



Sobol and GJR-GARCH for Dominant Factors of European Option Prices in Blue-Chip Index Constituent Stocks

Nathania Azzah Mu'azara and Rudianto Artiono*

Mathematics Department, Universitas Negeri Surabaya, Surabaya, 60231, Indonesia

Abstract

Conditional volatility-based option pricing research generally emphasizes improving volatility inputs, but has not yet fully explained the relative contribution of each parameter to variation in simulated option-pricing responses for Indonesian index stocks. This study integrates Black–Scholes, GJR-GARCH asymmetric volatility, and Sobol global sensitivity analysis to identify dominant factors in a model-generated European call-option response for verified constituents of an Indonesian blue-chip index, denoted Index X. The sample consists of 19 constituents that remained consistent from February 2022 to December 2025. Logarithmic returns are tested for stationarity and ARCH effects, conditional volatility is estimated using GJR-GARCH(1,1), and the Black–Scholes response is evaluated with five independently and uniformly distributed inputs: moneyness, maturity, risk-free rate, volatility multiplier, and proportional transaction cost. The results show that all returns are stationary and most stocks exhibit significant ARCH effects. Under the baseline domain, maturity and moneyness are the leading factors, while the volatility multiplier is secondary and the risk-free rate and transaction cost have smaller contributions. The ranking remains contract-dominated under a wider scenario domain and under a Student- t innovation check, although the magnitudes and ordering of maturity and moneyness can change. The contribution of this research is sensitivity decomposition of a specified Black–Scholes response with a GJR-GARCH-based volatility input, not evidence of improved pricing accuracy against observed option prices. Issuer and index identities are masked in the manuscript to prevent issuer-specific model outputs from being misconstrued as investment recommendations or comparative assessments; a confidential identity crosswalk is retained for editorial verification.

Keywords: Black–Scholes; GJR-GARCH; blue-chip index; European options; Sobol sensitivity analysis.

Copyright © 2026 by Authors, Published by CAUCHY Group. This is an open access article under the CC BY-SA License (<https://creativecommons.org/licenses/by-sa/4.0>)

1. Introduction

European options are financial derivative contracts whose value depends on the price movement of an underlying asset and can be exercised only at maturity. In mathematical finance, the valuation of European options links stock price, strike price, maturity, interest rate, and volatility into a single option-pricing response. The Black–Scholes model remains a widely used analytical framework because it provides a closed-form solution and a clear relationship between market parameters and option values. However, the classical model assumes constant volatility and frictionless market conditions, whereas empirical stock returns generally show changing volatility,

*Corresponding author. E-mail: rudiantoartiono@unesa.ac.id

volatility clustering, asymmetry, and extreme movements. These characteristics make volatility modelling important when the underlying assets are traded in emerging markets [1, 2].

Several studies have extended option-pricing frameworks by incorporating conditional volatility models. Conditional GARCH approaches are used to update volatility when risk changes over time [3, 4]. Realized GARCH, stochastic volatility, implied-volatility evidence, and market-calibration approaches provide alternative volatility inputs [5–8]. In the Indonesian and regional context, option-pricing studies have also combined Black–Scholes with GARCH and GJR-GARCH volatility forecasting [9, 10]. These studies show that volatility specification matters, but they generally emphasize pricing accuracy, forecasting, or calibration rather than the relative contribution of the input factors in a specified pricing response.

The GJR-GARCH model is relevant because it permits an asymmetric response of volatility to negative and positive return shocks. Negative shocks may increase future volatility more strongly than positive shocks of the same magnitude, although this effect must be assessed empirically for each stock. Recent studies have applied GJR-GARCH and related asymmetric models in option pricing, volatility forecasting, and financial-risk modelling [11–13]. The present study therefore uses GJR-GARCH as a volatility-input model, while explicitly distinguishing that role from a full horizon-consistent GJR-GARCH option-pricing model.

This study focuses on a formally evaluated Indonesian blue-chip index, denoted Index X. Index X is selected because it comprises relatively liquid and large-capitalization stocks, making its constituents relevant underlying assets for a computational option-response experiment. The official evaluation announcements also make the constituent-selection rule traceable. The verified sample contains 19 stocks that remained constituents throughout the available February 2022–December 2025 evaluation window. Previous studies on Indonesian stock indices and blue-chip assets have examined volatility, risk, portfolio performance, and forecasting [14–16]. Other Indonesian studies show the relevance of GARCH-family models for market volatility and risk [17, 18]. However, fewer studies determine which inputs dominate the variation of a simulated European option response when a GJR-GARCH-based volatility input is used.

The index and issuer identities are masked in the main manuscript as Index X and Stock A–Stock S. This is not intended to conceal the public origin of the data. The reason is that the study estimates model-based volatility and sensitivity measures, not issuer fundamentals, credit quality, or investment performance; displaying company names beside high volatility, residual ARCH, or dominant-factor rankings could encourage an issuer-level or investment interpretation that the design cannot support. Masking therefore keeps the scientific claim at the level of the response mechanism and avoids reputational or recommendation-like inferences. To preserve auditability, a complete identity crosswalk and the official constituent evidence are retained in a confidential reproducibility supplement that can be supplied to the editor and reviewers.

Global sensitivity analysis is useful for this purpose. A one-at-a-time approach changes one parameter while holding the other parameters fixed and therefore does not fully describe interactions. Sobol analysis decomposes output variation into first-order contributions and total-order contributions that include interactions. In an option-pricing response, it can assess the relative roles of moneyness, maturity, volatility, interest rate, transaction cost, and their interactions [19, 20]. The approach is supported by work on quasi-random sequences and global sensitivity analysis [21, 22].

Three research streams therefore remain insufficiently integrated. The first replaces constant volatility with GARCH-type inputs in option pricing. The second applies GARCH-family models to Indonesian financial data. The third evaluates input importance through global sensitivity analysis. This study integrates these streams for verified Index X constituents through a Black–Scholes response with GJR-GARCH-based volatility and a Sobol decomposition. The scientific object is the variation of the model-generated response under an explicit scenario domain, not an observed option-market price.

The study develops a computational framework combining GJR-GARCH estimation, Black–

Scholes European call-option valuation, and Sobol global sensitivity analysis. The five uncertain factors are moneyness, maturity, risk-free rate, volatility multiplier, and proportional transaction cost. Each factor is treated as an independent uniform input over its stated interval. This distributional assumption is important because Sobol indices depend on both the response function and the probability law assigned to the inputs.

The contribution is threefold. First, the study provides an auditable Index X stock-level response experiment based on a conditional-volatility input. Second, it ranks contract and volatility factors through first-order and total-order Sobol indices rather than through local sensitivity alone. Third, it reports convergence, cross-stock bootstrap variability, model robustness, and wider-domain robustness. The paper does not claim that the proportional transaction-cost loading is a general frictional option-pricing mechanism, nor that the simulated prices improve on observed market prices.

The remainder of this article is organized as follows. Section 2 describes the Index X sample, the GJR-GARCH volatility input, the Black–Scholes response, the controlled transaction-cost loading, the independent-uniform Sobol design, and the robustness checks. Section 3 presents the empirical results and their interpretation. The final section summarizes the conclusions, limitations, and directions for future work.

2. Research Method

The methodology is organized as a sequential computational framework that links verified sample construction to volatility modelling, option-response formulation, and global sensitivity analysis. The subsections below first describe the research design and data, then present return diagnostics and the GJR-GARCH specification, formulate the Black–Scholes response and transaction-cost loading, explain the Sobol design, and finally document the reproducibility and robustness procedures that support the analysis.

2.1. Research Design and Data

This study is a computational mathematical-finance study. Its objective is not to claim improved pricing accuracy against observed option quotations, but to decompose the variation of a model-generated European call-option response. The workflow consists of official Index X constituent verification, adjusted-price processing, return diagnostics, GJR-GARCH volatility estimation, Black–Scholes response evaluation, and Sobol global sensitivity analysis.

Index X is selected because it is an officially evaluated group of relatively liquid and large-capitalization Indonesian stocks. This makes the index a traceable and market-relevant universe for a controlled option-response experiment, while the official evaluation announcements provide an auditable sampling basis. The final sample contains 19 constituents that remained present in every verified Index X evaluation announcement overlapping the February 2022–December 2025 window; they are denoted Stock A through Stock S. Historical adjusted closing prices are obtained from a public financial-data provider. The consistent-constituent rule creates potential survivorship bias because stocks entering or leaving Index X during the window are excluded; this is accepted to obtain a balanced, consistently verified panel.

The masking protocol is retained for a substantive reporting reason. The stock-level quantities in this study are outputs of a deliberately simplified response experiment and should not be read as issuer-quality, risk-rating, or investment-performance comparisons. Associating named issuers with extreme volatility, residual ARCH, or sensitivity ranks would create a realistic risk of interpretation beyond the model’s evidential scope. Masking the names therefore limits the claim to the mathematical mechanism being studied and reduces recommendation-like or reputational inference. Reproducibility is not abandoned: a confidential crosswalk from Stock A–Stock S to the public source identifiers, together with the Index X identity and official constituent documents, is maintained for verification by the editor and reviewers.

The February 2022 starting point is the earliest period for which the official Index X files used in the verification procedure could be consistently extracted and cross-checked. This choice prioritizes constituent verification and comparability over a longer but less consistently documented window. Let S_t denote the adjusted closing price at trading day t . The daily logarithmic return is

$$r_t = \ln \left(\frac{S_t}{S_{t-1}} \right). \quad (1)$$

Eq. (1) transforms the adjusted price series into the return series used in all subsequent stationarity, ARCH, and volatility analyses. The complete sequence of data preparation, volatility estimation, response construction, and sensitivity analysis is summarized in Fig. 1.



Fig. 1: Black–Scholes–GJR-GARCH–Sobol research design

2.2. Preliminary Tests and GJR-GARCH Model

Descriptive statistics are computed for each return series. The Augmented Dickey–Fuller test evaluates stationarity, and the ARCH-LM test evaluates conditional heteroskedasticity. Stock C is retained even though its preliminary ARCH-LM result is not significant at 5%, because it satisfies the same official constituent rule as the other stocks; this exception is reported rather than used to alter the common modelling pipeline.

The conditional-mean relation is given in Eq. (2), while Eq. (3) defines the asymmetric conditional-variance recursion:

$$r_t = \mu + \varepsilon_t, \quad \varepsilon_t = \sigma_t z_t, \quad (2)$$

and

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \gamma I_{\{\varepsilon_{t-1} < 0\}} \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2. \quad (3)$$

The baseline specification uses a constant conditional mean and normally distributed standardized innovations. GJR-GARCH is chosen as a common flexible specification because it nests the symmetric response when $\gamma = 0$, permits leverage asymmetry when the data support it, and allows the same diagnostic pipeline to be applied without selecting a different model after observing each stock's significance result. The choice is evaluated through parameter significance, residual ARCH tests, persistence, a symmetric-GARCH AIC comparison, and a Student- t innovation check. The persistence measure is $\rho = \alpha + \beta + \gamma/2$, with covariance-stationarity requiring $\rho < 1$. The one-step-ahead daily volatility forecast is converted to an annualized base volatility by Eq. (4):

$$\sigma_{\text{ann}} = \sigma_{\text{daily}} \sqrt{252}. \quad (4)$$

This annualized value is held fixed as the base volatility inside each contract scenario. The reason for using one common as-of-date volatility signal is to isolate the Sobol contribution of the contract inputs under an identical information set; allowing the volatility forecast itself to evolve with T would make volatility horizon-dependent and mix the maturity effect with variance dynamics. This design is suitable for a controlled sensitivity experiment, but it is a Black–Scholes response with a GJR-GARCH-based volatility input, not a full GJR-GARCH option-pricing model. It is not equivalent to a multi-step forecast or an expected average conditional variance over a maturity of up to one year. A horizon-consistent construction is the appropriate extension when the objective changes from factor isolation to market-consistent valuation across maturities.

Post-estimation ARCH-LM tests are applied to standardized residuals. As robustness checks, the same return series are re-estimated with Student- t innovations, and the normal-innovation GJR-GARCH is compared with a symmetric GARCH(1,1) specification using AIC. The robustness outputs are generated by `scripts/15_article2_reviewer_robustness.py` and do not replace the baseline results.

2.3. Black–Scholes Response and Transaction-Cost Loading

For each Sobol scenario, define the moneyness ratio as $q = K/S_0$ and the effective volatility as $\sigma^* = m_\sigma \sigma_{\text{ann}}$, where m_σ is the volatility multiplier. The symbols q and m_σ are deliberately distinct: q determines the relative strike through $K = qS_0$, whereas m_σ scales only volatility. The European call value is then computed with the effective scenario volatility in Eq. (5):

$$C_{\text{BS}} = S_0 N(d_1) - K e^{-rT} N(d_2), \quad (5)$$

where the standardized terms are defined by Eq. (6):

$$d_1 = \frac{\ln(S_0/K) + (r + \frac{1}{2}(\sigma^*)^2)T}{\sigma^* \sqrt{T}}, \quad d_2 = d_1 - \sigma^* \sqrt{T}. \quad (6)$$

To include a controlled transaction-cost factor without claiming a general frictional pricing model, the simulated Sobol response is defined explicitly in Eq. (7) as

$$Y = C_{BS}(S_0, K, T, r, \sigma^*)(1 + \tau), \quad (7)$$

where τ is a proportional loading. The loading is retained because the research question requires a transparent, bounded cost perturbation while preserving the closed-form Black–Scholes response and a separately identifiable Sobol input. It is a scenario surcharge, not a structural claim about hedging costs. Transaction costs in option pricing generally affect replication, hedging, or boundary conditions and cannot generally be represented by this multiplication. Accordingly, the small transaction-cost Sobol index is partly induced by the multiplicative form and selected $[0, 0.02]$ range and is interpreted only as a property of this experiment, not as a general conclusion that transaction costs are unimportant.

2.4. Sobol Global Sensitivity Analysis

The uncertain inputs are $X = (X_1, \dots, X_5)$, consisting of moneyness $q = K/S_0$, maturity T , risk-free rate r , volatility multiplier m_σ , and transaction cost τ . Each input is assumed to be independent and uniformly distributed over the corresponding interval. Uniform distributions are used because the experiment specifies plausible bounds but has no defensible empirical density for hypothetical option contracts; equal weighting avoids imposing an unsupported concentration around a preferred scenario and makes the conditional interpretation transparent. The symbols, units, and baseline ranges are reported in Table 1.

Table 1: Independent-uniform input domain for Sobol analysis

Input factor	Symbol	Uniform range
Moneyness	$q = K/S_0$	$[0.90, 1.10]$
Maturity	T	$[1/12, 1]$ year
Risk-free rate	r	$[0.04, 0.07]$
Volatility multiplier	m_σ	$[0.75, 1.25]$
Transaction cost	τ	$[0.00, 0.02]$

The use of a policy-rate interval for r is a scenario choice for a computational experiment. The interval supplies a documented domestic monetary-rate scale and a common range across stocks, which supports controlled comparison. A maturity-matched government yield would be conceptually more direct for market valuation; therefore the policy rate is not presented as an exact tradable discount curve. The factor ranking is conditional on the independent-uniform domain and is not an empirical joint-distribution result.

For each scenario, $K = qS_0$ and $\sigma^* = m_\sigma \sigma_{\text{ann}}$ are used in Eqs. (5)–(7). With $N = 2^{10} = 1024$ and five factors, $N(k + 2) = 7168$ model evaluations are required per stock. First-order and total-order indices are estimated using the Sobol sequence, and stock-level indices are aggregated by arithmetic mean across the 19 constituents.

Numerical convergence is assessed at $N = 512, 1024$, and 2048 . Bootstrap intervals use 5,000 resamples with seed 20260703. These intervals describe cross-stock variability in the deliberately selected 19-stock panel; they are not sampling-error intervals for a single Sobol estimator and are not population-inference intervals. Numerical stability is assessed separately by the convergence comparison.

As an additional domain check, the wider domain is $K/S_0 \in [0.85, 1.15]$, $T \in [1/12, 1.5]$, $r \in [0.03, 0.08]$, $m_\sigma \in [0.50, 1.50]$, and $\tau \in [0, 0.03]$. The Student- t check propagates Student- t GJR-GARCH forecasts through the same baseline Sobol domain. These checks test whether the qualitative ranking is tied to one innovation distribution or one narrow interval choice.

2.5. Reproducibility

Computations use Python, `pandas`, `statsmodels`, `arch`, and `SALib`. The Sobol seed is 42 and the bootstrap seed is 20260703. The revised submission reports the 19 masked labels, observation window, input distributions, parameter ranges, model assumptions, and robustness outputs. The processed data, scripts, and generated tables use the same masked labels. The confidential identity crosswalk and public-source verification record can be supplied to the editor and reviewers; raw market data remain subject to the provider’s terms.

3. Results and Discussion

This section reports results for the 19 verified Index X constituents. The interpretation concerns variation in the specified simulated Black–Scholes response, not observed option-market prices. The results are organized around data diagnostics, volatility estimation, response construction, Sobol ranking, and robustness checks.

3.1. Verified Index X Sample and Return Characteristics

The processed prices cover 2022-02-02 to 2025-12-30. Each stock contains 937 adjusted closing-price observations and 936 logarithmic returns, with no missing values in the final panel. The sample is reported as Stock A–Stock S under the masking protocol. Stock H has the highest return standard deviation, 0.036718. Stock A has the highest kurtosis, 15.958532, followed by Stock O, 12.758002. These differences support stock-specific conditional-volatility inputs rather than one common constant-volatility assumption [17, 18].

3.2. Preliminary Tests and GJR-GARCH Diagnostics

The ADF test rejects the unit-root hypothesis for all 19 return series at 5%. The ARCH-LM test detects significant ARCH effects in 18 of 19 stocks; Stock C is the only exception. Stock C is retained because it satisfies the same official Index X constituent criterion, and applying the common pipeline avoids data-dependent sample selection. Post-estimation ARCH remains significant for Stock A only.

Table 2 consolidates these preliminary and post-estimation diagnostic outcomes before the detailed parameter results are presented.

Table 2: Summary of preliminary and post-estimation diagnostics

Diagnostic result	Number of stocks	Notes
Stationary return series	19 of 19	ADF rejects unit root at 5%
Significant ARCH effect	18 of 19	Stock C is the only exception
Residual ARCH after GJR-GARCH	1 of 19	Stock A remains significant at 5%
Verified Index X constituents	19 of 19	All satisfy the official rule

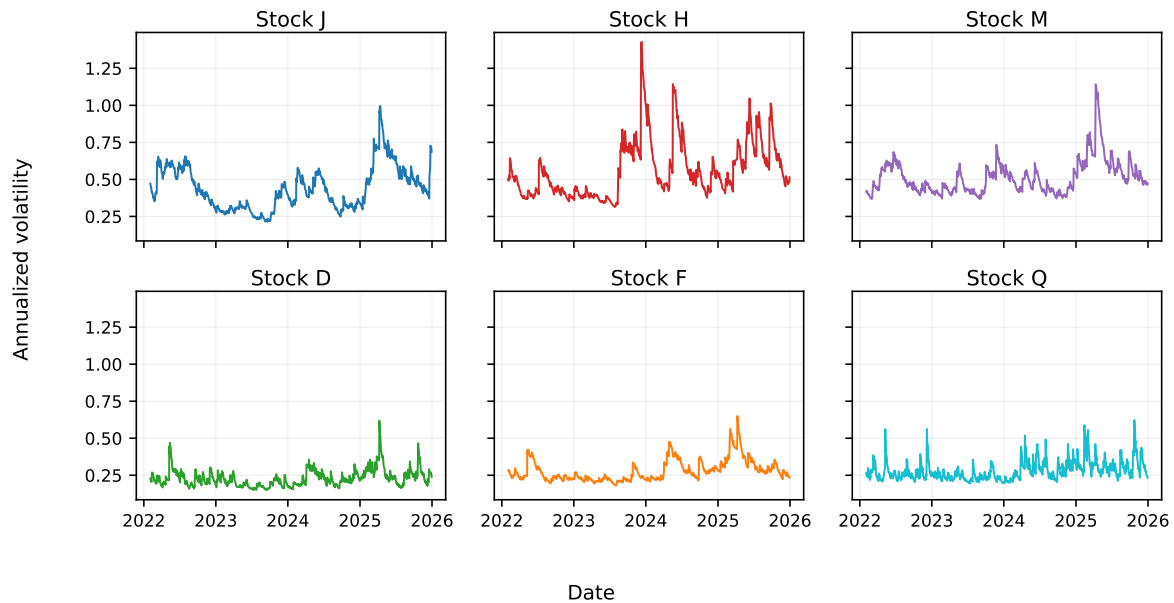
Table 3 reports the full baseline GJR-GARCH specification. The largest annualized forecast is for Stock J, 0.670147, followed by Stock H, 0.501156, and Stock M, 0.469784. The persistence values satisfy $\rho < 1$, but several are close to one and therefore indicate highly persistent volatility.

The signs of γ_1 are heterogeneous: 12 stocks have positive estimates, but only Stock E and Stock P have statistically significant positive asymmetry at 5%. Thus, the positive-asymmetry evidence supports retaining an asymmetric specification as a flexible model, not a claim that all Index X stocks exhibit a significant leverage effect. The baseline diagnostics show significance for ω in 4 stocks, α_1 in 7 stocks, and β_1 in all 19 stocks. A symmetric-GARCH comparison gives a lower AIC for GJR-GARCH in 12 of 19 stocks. This result supplies a concrete reason for retaining GJR-GARCH as the common specification: it improves AIC for the majority while preserving a uniform cross-stock pipeline and nesting the symmetric case. Stock A retains significant residual ARCH, so its forecast receives a cautious interpretation.

Table 3: Baseline GJR-GARCH estimation results and diagnostics

Stock	ω	α_1	γ_1	β_1	ρ	Forecast vol.	Post-ARCH p
Stock A	1.452625	0.156876	-0.083787	0.677778	0.792760	0.379482	0.027766
Stock B	0.044191	0.049731	0.004858	0.942497	0.994657	0.393695	0.828102
Stock C	0.700415	0.049405	0.136117	0.649605	0.767069	0.236699	0.954918
Stock D	0.117370	0.086825	0.050766	0.838092	0.950300	0.226803	0.565950
Stock E	0.074896	0.006310	0.093779	0.927286	0.980486	0.219705	0.711202
Stock F	0.063871	0.014294	0.073371	0.930020	0.980999	0.228244	0.429175
Stock G	0.361247	0.161583	-0.008665	0.747265	0.904517	0.227556	0.868069
Stock H	0.222910	0.077977	-0.046870	0.928355	0.982897	0.501156	0.667228
Stock I	0.332475	0.045290	0.090677	0.831514	0.922142	0.268779	0.285508
Stock J	0.045210	0.050408	0.018491	0.938815	0.998468	0.670147	0.896098
Stock K	0.068786	0.030530	0.046852	0.913593	0.967549	0.247795	0.673489
Stock L	0.079155	0.023704	0.042166	0.939788	0.984575	0.380287	0.633325
Stock M	0.235730	0.026555	0.040867	0.932802	0.979790	0.469784	0.212424
Stock N	0.464001	0.079500	-0.007327	0.800937	0.876773	0.272773	0.294273
Stock O	0.131356	0.112449	-0.031342	0.878827	0.975604	0.197550	0.644983
Stock P	0.236082	0.037767	0.120269	0.867179	0.965081	0.361240	0.871406
Stock Q	0.319971	0.084214	0.110289	0.769412	0.908770	0.224006	0.130027
Stock R	1.465693	0.122461	-0.104668	0.584301	0.654428	0.303241	0.993830
Stock S	0.335661	0.097749	-0.000363	0.855532	0.953099	0.392012	0.990593

Fig. 2 displays the six selected series used to illustrate the range of conditional-volatility paths; it does not display all 19 constituents.

**Fig. 2:** GJR-GARCH conditional annualized volatility of selected verified Index X stocks

3.3. Simulated Option-Response Interpretation

The response is evaluated using the Black-Scholes formula with the one-step-ahead annualized GJR-GARCH forecast held fixed across contract scenarios. Stock J tends to exhibit the widest response distribution because its baseline forecast volatility is the largest, while Stock R has the largest nominal price scale. Because moneyness is denoted by $q = K/S_0$, so that $K = qS_0$, the call response can be written as $C_{BS} = S_0 g(q, T, r, \sigma^*)$; thus S_0 changes nominal prices but should not by itself change normalized variance-based Sobol rankings. Liquidity is not an input to the response function. Therefore, stock-level differences are interpreted primarily through the

GJR-GARCH volatility input and interactions among response factors, not as causal effects of liquidity or absolute price scale.

The transaction-cost factor is a proportional loading only. Its small index is therefore expected to depend on both the loading form and the narrow $[0, 0.02]$ interval. The result cannot be generalized to a nonlinear frictional option-pricing model.

3.4. Sobol Global Sensitivity Results

The baseline aggregate results are shown in Table 4. Maturity has the largest total-order index, 0.468080, followed closely by moneyness, 0.455075. The difference is only 0.013005, so these two factors are better described as jointly dominant. Volatility multiplier ranks third, while risk-free rate and transaction cost are smaller within the selected independent-uniform domain.

Table 4: Baseline aggregate Sobol sensitivity indices

Rank	Factor	Mean S_1	Mean S_T
1	Maturity	0.460323	0.468080
2	Moneyness	0.453890	0.455075
3	Volatility multiplier	0.074240	0.082025
4	Risk-free rate	0.002828	0.003616
5	Transaction cost	0.000285	0.000225

Numerical convergence and the deliberately cross-stock bootstrap summary are reported in Table 5.

Table 5: Convergence and cross-stock bootstrap intervals for baseline total-order indices

Factor	$N = 512$	$N = 1024$	$N = 2048$	Bootstrap 95% interval
Maturity	0.467842	0.468080	0.468069	[0.416983, 0.523238]
Moneyness	0.454226	0.455075	0.454110	[0.384508, 0.521950]
Volatility multiplier	0.081555	0.082025	0.081969	[0.066956, 0.098304]
Risk-free rate	0.003611	0.003616	0.003627	[0.003034, 0.004168]
Transaction cost	0.000225	0.000225	0.000225	[0.000204, 0.000248]

The convergence values change only slightly from $N = 512$ to $N = 2048$, supporting numerical stability at $N = 1024$. The bootstrap intervals summarize heterogeneity across the deliberately selected 19-stock panel; they do not measure Sobol estimator sampling error. The overlap between the maturity and moneyness intervals reinforces the co-dominance interpretation.

Fig. 3 provides a visual comparison of the aggregate total-order indices reported in Table 4.

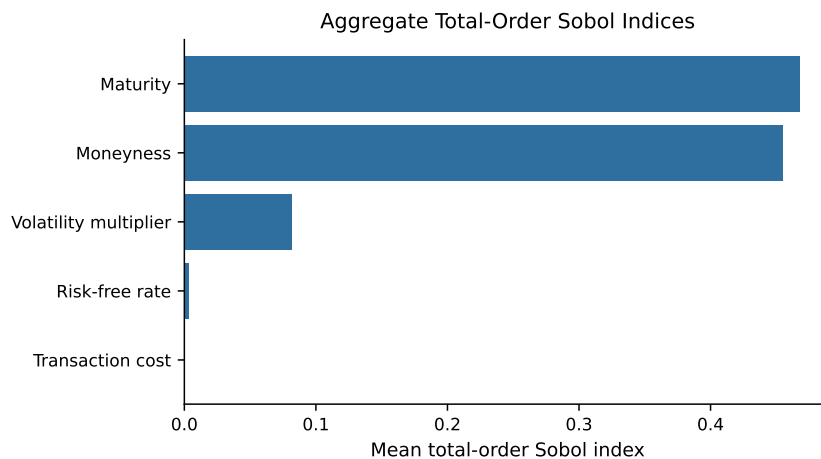


Fig. 3: Aggregate total-order Sobol indices of simulated option-response factors

The stock-level ranking also varies. Table 6 reports the dominant factor and corresponding total-order index for every masked constituent. Moneyness is dominant for Stock C, Stock D, Stock E, Stock F, Stock G, Stock I, Stock K, Stock N, Stock O, and Stock Q. Maturity is dominant for Stock A, Stock B, Stock H, Stock J, Stock L, Stock M, Stock P, Stock R, and Stock S. Since liquidity is not included in the response and S_0 only scales the response under $q = K/S_0$ moneyness, these differences should not be attributed causally to liquidity or spot-price scale. They are more plausibly associated with the stock-specific GJR-GARCH volatility input and factor interactions.

Table 6: Dominant factor by verified Index X stock

Stock	Dominant factor	S_T	Stock	Dominant factor	S_T
Stock A	Maturity	0.554094	Stock B	Maturity	0.567302
Stock C	Moneyness	0.579775	Stock D	Moneyness	0.599838
Stock E	Moneyness	0.614475	Stock F	Moneyness	0.596891
Stock G	Moneyness	0.598298	Stock H	Maturity	0.644603
Stock I	Moneyness	0.517935	Stock J	Maturity	0.712622
Stock K	Moneyness	0.557804	Stock L	Maturity	0.554863
Stock M	Maturity	0.625654	Stock N	Moneyness	0.510611
Stock O	Moneyness	0.661237	Stock P	Maturity	0.535920
Stock Q	Moneyness	0.605583	Stock R	Maturity	0.468142
Stock S	Maturity	0.565780			

Fig. 4 complements Table 6 by showing the complete stock-by-factor pattern of total-order indices.

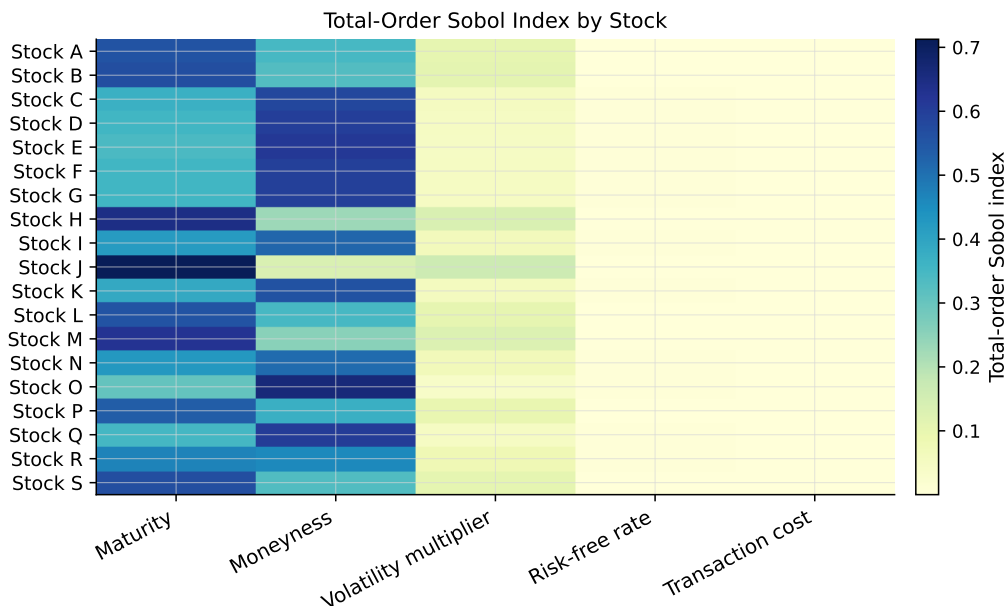


Fig. 4: Total-order Sobol index heatmap by verified Index X stock

3.5. Robustness Analysis

The wider independent-uniform domain produces mean total-order indices of 0.456509 for moneyness, 0.364628 for maturity, 0.193912 for volatility multiplier, 0.009555 for risk-free rate, and 0.000368 for transaction cost. Moneyness and maturity remain the two leading factors, although their order switches. This confirms contract dominance but also shows that magnitudes and ordering are conditional on the scenario domain.

The Student- t innovation check produces aggregate total-order indices of 0.489742 for maturity,

0.427795 for moneyness, 0.087998 for volatility multiplier, 0.003384 for risk-free rate, and 0.000234 for transaction cost. The Student- t specification has a lower AIC than the normal-innovation GJR-GARCH specification for all 19 stocks, and its forecast volatility is higher for 17 stocks. The two contract factors remain dominant, but the numerical indices change. This supports reporting the normal specification as the baseline while avoiding a claim that its exact Sobol magnitudes are universal.

Overall, the revised evidence supports a conditional conclusion: within the stated response, independent-uniform domain, and GJR-GARCH-based volatility input, maturity and moneyness are the leading contributors to simulated option-response variation. The result is not a claim about all transaction-cost models, all volatility distributions, or observed option prices.

4. Conclusion

This study developed a Black–Scholes–GJR-GARCH–Sobol framework for decomposing variation in a simulated European call-option response for 19 verified constituents of an Indonesian blue-chip index, denoted Index X. Index X was selected because its official evaluation process provides a traceable universe of relatively liquid and large-capitalization stocks. The observation window was February 2022 to December 2025, and the adjusted-price panel contained 937 observations and 936 logarithmic returns per stock.

All return series were stationary and 18 of 19 stocks exhibited significant ARCH effects. GJR-GARCH supplied stock-specific conditional-volatility inputs, but the one-step-ahead daily forecast annualized by $\sqrt{252}$ was held fixed across maturities to isolate contract-factor contributions under one information set. It should therefore be understood as a GJR-GARCH-based input to a Black–Scholes response, not as a horizon-consistent full GJR-GARCH option-pricing model. Stock A retained significant residual ARCH, while a symmetric-GARCH comparison produced a lower AIC for GJR-GARCH in 12 of 19 stocks. Positive asymmetry was statistically significant for only Stock E and Stock P, so the asymmetric specification is supported as a flexible common model rather than as a universal property of the sample.

Under the baseline independent-uniform domain, maturity and moneyness were jointly dominant, with total-order indices of 0.468080 and 0.455075. The volatility multiplier ranked third at 0.082025, while the risk-free rate and proportional transaction cost were 0.003616 and 0.000225. The wider-domain check kept moneyness and maturity as the two leading factors, although their order switched. The Student- t check also retained the two contract factors as the leading pair, while changing their magnitudes. These findings are therefore conditional on the response formulation, input distributions, parameter ranges, and volatility specification.

The transaction-cost result is deliberately limited. The response uses $C_{BS}(1+\tau)$ as a controlled proportional loading; it does not represent the general effects of transaction costs on hedging, replication, or boundary conditions. The small transaction-cost index is consequently a property of this loading and its selected range, not evidence that transaction costs are marginal in other option-pricing models.

The contribution of the study is sensitivity analysis of a model-generated response. It identifies how contract design variables and a GJR-GARCH-based volatility input contribute to simulated option-response variation, rather than demonstrating improved pricing accuracy against observed option prices. The bootstrap intervals describe cross-stock heterogeneity in the selected constituent set, whereas convergence checks address numerical stability of the Sobol estimator; the two should not be conflated.

The study is limited by its call-only response, one-step-ahead volatility input, independent-uniform Sobol design, proportional transaction-cost loading, selected Index X survivorship-based panel, and baseline normal innovations. The index and issuer labels are masked to prevent model diagnostics from being misread as issuer comparisons or investment recommendations; the confidential identity crosswalk is retained for editorial verification. Future research may use horizon-consistent multi-step or average-variance forecasts, nonlinear transaction-cost models,

alternative innovation distributions such as GED, dynamic constituent samples, correlated input distributions, and observed option-market data.

CRediT Authorship Contribution Statement

Nathania Azzah Mu'azara: Conceptualization, methodology, data collection, data curation, formal analysis, software implementation, validation, visualization, writing–original draft, and writing–review and editing. **Rudianto Artiono:** Supervision and validation.

Declaration of Generative AI and AI-assisted Technologies

No generative AI or AI-assisted technologies were used during the preparation of this manuscript.

Declaration of Competing Interest

The authors declare no competing interests.

Funding and Acknowledgements

This research received no external funding and this research is formally addressed to the Head of the Study Program and the Dean of the Faculty as part of the academic and institutional requirements.

Data and Code Availability

The Index X constituent data used to determine the final sample were obtained from official index evaluation announcements issued by the Indonesian stock exchange. Historical daily adjusted closing prices for the 19 constituents were obtained from a public financial-data provider for February 2, 2022, to December 30, 2025. The final dataset contains 19 verified stocks, reported as Stock A–Stock S, and 937 daily adjusted closing-price observations per stock. The processed data, scripts, parameter tables, robustness tables, and figures accompanying this revised manuscript use the same masked labels. A confidential crosswalk to the public identifiers and official constituent evidence is available to the editor and reviewers for reproducibility checks. Raw market data remain subject to the provider's terms.

The computational analysis was performed in Python using `pandas`, `statsmodels`, `arch`, and `SALib`. The reproducibility materials include the GJR-GARCH and Sobol scripts, processed return and price panels, random seeds 42 and 20260703, baseline result tables, Student-*t* and symmetric-GARCH robustness outputs, and the wider-domain check. The original market data remain subject to the terms and conditions of their provider.

References

- [1] Jean-Paul Chavas, Jian Li, and Linjie Wang. “Option Pricing Revisited: The Role of Price Volatility and Dynamics”. In: *Journal of Commodity Markets* 33 (2024), p. 100381. DOI: [10.1016/j.jcomm.2023.100381](https://doi.org/10.1016/j.jcomm.2023.100381).
- [2] Yawei Wu. “Options Pricing Comparison between the Black–Scholes Model and the Binomial Tree Model: A Case Study of American Equity Option and European-style Index Option”. In: *BCP Business & Management* 32 (2022), pp. 168–177. DOI: [10.54691/bcpbm.v32i.2885](https://doi.org/10.54691/bcpbm.v32i.2885).
- [3] Marcos Escobar-Anel, Javad Rastegari, and Lars Stentoft. “Option Pricing with Conditional GARCH Models”. In: *European Journal of Operational Research* 289.1 (2021), pp. 350–363. DOI: [10.1016/j.ejor.2020.07.002](https://doi.org/10.1016/j.ejor.2020.07.002).

- [4] Dong Hwan Oh and Yang-Ho Park. “GARCH Option Pricing with Volatility Derivatives”. In: *Journal of Banking & Finance* 146 (2023), p. 106718. DOI: [10.1016/j.jbankfin.2022.106718](https://doi.org/10.1016/j.jbankfin.2022.106718).
- [5] Zheng Fang and Jae-Young Han. “Realized GARCH Model in Volatility Forecasting and Option Pricing”. In: *Computational Economics* 66.5 (2025), pp. 3637–3657. DOI: [10.1007/s10614-024-10826-8](https://doi.org/10.1007/s10614-024-10826-8).
- [6] Tao Pang and Yang Zhao. “On GARCH and Autoregressive Stochastic Volatility Approaches for Market Calibration and Option Pricing”. In: *Risks* 13.2 (2025), p. 31. DOI: [10.3390/risks13020031](https://doi.org/10.3390/risks13020031).
- [7] Tianyi Wang, Sicong Cheng, Fangsheng Yin, and Mei Yu. “Overnight Volatility, Realized Volatility, and Option Pricing”. In: *Journal of Futures Markets* 42.7 (2022), pp. 1264–1283. DOI: [10.1002/fut.22330](https://doi.org/10.1002/fut.22330).
- [8] Xin-Jiang He and Sha Lin. “A Fractional Black–Scholes Model with Stochastic Volatility and European Option Pricing”. In: *Expert Systems with Applications* 178 (2021), p. 114983. DOI: [10.1016/j.eswa.2021.114983](https://doi.org/10.1016/j.eswa.2021.114983).
- [9] Fatimah Oktaviani and Emy Siswanah. “Option Pricing Using Modification of Black Scholes Merton Model with GJR-GARCH”. In: *CAUCHY: Jurnal Matematika Murni dan Aplikasi* 11.1 (2026), pp. 857–869. DOI: [10.18860/cauchy.v11i1.37426](https://doi.org/10.18860/cauchy.v11i1.37426).
- [10] Difa Tazkya, Rini Oktavia, and Intan Syahrini. “The Trinomial Tree Method in Pricing European Gold Option with Volatility Forecasting Using the GARCH (1,1) Model”. In: *ZERO: Jurnal Sains, Matematika dan Terapan* 9.1 (2025), p. 277. DOI: [10.30829/zero.v9i1.25455](https://doi.org/10.30829/zero.v9i1.25455).
- [11] Xinya Li. “The Pricing of CSI 300 ETF Options with GJR-GARCH Model”. In: *Operations Research and Fuzziology* 14.1 (2024), pp. 846–855. DOI: [10.12677/orf.2024.141078](https://doi.org/10.12677/orf.2024.141078).
- [12] Christophe Chorro and Rahantamialisoa H. Fanirisoa Zazaravaka. “Discriminating Between GARCH Models for Option Pricing by Their Ability to Compute Accurate VIX Measures”. In: *Journal of Financial Econometrics* 20.5 (2021), pp. 902–941. DOI: [10.1093/jjfinec/nbaa042](https://doi.org/10.1093/jjfinec/nbaa042).
- [13] Sarula Bai, Jaewon Jung, and Shun Li. “The Spillover Effects of Market Sentiments on Global Stock Market Volatility: A Multi-Country GJR-GARCH-MIDAS Approach”. In: *Journal of Risk and Financial Management* 17.12 (2024), p. 569. DOI: [10.3390/jrfm17120569](https://doi.org/10.3390/jrfm17120569).
- [14] Anggas Arden Arya Satya and Mahatir Muhammad. “Volatility Drivers in Islamic Stock Markets: The Case of IDX30 Index Firms in Indonesia”. In: *ITQAN: Journal of Islamic Economics, Management, and Finance* 5.1 (2026), pp. 94–107. DOI: [10.57053/itqan.v5i1.151](https://doi.org/10.57053/itqan.v5i1.151).
- [15] Siti Maysaroh and Anita Handayani. “Leverage, Dividend Policy, and Profitability Effects on IDX30 Stock Price Volatility During 2019–2023”. In: *Jurnal Ilmiah Manajemen Kesatuan* 13.2 (2025), pp. 1051–1060. DOI: [10.37641/jimkes.v13i2.3159](https://doi.org/10.37641/jimkes.v13i2.3159).
- [16] Sidiq Nuriksan and Nikmah. “Financial News Sentiment and Investor Confidence: Determinants of Stock Price Volatility in IDX30”. In: *Journal of Humanities Social Sciences and Business (JHSSB)* 4.3 (2025), pp. 493–506. DOI: [10.55047/jhssb.v4i3.1648](https://doi.org/10.55047/jhssb.v4i3.1648).
- [17] Dery Westryananda Putra, Sri Hasnawati, and Muslimin. “Ramadan Effect and Volatility Risk by GARCH Model: Evidence in Indonesia Stock Market”. In: *Jurnal Bisnis dan Manajemen* 17.2 (2021), pp. 73–82. DOI: [10.23960/jbm.v17i2.220](https://doi.org/10.23960/jbm.v17i2.220).

- [18] I Made Adhi Dharmaningrat, Helena Margaretha, and Kie Van Ivanky Saputra. “Predicting the Volatility of Jakarta Composite Index Using GARCH and LSTM with Volume-Up Strategy Approach”. In: *Journal of Information Systems Engineering and Business Intelligence* 11.3 (2025), pp. 311–322. DOI: [10.20473/jisebi.11.3.311-322](https://doi.org/10.20473/jisebi.11.3.311-322).
- [19] Stefano Scoleri, Marco Bianchetti, and Sergei Kucherenko. “Application of Quasi Monte Carlo and Global Sensitivity Analysis to Option Pricing and Greeks: Finite Differences vs. AAD”. In: *Wilmott* 2021.116 (2021), pp. 66–83. DOI: [10.1002/wilm.10972](https://doi.org/10.1002/wilm.10972).
- [20] Yin Gao and Miao Tian. “Pricing Problem and Sensitivity Analysis of Knock-in External Barrier Options Based on Uncertain Stock Model”. In: *Chaos, Solitons & Fractals* 187 (2024), p. 115356. DOI: [10.1016/j.chaos.2024.115356](https://doi.org/10.1016/j.chaos.2024.115356).
- [21] Xifu Sun, Barry Croke, Stephen Roberts, and Anthony Jakeman. “Investigation of Determinism-Related Issues in the Sobol Low-Discrepancy Sequence for Producing Sound Global Sensitivity Analysis Indices”. In: *ANZIAM Journal* 62 (2021), pp. C84–C97. DOI: [10.21914/anziamj.v62.16094](https://doi.org/10.21914/anziamj.v62.16094).
- [22] Maarten Arnst, Christian Soize, and Kevin Bulthuis. “Computation of Sobol Indices in Global Sensitivity Analysis from Small Data Sets by Probabilistic Learning on Manifolds”. In: *International Journal for Uncertainty Quantification* 11.2 (2021), pp. 1–23. DOI: [10.1615/Int.J.UncertaintyQuantification.2020032674](https://doi.org/10.1615/Int.J.UncertaintyQuantification.2020032674).