



Marine Insurance: Frequency–Severity Claim Modeling and the Effect of Delay on Ruin Probability

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Abstract

Reporting dates do not identify claim payment times. We quantify a reporting-based payment-time assumption's effect on finite-horizon cash ruin in an idealized marine portfolio. The freMarine calibration comprises 1,109 positive-payment snapshot claims over 41 occurrence months, with negative-binomial monthly counts, finite-mixture severity, and empirical zero-inflated reporting lags. Refitted-bootstrap diagnostics assess distributional adequacy; conflicting severity diagnostics and rejected parametric lag fits remain explicit. Event-driven simulation couples immediate and proxy payments on the same occurrences and severities, with constant aggregate premiums and an empty initial system. For 250,000 paired paths over 60 months at initial surplus 2,000 EUR thousands, cash-ruin estimates are 0.2951/0.2363 for immediate/proxy payment. The paired difference is -0.0588 (95% conditional Monte Carlo interval $[-0.0597, -0.0579]$); mean outstanding payments are 718.79 EUR thousands. Nonnegative delay mechanically favors coupled cash surplus without discharging liabilities. Unobserved payment histories, opening claims, and policyholder behavior preclude claims of predictive superiority, a preferred handling policy, or regulatory capital needs.

Keywords: Delayed claims; Marine insurance; Mixture distribution; Ruin probability.

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1. Introduction

Marine claim cash flows depend on occurrence frequency, severity, and payment timing. Finite-horizon cash ruin means negative cash surplus within a specified period, not ultimate ruin [1] or an assessment of all outstanding liabilities. Calibration and simulation must align: fitting overdispersed monthly counts as negative binomial and then using only homogeneous Poisson arrivals changes the model. Classical Cramér–Lundberg specifies compound Poisson claims and deterministic premiums; it does not require light-tailed severity by definition. Positive exponential moments are required for particular adjustment-coefficient arguments and exponential ruin bounds [2].

Frequency–severity studies address motor portfolios [3, 4], marine liability losses [5], and oil-spill compensation with different data and models [6]; delayed-claim studies examine outflow timing in surplus processes [7, 8]. In a distinct two-claim-type model, Saruan and Effendie [9]

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use an FGM copula for dependence between inter-claim times and amounts, deriving ruin results for an exponential-severity illustration. The present analysis identifies no two claim types and estimates no such dependence; it instead calibrates observed positive payments for a conditional cash-timing scenario, not actual settlement practice.

Occurrence, reporting, settlement, and payment are distinct: reporting lag ends at insurer notification, settlement resolves the claim, and payments may occur in instalments. A reported but unpaid claim is not incurred but not reported (IBNR) merely because it remains unpaid [10]. Individual paid-incurred reserving uses reporting, payment, and incurred-loss histories [11]; observation-delay [12] and marked-Cox [13] models address unreported claims, with the latter jointly representing reporting delay and hidden IBNR counts. These targets differ from scheduling payments for selected positive-paid claims. freMarine provides occurrence/reporting dates and snapshot paid amounts, not payment histories [14]; reporting lags therefore do not identify a payment-delay distribution.

The gap addressed is the lack of an aligned, auditable cash-ruin comparison for this positive-payment marine snapshot that respects monthly overdispersion and distinguishes reporting from payment. We ask how replacing immediate payment with a reporting-based proxy changes finite-horizon cash ruin, holding occurrence-severity laws, premiums, initial surplus, and horizon fixed. The computational contribution is a paired event-driven comparison with monthly negative-binomial arrivals, a fully specified severity mixture, separate paid/outstanding accounting, and sensitivity analysis. It quantifies a conditional timing assumption, not a new general ruin theorem, an IBNR estimator, or an estimate of handling benefits, reputation, lapse, or predictive superiority.

2. Methods

The analysis calibrates counts, severities, and reporting lags on a common snapshot population, then compares two payment-time rules on shared simulation paths. Calibration, event generation, and archival reconciliation use Python with NumPy, Pandas, SciPy, and Matplotlib.

2.1. Data

The freMarine dataset records 1,274 French marine claims occurring in January 2003–June 2006, with status documented at the end of June 2006 [14]. Monetary values are rescaled and contain added noise; this is a historical illustration, not a current market-wide loss calibration. We use *ClaimPaid* (EUR thousands), *OccurDate*, *ReporDate*, and *ClaimStatus*; payment dates and policy exposure are unavailable.

The common window [1 January 2003, 1 June 2006) comprises 41 calendar months. Its single June occurrence is excluded because the terminal month is sparsely observed, not because earlier months are demonstrably complete. The maximum reporting date, 13 June 2006, is not a verified extraction cutoff. Within this window, 1,109 positive-payment claims enter all three calibrations; 164 zero-payment claims are excluded, including 72 settled and 92 non-settled claims. Non-settled zeros are not structural zero ultimate losses. The modeling variables are complete; missing vessel characteristics and deductibles do not determine inclusion.

The estimand concerns positive payments observed at the snapshot, not the ultimate loss process. Of the retained claims, 1,029 are settled and 80 non-settled; the latter may develop further, whereas settled-only selection changes the population. *ClaimCharge* is diagnostic, not a verified ultimate payment: its relationship to paid amounts is nonuniform. Monthly counts reindex this same sample over all 41 months, including zero-count months if present. Reporting lag is

$$\text{Delay}_i = \text{ReporDate}_i - \text{OccurDate}_i. \quad (1)$$

There are 110 zero and 999 positive day differences, with no negative lags. These reporting lags (Eq. (1)) are not settlement or payment lags. Table 1 gives the sample partitions and

distinguishes full from positive lags.

Table 1: Descriptive statistics of the common positive-payment snapshot sample. Severity is in EUR thousands, reporting lag in days, and counts per month

Statistic	Severity	Lag (all)	Lag (positive)	Monthly count
n	1109	1109	999	41
Mean	24.754	31.796	35.297	27.049
Std	147.876	51.688	53.315	7.978
Min	0.100	0.000	1.000	4.000
Q1	1.400	6.000	8.000	22.000
Median	4.300	15.000	18.000	28.000
Q3	11.200	38.000	42.000	32.000
Max	3667.000	594.000	594.000	49.000
Skew.	17.138	4.437	4.315	0.031
Excess kurt.	366.635	28.322	26.550	1.186

All records in the source dataset: 1274; Outside the common occurrence window (excluded): 1; Records in the common occurrence window: 1273; Zero observed payments within the window (excluded): 164; Primary sample: positive observed payments: 1109; Settled claims in the primary sample: 1029; Non-settled claims in the primary sample: 80; Zero reporting delays in the primary sample: 110; Positive reporting delays in the primary sample: 999; Monthly occurrence-count observations: 41. Status and lag counts are separate partitions, not sequential exclusions.

Development diagnostics compare windows ending in December 2004, June 2005, December 2005, and May 2006, and the settled-only positive-payment subset. They cannot identify unreported claims or separate development from exposure/risk changes. The primary 41-month illustration is retained for comparability, not asserted to be an unbiased prospective calibration.

2.2. Calibration and Model Assessment

Poisson and negative-binomial (NB) monthly counts are fitted by maximum likelihood [2]. The Poisson estimate is the sample mean; the NB likelihood profiles $r > 0$ with $q = \bar{K}/(\bar{K} + r)$, checked independently by joint transformed-parameter optimization. The convention in Eq. (7) makes q a failure probability; SciPy receives $1 - q$. Relative comparison uses

$$\text{AIC} = -2\ell + 2k, \quad \text{BIC} = -2\ell + k \ln n, \quad (2)$$

where ℓ is maximized log-likelihood, k the free-parameter count, and n the relevant sample size (41 for counts). Lower criteria in Eq. (2) do not establish absolute adequacy. For a discrepancy statistic T_{obs} , parametric-bootstrap draws from the fitted law re-estimate all free parameters and recompute the statistic; with B successful refits,

$$p_{\text{boot}} = \frac{1 + \sum_{b=1}^B \mathbf{1}\{T_b \geq T_{\text{obs}}\}}{B + 1}. \quad (3)$$

Count assessment uses Eq. (3) with $B = 1,999$ and Pearson/discrete-KS statistics. Adjacent integer bins merge left to right until fitted expected counts reach five, with the final remainder merged backward; each refit repeats this rule. Bin counts, descriptive degrees of freedom, failed refits, seeds, and Monte Carlo standard errors are archived. The candidate favored by both information criteria is primary; a mean-matched Poisson law remains the comparator.

Severity candidates are exponential, gamma, lognormal, Weibull, Lomax (Pareto II), Lindley, and Burr XII, with zero location and support $x \geq 0$. Gamma uses shape a and scale s . Lognormal μ, σ describe $\log Y$ (SciPy scale e^μ), with MLEs $\hat{\mu} = n^{-1} \sum_i \log Y_i$ and $\hat{\sigma}^2 = n^{-1} \sum_i (\log Y_i - \hat{\mu})^2$, without a degrees-of-freedom correction. Lomax has $F(x) = 1 - (1 + x/s)^{-\alpha}$ and mean $s/(\alpha - 1)$ only for $\alpha > 1$; its scale is not arrival rate λ . Burr XII has three free parameters (c, k, s) and finite mean only if $ck > 1$.

Fitted-data KS and AD are discrepancy statistics, not distribution-free tests. Single-family assessments use $B = 999$ full refits. Since 71.6% of severities are tied on a 0.1 EUR-thousand grid, bootstrap draws are rounded to that grid and bounded below by 0.1 before refitting. AD is primary at $\alpha = 0.05$ to emphasize tail discrepancies; KS is secondary. Selection minimizes BIC among finite-mean candidates with AD $p_{\text{boot}} \geq 0.05$. If none qualifies, no single family is declared adequate; the threshold is not relaxed.

Full-support mixtures allow heterogeneity but remain vulnerable to tail misspecification [15]; they differ from threshold composites such as Cooray and Ananda's lognormal–Pareto model [16]. Their density is

$$f(x) = \sum_{k=1}^K \pi_k f_k(x; \theta_k), \quad \sum_{k=1}^K \pi_k = 1, \quad (4)$$

In Eq. (4), f_k is a component density with parameters θ_k and nonnegative weight π_k . We compare four pure-lognormal mixtures ($K = 2, 3, 4, 5$) and ten unique three-component multisets of gamma, lognormal, Lomax, and Weibull. Family permutations are one candidate; quantile-block assignments supply starts. Each observed-data fit jointly optimizes weights and parameters by L-BFGS-B under finite bounds, using 12 distinct starts from block rotations, Dirichlet weights, and log-scale jitter. It is not EM: initial/final log-likelihoods, iterations, function evaluations, stop reasons, and degeneracy flags replace an M-step status in the archive. Repeated-family components are ordered by mean only for reporting.

In ascending observed-fit BIC order, mixtures receive a $B = 199$ refitted bootstrap pilot; the first with AD $p_{\text{boot}} \geq 0.05$ is confirmed with $B = 999$, moving to the next rank if rejected. Synthetic severities are grid-rounded before two-start refits; KS remains secondary. Mean-error tolerances of 5%, 10%, and 20% assess diagnostic sensitivity only, not acceptance or tail adequacy.

For reporting lags, $p_0 = 110/1109 = 0.09919$ and a proper positive-part law H_+ give

$$\Pr(W = 0) = p_0, \quad \Pr(W \leq w) = p_0 + (1 - p_0) \Pr(W_+ \leq w), \quad w > 0,$$

with $E[W] = (1 - p_0)E[W_+]$ when finite. Lognormal, gamma, and Weibull fit only the 999 positive lags; their parameters and means are conditional, with the same scale conventions as severity. Ties affect 85.3% of positive lags, so $B = 999$ bootstrap samples are rounded to nearest days, bounded below by one, and refitted for primary AD and secondary KS. None passes the rule. Consequently H_+ is the empirical mass function on 147 distinct positive day values, with observed relative frequencies and days divided by 365.25/12. It preserves the support and mean, not a parametric adequacy claim, and cannot exceed the observed 594-day maximum.

2.3. Occurrence, Payment, and Cash Surplus

Time is in model months and money in EUR thousands; $365.25/12 = 30.4375$ days per month is a conversion convention, not an exposure adjustment to calendar-month counts. Fix $u \geq 0$ and $T > 0$. There are no occurrences at or before zero and no inherited outstanding claims, hence $U(0) = u$: the portfolio starts empty, not mature. Let $0 < T_1 < T_2 < \dots$ be locally finite occurrences, $N(t) = \sum_i \mathbf{1}(T_i \leq t)$, and attach finite amounts $Y_i \geq 0$ and lags $W_i \geq 0$. Payment accounting is

$$P_i = T_i + W_i, \quad S_{\text{pay}}(t) = \sum_{i=1}^{N(t)} Y_i \mathbf{1}(P_i \leq t), \quad (5)$$

$$U(t) = u + C(t) - S_{\text{pay}}(t), \quad C(t) = ct, \quad t \geq 0. \quad (6)$$

Nonnegative lags prevent an occurrence after t from paying by t ; payments neither advance nor pause the occurrence clock and may reorder claims.

Primary monthly counts $K_m = N(m) - N(m-1)$ in $(m-1, m]$ are independent NB variables, with $r > 0$, $0 < q < 1$, and

$$\begin{aligned} \Pr(K_m = k) &= \frac{\Gamma(k+r)}{\Gamma(r)k!} (1-q)^r q^k, \quad k = 0, 1, \dots, \\ \lambda = E[K_m] &= \frac{rq}{1-q}, \quad \text{Var}(K_m) = \lambda + \frac{\lambda^2}{r}. \end{aligned} \quad (7)$$

Here λ is a monthly mean, not a fixed NB Poisson intensity. Conditional on counts, independent uniform times in each month are sorted, independently across months. Equivalently, independent monthly Gamma intensities have shape r and scale $q/(1-q)$, with a conditionally Poisson process constant within each month; intensity is redrawn monthly, not once per horizon. For noninteger T , generate through $\lceil T \rceil$ and retain $T_i \leq T$. The comparator is homogeneous Poisson with intensity λ . Independence and within-month uniformity are assumptions beyond a marginal NB fit; temporal diagnostics and fitted parameters appear in Section 3.2. Crossing the two occurrence laws with immediate/proxy payment gives four scenarios; only Poisson/no-delay is classical Cramér–Lundberg here.

Severities are IID from the positive-payment law G , with $\mu_G = E[Y_i] > 0$ finite. Mixtures require finite component means and valid weights, giving $\mu_G = \sum_k \pi_k E[Y_i | k]$. Reporting-based scenario lags follow the law already calibrated above:

$$W_i = \begin{cases} 0, & \text{with probability } p_0, \\ W_{i,+}, & W_{i,+} \sim H_+, \text{ with probability } 1 - p_0. \end{cases} \quad (8)$$

The law in Eq. (8) has positive mean 35.2973 days; the full mean is 31.7962 days, or 1.04464 months. Each snapshot amount is paid once at occurrence plus this lag, setting additional report-to-payment time to zero. This assumption identifies neither actual payment/settlement dates or instalments nor ultimate open-claim losses. Pairs (Y_i, W_i) are IID across claims, severity and lag are independent within each pair, and all pairs are independent of the occurrence process and monthly intensities; these dependencies are assumed, not established by the audit. Immediate payment sets $W_i = 0$ on the same occurrences and severities, sharing u, c, T and the empty initial state.

Both mean-matched occurrence laws use constant aggregate premium income

$$c = (1 + \theta)\lambda\mu_G, \quad \theta = 0.30, \quad (9)$$

based on the sampled severity mean, not an unadjusted empirical substitute. With $\mu_G = 25.4201$, Eq. (9) gives $c = 893.8568$ EUR thousand/month. Loading is relative to mean incurred amounts $\lambda\mu_G$, not necessarily finite-horizon paid amounts in an empty system. It is not individual claimant income: instalments, renewals, non-payment, post-claim lapse, or feedback to premiums/exposure are unmodeled. Expenses, reinsurance, investment returns, reputation, and exposure changes are excluded. Lowering aggregate income with fixed claims is a revenue-shortfall stress, not an estimated lapse model. Positive loading establishes neither sufficient empirical capital nor an infinite-horizon bound. A positive-weight nondegenerate lognormal component has $E[e^{aY}] = \infty$ for every $a > 0$, despite finite mean; no positive Lundberg exponent or exponential bound is imported. Bounded finite-horizon ruin indicators need no exponential moment, while premium and expected outstanding amounts require appropriate moments.

The payment process is right-continuous; surplus is checked after summing simultaneous payments. Nonnegative amounts make this sufficient to detect negative intermediate surplus at that timestamp; between payments surplus increases at $c > 0$. Payments exactly at T count, whereas $T_i \leq T < P_i$ remain outstanding, without forced terminal payment. For $t \leq T$,

$$O_N(t) = \sum_{i=1}^{N(t)} \mathbf{1}(P_i > t), \quad O_Y(t) = \sum_{i=1}^{N(t)} Y_i \mathbf{1}(P_i > t). \quad (10)$$

These nominal diagnostics (Eq. (10)), not ultimate reserves, satisfy

$$S_{\text{pay}}(t) + O_Y(t) = \sum_{i=1}^{N(t)} Y_i.$$

Generate occurrences and marks through T even after ruin to avoid distorting horizon diagnostics. Cash ruin is

$$\begin{aligned} \tau(u) &= \inf\{t \geq 0 : U(t) < 0\}, \quad \inf \emptyset = \infty, \\ \psi(u, T) &= \Pr\{\tau(u) \leq T\} = \Pr\left\{\inf_{0 \leq t \leq T} U(t) < 0\right\}. \end{aligned} \quad (11)$$

Zero surplus is not ruin; $T, T_i, \tau(u)$ mean horizon, occurrence times, and ruin time. This target is neither ultimate ruin nor surplus net of all outstanding liabilities.

Yuen and Guo's compound-binomial model has discrete by-claim delay [7]; Yuen, Guo, and Ng's continuous-time model uses compound-Poisson main claims and exponential by-claim delay [8]. Removing immediate main-claim payments means setting $X_i \equiv 0$ in that accounting, not identifying two observed claim types here. Equations (5)–(6) adapt the accounting with NB occurrences, empirical reporting-based lags, and a finite-horizon target, without transferring their Lundberg results.

2.4. Monte Carlo Simulation and Verification

Generate the complete occurrence schedule through T , draw severity/lag marks, form payment times independently of the next occurrence, and sort payments due by T . Update one portfolio surplus at payment events; post-horizon payments remain outstanding. Both timing rules use shared occurrences and severities. For independent paths,

$$\hat{\psi}(u, T) = \frac{1}{n_{\text{sim}}} \sum_{j=1}^{n_{\text{sim}}} \mathbf{1}\{\tau_j(u) \leq T\}. \quad (12)$$

Correct event generation and independence give convergence for fixed parameters, u, T as $n_{\text{sim}} \rightarrow \infty$; within-path timing scenarios are paired, not independent. Production uses $T = 60$, $u = 0, 500, \dots, 5000$ and 250,000 paths per frequency law after a 10,000-path pilot. Scenario intervals are 95% Wilson intervals. With $D_j = \mathbf{1}\{\tau_{j,\text{delay}} \leq T\} - \mathbf{1}\{\tau_{j,\text{no delay}} \leq T\}$, the paired interval is $\bar{D} \pm 1.96s_D/\sqrt{n_{\text{sim}}}$, with shared-path sample standard deviation s_D . Relative changes divide by a positive no-delay estimate only. Intervals quantify conditional Monte Carlo uncertainty, excluding parameter and model-selection uncertainty.

The two 125,000-path production chunks use Poisson seeds 10 and 21, and NB seeds 30 and 41. The recorded environment is Python 3.11.15, NumPy 2.3.5, Pandas 2.2.3, SciPy 1.16.3, and Matplotlib 3.10.0. Deterministic edge cases, an independently coded payment evaluator, and count checks validate event ordering. All 500,000 paths are archived; aggregate counts and outstanding amounts are reconstructed from these outputs.

3. Results and Discussion

Building on the modeling framework, this section connects the observed claim characteristics to finite-horizon cash ruin. We first examine the retained sample and assess frequency, severity, and reporting-lag models to specify the simulation inputs. These inputs support the comparison of immediate and reporting-based proxy payments, followed by simulation validation and sensitivity analyses to clarify the findings and their limitations.

3.1. Descriptive Statistics and Exploratory Claims Analysis

A few large snapshot payments dominate the severity tail (Table 1 and Fig. 1). Monthly counts have mean 27.049 and sample variance 63.648, a variance-to-mean ratio of 2.353: descriptive

overdispersion, not validation of a stationary arrival model. Full and positive-lag summaries in the table describe reporting, not settlement.

Counts and monthly mean reporting lags vary over time without establishing stationarity. In May 2006, only 4 of 22 recorded occurrences have positive payments and 18 claims are non-settled, warranting caution about snapshot selection and development.

The January 2003–December 2004 window retains 703 positive-payment claims over 24 months: monthly count mean 29.292, severity mean 32.497 EUR thousands, and full reporting-lag mean 36.147 days, versus 27.049, 24.754, and 31.796 over 41 months. Settled-only selection retains 1,029 claims with mean severity 18.306. Differences may reflect payment development, settlement selection, exposure changes, or large-loss variability, not uniquely reporting truncation.

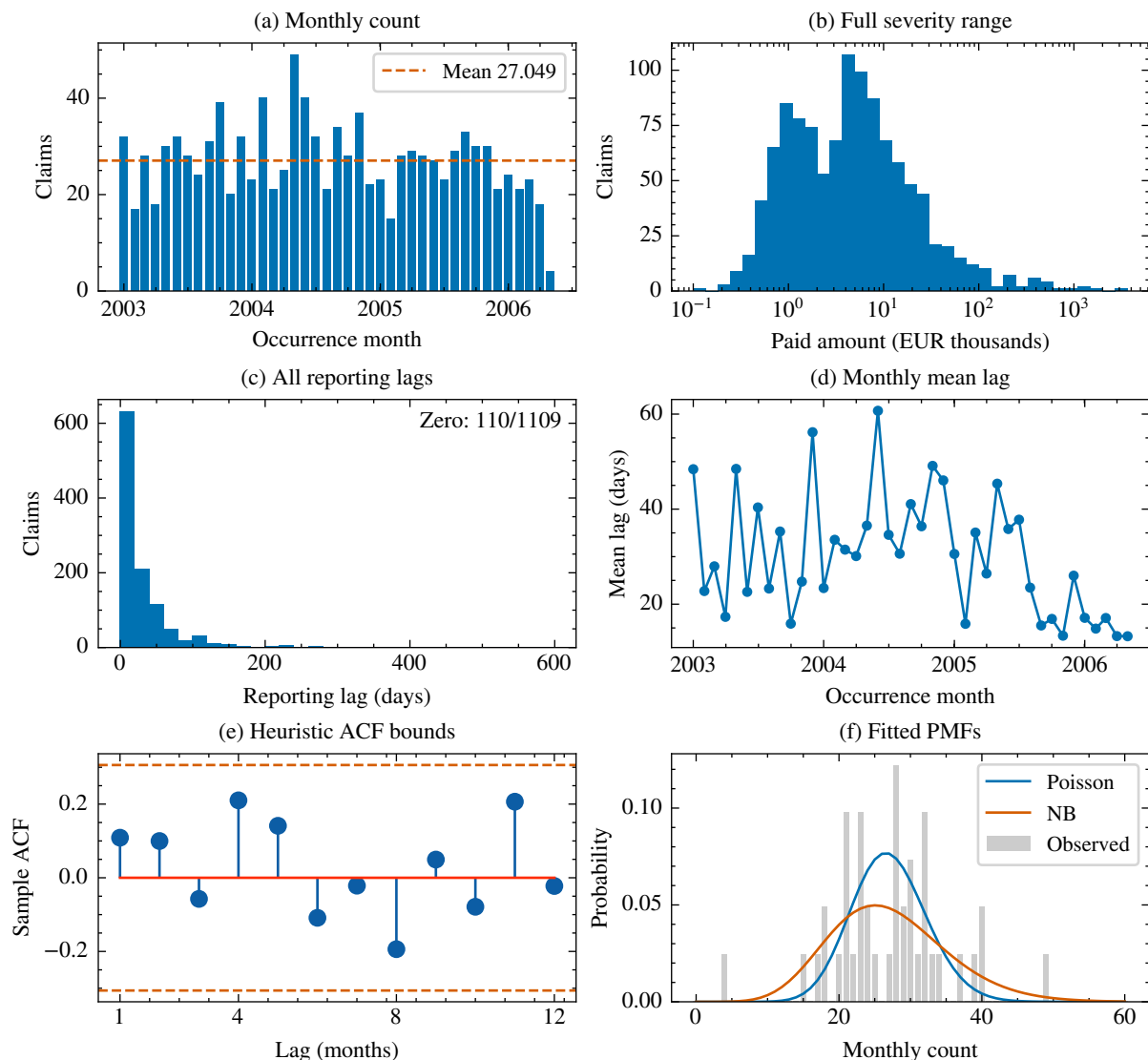


Fig. 1: Six unique sample/frequency panels: (a) counts and fitted mean; (b) full paid-amount range on a log axis; (c) all reporting lags, including zeros; (d) monthly mean lag; (e) ACF through lag 12 with heuristic bounds $\pm 1.96/\sqrt{41}$; (f) observed frequencies and fitted PMFs. Histograms use all retained observations and count heights. Bounds do not establish independence/stationarity; May 2006 has four positive payments and may be incompletely developed.

Heavy, long tails in marine liability losses [5] motivate flexible severity models, but that different portfolio provides context, not validation of the freMarine tail.

3.2. Claim Frequency Modeling

NB improves maximized log-likelihood from -155.239 to -144.939 , lowering AIC/BIC by 18.601/16.887 relative to Poisson (Table 2). It is the primary marginal model; Poisson remains mean-matched. With $B = 1,999$, NB Pearson/discrete-KS bootstrap p -values are 0.955/0.160, and Poisson's are 0.102/0.188. Neither is rejected at 5%; relative fit, not absolute falsification, motivates the choice. No refit failed.

Table 2: Frequency, single-severity, and positive-reporting-lag fits with refitted bootstrap diagnostics. AIC/BIC compare relative fit, not absolute adequacy

Family	Parameters	AIC	BIC	KS (p)	Pearson/AD (p)	Mean
A: Monthly frequency (41 months)						
Negative binomial	$r = 18.1416$ $q = 0.5986$	293.878	297.305	0.1373 (0.160)	0.921 (0.955)	–
Poisson	$\lambda = 27.0488$	312.479	314.192	0.1577 (0.188)	7.724 (0.102)	–
B: Severity (1109 positive payments; EUR thousands)						
Burr XII*	$c = 1.6529$ $k = 0.4843$ $s = 1.8654$	7378.67	7393.70	0.0564 (0.001)	4.679 (0.001)	undefined
Lognormal	$\mu = 1.5224$ $\sigma = 1.4976$	7423.67	7433.69	0.0537 (0.001)	6.577 (0.001)	14.066
Lomax (Pareto II)	$\alpha = 1.1977$ $s = 5.4872$	7449.75	7459.77	0.0831 (0.001)	8.951 (0.001)	27.757
Weibull	$a = 0.5597$ $s = 10.1262$	7870.00	7880.02	0.1517 (0.001)	42.797 (0.001)	16.787
Gamma	$a = 0.3915$ $s = 63.2268$	8343.85	8353.87	0.2102 (0.001)	99.879 (0.001)	24.754
Exponential	$s = 24.7543$	9337.56	9342.57	0.4099 (0.001)	464.126 (0.001)	24.754
Lindley	$\theta = 0.07788$	11632.92	11637.93	0.5321 (0.001)	1031.108 (0.001)	24.754
C: Positive reporting lag (999 observations; days)						
Lognormal	$\mu = 2.8589$ $\sigma = 1.2286$	8962.55	8972.37	0.0520 (0.033)	2.466 (0.015)	37.102
Weibull	$a = 0.8504$ $s = 31.9641$	9067.04	9076.85	0.0773 (0.007)	10.747 (0.001)	34.765
Gamma	$a = 0.8370$ $s = 42.1690$	9100.36	9110.18	0.0818 (0.001)	12.784 (0.001)	35.297

KS and Pearson/AD entries are statistic (bootstrap p). Panel A uses Pearson, panels B/C use AD as primary; KS is secondary. Count $B = 1999$, with refitting and repeated bin merging: NB bins (df) 7 (4); Poisson 6 (4). Continuous $B = 999$ with refitting and observation-grid rounding (severity 0.1; lag one day, minimum one). Burr XII $\hat{c}\hat{k} = 0.8005 < 1$: mean undefined. Minimum continuous bootstrap $p = 0.001$. Lag means are conditional on $W > 0$; full mean is $(1 - 110/1109)E[W_+]$. No single severity or positive-lag candidate passes AD at 5%.

The fitted NB parameters in Table 2 imply monthly mean 27.049 and variance 67.378, with SciPy success probability 0.4014. Simulation uses the independent monthly, conditional-uniform process in Section 2.3, not one horizon-wide Gamma intensity or exponential interarrivals based on the NB mean.

Across 12 lags, the largest sample ACF is 0.210 at lag four, below the heuristic bound 0.306. This does not establish independence or stationarity. The slope is -0.178 claims per month index and the final count is four; development, exposure, or random variation may contribute. Window sensitivities examine this concern. Count variance alone establishes neither the direction nor magnitude of required capital.

3.3. Claim Severity Modeling

All seven single severity families fail adequacy (Table 2): AD-bootstrap $p = 0.001$, the attainable minimum, with KS agreeing. Burr XII's free scale improves AIC over the earlier unit-scale fit,

but $\hat{c}\hat{k} = 0.8005 < 1$ leaves its mean undefined. Lognormal has the lowest BIC among finite-mean families, not an adequate selected law; this motivates, but does not validate, mixtures.

LN+LN+W has the smallest BIC among 14 mixtures (Table 3); 3 of 12 starts lie within 0.001 log-likelihood of the best nondegenerate fit, limiting stability claims to this search. Pilot ($B = 199$) AD/KS p -values are 0.125/0.040; final ($B = 999$) values are 0.103/0.034, with no failed refits. The primary AD rule selects the model, while secondary KS rejection signals residual mismatch.

Table 3: All 14 unique severity-mixture candidates, ranked by BIC. G, LN, L and W denote gamma, lognormal, Lomax and Weibull

Rank	Model	k	AIC	BIC	Mean error (%)	Stable	AD p	KS p
1	LN+LN+W	8	7294.55	7334.64	2.69	yes	0.103	0.034
2	LN+L+W	8	7295.39	7335.48	38.50	yes	–	–
3	G+LN+L	8	7296.99	7337.08	91.54	yes	–	–
4	G+LN+W	8	7299.88	7339.97	2.46	yes	–	–
5	LN ₃	8	7301.73	7341.82	0.42	yes	–	–
6	LN+LN+L	8	7302.13	7342.22	6.10	no	–	–
7	G+G+LN	8	7302.43	7342.52	0.68	yes	–	–
8	LN ₄	11	7292.23	7347.36	6.26	yes	–	–
9	G+G+L	8	7307.28	7347.37	20.50	yes	–	–
10	G+L+W	8	7310.09	7350.18	19.19	yes	–	–
11	L+W+W	8	7316.32	7356.41	11.05	yes	–	–
12	LN ₅	14	7297.21	7367.37	0.37	no	–	–
13	LN ₂	5	7355.43	7380.48	39.45	no	–	–
14	LN+L+L	8	7340.58	7380.67	131.60	yes	–	–

Family permutations are collapsed. – means not computed under the sequential BIC-ordered rule, not a nonsignificant test. Selected LN+LN+W: $B = 999$, AD primary, KS secondary. Stable requires at least two nondegenerate converged starts within 0.001 log-likelihood of the optimum. Mean error is diagnostic, not an adequacy criterion.

Table 4 specifies exact component parameters, means, and weighted contributions for reconstructing the sampler. Its mean 25.4201 EUR thousands exceeds the empirical 24.7543 by 2.69%. Diagnostic tolerances of 5%, 10%, and 20% retain the same BIC winner without altering the AD rule.

Table 4: Exact LN+LN+W sampler specification. Monetary scale, component means and weighted contributions are in EUR thousands

Component	Family	Weight	Parameters	Component mean	Weighted contribution
1	Lognormal	0.663372	$\mu = 1.669402$ $\sigma = 1.029409$	9.0182	5.9824
2	Lognormal	0.126467	$\mu = 3.540455$ $\sigma = 1.722998$	152.1422	19.2410
3	Weibull	0.210161	$a = 2.801296$ $s = 1.050904$	0.9358	0.1967

Tail agreement is incomplete: ratios in panel (d) of Fig. 2 are near one at 0.90, 0.95, and 0.99, but reach 1.192 at 0.975 and 1.221 at 0.995. These departures may affect ruin; AD non-rejection does not establish that they are negligible.

The components are latent distributional devices, not incident or vessel-class groups; neither variable identified membership. Operational interpretations would be speculative.

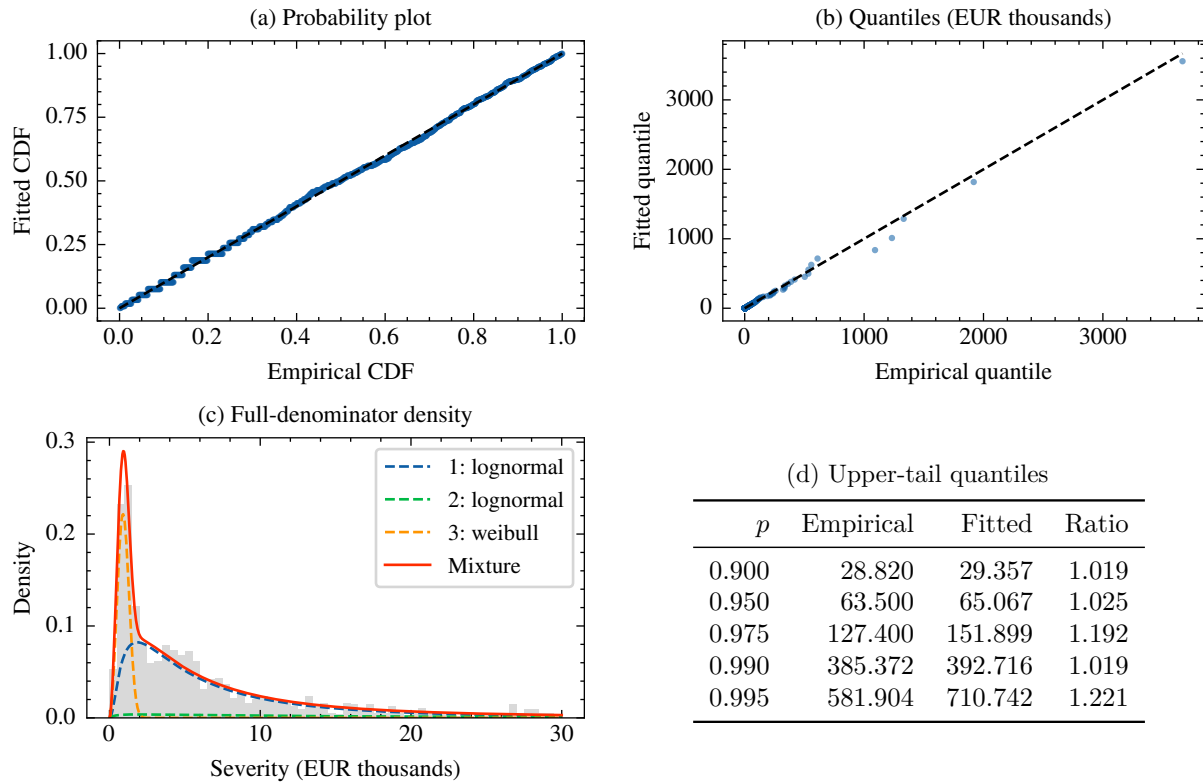


Fig. 2: Selected LN+LN+W severity diagnostics: (a) PP and (b) QQ plots for all 1109 payments; (c) weighted component densities and histogram over 0–30 EUR thousands (1004/1109, 90.5% shown); (d) five exact empirical/fitted upper quantiles and ratios. Histogram uses the full-sample denominator; displayed area is less than one. Quantiles are in EUR thousands. Primary AD does not reject, but secondary KS and tail mismatch remain.

3.4. Claim Delay Modeling

No parametric positive-lag family qualifies (Table 2). Lowest-AIC/BIC lognormal is rejected by KS/AD ($p = 0.033/0.015$); Weibull and gamma reject more strongly. No refit failed. Figure 3 describes these rejected fits, not evidence overriding the primary rule.

Lognormal’s positive/full means, 37.102/33.422 days, belong to a rejected approximation and are not simulation inputs. The empirical zero-inflated sampler has means 35.2973/31.7962 days (full mean 1.04464 months); its probabilities sum to one on the observed 147-point positive support. One million draws give zero fraction 0.09893 and full mean 1.04409 months: an implementation check, not external validation. Its bounded reporting-lag support does not identify payment deferral.

3.5. Ruin Probability Estimates Under Delayed Claims

Table 5 gives shared inputs; Table 6 and Fig. 4 report the independently seeded production comparison over 60 months. In these displays, $\hat{\psi}_T(u) = \hat{\psi}(u, T)$ from Eqs. (11)–(12). Archived paths, calibration, and seeds reconstruct counts and outstanding amounts; smaller sensitivity runs remain separate.

The analytical sampler mean, not empirical mean 24.7543, gives monthly pure/loaded premiums 687.5822/893.8568 EUR thousands, with $c/(\lambda\mu_G) - 1 = 0.30$ to floating-point precision. The coupled comparison changes payment timing only; it is conditional, not a causal estimate of slower real-world handling.

At $u = 2000$ EUR thousands, NB immediate/proxy cash-ruin estimates are 0.295064/0.236288 (73766/59072 ruins; MCSE 0.000912/0.000850). The paired proxy-minus-immediate difference is -0.058776 (-5.88 percentage points), MCSE 0.000470, with 95% interval $[-0.059698, -0.057854]$

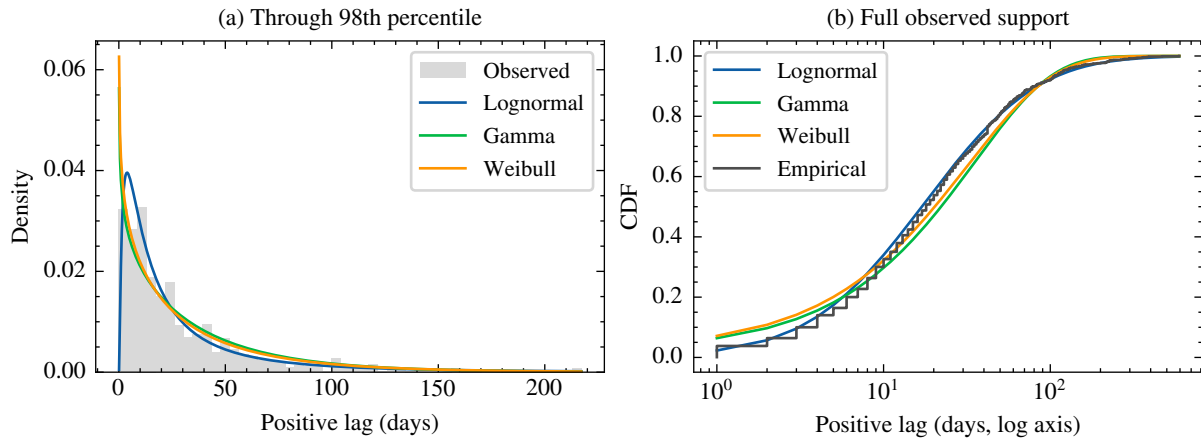


Fig. 3: Positive-reporting-lag diagnostics for three rejected parametric fits. (a) Through the 98th percentile, 217.12 days (979/999, 98.0% shown), with the full positive-sample histogram denominator and unconditional positive densities. (b) CDFs on the full observed support of 999 lags. These plots do not override refitted-bootstrap rejection.

Table 5: Inputs shared by both timing scenarios. The lag law is empirical resampling, not a validated parametric payment-delay fit

Quantity	Source / unit	Value
A: Common frequency, severity and premium inputs		
(r, q)	Fitted negative binomial frequency model	(18.1416, 0.5986)
$\lambda = E[K_m]$	NB/Poisson common monthly mean	27.0488 claims/month
G (severity of Y_i)	Selected two-lognormal-plus-Weibull mixture	Tables 3–4
μ_G	Mean of selected LN+LN+W mixture	25.4201 EUR thousand/claim
θ	Expected-value safety loading	0.30
c	$(1 + \theta)\lambda\mu_G$	893.8568 EUR thousand/month
$\hat{p}_0$	Empirical atom at zero reporting lag	110/1109 = 0.09919
H_+	Empirical positive reporting-lag resampling	147 support points
$E[W]$	Full reporting-lag mean	31.7962 days = 1.04464 months
B: Empirical reporting-lag specification		
Zero-lag atom	probability	110/1109=0.099188
Positive observations	claims	999
Distinct positive support points	days	147
Conditional positive mean	days	35.297297
Full mean including zero atom	days	31.796213
Maximum resampled positive lag	days	594

Panel A source descriptions and panel B units apply row by row. Both displayed zero-atom precisions are retained. Severity sampler: Table 4; monthly pure premium 687.5822 EUR thousands. Positive lags use observed relative frequencies on their support; reporting days are divided by 30.4375. Actual payment dates are unobserved.

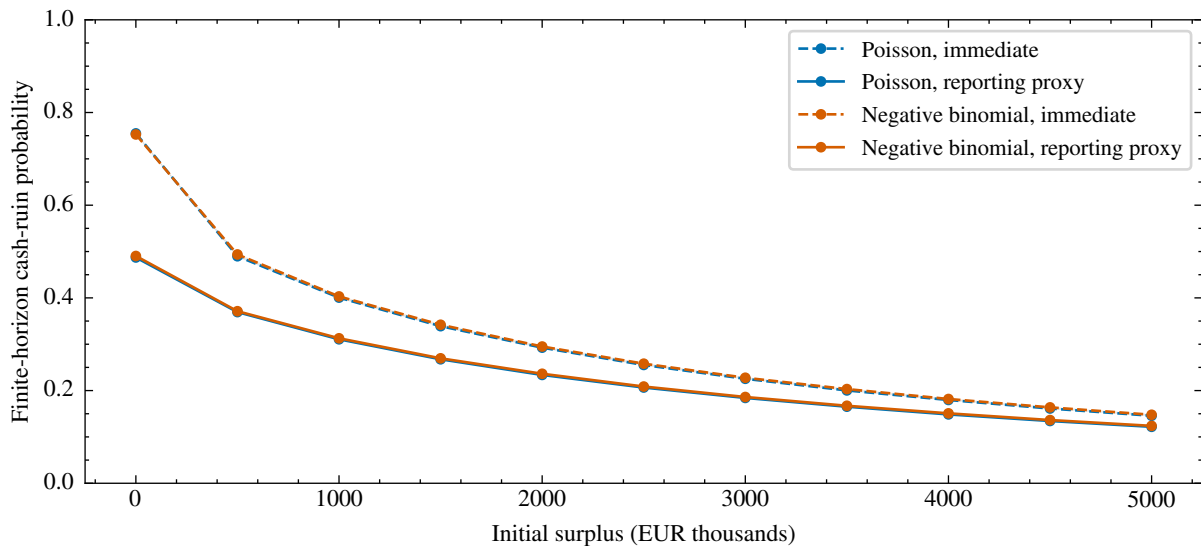
and relative change -19.92% . Mean-matched Poisson gives $0.292596/0.233716$ and difference -0.058880 ; the frequency specifications remain distinct.

On coupled paths, nonnegative claims paid no earlier give $S_{\text{pay, delay}}(t) \leq S_{\text{pay, no delay}}(t)$ and $U_{\text{delay}}(t) \geq U_{\text{no delay}}(t)$ for every t , under fixed premiums and an empty initial system. Delayed ruin is a subset of immediate-payment ruin; the contribution is calibrated magnitude/sensitivity,

Table 6: Production cash ruin at representative initial-surplus levels ($N = 250000$ per frequency, $T = 60$ months, $\theta = 30\%$). Counts are exact; probabilities and 95% Wilson intervals use four decimals

Frequency	Timing	u	Count	$\hat{\psi}_T(u)$	95% CI
Poisson	No delay	0	188736	0.7549	[0.7533, 0.7566]
Poisson	Reporting-delay proxy	0	121790	0.4872	[0.4852, 0.4891]
Poisson	No delay	2000	73149	0.2926	[0.2908, 0.2944]
Poisson	Reporting-delay proxy	2000	58429	0.2337	[0.2321, 0.2354]
Poisson	No delay	5000	36420	0.1457	[0.1443, 0.1471]
Poisson	Reporting-delay proxy	5000	30447	0.1218	[0.1205, 0.1231]
Negative binomial	No delay	0	188153	0.7526	[0.7509, 0.7543]
Negative binomial	Reporting-delay proxy	0	122603	0.4904	[0.4885, 0.4924]
Negative binomial	No delay	2000	73766	0.2951	[0.2933, 0.2969]
Negative binomial	Reporting-delay proxy	2000	59072	0.2363	[0.2346, 0.2380]
Negative binomial	No delay	5000	36956	0.1478	[0.1464, 0.1492]
Negative binomial	Reporting-delay proxy	5000	30964	0.1239	[0.1226, 0.1252]

u is in EUR thousands. All 11 capital levels remain plotted; conditional MCSE and paired-difference uncertainty are reported in the main text and archived results.

**Fig. 4:** Production cash ruin $\psi(u, 60)$ for all 11 capital levels $u = 0$ –5000 EUR thousands (0–5 million EUR), from the same T11 all-grid results as Table 6. Bands: pointwise 95% Wilson intervals conditional on fitted parameters. Timing paths are paired within each mean-matched frequency model; paired uncertainty is given in the text.

not novel ordering. Across the 11-point grid, absolute and relative reductions decrease: NB differences at $u = 0$ and 5000 are -0.262200 (34.84%) and -0.023968 (16.21%). No production cell has zero ruins; a zero count would require an interval upper bound, not a zero-risk claim.

Cash ordering does not discharge obligations: at month 60, NB proxy payments outstanding average 718.79 EUR thousands across 28.23 claims (amount median 428.21; 90th percentile 1412.59). Overlapping waits retain one premium stream *ct*. Immediate payment is a conservative timing comparator only for this coupled cash metric. Neither scenario is validated against insurer surplus/ruin; lower cash ruin establishes neither lower ultimate loss or liability-inclusive risk nor predictive accuracy, lower regulatory capital, or an optimal capital level.

3.5.1. Simulation validation and sensitivity analysis

The 10000-path pilot has at least 1258 ruins per main cell, supporting 250000-path production. Each supporting sensitivity uses 10000 paths on the 11-point capital grid. Shared horizon paths

generated through 120 months give nondecreasing counts at 12, 36, 60, and 120 months, as the ruin event requires.

At $u = 2000$, increasing loading from 0.10 to 0.50 lowers delayed ruin from 0.3886 to 0.1558 (Table 7; Fig. 5). Extending the horizon from 12 to 120 months raises it from 0.1315 to 0.2570, still finite-time, with timing differences near six percentage points. Removing the zero-lag atom changes 0.2378 to 0.2271. These one-factor experiments exclude joint parameter uncertainty.

Hypothetical additional payment lags of 0.5/1 month lower delayed estimates to 0.2117/0.1883; they are not estimated from freMarine. Conversely, fixed-claims premium shortfalls of 10%/20% raise estimates to 0.3236/0.4551. The shortfall experiment's 0.2374 baseline is separate from production 0.236288. These are cash-flow stresses, not estimated lapse responses.

Table 7: One-factor-at-a-time sensitivities at $u = 2000$ under NB frequency ($N = 10000$ per specification). Paired difference: reporting proxy minus immediate payment

Experiment	Specification	$\hat{\psi}_{T,\text{delay}}$	Paired difference
E2: loading	$\theta = 0.10$	0.3886	-0.0661
	$\theta = 0.30$	0.2334	-0.0586
	$\theta = 0.50$	0.1558	-0.0455
E3: horizon	$T = 12$	0.1315	-0.0616
	$T = 36$	0.2047	-0.0641
	$T = 60$	0.2321	-0.0634
	$T = 120$	0.2570	-0.0615
E4: delay definition	Atom at zero	0.2378	-0.0551
	Positive-only	0.2271	-0.0658
E5: added payment lag	$d = 0$ month	0.2374	-0.0567
	$d = 0.5$ month	0.2117	-0.0824
	$d = 1$ month	0.1883	-0.1058
E6: revenue shortfall	$\delta = 0.00$	0.2374	-0.0548
	$\delta = 0.10$	0.3236	-0.0615
	$\delta = 0.20$	0.4551	-0.0640
E8: calibration	24 months	0.2688	-0.0911
	30 months	0.2435	-0.0542
	36 months	0.2363	-0.0629
	41 months	0.2384	-0.0568
	Settled-only 41 months*	0.1246	-0.0441

Separate smaller runs: experiment-specific baselines are not the primary $N = 250000$ production sample. *Settled-only 41 months is a stress specification: neither frequency candidate passes its Pearson bootstrap check (NB $p = 0.004$).

Refitting frequency, all 14 severity candidates, and empirical lags at 24/30/36/41 months and settled-only 41 months gives model severity means 18.61–47.41 and 30%-loaded monthly rates 607.11–1752.52 EUR thousands. Window delayed estimates range 0.2363–0.2688. The 30-month LN+Lomax+Weibull alternative meets the AD threshold ($p = 0.080$), but 143 of 1142 attempted refits fail, making it numerically fragile. Settled-only ruin 0.1246 is not superior calibration: its count model fails adequacy (Pearson $p = 0.004$), and settlement is post-occurrence selection.

No alternative parametric lag is simulated because gamma, Weibull, and lognormal fail primary AD. No opening-backlog experiment is run: pre-2003 occurrences, opening liabilities, and residual reporting/payment lags are unobserved. Empty-system results therefore cannot transfer directly to a mature insurer.

The records contain neither actual payment histories nor service/reputation outcomes, so the proxy ranks no handling policies, recommends no payment deferral, and supplies no service-time benchmark or prediction beyond the horizon. Premium cessation, non-payment, and renewal/lapse responses to claims are unmodeled. Revenue stresses with fixed claims estimate neither shortfall occurrence probabilities nor joint lapse effects on exposure and counts; those

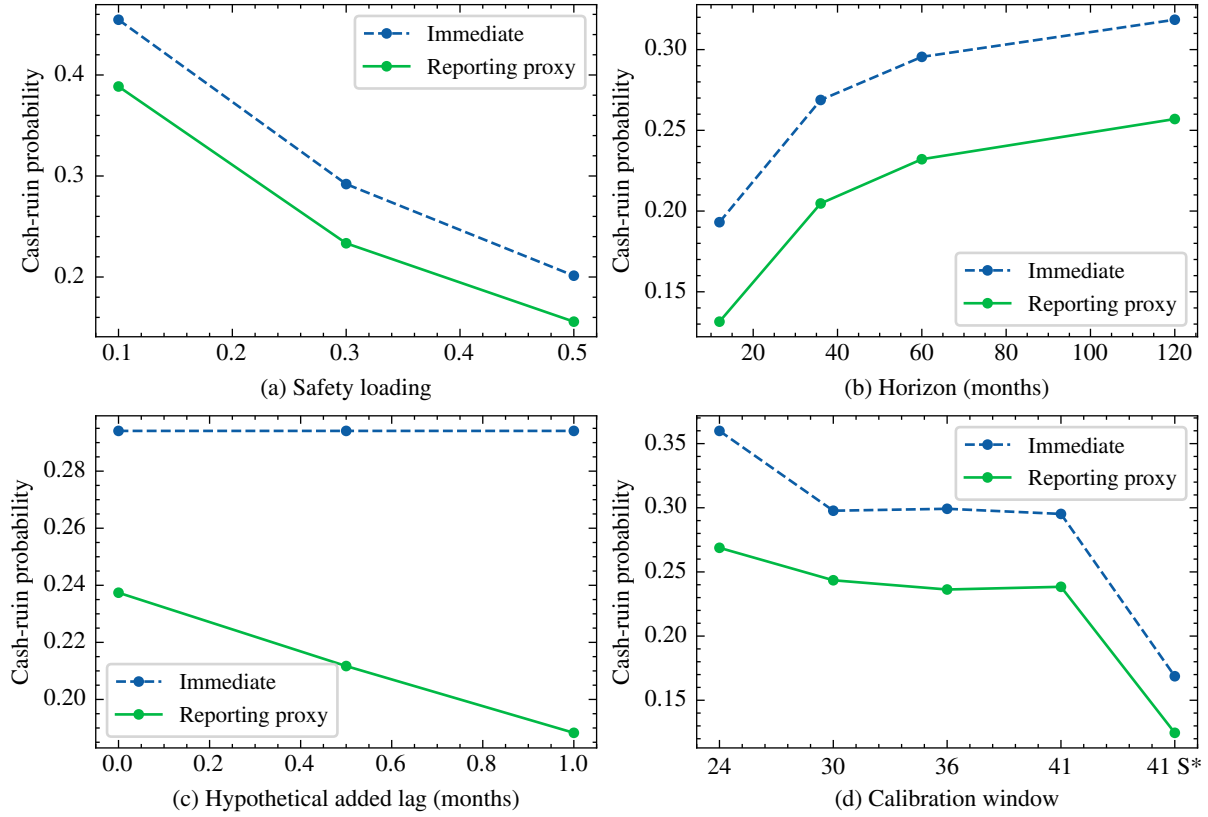


Fig. 5: Selected NB sensitivities at $u = 2000$, each using $N = 10000$ paths. Panels show loading, horizon, hypothetical added payment lag, and calibration window. “41 S*” is settled-only: its failed frequency adequacy check precludes treating it as a validated alternative. All 20 specifications and paired differences are in Table 7.

require policy-level premium, renewal, exposure, and claim histories.

Historical noisy/rescaled payments, open-claim development, zero-payment exclusions, and window-dependent selection limit the snapshot estimand (Section 2.1). The single-payment reporting proxy and independent monthly counts/marks, constant premium/exposure, and empty-state assumptions limit transfer; expenses, reinsurance, and investment returns are omitted. Capital-threshold/injection rules [17] and reinsurance-treaty comparisons [18] concern different surplus mechanisms, not this timing experiment. There is no external validation. Wilson/paired intervals cover simulation error conditional on fitted inputs, not parameter or model-selection uncertainty; cash ruin establishes neither long-run solvency nor liability-inclusive capital adequacy.

4. Conclusion

The aligned positive-payment calibration quantifies a reporting-based timing assumption, not an operational benefit from delaying claims. With occurrences independent of scheduling, nonnegative amounts, fixed aggregate premiums, and an empty initial system, delayed cash surplus is no lower on coupled paths; payments after the horizon remain outstanding. At $u = 2000$ EUR thousands, NB 60-month cash-ruin estimates are 0.2951/0.2363 for immediate/proxy payment, with paired difference -0.0588 (95% conditional Monte Carlo interval $[-0.0597, -0.0579]$). The proxy leaves mean outstanding amounts of 718.79 EUR thousands across 28.23 claims. Lower modeled cash ruin establishes neither lower ultimate liabilities nor lower regulatory capital needs.

The magnitude varies with loading, horizon, hypothetical additional lag, premium shortfall, and calibration window; settled-only frequency fails adequacy. Historical modified snapshot payments, independent marks, and constant premium/exposure limit transfer, while the intervals

exclude parameter and model uncertainty. Payment and claim-development histories are needed to assess actual cash timing; policy-level premium, renewal, and exposure histories are needed to examine post-claim cessation. Actual payment dates, lapse, reputation, and ultimate development remain unobserved, precluding a preferred handling policy or long-term solvency conclusion.

CRedit Authorship Contribution Statement

Sandy Salomo Saruan: Conceptualization, Writing–Original Draft Preparation, Visualization, Investigation, Resources, Software. **Adhitya Ronnie Effendie:** Methodology, Writing–Review & Editing, Supervision, Validation. **Rana Sahilah:** Draft Preparation, Computation & Visualization.

Declaration of Generative AI and AI-assisted technologies

In preparing this manuscript, the authors employed the assistance of Claude Sonnet 5 Thinking (Anthropic) to assist only with writing support, proofreading, and editing support. All results, content, and methodology originate from the authors own original research, and the authors take full responsibility for the content of this manuscript.

Declaration of Competing Interest

The authors declare no competing interests.

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Data and Code Availability

The freMarine dataset analyzed in this study is publicly available through the CASdatasets package [14].

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