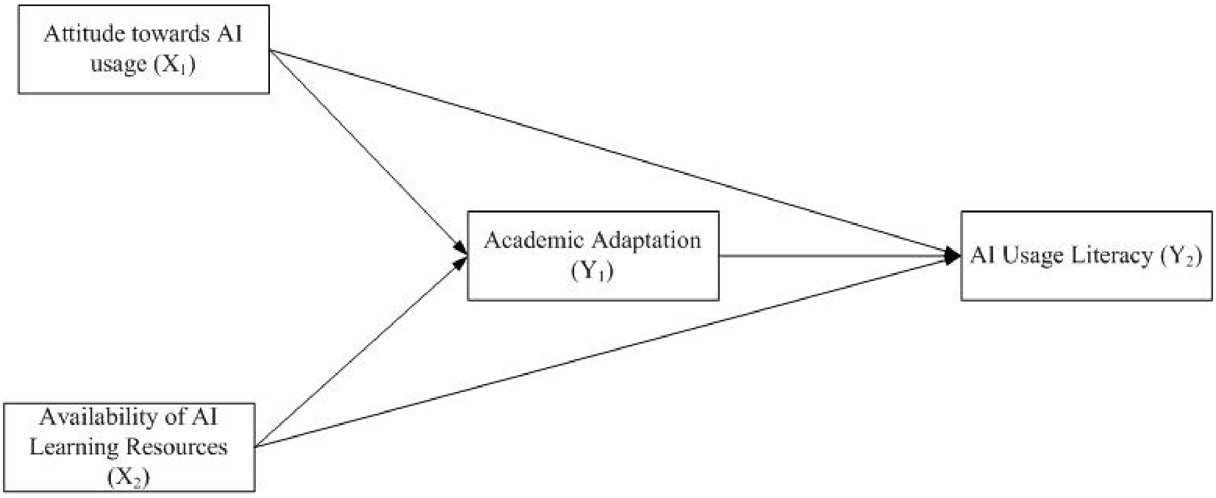

Fig. 1: Research Model

Prior to the analysis, instrument quality was assessed using corrected item-total correlation (CITC) for validity and Cronbach's Alpha for reliability. Detailed results are provide in the Appendix. All indicators satisfied the validity criterion (CITC 0.30), and all constructs demonstrated satisfactory reliability, with Cronbach's Alpha values ranging from 0.782 to 0.887.

2.1. Ramsey RESET

The Ramsey RESET test was first introduced by Ramsey in 1969 and has since become one of the most widely used methods for testing the linearity assumption in regression models. According to Gujarati [14], the Ramsey RESET test is carried out through a few systematic stages as follows:

1. The first stage is to form the initial regression equation, which is expressed as Eq. (1)

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i, \quad i = 1, 2, \ldots, n \tag{1}$$

The parameters in the first regression equation are estimated using the Ordinary Least Squares (OLS) method, yielding

$$\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i. \tag{2}$$

Based on Eq. (2), the coefficient of determination from the first regression is calculated and denoted by R^2_{first}.

2. The second stage is to form an augmented regression equation by introducing additional variables based on powers of the fitted values obtained from the first regression. The second regression equation is expressed as Eq. (3)

$$Y_i = \beta_0 + \beta_1 X_i + \gamma_1 \hat{Y}_i^2 + \gamma_2 \hat{Y}_i^3 + \varepsilon_i, \quad i = 1, 2, \ldots, n. \tag{3}$$

The parameters are then estimated, resulting in Eq. (4)

$$\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i + \hat{\gamma}_1 \hat{Y}_i^2 + \hat{\gamma}_2 \hat{Y}_i^3. \tag{4}$$

Subsequently, the coefficient of determination from the second regression is calculated and denoted by R^2_{second}.

3. The final stage is hypothesis testing. The null hypothesis states that all additional coefficients are equal to zero:

$$H_0 : \gamma_1 = \gamma_2 = 0, \tag{5}$$

against the alternative hypothesis

$$H_1 : \text{at least one of } \gamma_1 \text{ or } \gamma_2 \neq 0. \tag{6}$$

The test statistic is given by Eq. (7)

$$F = \frac{\left(R^2_{\text{second}} - R^2_{\text{first}}\right)/2}{\left(1 - R^2_{\text{second}}\right)/(n-m)}. \tag{7}$$

where n denotes the number of observations and m denotes the number of parameters in the second regression model. The decision rule is to reject H_0 if the p-value is less than the significance level α. Rejection of H_0 indicates that the regression model does not satisfy the linearity assumption. The validity of the F-test relies on the nested-model structure, where the augmented model contains all terms from the initial model together with additional specification-testing terms.

2.2. Semiparametric Truncated Spline Path Analysis

Semiparametric regression combines parametric and nonparametric components, allowing some relationships to be specified explicitly while others are estimated flexibly from the data. This approach is particularly useful when certain predictor effects exhibit nonlinear patterns that cannot be adequately represented by conventional linear models [19].

In this study, the semiparametric path model consists of two structural equations representing the mediating variable Y_1 and the final endogenous variable Y_2. The general form of the semiparametric path model can be written as Eqs. (22) and (23) [20]:

$$Y_{1i} = X\beta + f(X_{ri}) + \varepsilon_{1i}, \tag{8}$$
$$Y_{2i} = X\beta + f(X_{ri}) + f(Y_{1i}) + \varepsilon_{2i}, \quad i = 1, 2, \ldots, n, \tag{9}$$

where $X\beta$ denotes the parametric component, $f(\cdot)$ denotes the nonparametric component,ε_{1i} and ε_{2i} are random error terms.

A general semiparametric regression model can be expressed as (10)

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \cdots + \beta_k X_{ki} + f(X_i) + \varepsilon_i. \tag{10}$$

The nonparametric regression function $f(X_i)$ can be approximated using a truncated spline function, which is expressed as Eq. (11)

$$\hat{f}(X_i) = \hat{\beta}_0 + \sum_{j=1}^{p} \hat{\beta}_j X_i^j + \sum_{k=1}^{q} \hat{\beta}_{p+k} \left(X_i - K_k\right)_+^p . \tag{11}$$

The truncated spline basis function is defined as Eq. (12)

$$(X_i - K_k)_+^p = \begin{cases} (X_i - K_k)^p, & X_i \geq K_k, \\ 0, & X_i < K_k. \end{cases} \tag{12}$$

where K_k denotes the k-th knot point. Throughout this study, truncated spline terms are represented using the basis function $(X_i - K_k)_+^p$.

2.3. Data Generation in Study Simulation

The simulation study was conducted to evaluate the ability of the proposed modified multigroup Ramsey RESET to correctly identify predefined semiparametric relationship structures under various multigroup scenarios. To maintain realistic data characteristics, parameter values and distributional properties were informed by an empirical AI literacy dataset, while the relationship forms were generated according to predefined simulation scenarios.

A two-group multigroup design was used with a total sample size of n=83, consisting of 42 observations in Group 1 and 41 observations in Group 2. The multigroup structure was modeled using dummy-variable interactions, allowing all groups to be analyzed simultaneously within a single integrated model.

The simulation model was constructed based on a research path diagram consisting of two exogenous variables, Attitude towards AI usage (X_1) and AI Learning Resources (X_2), one intervening variable, Academic Adaptation (Y_1), and one endogenous variable, AI Usage Literacy (Y_2). Path coefficient magnitudes were informed by empirical estimates, whereas the functional forms followed predefined simulation scenarios consisting of linear and single-knot truncated spline relationships.

The simulation study considered 16 multigroup scenarios obtained from all combinations of linear, truncated linear spline, truncated quadratic spline, and truncated cubic spline relationship structures across the two groups. For each scenario, coefficient values were obtained by fitting the corresponding semiparametric multigroup path model to the empirical AI literacy dataset. This approach preserved realistic parameter magnitudes while ensuring consistency between the data-generating mechanism and the specification being evaluated.

1. Determine the simulation sample size as $n = 83$, consisting of 42 observations in Group 1 and 41 observations in Group 2.
2. Construct the semiparametric multigroup path model based on the research framework and dummy-variable interaction structure.
3. Define the sixteen simulation scenarios representing all combinations of linear, truncated linear spline, truncated quadratic spline, and truncated cubic spline relationships across the two groups.
4. Estimate scenario-specific coefficient values and determine the error variance using the empirical AI literacy dataset. The knot location was selected using the minimum GCV criterion and subsequently fixed throughout the simulation study to ensure comparability across simulation replications.

5. Generating exogenous variables from uniform distributions bounded by the minimum and maximum values observed in the empirical AI literacy dataset. This approach was adopted to preserve the observed range of the empirical data while allowing random variation within realistic limits.
6. Generate multivariate normal error terms with $E(\varepsilon) = 0$ and $Var(\varepsilon) = \mathbf{\Sigma}$, assuming homogeneous variance across groups ($\sigma_1^2 = \sigma_2^2$). The error variance was determined using an MSE value of 0.333 obtained from the empirical model.
7. Generate the intervening and endogenous variables according to the specified data-generating mechanism and apply the modified multigroup Ramsey RESET to evaluate whether the null model corresponding to the true specification was retained while competing misspecified alternatives were rejected.

Model performance was evaluated based on the ability of the proposed procedure to retain the null hypothesis when it corresponded to the true data-generating specification and to reject competing misspecified alternatives. Evaluation metrics included p-values, the Correct-to-Incorrect p-value Ratio, Dominance Ratio, and Accuracy.

It should be noted that the empirical dataset was used solely to inform realistic parameter settings and distributional characteristics. The statistical performance of the proposed method was evaluated through simulation scenarios specified independently of the empirical application. The empirical dataset was subsequently used only to illustrate the practical implementation of the proposed method.

2.4. Performance Evaluation Criteria

Because the true data-generating specification is known in the simulation study, the performance of the proposed procedure was evaluated according to its ability to correctly identify that specification among competing alternatives. Three evaluation criteria were used: the Correct-to-Incorrect P-value Ratio, the Dominance Ratio, and Accuracy. These measures assess whether the specification corresponding to the data-generating mechanism tends to produce larger p-values than competing misspecified models. The use of relative p-value comparisons was intended solely for simulation-based performance assessment and should not be interpreted as proof that a model is correct.

The Correct-to-Incorrect P-value Ratio compares the p-value associated with the data-generating specification to the average p-value of competing misspecified models. Larger values indicate greater separation between correctly specified and misspecified models and therefore better discriminatory performance. The ratio is defined as Eq. (13).

$$R_i = \frac{p_{\text{correct}}}{\bar{p}_{\text{incorrect}}} \tag{13}$$

The Dominance Ratio evaluates the extent to which the p-value associated with the data-generating specification exceeds that of the strongest competing alternative. Values greater than one indicate that the correctly specified model produces a larger p-value than any misspecified model, whereas values close to one suggest that at least one alternative specification exhibits similar behavior. The Dominance Ratio is defined as Eq. (14).

$$DR_i = \frac{p_{\text{correct}}}{max(p_{\text{incorrect}})} \tag{14}$$

Accuracy is defined as the percentage of simulation replications in which the specification corresponding to the data-generating mechanism produces the largest p-value among all candidate models. This measure evaluates how consistently the proposed procedure recovers the known simulation pattern. Higher accuracy values indicate more reliable identification of the true specification. The accuracy formula can be written as Eq. (15)

$$Accuracy = \frac{N_{\text{correct}}}{N_{\text{rep}}} \times 100\% \tag{15}$$

where N_{correct} denotes the number of simulation replications in which the model corresponding to the true simulation pattern yields the largest p-value among all candidate models, and N_{rep} represents the total number of simulation replications.

These measures provide complementary perspectives for evaluating the ability of the proposed procedure to distinguish the true simulation specification from competing alternatives. The Correct-to-Incorrect p-value Ratio measures average separation, the Dominance Ratio measures superiority over the strongest competing model, and Accuracy measures the consistency of correct specification identification across simulation replication.

3. Results and Discussion

This section presents the research results and a comprehensive discussion on the development of the modified multigroup Ramsey RESET in a semiparametric multigroup path model. The discussion focuses on the method's ability to simultaneously identify linear and truncated spline relationships using a dummy-variable interaction approach. Additionally, this section explains the method's performance evaluation results from simulation studies and its application to empirical data on students' AI literacy, demonstrating the ability of the proposed method to identify model specifications across various relationship patterns between variables.

3.1. Modified Multigroup Ramsey RESET

The modified multigroup Ramsey RESET method used in this study was developed based on the fundamental principles of Ramsey RESET, which involve comparing the assumed correct model with alternative models to detect model specification errors. However, in this study, the procedure was modified to evaluate hypothesized linear and truncated-spline specifications within a multigroup semiparametric path model and to assess their relative compatibility with the observed data. The development was carried out using a dummy-variable interaction approach, so that all groups were analyzed within a single integrated model system without separating the data by group. The steps for modifying the Ramsey RESET are as follows.

1. Identifying candidate semiparametric multigroup models
   The first step is to identify all candidate models to be tested. In this study, four relationship forms were used: the Linear (L) model, Truncated Linear Spline (TLS), Truncated Quadratic Spline (TQS), and Truncated Cubic Spline (TCS) models. The multigroup approach used in this study comprised two groups, yielding 16 model combinations that show in **??**.
   Each combination in **??** describes a different form of relationship within each group in the semiparametric multigroup model. To ensure the validity of the modified Ramsey RESET framework, all alternative specifications were formulated as augmented versions of the reference model. Additional polynomial and truncated spline components were introduced gradually, allowing the comparisons between the reference and alternative models to satisfy the nested model principle required for F-test, based hypothesis testing. Consequently, each comparison evaluates significant improvement over the simpler reference specification.
2. Specifying the null hypothesis model H_0
   At this stage, one candidate model is treated as the reference model H_0 and compared against competing candidate specifications. For example, it is assumed that the linear-linear model fits the data, so a linear-linear model is formulated and estimated using a dummy-variable interaction-based multigroup approach, ensuring that all groups remain within a single system of equations. The linear-linear model can be written as Eq. (16).

$$Y_i = \beta_0 + \beta_1 D_i + \beta_2 X_i + \beta_3 X_i D_i + \varepsilon_i. \tag{16}$$

3. Formulating the alternative model H_1
After determining the base model to be used, the next step is to compare that model with all other candidate models, which are treated as alternative models H_1. For example, the linear-linear model is compared step by step with the linear-truncated linear spline as shown in Eq. (17), whereas the complete set of 16 specifications is provided in Appendix A.3.

$$Y_i = \beta_0 + \beta_1 D_i + \beta_2 X_i + \beta_3 X_i D_i + \delta_1 D_i (X_i - K_1)_+ + \varepsilon_i. \tag{17}$$

The truncated spline basis functions are defined as Eq. (18)

$$(X_i - K_1)_+ = \begin{cases} (X_i - K_1), & X_i > K_1, \\ 0, & X_i \leq K_1, \end{cases} \tag{18}$$

The knot location was selected using the minimum GCV criterion. Candidate knot locations were evaluated for each spline specification, and the knot producing the smallest GCV value was retained for both the empirical application and simulation study.
For other alternative models, nonparametric components are added as required. The addition of these components aims to test whether the alternative model provides a more appropriate specification compared to the baseline model.

4. Conducting hypothesis testing
The hypotheses for the modified multigroup Ramsey RESET test are formulated as equations (19) and (20).

$$H_0 : \delta_1 = \delta_2 = \cdots = \delta_p = 0, \tag{19}$$

versus

$$H_1 : \exists\, j \in \{1, 2, \ldots, p\} \text{ such that } \delta_j \neq 0. \tag{20}$$

The parameter δ_j is the augmentation coefficient in the modified Ramsey RESET, representing the additional functional components of the alternative model. This parameter is used to detect whether there are additional relationship structures beyond those assumed by the baseline model in the null hypothesis.
Testing decisions are made based on the p-value obtained from the F-test statistic calculation using Eq. (21)

$$F = \frac{\left(R^2_{\text{second}} - R^2_{\text{first}}\right)/p}{\left(1 - R^2_{\text{second}}\right)/(n - m)}. \tag{21}$$

where R^2_{first} is coefficient of determination of the baseline model, R^2_{second} is coefficient of determination of the alternative model, p is number of additional parameters in the alternative model, n is number of observations, and m is number of parameters in the alternative model.
If the resulting p-value exceeds the significance level $\alpha = 0.05$, there is insufficient evidence that the alternative model provides a statistically significant improvement over the reference model. Conversely, a p-value less than the significance level indicates that the additional components included in the alternative model provide a statistically significant improvement over the reference specification.
In the empirical application, specification decisions are based on this hypothesis-testing framework. In contrast, for simulation studies where the data-generating specification is known, the relative magnitude of p-values across competing models is additionally used to evaluate the ability of the proposed procedure to recover the generating relationship structure.

5. Repeating the testing for all candidate models
   Each candidate specification is subsequently treated as the reference model and compared with all remaining alternatives. Repeting this process for all candidate specifications produces a complete pairwise comparison matrix.
6. Performance evaluation of the modified multigroup Ramsey RESET
   Under each simulation scenario, performance was evaluated according to the ability of the proposed procedure to recover the known data-generating specification. This assessment was conduct using the Correct-to-Incorrect p-value Ratio, Dominance Ratio, and Accuracy measures described in the previous subsection.

3.2. Study Simulation Results

This section presents the performance evaluation results of the proposed multigroup Ramsey RESET test through a simulation study. The main focus of the analysis is to assess the test's ability to retain the null hypothesis under correct specification conditions and detect specification errors when the functional relationships differ between groups. The interpretation of the simulation results is based on two complementary principles. First, a p-value greater than 0.05 indicates that the candidate specification is not rejected relative to a competing alternative specification. Second, because the data-generating specification is known, the relative magnitude of p-values is used to evaluate the ability of the proposed procedure to recover the underlying relationship structure.

Table 1: Performance of the Multigroup Ramsey RESET Test

Pattern of Simulation	Num of Rep	P-value for Ramsey and Modify Ramsey						
		L-L	L-TLS	L-TQS	L-TCS	TLS-L	TQS-L	TCS-L
L-L	1000	0.9131	0.0451	0.0903	0.0213	0.0443	0.0241	0.0218
L-TLS	1000	0.0462	0.6659	0.0272	0.0225	0.6371	0.0282	0.0229
L-TQS	1000	0.0453	0.0274	0.6134	0.0247	0.0261	0.7152	0.0232
L-TCS	1000	0.0443	0.0252	0.0303	0.6776	0.0250	0.0305	0.7014
TLS-L	1000	0.0446	0.6135	0.0324	0.0276	0.6206	0.0274	0.0225
TQS-L	1000	0.0456	0.0312	0.6131	0.0235	0.0277	0.5841	0.0201
TCS-L	1000	0.0451	0.0267	0.0273	0.6307	0.0234	0.0282	0.6385

Table 2: Performance of the Multigroup Ramsey RESET Test for Truncated Spline Condition

Pattern of Simulation	Num of Rep	P-value for Ramsey and Modify Ramsey								
		TLS-TLS	TLS-TQS	TLS-TCS	TQS-TLS	TQS-TQS	TQS-TCS	TCS-TLS	TCS-TQS	TCS-TCS
TLS-TLS	1000	0.6893	0.0309	0.0218	0.0305	0.6268	0.0272	0.0250	0.0272	0.8024
TLS-TQS	1000	0.0265	0.6479	0.0330	0.7060	0.0297	0.0278	0.0305	0.0287	0.0207
TLS-TCS	1000	0.0319	0.0289	0.6651	0.0278	0.0288	0.0275	0.6573	0.0285	0.0290
TQS-TLS	1000	0.0290	0.7451	0.0295	0.6747	0.0277	0.0265	0.0331	0.0318	0.0279
TQS-TQS	1000	0.0276	0.0257	0.0275	0.0310	0.6960	0.0284	0.0279	0.0299	0.0294
TQS-TCS	1000	0.0286	0.0303	0.0286	0.0293	0.0309	0.7160	0.0288	0.7247	0.0304
TCS-TLS	1000	0.0293	0.0269	0.6737	0.0288	0.0272	0.0308	0.6945	0.0280	0.0297
TCS-TQS	1000	0.0289	0.0302	0.0321	0.0302	0.0301	0.7002	0.0302	0.6765	0.0296
TCS-TCS	1000	0.0259	0.0300	0.0293	0.0281	0.0316	0.0302	0.0270	0.0306	0.6936

Based on Table 1, the L-L simulation scenario produced the clearest result, with the L-L specification yielding the largest average p-value (0.9131), whereas competing spline specifications generated substantially smaller values. This indicates that the proposed procedure successfully retained the specification corresponding to the true data-generating mechanism.

Similar patterns were observed across most mixed and spline-based simulation scenarios on Table 2. In general, specifications corresponding to the data-generating mechanism produced the largest or among the largest average p-values. However, several competing spline specifications also generated relatively large p-values, particularly when they shared similar functional characteristics with the generating model. This suggests that discrimination becomes more challenging when alternative spline forms exhibit comparable flexibility.

The strongest separation occurred under homogeneous relationship structures, particularly L-L, TQS-TQS, and TCS-TCS, where the corresponding specifications consistently produced substantially larger p-values than competing alternatives. In contrast, mixed spline scenarios tended to generate closer p-values because multiple spline forms were capable of approximating similar relationship patterns.

It should be emphasized that the relative p-value comparisons reported in this simulation study were used exclusively for performance evaluation because the true data-generating specification was known.

Table 3: Performance Evaluation of the Modified Multigroup Ramsey RESET

No	Pattern of Simulation	Performance Evaluation Criteria		
		Correct-to-Incorrect p-value Ratio	Dominance Ratio	Accuracy (%)
1	L-L	42.5148	20.2516	100.0
2	L-TLS	10.9498	1.0451	52.0
3	L-TQS	9.2119	0.8576	40.4
4	L-TCS	10.2458	0.9662	49.8
5	TLS-L	10.3054	1.0116	58.3
6	TQS-L	9.7724	0.9527	61.2
7	TCS-L	10.4461	1.0124	50.0
8	TLS-TLS	6.0362	0.8591	30.0
9	TLS-TQS	9.6331	0.9177	41.0
10	TLS-TCS	10.1951	1.0120	48.0
11	TQS-TLS	9.5033	0.9056	38.1
12	TQS-TQS	30.6812	22.4618	100.0
13	TQS-TCS	10.2605	0.9880	41.9
14	TCS-TLS	10.5554	1.0309	46.9
15	TCS-TQS	9.9128	0.9661	41.0
16	TCS-TCS	29.9302	21.9593	100.0

The results of the evaluation using the Correct-to-Incorrect P-value Ratio, Dominance Ratio, and Accuracy on Table 3 indicate that the modified multigroup Ramsey RESET is able to distinguish between competing model specifications across a range of multigroup semiparametric relationship patterns, although its performance varies across simulation scenarios.

The Correct-to-Incorrect P-value Ratio exceeded one in all simulation scenarios, indicating that the specification corresponding to the data-generating mechanism consistently produced larger average p-values than competing alternatives. The largest ratios were observed for the L-L, TQS-TQS, and TCS-TCS scenarios, suggesting strong separation between the generating specification and competing models. Similar patterns were observed for the Dominance Ratio, although several mixed spline scenarios produced values close to or below one, indicating that at least one competing specification occasionally generated a p-value comparable to or larger than that of the generating model.

Accuracy results further support these findings. Perfect recovery (100%) was achieved under the L-L, TQS-TQS, and TCS-TCS scenarios, whereas mixed relationship structures produced substantially lower accuracies ranging from 30% to 61.2%. These results suggest that the proposed procedure is particularly effective when both groups share the same underlying relationship structure, but discrimination becomes more difficult when competing spline specifications possess similar levels of functional flexibility.

Overall, the modified multigroup Ramsey RESET demonstrates the ability to distinguish between competing model specifications, particularly under homogeneous relationship structures. The strongest performance was observed for the L-L, TQS-TQS, and TCS-TCS scenarios, which exhibited large Correct-to-Incorrect p-value Ratios, high Dominance Ratios, and perfect accuracy. However, performance became less stable in several mixed spline scenarios, where competing specifications often produced similar p-value patterns because of their comparable functional flexibility. These findings indicate that the proposed method is capable of recovering the underlying specification across a wide range of multigroup semiparametric scenarios, with the strongest performance observed under homogeneous relationship structures. Performance decreases in several mixed spline conditions because competing spline specifications may approximate similar functional patterns, resulting in less pronounced separation among candidate models.

3.3. Semiparametric Truncated Spline Path Model in Empirical Data

In the empirical application, the Linear-Linear (L-L) specification was used as the initial reference model H_0. The modified multigroup Ramsey RESET was then employed to evaluate whether alternative truncated spline specifications provided a statistically meaningful improvement over the baseline linear structure for each relationship in the path model.

A summary of the multigroup linearity assumption test results for each relationship is presented in **??**. Starting from the Linear-Linear reference specification, the modified multigroup Ramsey RESET was used to evaluate alternative truncated spline specifications for each relationship. The selected specifications correspond to the alternative models that provided a statistically significant improvement over the baseline Linear–Linear specification or, when no significant improvement was detected, the baseline linear specification was retained. For brevity, only the selected specifications are reported in the main text, whereas the complete results of all candidate-model comparisons are presented in Appendix A.4.

Table 4: Selected Candidate Specifications Based on the Modified Multigroup Ramsey RESET Test

Relationship Variables	Alternative Model	p-value	Decision	Selected Model
$X_1 \to Y_1$	L–TLS	0.0126	Rejected H_0	L–TLS
$X_2 \to Y_1$	TLS–L	0.0905	Accepted H_0	L–L
$X_1 \to Y_2$	TLS–L	0.0296	Rejected H_0	TLS–L
$X_2 \to Y_2$	TLS–L	0.0197	Rejected H_0	TLS–L
$Y_1 \to Y_2$	L–TLS	0.0103	Rejected H_0	L–TLS

The selected specifications correspond to the alternative models that provided a statistically significant improvement over the baseline Linear–Linear specification or, when no significant improvement was detected, the baseline linear specification was retained. The selected specifications correspon to the candidate models that were not rejected by the modified multigroup Ramsey RESET and were therefore considered more appropriate that the competing alternatives for representing the observed relationship patterns.

The empirical analysis treated the Linear–Linear (L-L) specification as the baseline model and evaluated alternative truncated spline specifications using the modified multigroup Ramsey RESET. Based on **??**, the relationship between Attitude towards AI usage X_1 and Academic Adaptation Y_1, as well as the relationship between Academic Adaptation Y_1 and AI Usage Literacy Y_2, the Linear–Truncated Linear Spline (L–TLS) specification was selected, indicating a linear relationship in Group 1 and a truncated linear spline relationship in Group 2. In contrast, the relationships between Attitude towards AI X_1 and AI Usage Literacy Y_2, and between AI Learning Resources X_2 and AI Usage Literacy Y_2, were represented by the Truncated Linear spline-Linear (TLS-L) specification, indicating nonlinear behavior in Group 1 and linear behavior in Group 2. Meanwhile, the relationship between AI Learning Resources X_2 and Academic Adaptation Y_1 remained adequately represented by a linear specification in both groups. These results suggest that nonlinear relationship patterns are present in both groups, although they occur on different paths within the multigroup AI literacy model. The model equations derived from these results are as Eq. (22) and Eq. (23).

$$\hat{f}_{1i} = 2.541 - 3.893D_i + 0.144x_{1i} + 1.320x_{1i}D_i - 1.737D_i(x_{1i} - 2.959) \\ + 0.129x_{2i} - 0.020x_{2i}D_i \tag{22}$$

$$\hat{f}_{2i} = 11.572 - 1.103D_i - 1.752x_{1i} + 2.112x_{1i}D_i + 1.848(1 - D_i)(x_{1i} - 2.251) \\ - 1.636x_{2i} + 2.056x_{2i}D_i + 2.062(1 - D_i)(x_{2i} - 2.463)_+ - 0.093y_{1i} \\ - 3.525y_{1i}D_i + 3.881D_i(y_{1i} - 2.655) \quad (23)$$

with dummy variable can defined as Eq. (24)

$$D_i = \begin{cases} 0, & \text{Group 1,} \\ 1, & \text{Group 2.} \end{cases} \quad (24)$$

To provide information on estimation uncertainty, the standard errors and p-values associated with the coefficients of the final semiparametric multigroup path model are presented in Table 5. These statistics are reported for completeness, although the primary objective of the empirical application is specification identification rather than hypothesis testing of individual coefficients.

Table 5: Parameter Estimates, Standard Errors, and p-values of the Final Semiparametric Multigroup Path Model

Parameter	Estimate	Std. Error	p-value
Panel A: Academic Adaptation (Y_1)			
Intercept	2.541	0.217	< 0.001
D_i	-3.893	0.519	< 0.001
X_1	0.144	0.060	0.021
X_1D_i	1.320	0.163	< 0.001
$D_i(X_1 - 2.959)_+$	-1.737	0.185	< 0.001
X_2	0.129	0.050	0.016
X_2D_i	-0.021	0.067	0.380
Panel B: AI Usage Literacy (Y_2)			
Intercept	11.572	0.615	< 0.001
D_i	-1.103	5.804	0.398
X_1	-1.752	0.130	< 0.001
X_1D_i	2.112	0.138	< 0.001
$(1 - D_i)(X_1 - 2.251)_+$	1.849	0.189	< 0.001
X_2	-1.636	0.183	< 0.001
X_2D_i	2.056	0.185	< 0.001
$(1 - D_i)(X_2 - 2.463)_+$	2.062	0.292	< 0.001
Y_1	-0.093	0.056	0.129
Y_1D_i	-3.525	2.165	0.078
$D_i(Y_1 - 2.655)_+$	3.881	2.176	0.059

As shown in Table 5, most coefficients associated with the selected linear and truncated spline components are statistically significant, although several group-specific linear effects remain nonsignificant. These results are reported as supplementary information, whereas the primary focus of the empirical application remains the identification of relationship specifications.

The knot locations were determined using the Generalized Cross Validation (GCV) criterion, where the knot configuration yielding the smallest GCV value was selected for each truncated spline relationship. The corresponding GCV values and candidate knot evaluations are reported in Appendix A.5 as a sensitivity assessment of knot selection.

The first structural equation indicates that the relationship between X_1 and Y_1 follows an L–TLS specification, implying a change in slope after the knot point of 2.959 for Group 2, whereas the relationship between X_2 and Y_1 remains linear in both groups. This suggests that the effect of attitudes toward AI usage varies across levels of AI readiness, while the contribution of AI learning resources remains relatively stable.

The second structural equation shows that the relationships between X_1 and Y_2 and between X_2 and Y_2 follow TLS–L specifications, indicating nonlinear effects in Group 1 and linear effects in Group 2. In contrast, the relationship Y_1 and Y_2 follows an L–TLS specification, implying a change in slope after the knot point of 2.617 for Group 2. These findings suggest that several determinants of AI usage literacy exhibit group-specific nonlinear behavior.

Overall, the empirical application demonstrates that the modified multigroup Ramsey RESET provides evidence that different relationship structures may exist across AI readiness groups within a single semiparametric path model. Several relationships were adequately represented by linear functions, whereas others required truncated spline specifications to capture changes in relationship patterns. These findings illustrate the usefulness of the proposed method for detecting heterogeneous functional forms across groups without requiring separate model estimation.

3.4. Discussion

The simulation results indicate that the modified multigroup Ramsey RESET is capable of distinguishing between competing linear and truncated-spline specifications within semiparametric multigroup path models. When the data-generating specification was known, the proposed procedure generally assigned larger p-values to the generating specification than to competing alternatives, indicating its ability to recover the underlying relationship structure under a variety of simulation scenarios.

The strongest performance was observed under homogeneous relationship structures, particularly in the L-L, TQS-TQS, and TCS-TCS scenarios. These conditions produced substantially larger Correct-to-Incorrect P-value Ratios and Dominance Ratios, together with perfect Accuracy values. This finding suggests that specification identification becomes more stable when both groups share the same underlying functional form. Similar observations have been reported in semiparametric modeling studies, where consistency in functional structure generally improves estimation stability and model discrimination performance [21].

In contrast, several mixed spline scenarios produced lower Accuracy values and Dominance Ratios close to, or slightly below, one. These results indicate that the proposed method may encounter difficulty distinguishing between competing specifications when alternative spline functions generate similar response patterns. Such behaviour is expected because truncated linear, quadratic, and cubic spline functions can exhibit comparable shapes within restricted data ranges, particularly when the available observations are limited around the knot region [9]. Therefore, although the proposed method remains informative under mixed relationship conditions, its discriminatory performance is less stable than under homogeneous specifications.

The empirical application demonstrates that several relationships within the AI literacy model cannot be adequately represented by purely linear functions. Truncated spline structures were identified for the relationships between Attitude towards AI Usage and Academic Adaptation, between Attitude towards AI Usage and AI Usage Literacy, between AI Learning Resources and AI Usage Literacy, and between Academic Adaptation and AI Usage Literacy. These findings suggest that the effects of several explanatory variables vary across different ranges of the predictor values and differ between AI readiness groups. Such results are consistent with previous studies indicating that adaptation to AI technologies and AI-related learning behaviours often develop in nonlinear ways because of differences in technological readiness, learning experiences, and patterns of AI utilization [22].

A major contribution of this study is the development of a multigroup semiparametric specification-testing framework based on dummy-variable interactions. The proposed approach enables simultaneous evaluation of alternative linear and truncated-spline specifications across groups within a single integrated model, thereby avoiding separate group-specific analyses and facilitating direct comparison of competing relationship structures.

Nevertheless, the proposed method should be viewed primarily as a specification-testing procedure, while its model-selection role is limited to comparing candidate specifications considered in the analysis. In the simulation study, larger p-values were used as indicators of compatibility between candidate specifications and the underlying data-generating mechanism. However, a large p-value does not guarantee that a model is correct; it only indicates insufficient evidence against the hypothesized specification relative to the competing alternatives considered in the analysis. Consequently, the proposed procedure is most appropriately used as a tool for evaluating and comparing candidate semiparametric specifications rather than as proof of model correctness.

The empirical application should also be interpreted with several practical considerations. First, the group classification was based on the mean AI readiness score because no established theoretical threshold was available. Second, knot locations were selected using the minimum GCV criterion and were subsequently examined through candidate-knot comparisons reported in Appendix A.5. Third, the empirical sample consisted of only 83 respondents, which may limit the stability of spline estimation around knot regions. Consequently, the empirical results should be viewed as an illustration of the proposed procedure rather than as definitive evidence regarding the underlying AI literacy relationships.

Overall, the results demonstrate that the modified multigroup Ramsey RESET provides a useful alternative for assessing linear and truncated-spline specifications in multigroup semiparametric path models. The method performs particularly well under homogeneous relationship structures, whereas discrimination becomes more challenging in several mixed spline scenarios where competing specifications exhibit similar functional characteristics. Future studies may consider larger sample sizes, alternative grouping strategies, and multiple-knot spline specifications to further evaluate the robustness of the proposed framework.

4. Conclusion

This study developed a modified multigroup Ramsey RESET for evaluating linear and truncated-spline specifications within semiparametric multigroup path models using a dummy-variable interaction framework. In simulation settings where the data-generating specification was known, the proposed procedure generally assigns larger p-values to the generating specification than to competing alternatives, indicating its ability to recover the underlying relationship structure. The strongest performance was observed under homogeneous relationship structures, whereas several mixed spline scenarios produced lower Accuracy values and Dominance Ratios close to one, indicating more limited discriminatory performance when competing spline specifications exhibit similar functional characteristics.

The empirical application to AI literacy data showed that some relationships were adequately represented by linear functions, whereas others required truncated spline specifications. These findings illustrate the usefulness of the proposed procedure for detecting potential linear and truncated-spline relationship structures within a single integrated multigroup framework.

This study is limited to linear and truncated spline specifications with a single knot and a two-group multigroup structure. Future research may extend the proposed framework to more complex semiparametric models, alternative knot configurations, larger multigroup settings, and different application domains.

CRediT Authorship Contribution Statement

M. Dziqri Nur Rohiim: Data Curation, Software, Formal Analysis, Visualization, Writing–Original Draft Preparation. **Adji Achmad Rinaldo Fernandes:** Conceptualization, Methodology, Validation, Supervision, Writing–Review & Editing. **Achmad Efendi:** Methodology, Validation, Investigation, Supervision, Writing–Review & Editing. **Mujiono:** Investigation, Resources, Data Curation, Writing–Review & Editing. **Kamelia Hidayat:** Conceptualization, Methodology, Software, Validation, Formal Analysis, Visualization, Writing–Review & Editing.

Declaration of Generative AI and AI-assisted technologies

During the preparation of this manuscript, the authors used generative artificial intelligence (AI) tools to assist with language refinement, grammar checking, and improving the clarity of the manuscript. The authors carefully reviewed and edited all AI-generated content and take full responsibility for the accuracy, integrity, and originality of the final manuscript. No AI tools were used for data collection, data analysis, interpretation of results, or scientific decision-making.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data and Code Availability

The dataset used in this study is available from the corresponding author upon reasonable request. Researchers interested in accessing the data for verification, replication, or further research purposes may contact the corresponding author directly. The analytical procedures and code used in this study can also be provided upon request to facilitate reproducibility of the reported results. The appendices referenced throughout this manuscript are provided separately as Supplementary Material and can be accessed via the following repository link: https://github.com/dziqrinur02-boop/ramsey-reset-spline-simulation/blob/b6bddd407bc0b3e39fb69b23f5a3f4522f72a62f/Appendix.md.

References

[1] David Ruppert, M. P. Wand, and R. J. Carroll. *Semiparametric Regression*. Cambridge: Cambridge University Press, 2003. https://books.google.co.id/books?id=Y4uEvXFP2voC.

[2] B. Chen, B. Wang, Z. Xiao, and Y. Yi. "Improved Estimation of Semiparametric Dynamic Copula Models with Filtered Nonstationarity". In: *Journal of Econometrics* (2024). DOI: https://doi.org/10.1016/j.jeconom.2024.105739.

[3] Trevor J. Hastie. "Generalized Additive Models". In: *Statistical Models in S*. Chapman and Hall, 2017, pp. 249–307. DOI: https://doi.org/10.1201/9780203738535.

[4] Randall L. Eubank. *Nonparametric Regression and Spline Smoothing*. 2nd ed. New York: Marcel Dekker, 1999. DOI: https://doi.org/10.1201/9781482273144.

[5] Carl de Boor. *A Practical Guide to Splines*. New York: Springer, 2001. https://www.stat.cmu.edu/~brian/valerie/617-2022/week07/spline%20references/pdfcookie.com_a-practical-guide-to-splines.pdf.

[6] Joseph F. Hair, William C. Black, Barry J. Babin, and Rolph E. Anderson. *Multivariate Data Analysis*. 8th ed. Hampshire: Cengage Learning, 2019. https://books.google.co.id/books?id=0R9ZswEACAAJ.

[7] Bengt Muthén and Tihomir Asparouhov. "Recent Methods for the Study of Measurement Invariance with Many Groups". In: *Structural Equation Modeling* 25.4 (2018), pp. 637–664. DOI: 10.1080/10705511.2018.1427898.

[8] Rick H. Hoyle. *Handbook of Structural Equation Modeling*. 2nd ed. New York: Guilford Press, 2023. https://books.google.co.id/books?id=P16bEAAAQBAJ.

[9] Simon N. Wood. *Generalized Additive Models: An Introduction with R*. 2nd ed. Chapman and Hall/CRC, 2022. DOI: https://doi.org/10.1201/9781315370279.

[10] Alan Agresti. *Statistical Methods for the Social Sciences*. 5th ed. Boston: Pearson, 2018. https://books.google.co.id/books?id=AS9EtAEACAAJ.

[11] James B. Ramsey. "Tests for Specification Errors in Classical Linear Least-Squares Regression Analysis". In: *Journal of the Royal Statistical Society: Series B* 31.2 (1969), pp. 350–371. DOI: https://doi.org/10.1111/j.2517-6161.1969.tb00796.x.

[12] Jeffrey M. Wooldridge. *Introductory Econometrics: A Modern Approach*. 7th ed. Boston: Cengage Learning, 2020. https://books.google.co.id/books?id=tXqAzwEACAAJ.

[13] John Fox. *Applied Regression Analysis and Generalized Linear Models*. 3rd ed. Thousand Oaks: Sage Publications, 2016. https://books.google.co.id/books?id=3wrwCQAAQBAJ.

[14] Damodar N. Gujarati and Dawn C. Porter. *Basic Econometrics*. 5th ed. New York: McGraw-Hill, 2009. https://books.google.co.id/books?id=6l1CPgAACAAJ.

[15] Russell Davidson and James G. MacKinnon. *Econometric Theory and Methods*. New York: Oxford University Press, 2004. https://books.google.co.id/books?id=vqkap8AwAy0C.

[16] M. R. H. Nurdin, A. A. R. Fernandes, E. Sumarminingsih, and M. O. Ullah. "Development of Ramsey RESET to Identify the Polynomials Order of Smoothing Spline with Simulation Study". In: *JTAM (Jurnal Teori dan Aplikasi Matematika)* 9.1 (2025). DOI: 10.31764/jtam.v9i1.26785.

[17] M. Z. Hidayatulloh, Solimun, A. A. R. Fernandes, A. F. Rizqia, and F. H. Junianto. "Modified Ramsey RESET in Combined Truncated Spline–Fourier Nonparametric Path Analysis on Waste Management Behavior". In: *CAUCHY: Jurnal Matematika Murni dan Aplikasi* 10.2 (2025), pp. 1293–1308. DOI: https://doi.org/10.18860/cauchy.v10i2.37239.

[18] Kamelia Hidayat, A. A. R. Fernandes, A. Iriany, and M. Z. Hidayatulloh. "Evaluating Double Resampling through Simulation and Application in Semiparametric Truncated Spline for Waste Economic Value". In: *ZERO: Jurnal Sains, Matematika dan Terapan* 9.3 (2025), pp. 857–870. DOI: http://dx.doi.org/10.30829/zero.v9i3.26773.

[19] A. A. R. Fernandes et al. *Analisis Regresi dalam Pendekatan Fleksibel: Ilustrasi dengan Paket Program R*. Universitas Brawijaya Press, 2021. https://books.google.co.id/books?id=SKFgEAAAQBAJ.

[20] A. A. R. Fernandes and Solimun. *Metode Analisis Data Penelitian: Pendekatan Regresi*. Malang: Universitas Brawijaya Press, 2022. https://books.google.co.id/books?id=nGmuEAAAQBAJ.

[21] Ludwig Fahrmeir, Thomas Kneib, Stefan Lang, and Brian Marx. *Regression: Models, Methods and Applications*. 2nd ed. Berlin: Springer, 2023. https://books.google.co.id/books?id=j_IPzwEACAAJ.

[22] Xiaoming Zhai et al. "A Review of Artificial Intelligence in Education from 2010 to 2020". In: *Complexity* 2021 (2021), pp. 1–18. DOI: https://doi.org/10.1155/2021/8812542.