



Modified Ramsey RESET and Double-Resampling Bootstrap for Semiparametric Path Modeling of Financial Literacy

Fachira Haneinanda Junianto¹, Adji Achmad Rinaldo Fernandes^{2*}, Solimun², Ani Budi Astuti², and Muhammad Hisyam Lee³

¹*Department of Mathematics, Brawijaya University, Malang, 65145, Indonesia*

²*Department of Statistics, Brawijaya University, Malang, 65145, Indonesia*

³*Faculty of Science, Universiti Teknologi Malaysia, Skudai, 81310, Johor, Malaysia*

Abstract

Financial literacy is a multidimensional construct that is becoming increasingly important in the digital age, yet modeling its determinants often poses challenges when some of the relationships among variables are nonlinear and the normality assumption is not met. This study compared the standard bootstrap and the double-resampling bootstrap within a semiparametric path analysis framework based on truncated splines, applied to the estimation of indirect (mediation) effects in financial literacy modeling. Primary data were collected from 100 college students in Malang City using purposive sampling, with technology access (X_1) as the exogenous variable, financial knowledge (Y_1) as the mediator, and financial literacy (Y_2) as the response variable. The Ramsey RESET linearity test indicated that the relationship between technology access and financial knowledge was nonlinear, and it was therefore modeled using a one-knot linear truncated spline (minimum GCV of 0.155), while the other paths were linear. A Monte Carlo study at four sample sizes ($n = 50, 100, 200$, and 1000) with $R = 400$ replications per cell evaluated both procedures on coverage probability, interval length, test size, and power in addition to bias, RMSE, and relative efficiency. Two formulations of the double-resampling standard error are distinguished. Under the $\sqrt{B_2}$ -scaled formulation, the formulation that reproduces the empirical results reported here, the relative efficiency equals the number of second-stage repetitions B_2 exactly, at every coefficient and every sample size, which follows as an algebraic identity rather than as a property of the estimator. The resulting intervals were shorter by a factor of $1/\sqrt{B_2}$ but attained an empirical coverage of 0.387 against a nominal 0.95, and the associated test rejected a true null hypothesis in 0.612 of replications against a nominal 0.05; neither quantity improved as n rose from 50 to 1000. Under the averaged-estimate formulation, that is, the definition as written, calibration was restored (coverage 0.950, test size 0.036) and the efficiency advantage was no longer present (relative efficiency 0.92). The standard bootstrap was well calibrated throughout (coverage 0.939, test size 0.046) and was therefore adopted as the basis for inference in this model. Technology access had a significant direct effect on financial literacy under both procedures; the indirect effect through financial knowledge was positive on both sides of the knot and was established at the 5% level under the standard bootstrap applied directly to the empirical estimates ($t_{\text{calc}} = 4.523, p < 0.001$ below the knot; $t_{\text{calc}} = 3.215, p = 0.002$ above it), even though the calibrated procedure attained a simulated power of only 0.520 below the knot and 0.355 above it for effects of the estimated size at $n = 100$.

Keywords: Double Resampling Bootstrap; Financial Literacy; Mediation Analysis; Semiparametric Path Analysis; Truncated Spline.

*Corresponding author. E-mail: fernandes@ub.ac.id

1. Introduction

Financial literacy is a multidimensional competency that shapes the quality of financial decision-making, particularly in a digital ecosystem that requires the ability to access, understand, and use financial information [1]. Cross-national surveys show that literacy levels remain low and uneven across segments of society [2]. In the digital context, technology access and financial knowledge are interrelated factors that often form nonlinear and mediated relationships, which path analysis represents by estimating direct, indirect, and total effects within a single model. Parametric path analysis, however, assumes linearity and normality of residuals, assumptions that socioeconomic data often fail to meet; recent work addresses this by other means, as in the copula-based estimation of causal effects in path models developed by Ali et al. [3]. Semiparametric path analysis instead combines parametric and nonparametric components, modeling linear and nonlinear relationships simultaneously without sacrificing interpretability [4, 5].

The nonparametric component used here is the truncated spline, a segmented polynomial connected at knot points [6], whose number and location are selected by the Generalized Cross-Validation (GCV) criterion [7, 8]. Applications have expanded to mediating variables [9, 10] and to other response types, such as binary outcomes [11]. Which component applies to a given path is decided by testing rather than assumption, and the Ramsey RESET is an established test of functional-form misspecification [12]: a parametric specification is appropriate where the test identifies a known form, and the truncated spline suits the case where the form remains unidentified, since it represents a nonparametric pattern without discarding the linear structure of the variable [13].

Inference is the harder stage. Testing indirect (mediation) effects when normality fails and the sample is limited strains classical asymptotic approaches, which tend to produce unstable estimates and biased intervals. The bootstrap is the usual alternative and has been developed specifically for mediation [14]. A double-resampling extension applies resampling at two hierarchical levels, on the argument that the second stage calibrates the distribution produced by the first and thereby improves the accuracy of the resulting coverage and test levels [15]. Whether a given implementation delivers that improvement, or only a narrower standard error, is the question this study takes up. Evidence within truncated spline semiparametric path analysis remains limited: most studies report point estimates without two-stage resampling-based tests, and without validating performance against known population values.

Two earlier studies by substantially the same authors are the closest antecedents, and the distinction from them requires an explicit statement. Junianto et al. [16] proposed a Monte Carlo-based Bayesian path analysis for indirect effects on financial literacy, and specified the population parameters adopted in the simulation design used here. Junianto et al. [17] developed a semiparametric truncated spline path analysis with *jackknife*-based double resampling; the same group has also worked on flexible structural modeling in applied business settings [18]. The jackknife is deterministic, proceeding by systematic deletion, so the resampling distribution it generates is fixed by the sample rather than drawn from it. That rigidity bites hardest precisely where this study operates: an indirect effect is a product of coefficients, and products have asymmetric, non-normal sampling distributions that a deterministic scheme approximates poorly.

This study therefore develops and evaluates a *bootstrap*-based double-resampling inference within the same framework. Three features distinguish it from [17]: the functional form of each path is selected from the data by a sequential Modified Ramsey RESET procedure, so that modeling $X_1 \rightarrow Y_1$ nonparametrically is an outcome of testing rather than an assumption; the indirect effect is evaluated separately on either side of the knot, which requires the direct, indirect, and total effects to be defined piecewise; and the reported efficiency of double resampling

is interrogated rather than only reported, identifying the dependence of relative efficiency on the number of second-stage repetitions as a property of the design. The study accordingly compares the two procedures, estimates the direct, indirect, and total effects among technology access, financial knowledge, and financial literacy, and validates both procedures by Monte Carlo simulation on bias, RMSE, relative efficiency, coverage, interval length, test size, and power at four sample sizes ($n = 50, 100, 200$, and 1000). The application to Indonesian student financial literacy is the empirical setting, not a methodological claim.

2. Methods

This section describes the methodological framework used in the study, including data sources, measurement instruments, sampling procedures, and analysis techniques applied to construct and test a semiparametric path analysis model using a truncated spline approach and double-resampling bootstrap significance testing.

2.1. Research Design

This study employs a quantitative approach with a computational-statistical design that combines empirical data analysis and simulation studies. The quantitative approach was chosen because the primary objective of the study is to compare the performance of two inference methods, standard bootstrap resampling and double bootstrap resampling, within the framework of semiparametric truncated spline path analysis, which requires objective numerical measurements and statistical testing. The empirical component consists of a cross-sectional survey of respondents, while the computational component, in the form of Monte Carlo simulations, is used to validate the methods' performance against known population parameter values (ground truth). In line with the concept of a data-driven approach proposed by Astuti [19], the analysis focuses on the empirical patterns emerging from the data so that model estimates are more adaptive, objective, and accurate.

2.2. Data Sources and Variables

This study uses primary data collected through structured questionnaires designed based on the research indicators. The study population consists of college students in Malang, with a sample of 100 respondents selected using purposive sampling. The sample selection criteria included being at least 18 years old, having access to digital technology such as smartphones or the internet, and having experience in daily financial management activities. College students were chosen because they are a group with a high level of access to technology but varying levels of financial literacy, making them relevant for testing the relationships between variables.

The 100 respondents fall within the range of 100 to 200 observations that Hair et al. [5] describe as a common minimum for path modeling. This is reported as context for the design rather than as evidence that the sample is adequate for the model estimated here. Because respondents were selected purposively against fixed eligibility criteria rather than by probability sampling, the results characterize the group of students studied; they are not presented as estimates for the college student population of Malang or of Indonesia.

The three variables in this study were measured through reflective indicators, summarized in Table 1. Technology access (X_1) was measured through three indicators [20]; financial knowledge (Y_1) through respondents' understanding of finance, also on three indicators [21]; and financial literacy (Y_2) through respondents' understanding of banking, likewise on three indicators [22]. Each indicator was measured by two questionnaire items, giving eighteen items in total. Every item was scored on a five-point Likert scale ranging from 1 (strongly disagree) to 5 (strongly agree). Indicator scores were obtained as the mean of their two constituent items, and each construct score as the mean of its three indicator scores, so that every indicator contributes equally to the construct.

The questionnaire items have two distinct origins, and the two are separated here because they carry different implications for the measurement evidence. The technology access items and the financial knowledge items were adapted from previously published instruments, namely the digital financial access items of Lestari et al. [20] and the financial knowledge items of Adiputra et al. [21] respectively. The financial literacy items, by contrast, were written for this study. Remund [22] sets out a conceptual account of what financial literacy comprises but does not publish a questionnaire instrument, so the three financial literacy indicators in Table 1 were developed by the research team from that conceptual definition rather than adapted from an existing item set. Because one of the three construct measures is newly developed and the other two are adapted, the validity and reliability evidence reported in Table 2 was computed on the present sample for all three constructs rather than assumed from the source studies.

Table 1: Constructs and measurement indicators

Role	Construct	Code	Indicator
Exogenous	Technology access (X_1)	$X_{1.1}$	Smartphone and internet access
		$X_{1.2}$	Internet use for financial information
		$X_{1.3}$	Digital financial service
Endogenous	Financial knowledge (Y_1)	$Y_{1.1}$	General knowledge of finance
		$Y_{1.2}$	Knowledge of insurance
		$Y_{1.3}$	Knowledge of savings
	Financial literacy (Y_2)	$Y_{2.1}$	Investment knowledge
		$Y_{2.2}$	Credit awareness
		$Y_{2.3}$	Knowledge of banking products

The measurement model was assessed before estimating the path model. Convergent validity was evaluated through indicator loadings, corrected item-total correlations, Cronbach's alpha, composite reliability (CR), and the average variance extracted (AVE). The results are reported in Table 2.

Table 2: Validity and reliability of the measurement model

Construct	Code	Loading	Item-total	α	CR	AVE
Technology access (X_1)	$X_{1.1}$	0.739	0.524	0.840	0.903	0.758
	$X_{1.2}$	0.924	0.803			
	$X_{1.3}$	0.935	0.825			
Financial knowledge (Y_1)	$Y_{1.1}$	0.682	0.620	0.646	0.818	0.602
	$Y_{1.2}$	0.869	0.600			
	$Y_{1.3}$	0.767	0.428			
Financial literacy (Y_2)	$Y_{2.1}$	0.675	0.603	0.644	0.761	0.515
	$Y_{2.2}$	0.760	0.628			
	$Y_{2.3}$	0.717	0.687			

Loadings are first principal component loadings of the indicator correlation matrix. α , CR, and AVE are computed at the indicator level. Cronbach's alpha computed at the item level is 0.890, 0.673, and 0.637 for X_1 , Y_1 , and Y_2 respectively.

All loadings exceed 0.50 and all corrected item-total correlations exceed 0.30, indicating that each indicator contributes meaningfully to its construct. Composite reliability is above the 0.70 threshold for all three constructs, and the average variance extracted exceeds 0.50 in every case, so convergent validity is supported. Cronbach's alpha is 0.840 for X_1 and approximately 0.64 for Y_1 and Y_2 ; the latter two fall below the customary 0.70 benchmark but remain within the 0.60–0.70 range that Hair et al. [5] regard as acceptable in exploratory research, and the corresponding composite reliabilities, which do not assume equal indicator weights, are comfortably above 0.70.

This study was reviewed and approved by the Research Ethics Committee for Social and Behavioural Sciences (*Komite Etik Riset Ilmu Sosial dan Perilaku*, KERIS-P) of Universitas Brawijaya, which assessed the protocol as being of low ethical risk. The research took the form of a non-interventional, minimal-risk social survey: no experiment was conducted on human participants, and the study involved no medical or biomedical procedure, no physical intervention, and no collection of sensitive personal or legal information. Consent was obtained from every participant: in writing where respondents were willing to sign, and verbally where they preferred not to, either because they wished to remain anonymous or because they were approached directly in the field. No personally identifiable information was recorded, and all responses were treated as confidential and used solely for academic purposes.

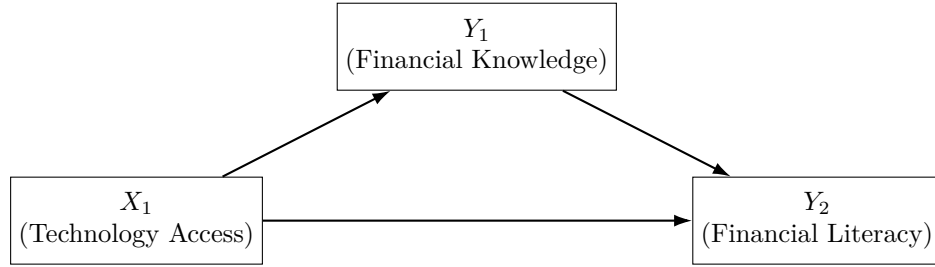


Fig. 1: Research model of technology access, financial knowledge, and financial literacy

The research model is shown in Fig. 1. It comprises three variables: technology access (X_1) as the exogenous variable, financial knowledge (Y_1) as the mediating variable, and financial literacy (Y_2) as the endogenous variable. The model represents the direct and indirect effects of technology access on financial literacy through financial knowledge. These effects are estimated with a truncated spline semiparametric path approach, which accommodates linear and nonlinear relationships within the same model.

2.3. Modified Ramsey RESET Linearity Test

Whether the relationship between an exogenous and an endogenous variable is linear can be established by testing. Ramsey [12] proposed the regression specification error test (RESET) for this purpose, and it remains a standard test of functional-form misspecification. The Modified Ramsey RESET extends that test to the case in which linearity has already been rejected: the next stage asks whether the relationship is quadratic, and rejection at that stage indicates a form that neither of the two parametric candidates identifies, so the path is treated as nonparametric. Throughout this subsection the coefficients $\beta_0, \beta_1, \beta_2, \beta_3$, and β_4 belong to the auxiliary test regressions and are local to the testing procedure; they are not the structural path coefficients $\beta_{11}, \beta_{21}, \beta_{12}$, and β_{22} defined in Eq. (12). The quadratic stage tests the following hypothesis:

$$\begin{aligned}
 H_0 : \beta_{k+2} = \beta_{k+3} = 0, \quad y_i = f(x_i) \text{ is a quadratic function; versus} \\
 H_1 : \text{there is one } \beta_{j+k+1} \neq 0, \quad y_i = f(x_i) \text{ is not a quadratic function.}
 \end{aligned}$$

The quadratic stage proceeds as follows. The auxiliary regression of Y_{1i} on X_{1i} and X_{1i}^2 is given in Eq. (1), with its coefficient of determination R_1^2 in Eq. (2).

$$\hat{Y}_{1i} = \hat{\beta}_0 + \hat{\beta}_1 X_{1i} + \hat{\beta}_2 X_{1i}^2 \quad (1)$$

$$R_1^2 = 1 - \frac{\sum_{i=1}^n (Y_{1i} - \hat{Y}_{1i})^2}{\sum_{i=1}^n (Y_{1i} - \bar{Y}_1)^2} \quad (2)$$

The regression is then augmented with $\hat{Y}_{1i}^2$ and $\hat{Y}_{1i}^3$, giving the model in Eq. (3), its fitted values in Eq. (4), and the new coefficient of determination R_2^2 in Eq. (5).

$$Y_{1i}^* = \beta_0^* + \beta_1^* X_{1i} + \beta_2^* X_{1i}^2 + \beta_3 \hat{Y}_{1i}^2 + \beta_4 \hat{Y}_{1i}^3 + \varepsilon_i^* \quad (3)$$

$$\hat{Y}_{1i}^* = \hat{\beta}_0^* + \hat{\beta}_1^* X_{1i} + \hat{\beta}_2^* X_{1i}^2 + \hat{\beta}_3 \hat{Y}_{1i}^2 + \hat{\beta}_4 \hat{Y}_{1i}^3 \quad (4)$$

$$R_2^2 = 1 - \frac{\sum_{i=1}^n (Y_{1i} - \hat{Y}_{1i}^*)^2}{\sum_{i=1}^n (Y_{1i} - \bar{Y}_1)^2} \quad (5)$$

The added terms are tested by $H_0 : \beta_3 = \beta_4 = 0$ against H_1 : at least one $\beta_j \neq 0$, $j = 3, 4$, using the F statistic in Eq. (6). If p -value $< \alpha$ then H_0 is rejected, the relationship is not quadratic, and the functional form is examined at the next stage; otherwise it is treated as quadratic and the sequence terminates.

$$F = \frac{(R_2^2 - R_1^2) / m}{(1 - R_2^2) / (n - p - 1 - m)} \sim F_{m, n-p-1-m} \quad (6)$$

If the quadratic form is also rejected, the procedure continues to the truncated spline stage, which repeats the same four steps with the quadratic term replaced by a one-knot truncated spline basis. The auxiliary regression and its coefficient of determination are given in Eq. (7), the augmented regression and its fitted values in Eq. (8), and the corresponding R_2^2 in Eq. (9).

$$\hat{Y}_{1i} = \hat{\beta}_0 + \hat{\beta}_1 X_{1i} + \hat{\beta}_2 (X_{1i} - K)_+, \quad R_1^2 = 1 - \frac{\sum_{i=1}^n (Y_{1i} - \hat{Y}_{1i})^2}{\sum_{i=1}^n (Y_{1i} - \bar{Y}_1)^2} \quad (7)$$

$$\begin{aligned} Y_{1i}^* &= \beta_0^* + \beta_1^* X_{1i} + \beta_2^* (X_{1i} - K)_+ + \beta_3 \hat{Y}_{1i}^2 + \beta_4 \hat{Y}_{1i}^3 + \varepsilon_i^* \\ \hat{Y}_{1i}^* &= \hat{\beta}_0^* + \hat{\beta}_1^* X_{1i} + \hat{\beta}_2^* (X_{1i} - K)_+ + \hat{\beta}_3 \hat{Y}_{1i}^2 + \hat{\beta}_4 \hat{Y}_{1i}^3 \end{aligned} \quad (8)$$

$$R_2^2 = 1 - \frac{\sum_{i=1}^n (Y_{1i} - \hat{Y}_{1i}^*)^2}{\sum_{i=1}^n (Y_{1i} - \bar{Y}_1)^2} \quad (9)$$

The hypothesis at this stage is

$H_0 : \beta_{k+2} = \beta_{k+3} = 0$, $y_i = f(x_i)$ is a nonparametric one-knot truncated spline; versus

H_1 : there is one $\beta_{j+k+1} \neq 0$, $y_i = f(x_i)$ is not a nonparametric one-knot truncated spline,

tested by the same F statistic, given in Eq. (10). Rejection leaves the form unidentified, so that a one-knot quadratic truncated spline may be considered; otherwise the one-knot linear truncated spline is adopted and the sequence terminates.

$$F = \frac{(R_2^2 - R_1^2) / m}{(1 - R_2^2) / (n - p - 1 - m)} \sim F_{m, n-p-1-m} \quad (10)$$

2.4. Semiparametric Truncated Spline Path Analysis

The semiparametric approach is the most flexible alternative when some relationships are linear and others are nonlinear. Semiparametric path analysis is a combination of parametric and nonparametric paths that combines the advantages of both. According to Fernandes and Solimun [4], this approach is used to model the relationship between variables where part of the relationship curve is known based on the results of linearity tests using Ramsey RESET (for linear relationships) and modified Ramsey RESET, while relationships whose curve forms are unknown are modeled nonparametrically using the truncated spline approach. The semiparametric path analysis equation model can generally be written as in Eq. (11).

$$y = \mathbf{X}\beta + \sum_{g=1}^{s-1} f_{\tilde{g}i} (Y_{1i}) + \varepsilon_i, \quad i = 1, 2, \dots, n \quad (11)$$

Applying Eq. (11) to the present model gives a semiparametric truncated spline path equation in which the path from technology access (X_1) to financial knowledge (Y_1) is modeled nonparametrically with a truncated spline, whereas the paths $X_1 \rightarrow Y_2$ and $Y_1 \rightarrow Y_2$ are modeled parametrically as linear relationships. This specification follows directly from the linearity tests reported in Section 3. The semiparametric path equation with a linear polynomial degree and one knot is presented in Eq. (12) [17, 23].

$$\begin{aligned}\hat{Y}_{1i} &= \hat{\beta}_{01} + \hat{\beta}_{11}X_{1i} + \hat{\beta}_{21}(X_{1i} - K)_+ \\ \hat{Y}_{2i} &= \hat{\beta}_{02} + \hat{\beta}_{12}X_{1i} + \hat{\beta}_{22}Y_{1i}\end{aligned}\quad (12)$$

where the truncated spline basis is defined as in Eq. (13) and K denotes the knot point on X_1 .

$$(X_{1i} - K)_+ = \begin{cases} X_{1i} - K, & X_{1i} > K \\ 0, & X_{1i} \leq K \end{cases} \quad (13)$$

Under this specification, $\hat{\beta}_{11}$ is the slope of $X_1 \rightarrow Y_1$ below the knot, $\hat{\beta}_{11} + \hat{\beta}_{21}$ is the slope above the knot, and $\hat{\beta}_{21}$ is the change in slope at the knot. The coefficients $\hat{\beta}_{12}$ and $\hat{\beta}_{22}$ are the direct effects of technology access and financial knowledge on financial literacy, respectively. The same notation is used throughout the Results and the Monte Carlo simulation.

2.5. Double-Resampling Bootstrap

Resampling obtains the empirical distribution of a parameter by repeated sampling with replacement from the available data, which yields variance, standard-error, and confidence-interval estimates without invoking normality. Its behavior under departures from the classical assumptions has been examined in the econometric literature [24] and, for mediation specifically, in [14]. The double-resampling bootstrap, introduced by Beran [25], regenerates data from an existing bootstrap sample: each of the B_1 first-stage samples is itself resampled B_2 times, so the total number of test statistics to be computed is $B_1 + B_1B_2$.

The resampling unit is the individual respondent: each first-stage sample is drawn by sampling n respondent records with replacement from the original data, and the full model in Eq. (12) is re-estimated on every resample with the knot K held fixed at the value selected by GCV on the original sample. Indexing the first-stage samples by $b = 1, 2, \dots, B_1$ and the second-stage samples drawn from within resample b by $r = 1, 2, \dots, B_2$, let $\hat{\beta}^{**}(b, r)$ denote the estimate obtained from second-stage sample (b, r) . The double-resampling estimate for first-stage sample b is the mean over its second-stage replicates, as in Eq. (14).

$$\hat{\beta}^*(b) = \frac{1}{B_2} \sum_{r=1}^{B_2} \hat{\beta}^{**}(b, r), \quad \hat{\beta}^*(\cdot) = \frac{1}{B_1} \sum_{b=1}^{B_1} \hat{\beta}^*(b) \quad (14)$$

The double bootstrap standard error is then the standard deviation of these B_1 averaged estimates, as in Eq. (15).

$$\hat{se}_{bb}^{\text{avg}} = \sqrt{\frac{1}{B_1 - 1} \sum_{b=1}^{B_1} [\hat{\beta}^*(b) - \hat{\beta}^*(\cdot)]^2} \quad (15)$$

Averaging the B_2 second-stage replicates removes second-stage sampling noise from $\hat{\beta}^*(b)$ but leaves the first-stage variation across b intact, so Eq. (15) does not by itself shrink the standard error by $\sqrt{B_2}$; such a reduction arises only if the first-stage standard error is divided by $\sqrt{B_2}$ directly, which is a distinct operation on the same replicates. Both are described as double-resampling standard errors in applied work, and because they behave differently they are distinguished here by name and symbol: Eq. (15) is the *averaged-estimate formulation* $\hat{se}_{bb}^{\text{avg}}$, and $\hat{se}_{bb}^{\sqrt{B_2}} = \hat{se}_b / \sqrt{B_2}$ is the *$\sqrt{B_2}$ -scaled formulation*. They are not interchangeable, and

Section 3 reports each separately, because the distinction determines whether the resulting intervals retain their nominal coverage. Where a statement applies to both, the unqualified term *double-resampling bootstrap* refers to the two-stage scheme itself rather than to either standard error.

Hypothesis testing then uses the statistic in Eq. (16), in which $\widehat{se}_{bb}$ denotes whichever of the two formulations is in use, referred to a t distribution with $n - p - 1$ degrees of freedom. Confidence intervals are constructed by the percentile method from the B_1 values of $\widehat{\beta}^*(b)$.

$$t_{\text{calc}} = \frac{\widehat{\beta}^*(\cdot)}{\widehat{se}_{bb}} \quad (16)$$

For the empirical analysis the resampling counts were $B_1 = 1000$ and $B_2 = 500$, with estimation carried out in RStudio. No random seed was fixed for the empirical run; the simulation fixes the seed 20260912 and is reproducible exactly. The Monte Carlo study uses $B_2 = 15$ for reasons of computational cost set out in Subsection 2.6. That choice is conservative with respect to the conclusions drawn in Section 3, since the distortion documented there grows with B_2 , so a larger second-stage count would widen rather than narrow the gap between nominal and attained coverage.

If the p -value generated through the double bootstrap method is smaller than the significance level α , then the null hypothesis H_0 is rejected. Whatever inferential benefit the second stage provides, its cost is computational: each first-stage bootstrap sample requires the second-stage sample to be regenerated B_2 times, so the total number of test statistic evaluations becomes $B_1 + B_1 B_2$. The cost matters here because the distribution of $\widehat{\tau}_b^{**}$ need not be independent of $\widehat{\tau}_b^*$ [24], so the second stage cannot be treated as a routine refinement of the first. The two methods are commonly compared through the relative efficiency measure, which is formulated as in Eq. (17).

$$RE = \frac{\text{Var}(\beta_b)}{\text{Var}(\beta_{bb})} \quad (17)$$

A relative efficiency value greater than one indicates that the double-resampling procedure produces estimates with a smaller estimated variance than the standard bootstrap. Two mechanisms may contribute to this. The first is inferential: prepivoting the first-stage statistic through a second round of resampling improves the asymptotic accuracy of the resulting coverage and test levels [25], and fast implementations make that iteration computationally practical [15]. The second is arithmetic: because the double-resampling estimate is the mean of B_2 second-stage replicates, its variance is reduced by a factor of approximately B_2 by construction, irrespective of any inferential gain. These two mechanisms are not separable when B_2 is held fixed, and Section 3 returns to this point when interpreting the magnitude of the relative efficiency obtained in this study.

2.6. Monte Carlo Simulation Procedure

To validate the performance of both methods using known parameter values, a Monte Carlo simulation study was conducted to generate data following the structure of the semiparametric truncated spline model described above, using the following steps.

1. Set the population parameter values for the model structure under study, namely β_{11} and β_{21} for the nonparametric truncated spline relationship ($X_1 \rightarrow Y_1$) and β_{12} and β_{22} for the linear relationships ($X_1 \rightarrow Y_2$ and $Y_1 \rightarrow Y_2$), with the knot point K placed at the median of X_1 . These parameter values are set to resemble the coefficients from the empirical estimates reported in the Results section, so that the simulation conditions reflect the characteristics of the study data.
2. Generate the exogenous variable X_1 from a standard normal distribution, then form the truncated spline basis $(X_1 - K)_+ = \max(X_1 - K, 0)$.

3. Generate the errors ε_1 and ε_2 from a right-skewed non-normal distribution (standardized exponential with mean 0 and variance 1) to reflect a violation of the normality assumption.
4. Define the mediator variable $Y_1 = \beta_{11}X_1 + \beta_{21}(X_1 - K)_+ + \varepsilon_1$ and the response variable $Y_2 = \beta_{12}X_1 + \beta_{22}Y_1 + \varepsilon_2$.
5. Repeat the data generation for four sample sizes, namely $n = 50, 100, 200$, and 1000 , with $R = 400$ Monte Carlo replications in every cell. A second set of $R = 400$ replications is generated for each null scenario, in which the coefficient of interest is set to zero, so that empirical test size can be recorded alongside power.
6. For each dataset, perform a standard bootstrap with $B_1 = 300$ first-stage repetitions to obtain the first-stage distribution, and a double-resampling bootstrap by adding $B_2 = 15$ second-stage repetitions within each first-stage sample. The value of B_1 is the same in every cell.
7. Calculate the performance metrics for each coefficient in each scenario: bias, RMSE, bootstrap standard error and variance, relative efficiency, empirical coverage probability of the nominal 95% interval, mean interval length, empirical type I error rate, and empirical power.

The resampling counts used in the simulation differ deliberately from those used in the empirical analysis. The empirical analysis reported in [Section 3](#) uses $B_1 = 1000$ and $B_2 = 500$, a budget that is affordable for a single dataset. The simulation uses $B_1 = 300$ and $B_2 = 15$ in every cell, because it requires $B_1 + B_1B_2$ estimator evaluations in each of $R = 400$ replications, at four sample sizes and in each null scenario. The two runs answer different questions: the empirical run reports the analysis of the observed data, whereas the simulation characterizes the behavior of the same two procedures under known parameter values. The entire simulation was generated from a single seed, 20260912, and is exactly reproducible.

3. Results and Discussion

This section reports the respondent characteristics, the tests of the model assumptions, and the estimation of the truncated spline semiparametric path model. The discussion addresses the relationships among technology access, financial knowledge, and financial literacy in line with the research objectives. The section then compares the standard bootstrap with the double-resampling bootstrap. That comparison is reported on two dimensions: the variance of the resulting estimates, which is the criterion conventionally used, and the calibration of the resulting intervals and tests, which determines whether the significance statements can be relied upon when the normality assumption is not met.

3.1. Respondent Characteristics

The study drew on 100 respondents, all of them students in Malang City selected by purposive sampling, whose distribution is reported in [Table 3](#). Most were aged 18–20 (60%), the gender split was close to even, and the majority came from the social and economic sciences (55%). This composition is consistent with the research context of technology access and financial literacy among younger adults, and the range of academic backgrounds brings differing degrees of prior exposure to financial issues into the sample.

3.2. Linearity Test

The linearity assumption test in the path analysis model was conducted using Ramsey RESET and its modified version based on truncated splines. This test aims to identify the possibility of nonlinear relationships between variables, thereby determining the need for a semiparametric approach in the model estimation process. The linearity test results are shown in [Table 4](#).

The sequential procedure applied to the path $X_1 \rightarrow Y_1$ ran through four stages, each with

Table 3: Respondent characteristics

Characteristic	Category	Percentage
Age	18–20 years	60%
	21–23 years	35%
	> 23 years	5%
Gender	Male	48%
	Female	52%
Field of study	Social & economics	55%
	Science & technology	30%
	Others	15%

Table 4: Ramsey RESET linearity assessment

Variable relationship	<i>p</i> -value			
	Ramsey RESET	Modified Ramsey RESET quadratic	Modified Ramsey RESET linear truncated spline	Modified Ramsey RESET quadratic truncated spline
Technology access (X_1) → financial knowledge (Y_1)	0.019**	0.028**	0.363 ^{ns}	0.024**
Technology access (X_1) → financial literacy (Y_2)	0.735 ^{ns}	–	–	–
Financial knowledge (Y_1) → financial literacy (Y_2)	0.534 ^{ns}	–	–	–

** significant at $\alpha = 0.05$; ^{ns} not significant.

its own null hypothesis, and selected the functional form by the first null hypothesis that was *not* rejected at $\alpha = 0.05$. Linearity was rejected ($p = 0.019$), the quadratic form was rejected in turn ($p = 0.028$), and the one-knot linear truncated spline was then retained ($p = 0.363$), at which point the sequence terminated and that form was adopted. The fourth-stage result for the quadratic truncated spline ($p = 0.024$) is reported for completeness only and was not used for selection.

For the two remaining paths, the first-stage Ramsey RESET returned $p = 0.735$ for technology access (X_1) → financial literacy (Y_2) and $p = 0.534$ for financial knowledge (Y_1) → financial literacy (Y_2). Neither result rejects linearity, so both paths were treated as linear and the sequence was not continued for them. The model as a whole is therefore semiparametric: the path $X_1 \rightarrow Y_1$ was modeled with a one-knot linear truncated spline, and the paths $X_1 \rightarrow Y_2$ and $Y_1 \rightarrow Y_2$ were modeled parametrically. The Generalized Cross-Validation (GCV) results are reported in Table 5.

Table 5: Semiparametric truncated spline GCV result

Linear truncated spline	GCV
1 knot	0.155
2 knots	0.297
3 knots	0.369

The knot specification was selected in two steps. The first step fixed the number of knots. Table 5 reports the GCV attained by the linear truncated spline at one, two, and three knots, each evaluated at the knot placement that minimizes GCV for that number of knots. The minimum, 0.155, occurs at one knot, and GCV increases monotonically as knots are added. One knot was therefore adopted for the $X_1 \rightarrow Y_1$ path.

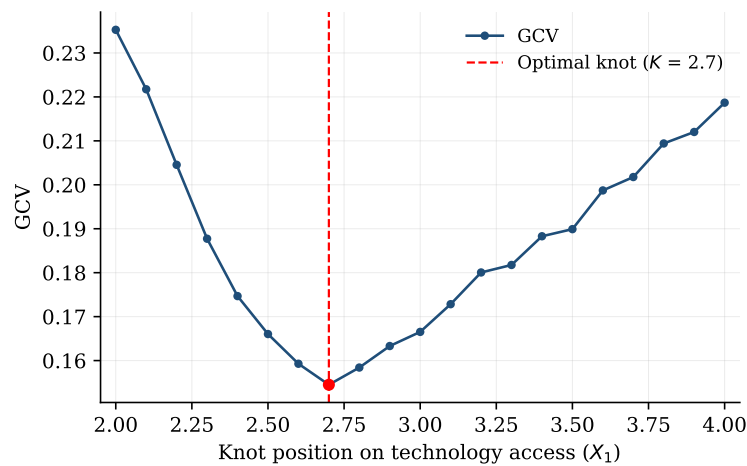


Fig. 2: GCV profile against the knot position on the nonparametric path from technology access to financial knowledge

The second step located that single knot. Fig. 2 plots GCV against the candidate knot position on the X_1 scale. The curve falls steeply to a single minimum and rises thereafter, so the criterion identifies one interior position rather than a flat region of equally good candidates. The vertical line marks the selected position, $K \approx 2.7$ on the technology access scale. The knot was held fixed at this value in all subsequent estimation and resampling, and it marks the point at which the marginal contribution of technology access to financial knowledge changes.

3.3. Semiparametric Truncated Spline Path Analysis

With the number and position of the knots fixed, the semiparametric path model was estimated. Because the $X_1 \rightarrow Y_1$ path is piecewise, the indirect effect takes a different value on each side of the knot: below it the product is $\hat{\beta}_{11}\hat{\beta}_{22} = 0.550 \times 0.314 = 0.173$, and above it $(\hat{\beta}_{11} + \hat{\beta}_{21})\hat{\beta}_{22} = 0.374 \times 0.314 = 0.117$. Neither quantity is $\hat{\beta}_{21}$ itself: $\hat{\beta}_{21} = -0.176$ is the change in the $X_1 \rightarrow Y_1$ slope at the knot, and it enters the indirect effect only through the second of the two products. Table 6 reports the direct effects, the two indirect effects, and the resulting total effects together.

Every path is positive. Access to technology is associated with higher financial knowledge both below the knot, with a coefficient of 0.550, and above it, with a coefficient of 0.374; the slope is smaller above the knot while remaining positive, so the change at the knot is a reduction in the marginal contribution rather than the loss of the relationship. Access to technology is also associated with higher financial literacy directly (0.367), as is financial knowledge (0.314). Under the standard bootstrap these two coefficients attain $p = 0.001$ and $p = 0.011$ respectively (Table 7), so both remain significant at the 5% level under the calibrated procedure, although the second does not reach the 1% level that the $\sqrt{B_2}$ -scaled column would suggest on its own.

For the two indirect effects, the $\sqrt{B_2}$ -scaled p -values are not read here as calibrated evidence; the simulation reported in Table 8 shows that this formulation attains a coverage of approximately 0.36 for these quantities at $n = 100$ and rejects a true null in about six replications in ten. Applying the standard bootstrap directly to each product instead gives $t_{\text{calc}} = 4.523$ ($p < 0.001$) below the knot and $t_{\text{calc}} = 3.215$ ($p = 0.002$) above it, so both indirect effects are established at the 5% level under the calibrated procedure. Mediation operates in the same direction in both segments, and the mediated portion is smaller above the knot than below it, which indicates a reduced marginal contribution at higher levels of technology access rather than the disappearance of the mediating pathway. The relation of this result to the power figures in Table 8 is taken up in the discussion following that table.

The total effect is accordingly 0.540 below the knot and 0.484 above it. The direct component is identical in the two segments, because only the $X_1 \rightarrow Y_1$ path carries a truncated spline, so the difference between the two totals comes entirely from the indirect component.

Table 6: Direct, indirect, and total effects

Effect	Coef.	Standard bootstrap			$\sqrt{B_2}$ -scaled
		SE	t_{calc}	p -value	p -value
<i>Direct</i>					
Technology access $\rightarrow$ financial knowledge ($X_1 < K$)	0.550				< 0.001
Technology access $\rightarrow$ financial knowledge ($X_1 > K$)	0.374				< 0.001
Technology access $\rightarrow$ financial literacy	0.367				< 0.001
Financial knowledge $\rightarrow$ financial literacy	0.314				< 0.001
<i>Indirect, through financial knowledge</i>					
Technology access $\rightarrow$ financial literacy ($X_1 < K$)	0.173	0.038	4.523	< 0.001	< 0.001
Technology access $\rightarrow$ financial literacy ($X_1 > K$)	0.117	0.037	3.215	0.002	< 0.001
<i>Total</i>					
Technology access $\rightarrow$ financial literacy ($X_1 < K$)	0.540				
Technology access $\rightarrow$ financial literacy ($X_1 > K$)	0.484				

The $\sqrt{B_2}$ -scaled column was obtained with the $\sqrt{B_2}$ -scaled double-resampling formulation at $B_1 = 1000$ and $B_2 = 500$; it is not read as calibrated evidence, and the standard bootstrap values for the same coefficients are reported in Table 7. For the indirect effects, the standard bootstrap SE, t_{calc} , and p -value were obtained by applying the resampling procedure underlying Eq. (16) directly to each product at $B_1 = 1000$: SE is the standard deviation of the B_1 bootstrap replicates of the product, and t_{calc} is referred to a t distribution with $n - p - 1$ degrees of freedom. Each total effect is the sum of the direct effect $\hat{\beta}_{12} = 0.367$ and the corresponding indirect effect, and is reported as a magnitude without a separate test. The calibration of the two formulations is assessed in Table 8.

3.4. Double-Resampling Bootstrap

Parameter significance testing was performed with both the standard bootstrap and the double-resampling bootstrap, so that the two procedures could be compared on the same data. The second procedure extends the conventional bootstrap by adding a second resampling stage within each first-stage sample. The results of that comparison are shown in Table 7.

Table 7: Comparison of bootstrap and double-resampling bootstrap results

Path	Coef.	p -value	
		Standard bootstrap	Double resampling, $\hat{se}_{bb}^{\sqrt{B_2}}$
β_{11}	0.550	< 0.001	< 0.001
β_{21}	-0.176	0.084	< 0.001
β_{12}	0.367	0.001	< 0.001
β_{22}	0.314	0.011	< 0.001

The double-resampling column was produced with the $\sqrt{B_2}$ -scaled formulation $\hat{se}_{bb}^{\sqrt{B_2}} = \hat{se}_b / \sqrt{B_2}$ at $B_1 = 1000$ and $B_2 = 500$, not with the averaged-estimate formulation of Eq. (15). The calibration of the two formulations is assessed below and reported in Table 8. The $\sqrt{B_2}$ -scaled p -values reported in Table 6 come from the same column.

In Table 7, every path in the model attains a smaller p -value under double resampling than under the standard bootstrap. The largest change occurs on β_{21} , the truncated spline increment of technology access $\rightarrow$ financial knowledge beyond the knot ($\hat{\beta}_{21} = -0.176$), where p falls from 0.084 to below 0.001.

This shift does not by itself indicate that the effect is significant. Both figures estimate the same quantity under two resampling schemes, and the two schemes differ only in how the standard error in the denominator is formed. The double-resampling standard error is computed from quantities that are averages of B_2 second-stage replicates, so it is smaller than the first-stage standard error whenever those averages are less dispersed than the first-stage estimates themselves. A reduction in the p -value that follows from a smaller denominator is therefore attributable to the way the standard error is constructed rather than to the evidence in the data,

and the direction of the change is determined in advance by that construction.

A Monte Carlo study with known population parameters was conducted to resolve this question, because the empirical comparison in Table 7 cannot. Data were generated from the semiparametric truncated spline model of Eq. (12) at four sample sizes ($n = 50, 100, 200$, and 1000) with $R = 400$ replications per cell, $B_1 = 300$, and $B_2 = 15$. In addition to bias, RMSE, and relative efficiency, the design records the quantities that determine whether an interval or a test is valid: empirical coverage probability of the nominal 95% interval, interval length, empirical type I error rate under a null in which the coefficient of interest is set to zero, and empirical power.

Table 8: Monte Carlo performance of the standard and double-resampling bootstrap

Path	Bias	RMSE	RE	CP		Size		Power	
				S	D	S	D	S	D
$n = 50$									
β_{11}	-0.0083	0.1669	15.00	0.928	0.438	0.055	0.593	0.863	0.980
β_{21}	+0.0249	0.2899	15.00	0.943	0.403	0.045	0.585	0.120	0.693
β_{12}	+0.0023	0.1529	15.00	0.935	0.395	0.045	0.645	0.718	0.960
β_{22}	+0.0014	0.2261	15.00	0.935	0.375	0.073	0.640	0.325	0.888
IE below	-0.0052	0.1343	15.00	0.935	0.380	0.035	0.628	0.158	0.860
IE above	+0.0103	0.1204	15.00	0.915	0.380	0.015	0.538	0.093	0.828
$n = 100$									
β_{11}	+0.0005	0.1118	15.00	0.945	0.393	0.055	0.625	0.988	1.000
β_{21}	-0.0029	0.1951	15.00	0.935	0.378	0.040	0.620	0.218	0.720
β_{12}	-0.0092	0.1000	15.00	0.943	0.403	0.058	0.628	0.913	0.995
β_{22}	+0.0067	0.1516	15.00	0.940	0.378	0.073	0.610	0.648	0.958
IE below	+0.0040	0.0925	15.00	0.933	0.358	0.060	0.600	0.520	0.958
IE above	+0.0026	0.0723	15.00	0.925	0.393	0.025	0.578	0.355	0.950
$n = 200$									
β_{11}	-0.0055	0.0800	15.00	0.925	0.355	0.053	0.668	1.000	1.000
β_{21}	+0.0083	0.1369	15.00	0.943	0.370	0.048	0.628	0.315	0.825
β_{12}	+0.0006	0.0734	15.00	0.958	0.365	0.055	0.675	0.998	1.000
β_{22}	-0.0011	0.0980	15.00	0.948	0.365	0.033	0.600	0.928	0.995
IE below	-0.0024	0.0602	15.00	0.935	0.403	0.033	0.598	0.888	0.995
IE above	+0.0006	0.0453	15.00	0.933	0.390	0.023	0.590	0.860	0.995
$n = 1000$									
β_{11}	+0.0004	0.0344	15.00	0.943	0.390	0.048	0.590	1.000	1.000
β_{21}	-0.0013	0.0576	15.00	0.950	0.433	0.048	0.600	0.845	0.995
β_{12}	-0.0006	0.0311	15.00	0.945	0.398	0.055	0.653	1.000	1.000
β_{22}	+0.0026	0.0434	15.00	0.950	0.383	0.050	0.605	1.000	1.000
IE below	+0.0016	0.0264	15.00	0.958	0.378	0.050	0.595	1.000	1.000
IE above	+0.0007	0.0197	15.00	0.935	0.385	0.038	0.595	1.000	1.000

S = standard bootstrap; D = double-resampling bootstrap; IE = indirect effect; RE = relative efficiency; CP = coverage probability.

A prior question had to be settled first. Eq. (15) defines the averaged-estimate formulation $\widehat{se}_{bb}^{\text{avg}}$ as the standard deviation of the B_1 averaged estimates, but at least two other readings appear in applied work: the standard deviation of the pooled B_1B_2 second-stage estimates divided by $\sqrt{B_2}$, and the $\sqrt{B_2}$ -scaled formulation $\widehat{se}_{bb}^{\sqrt{B_2}} = \widehat{se}_b/\sqrt{B_2}$. They are treated here as three distinct inference procedures rather than three ways of writing one, and applied to the same simulated data they yield relative efficiencies of 0.92, 7.3, and exactly 15. Only the $\sqrt{B_2}$ -scaled formulation reproduces the pattern in Table 7, so the results reported next use it, and the other two are taken up at the end of the subsection.

Under that formulation the relative efficiency is exactly B_2 rather than approximately B_2 :

Table 8 reports $RE = 15.00$ for all six coefficients at all four sample sizes. This follows from the definition, since $\widehat{se}_{bb} = \widehat{se}_b/\sqrt{B_2}$ implies $\text{Var}(\beta_{bb}) = \text{Var}(\beta_b)/B_2$, so the ratio in Eq. (17) reduces to B_2 identically. The quantity reflects the value assigned to B_2 rather than a property of the estimator, and setting $B_2 = 30$ would return 30. Its stability across coefficients and sample sizes, which might otherwise read as a dependable advantage, reflects an algebraic identity rather than a statistical property.

The consequences are visible in calibration. Averaged over the six coefficients, empirical coverage of the nominal 95% interval ranged only between 0.375 and 0.395 across the four sample sizes, against 0.939 for the standard bootstrap, so increasing n twentyfold left the deficit essentially unchanged. The level matches a closed-form prediction: dividing a correctly scaled standard error by $\sqrt{c}$ shortens the interval by that factor, so coverage becomes $2\Phi(1.96/\sqrt{c}) - 1$, which equals 0.387 at $c = B_2 = 15$; the simulation returned 0.387 averaged over all cells, and the mean interval-length ratio was 0.2582 against $1/\sqrt{15} = 0.2582$. Test size moved in the same proportion: the empirical type I error rate averaged 0.612 against the closed-form 0.613, while the standard bootstrap remained at 0.046. The apparent power gains should be read in this light. For β_{21} at $n = 50$, power rose from 0.120 to 0.693, an almost sixfold difference, but the type I error rate in the same cell was 0.585, so much of that difference reflects rejections that would also occur when the coefficient is zero. The same mechanism plausibly accounts for the change in the p -value of β_{21} in Table 7, which is therefore not interpreted as the detection of a genuine effect.

The two remaining formulations are reported for completeness. The averaged-estimate formulation, Eq. (15) implemented as written, was well calibrated, with coverage averaging 0.950 and type I error averaging 0.036, but its relative efficiency was 0.90 to 0.94, marginally below one: averaging the second-stage replicates removes second-stage noise while leaving intact the first-stage variation that the standard deviation across b measures. The intermediate formulation gave a relative efficiency near 7.3 and coverage near 0.54. Across the three, those returning a relative efficiency above one were miscalibrated by a corresponding amount, whereas the well-calibrated one showed no efficiency advantage.

Point estimation was unaffected by these differences, since bias and RMSE are properties of the point estimates that the two procedures share. The bias of the indirect effect below the knot moves from -0.0052 at $n = 50$ to $+0.0016$ at $n = 1000$, and its RMSE falls from 0.1343 to 0.0264, a ratio of 5.08 against the $\sqrt{20} = 4.47$ implied by a $1/\sqrt{n}$ rate. Four sample sizes under a single data-generating mechanism do not establish consistency, which is an asymptotic property requiring proof rather than simulation. These findings also bear on the substantive interpretation of the empirical results, and they bear on two distinct quantities that require separate statements.

The first is β_{21} , the change in the $X_1 \rightarrow Y_1$ slope at the knot. Its p -value in Table 7 falls from 0.084 under the standard bootstrap to below 0.001 under the $\sqrt{B_2}$ -scaled formulation, the largest change in the table. Because that reduction derives from a standard error that is smaller by a factor of $\sqrt{B_2}$, the significance obtained for β_{21} under the second procedure is not retained here. The value supported by the calibrated procedure is $p = 0.084$, so the estimated change of slope at the knot is not established at the 5% level in this sample. In substantive terms, the data are consistent with a mediated effect that is weaker above the knot than below it, but they do not establish that the two segments differ.

The second is the indirect effect itself, which is a different quantity. Its two values, 0.173 and 0.117, are products of coefficients, and Table 7 contains no row for either. What the simulation contributes here is the behavior of the two procedures for these products at the sample size actually available. At $n = 100$, the standard bootstrap attained a coverage of 0.933 for the indirect effect below the knot and 0.925 above it, with test sizes of 0.060 and 0.025, so it is calibrated for these quantities as well. Its power at the fitted parameter values, however, was 0.520 below the knot and 0.355 above it, so a calibrated procedure of this size would fail to detect an effect of this magnitude in a substantial share of samples the size of this one. Applying

the standard bootstrap directly to the empirical estimates nonetheless returned $t_{\text{calc}} = 4.523$ ($p < 0.001$) below the knot and $t_{\text{calc}} = 3.215$ ($p = 0.002$) above it (Table 6), so the calibrated procedure rejected the null hypothesis for both indirect effects in the sample actually collected. The mediating role of financial knowledge is accordingly reported as established at the 5% level for both segments; the moderate power at $n = 100$ means that a nonsignificant result would not by itself have ruled out an effect of this size, but it does not undo the significance obtained here. For the $\sqrt{B_2}$ -scaled formulation the corresponding figures are a coverage near 0.36 and a rejection rate near 0.60 under a true null, so its agreement with the calibrated result in this instance is not evidence that the formulation is trustworthy in general.

The direct effect of technology access on financial literacy ($\hat{\beta}_{12} = 0.367$) is significant under both procedures, at $p = 0.001$ under the standard bootstrap, and does not depend on any of these distinctions. Fig. 3 presents the same evidence in four panels. Panel (a) shows the relative efficiency of the $\sqrt{B_2}$ -scaled formulation fixed at B_2 across every sample size, with all six coefficients coinciding on a single line. Panel (b) shows the shorter double-resampling intervals that follow from that identity. Panels (c) and (d) show the calibration associated with those shorter intervals: coverage settles near 0.39 rather than 0.95, and the type I error rate near 0.61 rather than 0.05, in both cases flat in n . Taken together, the panels indicate that the reduction in interval length was accompanied by a corresponding loss of calibration at each of the sample sizes examined.

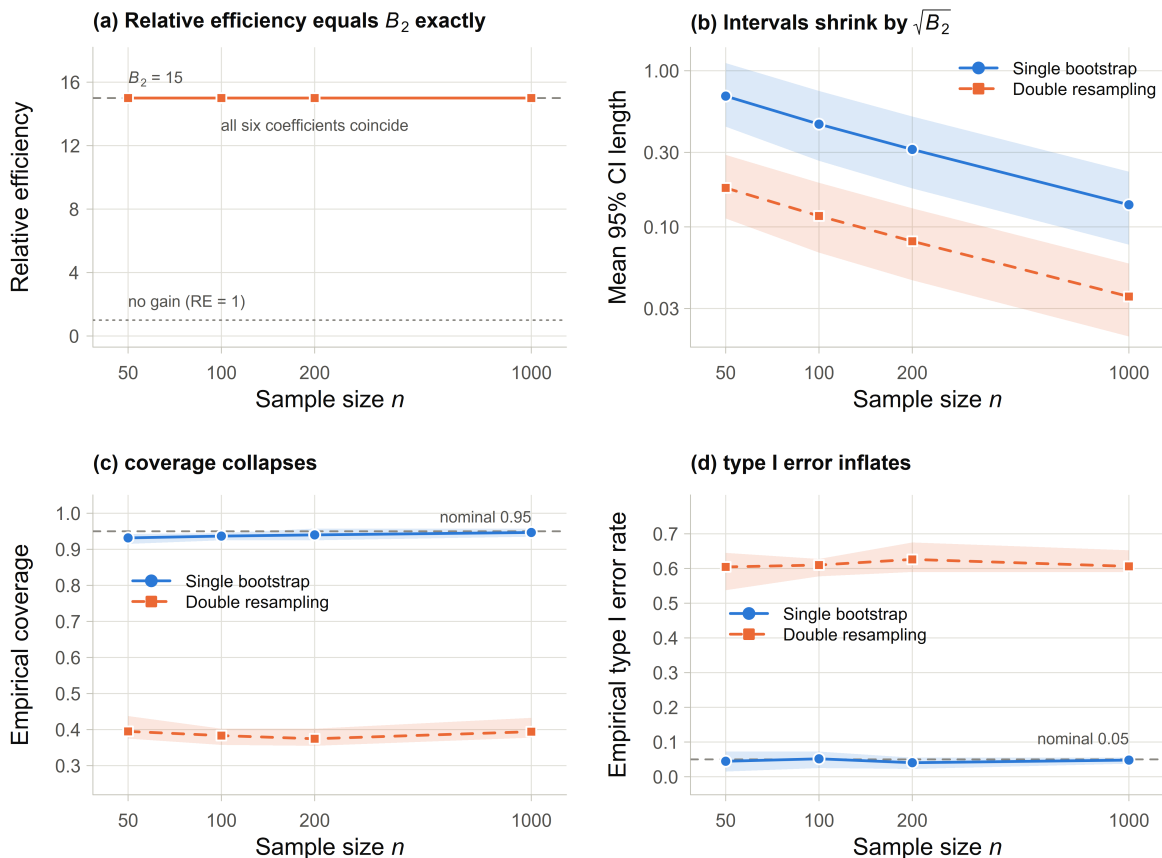


Fig. 3: Monte Carlo performance of the standard bootstrap versus the $\sqrt{B_2}$ -scaled double-resampling formulation across sample sizes. Panel (a) relative efficiency against B_2 ; panel (b) mean length of the nominal 95% interval; panel (c) empirical coverage against the nominal 0.95; panel (d) empirical type I error rate against the nominal 0.05. Shaded bands span the six model coefficients.

Placed against earlier methodological work, this result is consistent with the caution raised by MacKinnon [24] that the second-stage distribution need not be independent of the first, and

it illustrates one way in which the iterated bootstrap of Beran [25] may be misapplied: the second stage is intended to calibrate the first-stage distribution rather than to be averaged into a narrower one. Fast double bootstrap procedures [15] address the computational cost of iteration but assume the calibration step is performed correctly. Applications of truncated spline path analysis have reported point estimates without two-stage inference [9, 10]; the present study contributes an explicit account of what the second stage does and does not deliver within that modeling framework.

4. Conclusion

This study combined a semiparametric truncated spline path model with a double-resampling bootstrap significance test, and examined the efficiency gain conventionally attributed to the second resampling stage. The sequential Ramsey RESET retained a one-knot linear truncated spline on the path from technology access to financial knowledge ($p = 0.363$, H_0 not rejected) after rejecting linear and quadratic forms, while the two paths leading to financial literacy remained linear; the specification was corroborated by GCV.

The principal methodological finding turns on a distinction between two formulations of the same nominal quantity. Under the $\sqrt{B_2}$ -scaled formulation, which reproduces the empirical results reported here, the relative efficiency of double resampling equals B_2 exactly at every coefficient and sample size. This is an algebraic identity rather than a property of the estimator: the second-stage standard error is the first-stage standard error divided by $\sqrt{B_2}$, so the variance ratio in Eq. (17) reduces to B_2 by definition. The resulting intervals were shorter by $1/\sqrt{B_2}$ but covered the true value in 38.7% of replications against a nominal 95%, and the associated test rejected a true null in 61.2% against a nominal 5%, both close to their closed-form predictions and neither improving as n rose from 50 to 1000. Under the averaged-estimate formulation, Eq. (15) as written, calibration was restored (coverage 0.950, size 0.036) but the efficiency advantage disappeared (relative efficiency 0.92). No configuration delivered both, so the two formulations are separate inference procedures and results obtained under one should not be reported under the name of the other. A relative efficiency close to B_2 therefore warrants examination as a consequence of this arithmetic before being presented as a substantive result, and variance comparisons between resampling schemes are difficult to interpret unless accompanied by coverage and size. The standard bootstrap was well calibrated throughout (coverage 0.939, type I error 0.046) and was adopted as the basis for inference.

Three substantive statements should be distinguished. Technology access has a significant direct effect on financial literacy under both procedures ($\hat{\beta}_{12} = 0.367$, $p = 0.001$). The indirect effect through financial knowledge is positive on both sides of the knot, 0.173 below and 0.117 above, and the standard bootstrap establishes both at the 5% level ($t_{\text{calc}} = 4.523$, $p < 0.001$ below; $t_{\text{calc}} = 3.215$, $p = 0.002$ above); the simulation indicates that at $n = 100$ a calibrated procedure detects effects of this size in only about half of the samples below the knot and a third above it, so a nonsignificant result would not by itself have been informative, though this does not diminish the significance obtained. The change in slope at the knot, $\hat{\beta}_{21} = -0.176$, attains $p = 0.084$ and is not established, so the data do not support a claim that the mediated effect differs between segments.

Three limitations bound these conclusions. The sample of 100 respondents was drawn purposively, which restricts external validity, and the constructs were measured by questionnaire, so the estimates carry its measurement error. The simulation used population parameters from the empirical fit of this same study, evaluating performance around one configuration rather than a range of effect sizes, nonlinearities, knot positions, and error distributions. Comparisons were also made at unequal computational budgets, since double resampling requires $B_1 + B_1 B_2$ evaluations against B_1 . Future work could vary B_2 explicitly to test whether the identity reported here holds more generally, widen the parameter space, and evaluate whether a correctly implemented calibration step in the sense of Beran [25] recovers an advantage that the averaging

formulation does not.

CRedit Authorship Contribution Statement

Fachira Haneinanda Junianto: Conceptualization, Methodology, Software, Formal Analysis, Writing–Original Draft. **Adji Achmad Rinaldo Fernandes:** Conceptualization, Methodology, Validation, Writing–Review & Editing, Supervision. **Solimun:** Methodology, Validation, Supervision. **Ani Budi Astuti:** Data Curation, Visualization, Writing–Review & Editing. **Muhammad Hisyam Lee:** Methodology, Validation, Supervision.

Ethics Approval

This study was reviewed and approved by the Research Ethics Committee for Social and Behavioural Sciences (KERIS-P) of Universitas Brawijaya, which assessed the protocol as being of low ethical risk, under letter No. 11199/DST/UN10.F1101/B/TU/2026, protocol identifier KERIS-P/20260921-5007.

Declaration of Generative AI and AI-assisted technologies

No generative AI or AI-assisted technologies were used during the preparation of this manuscript.

Declaration of Competing Interest

The authors declare no competing interests.

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Data and Code Availability

The data and code supporting the findings of this study are available from the corresponding author upon reasonable request and subject to confidentiality agreements.

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