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Vine Copula-Based Multivariate Value-at-Risk for Extreme Rainfall Risk Assessment

Lailatul Maziyah Wildan Mufaridho*[1], Dewi Susanawati[1], and Wahyu Joko Saputro[2]

[1]*Department of Actuarial Science, Darunnajah University, Indonesia*
[2]*Department of Information Systems and Technology, Darunnajah University, Indonesia*

Abstract

Extreme rainfall occurring simultaneously across multiple locations can generate joint risks that are not adequately captured by univariate models assuming independence. This study develops an *R-vine copula*-based framework to model dependence among seven rainfall stations in DKI Jakarta and West Java, Indonesia, and quantify joint rainfall risk using multivariate Value-at-Risk (VaR). Station-specific rainfall distributions are modeled using a mixed discrete-continuous approach, followed by a probability integral transform to construct the R-vine dependence structure. Monte Carlo simulation is then used to generate multivariate rainfall scenarios and estimate VaR at the 95% and 99% confidence levels. The fitted R-vine consists of six tree levels and 21 pair-copulas, with Kendall's tau RMSE of 0.0935 between observed and simulated dependence. Based on 100,000 simulations, the multivariate VaR estimates are 46.0744 mm at the 95% level and 92.3355 mm at the 99% level, with bootstrap confidence intervals of 45.4340–46.7030 mm and 90.4392–94.6717 mm, respectively. Bootstrap intervals are obtained by resampling simulated aggregate rainfall values while keeping the fitted marginal and R-vine models fixed. ECDF and QQ plots show generally consistent distributional behavior, although some upper-tail discrepancies are observed. Kupiec testing provides an in-sample coverage assessment of station-level univariate VaR, with no significant departure from nominal exceedance rates. The 99% multivariate VaR should nevertheless be interpreted cautiously because only 193 complete observations are available, providing limited information on the far upper tail and potentially requiring extrapolation beyond the observed range. Overall, the proposed framework provides a flexible approach for quantifying joint rainfall risk through multivariate VaR while accounting for dependence among rainfall stations.

Keywords: Joint Extreme Rainfall; Vine Copula; Multivariate Value-at-Risk.

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1. Introduction

Extreme rainfall has become one of the major hydrometeorological hazards in Indonesia, particularly in the provinces of DKI Jakarta and West Java, which frequently experience floods and landslides. Located in the western part of Java, these provinces are strongly influenced by the Asian–Australian monsoon, the El Niño–Southern Oscillation (ENSO), and the Indian Ocean Dipole (IOD), resulting in considerable temporal and spatial variability in rainfall. In addition, the close geographical proximity between DKI Jakarta and West Java, together with

*Corresponding author. E-mail: lailatulmufaridho@darunnajah.ac.id

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the influence of regional atmospheric circulation and orographic effects, often leads to concurrent extreme rainfall events. In recent decades, climate change has further intensified the frequency and magnitude of these extreme rainfall events, increasing the occurrence of hydrometeorological disasters that cause substantial economic losses, infrastructure damage, environmental degradation, and social disruption. Consequently, reliable methods for assessing simultaneous extreme rainfall risk have become increasingly important for disaster mitigation and climate adaptation planning.

Extreme rainfall events are not merely local phenomena but frequently occur simultaneously across multiple rainfall stations due to shared climatic drivers and atmospheric circulation patterns. Such concurrent events may amplify hydrometeorological impacts because neighboring regions experience extreme rainfall within the same period. In DKI Jakarta and West Java, this phenomenon is particularly important because both provinces are climatologically connected and hydrologically linked through several major river basins. Therefore, analyzing rainfall risk independently at each observation station may overlook the complex dependence structure among regional rainfall extremes. A multivariate framework is therefore required to adequately characterize the dependence among multiple rainfall variables, particularly under extreme conditions. Furthermore, statistical analysis alone is insufficient to support regional disaster management. Integrating statistical modeling with a geographic information system (GIS) provides a more comprehensive understanding of the spatial distribution of extreme rainfall risk and facilitates regional decision-making.

Value-at-Risk (VaR) is one of the most widely used quantitative measures for estimating extreme risk at a specified confidence level [1]. Although originally developed in financial risk management, VaR has increasingly been applied in environmental studies to quantify the magnitude of rare events. Extending VaR to a multivariate framework, however, requires an appropriate model to describe complex dependence among multiple rainfall variables. Vine copulas provide a flexible dependence modeling framework by decomposing a high-dimensional multivariate distribution into a sequence of pair-copulas. Unlike conventional multivariate copulas that assume a single dependence structure, vine copulas allow different copula families to model each pairwise relationship, thereby accommodating heterogeneous, nonlinear, and tail dependence among rainfall stations. This flexibility makes vine copulas particularly suitable for modeling spatially distributed rainfall observations with varying dependence characteristics. Furthermore, integrating vine copula-based multivariate VaR with GIS enables spatial visualization of regional risk, thereby supporting the identification of areas vulnerable to simultaneous extreme rainfall events [2]. Although vine copulas have demonstrated superior performance in modeling complex multivariate dependence across various fields, their application to simultaneous extreme rainfall risk assessment in DKI Jakarta and West Java remains limited.

Previous studies have investigated extreme rainfall using various statistical and hydrological approaches. [3] employed Fuzzy C-Means clustering combined with Singular Value Decomposition (SVD) to classify rainfall regions in Indonesia. [4] examined the impact of climate change on extreme rainfall using descriptive statistical analysis based on rainfall intensity classifications. [5] applied Extreme Value Theory (EVT) to forecast extreme rainfall in Banten Province and reported a strong dependence among rainfall stations, with a correlation coefficient of 0.764. [6] analyzed dominant rainfall characteristics using correlation analysis and the Windrose method, while other studies have focused on flood susceptibility assessment using hydrological and numerical modeling techniques.

These studies have significantly improved the understanding of rainfall variability, rainfall prediction, and flood hazard assessment. Nevertheless, most previous research has concentrated on rainfall classification, prediction, or local hydrological analyses without explicitly modeling the complex dependence among multiple rainfall stations. The strong rainfall dependence reported by [5] indicates that inter-regional dependence should be incorporated into regional risk assessment. However, conventional correlation analysis primarily measures linear association and cannot

adequately capture nonlinear dependence, asymmetric dependence, or tail dependence that commonly characterize simultaneous extreme rainfall events. Moreover, traditional multivariate copulas may impose restrictive dependence assumptions that are insufficient for representing heterogeneous dependence structures observed across multiple rainfall stations.

Based on the existing literature, three research gaps remain. First, simultaneous extreme rainfall in DKI Jakarta and West Java has rarely been evaluated using multivariate risk measures. Second, the dependence among multiple rainfall stations has generally been analyzed using conventional correlation methods or single-family copulas rather than flexible vine copulas capable of representing heterogeneous nonlinear and tail dependence. Third, quantitative risk estimation has seldom been integrated with GIS to provide spatial representations of simultaneous extreme rainfall risk. Therefore, a comprehensive analytical framework that combines multivariate risk assessment, vine copula-based dependence modeling, and GIS visualization is still needed.

To address these research gaps, this study integrates vine copula-based multivariate Value-at-Risk (VaR) [7, 8] with GIS to quantify and visualize the risk of simultaneous extreme rainfall in DKI Jakarta and West Java. The vine copula model is employed to characterize the complex dependence structure among rainfall stations through pair-copula constructions, while GIS is used to illustrate the spatial distribution of the estimated risk. The novelty of this study lies in integrating vine copula-based multivariate Value-at-Risk with GIS to quantify and spatially visualize simultaneous extreme rainfall risk while explicitly accounting for heterogeneous nonlinear dependence among multiple rainfall stations. This integrated framework provides a more comprehensive regional risk assessment than conventional approaches that analyze rainfall independently or rely solely on correlation analysis. Accordingly, this study aims to estimate the marginal distributions of extreme rainfall, model the multivariate dependence structure among rainfall stations in DKI Jakarta and West Java using vine copulas, quantify simultaneous extreme rainfall risk through vine copula-based multivariate Value-at-Risk, and visualize the resulting spatial distribution of regional risk using GIS. The findings are expected to provide scientific support for hydrometeorological disaster mitigation, climate adaptation strategies, and evidence-based decision-making in regional rainfall risk management.

2. Some Concepts

This section introduces the main concepts and methodological components used in the analysis. It begins with rainfall data preparation and marginal distribution modeling, followed by pair-copula selection, R-vine dependence modeling, Value-at-Risk estimation, and spatial analysis. These components provide the theoretical and computational basis for the subsequent results and discussion.

2.1. Rainfall Data and Preprocessing

Let $X_i = \{x_{i1}, x_{i2}, \ldots, x_{in}\}$ denote the daily rainfall observations at station i, where $i = 1, 2, \ldots, d$. The multivariate rainfall dataset preserves the temporal correspondence among stations, with observations aligned by date. The rainfall data are first subjected to quality control. Records with invalid measurement codes, specifically 8888 and 9999, are treated as missing values. Duplicate observations recorded at the same station and date are consolidated before temporal alignment. Only observations with complete records across all stations are retained for subsequent multivariate modeling.

In this study, extreme rainfall refers to unusually high rainfall represented by the upper tail of the distribution. Univariate VaR characterizes extreme rainfall risk at each station, whereas multivariate VaR captures regional extreme rainfall risk by accounting for the dependence among stations. Unlike a peaks-over-threshold approach, no predefined threshold is used to extract extreme observations. Instead, the complete aligned daily rainfall series is used for marginal distribution estimation and dependence modeling, allowing both zero-rainfall and positive-rainfall observations to be retained. Because daily rainfall contains a probability mass at zero, each

station is modeled using a mixed discrete-continuous distribution, with the probability mass at zero separated from the continuous distribution of positive rainfall. The resulting marginal distributions are then transformed into pseudo-observations for the R-vine copula model.

2.2. Marginal Distribution Estimation

Each regional rainfall variable is modeled separately to estimate its marginal probability distribution. In copula modeling, the marginal distributions are estimated independently before constructing the joint dependence structure. This approach separates the marginal behavior of each rainfall variable from its dependence structure in accordance with Sklar's theorem [9]. For station i, let $p_{0,i} = P(X_i = 0)$ denote the probability of zero rainfall, and let $G_i(x;\theta_i)$ denote the conditional cumulative distribution function of positive rainfall. The mixed discrete-continuous marginal distribution is defined as

$$F_i(x) = \begin{cases} 0, & x < 0, \\ p_{0,i}, & x = 0, \\ p_{0,i} + (1 - p_{0,i})G_i(x;\theta_i), & x > 0. \end{cases} \tag{1}$$

where θ_i denotes the parameter vector of the selected continuous distribution.

Several candidate distributions are considered for the positive rainfall component, including the Gamma, Lognormal, Weibull, Burr, and Pearson Type III distributions. The unknown parameters are estimated using maximum likelihood estimation (MLE). Let $(x_{i1}^+, x_{i2}^+, \ldots, x_{in_i}^+)$ denote the positive rainfall observations at station i. The likelihood function [10] for the continuous component is

$$L(\theta_i) = \prod_{j=1}^{n_i} g\left(x_{ij}^+;\theta_i\right),$$

where $g(\cdot;\theta_i)$ denotes the probability density function of the candidate distribution. The corresponding log-likelihood function is

$$\ell(\theta_i) = \sum_{j=1}^{n_i} \log g\left(x_{ij}^+;\theta_i\right),$$

and the maximum likelihood estimator is obtained as

$$\hat{\theta}_i = \arg\max_{\theta_i} \ell(\theta_i).$$

The candidate marginal distributions are compared using the Akaike information criterion (AIC),

$$\mathrm{AIC} = -2\ell(\hat{\theta}_i) + 2k,$$

where k is the number of estimated parameters. The marginal distribution with the minimum AIC is selected, with the maximized log-likelihood used as supporting information.

Because daily rainfall contains a probability mass at zero and a continuous distribution for positive rainfall amounts [11], the standard continuous probability integral transform (PIT) is not directly applicable. A randomized PIT is therefore used to obtain continuous pseudo-observations for the copula model. For positive rainfall observations, the transformation is given by

$$U_i = p_{0,i} + (1 - p_{0,i})G_i\left(X_i;\hat{\theta}_i\right), \qquad X_i > 0.$$

For zero-rainfall observations, the probability mass at zero corresponds to the interval $(0, p_{0,i}]$. A randomized PIT is therefore applied by generating a random value within this interval:

$$U_i = V_i p_{0,i}, \qquad V_i \sim U(0,1), \qquad X_i = 0.$$

This distributional transformation maps zero-rainfall observations within their probability interval, while positive-rainfall observations are mapped to values above $p_{0,i}$. The resulting pseudo-observations are then used to construct the multivariate dependence structure using the R-vine copula [12].

2.3. Pair-Copula Family Selection and Parameter Estimation

After transforming the marginal variables to the uniform scale, the dependence structure is modeled using the vine copula framework. The vine structure is constructed through a pair-copula decomposition, where the joint multivariate distribution is represented by a sequence of conditional bivariate copulas. For each edge in the vine structure, several candidate copula families are considered. The unknown parameters of each candidate pair-copula are estimated using maximum likelihood estimation (MLE). Let

$$\mathbf{U} = (U_1, U_2, \ldots, U_d)$$

denote the pseudo-observations obtained from the randomized probability integral transform. The log-likelihood function of the vine copula model is defined as

$$\ell(\boldsymbol{\theta}) = \sum_{i=1}^{n} \sum_{e} \log c_{j_e,k_e|D_e} \left(u^{(i)}_{j_e|D_e}, u^{(i)}_{k_e|D_e}; \theta_e \right),$$

where $c_{j_e,k_e|D_e}$ denotes the conditional pair-copula density associated with edge e, D_e represents the conditioning set, and θ_e denotes the corresponding parameter vector. The parameter estimates are obtained by maximizing the log-likelihood function,

$$\hat{\boldsymbol{\theta}} = \arg\max_{\boldsymbol{\theta}} \ell(\boldsymbol{\theta}).$$

To identify the most suitable dependence model for each pair of variables, the Akaike information criterion (AIC) is computed as

$$\text{AIC} = -2\ell(\hat{\boldsymbol{\theta}}) + 2k,$$

where k is the number of estimated parameters in the candidate pair-copula model. The candidate copula family with the smallest AIC value is selected for each pair of variables in the vine structure. This data-driven procedure allows different copula families to be assigned to different pairs of variables, thereby providing a flexible representation of heterogeneous dependence patterns and tail dependence among extreme rainfall observations.

2.4. Vine Copula Model

According to Sklar's theorem, every multivariate cumulative distribution function can be uniquely decomposed into its marginal distributions and a copula function when the marginal distributions are continuous [9][13].

$$F(x_1, \ldots, x_d) = C\left(F_1(x_1), F_2(x_2), \ldots, F_d(x_d)\right),$$

where $F(\cdot)$ denotes the joint distribution, $F_i(\cdot)$ denotes the marginal distribution, and $C(\cdot)$ denotes the copula function.

Since this study focuses on simultaneous extreme rainfall, a vine copula framework is employed to model the complex multivariate dependence structure. Unlike conventional multivariate copulas, the vine copula decomposes a high-dimensional joint distribution into a sequence of bivariate copulas through a pair-copula construction (PCC) [14][15]. This approach provides greater flexibility in capturing heterogeneous dependence patterns and tail dependence among multiple rainfall variables.

The joint copula density is decomposed into a product of conditional bivariate pair-copula densities as [16]

$$c(u_1, \ldots, u_d) = \prod_{t=1}^{d-1} \prod_{e \in E_t} c_{j_e,k_e|D_e} \left(u_{j_e|D_e}, u_{k_e|D_e} \right), \tag{2}$$

where E_t denotes the set of edges in tree level t, $c_{j_e,k_e|D_e}$ denotes the conditional bivariate pair-copula density associated with edge e, and D_e denotes the conditioning set. The terms $u_{j_e|D_e}$ and $u_{k_e|D_e}$ are the conditional distribution values of variables j_e and k_e, respectively, given the variables in D_e.

2.5. Value-at-Risk

Value-at-Risk (VaR) is a quantile-based risk measure that represents the threshold value that is exceeded with probability $(1 - \alpha)$ at a given confidence level α. Although originally developed in financial risk management, VaR has been widely adopted to quantify extreme risks in environmental and hydrological applications because it provides an interpretable measure of rare events [17]. In the univariate setting, VaR is estimated separately for each rainfall station based on the empirical quantile of the observed rainfall data. Thus, for station i, the univariate VaR at confidence level α is defined as

$$\mathrm{VaR}_{\alpha,i} = Q_\alpha(X_i),$$

where $Q_\alpha(X_i)$ denotes the empirical α-quantile of the observed rainfall at station i. Accordingly, the univariate VaR represents the rainfall threshold at each station that is exceeded by approximately a fraction $(1 - \alpha)$ of the observed rainfall events.

Let $\mathbf{X} = (X_1, X_2, \ldots, X_d)$ denote the multivariate rainfall vector generated from the fitted R-vine copula model, where X_i represents the rainfall amount at the i-th station. To quantify the overall regional rainfall risk, the simulated multivariate rainfall vector is aggregated into a single regional rainfall indicator. This aggregation is analogous to the construction of a financial portfolio, in which multiple dependent risk components are combined into a single measure. In this study, each rainfall station represents one component of the regional rainfall system.

To evaluate the regional rainfall risk, an equally weighted aggregate rainfall variable is defined as

$$Y = \sum_{i=1}^{d} w_i X_i,$$

where w_i denotes the weight assigned to the i-th station. To characterize the collective behavior of rainfall across the monitoring stations, an equal-weight aggregation scheme is adopted, assigning the same weight to each station. This assumption ensures that no individual station is given greater importance in the regional aggregate and avoids introducing subjective weights when additional information, such as watershed characteristics, population exposure, or hydrological influence, is unavailable. Therefore, the weight for each station is defined as

$$w_i = \frac{1}{d}, \qquad i = 1, 2, \ldots, d,$$

where d denotes the total number of monitoring stations. Consequently, the aggregate rainfall variable becomes

$$Y = \frac{1}{d} \sum_{i=1}^{d} X_i.$$

The variable Y represents the average rainfall amount across the monitoring stations and serves as an aggregate indicator of regional rainfall conditions. The dependence and potential co-occurrence of high rainfall across stations are captured through the fitted R-vine copula model. The cumulative distribution function of the aggregate rainfall variable is given by

$$F_Y(y) = P(Y \leq y).$$

Accordingly, the VaR of the regional aggregate rainfall at confidence level α is defined as [18, 19]

$$\mathrm{VaR}_\alpha = \inf\{y \in \mathbb{R} : F_Y(y) \geq \alpha\}.$$

Here, the term multivariate VaR refers to a VaR measure derived from the joint multivariate rainfall distribution modeled by the R-vine copula, while the resulting VaR is a scalar threshold of the regional aggregate rainfall variable Y.

Since the distribution of Y induced by the R-vine copula generally has no closed-form expression, the VaR is estimated using Monte Carlo simulation. Let

$$Y^{(1)}, Y^{(2)}, \ldots, Y^{(N)}$$

denote the simulated aggregate rainfall values obtained from the fitted R-vine copula model. The estimated regional aggregate VaR is then computed as

$$\widehat{\mathrm{VaR}}_\alpha = Q_\alpha\left(Y^{(1)}, Y^{(2)}, \ldots, Y^{(N)}\right),$$

where $Q_\alpha(\cdot)$ denotes the empirical α-quantile of the simulated aggregate rainfall distribution.

2.6. Spatial Analysis

The estimated Value-at-Risk for each rainfall station was associated with its geographical coordinates,

$$R_i = (Lon_i, Lat_i, \mathrm{VaR}_i),$$

where Lon_i, Lat_i, and VaR_i denote the longitude, latitude, and estimated Value-at-Risk of the i-th station, respectively. For spatial visualization, the station-level VaR_{95} estimates were classified into three qualitative categories based on their empirical range and relative separation across stations. Values below 45 mm were classified as Low Risk, values between 45 and 58 mm as Moderate Risk, and values above 58 mm as High Risk. These boundaries were defined solely to facilitate comparative visualization of relative risk levels among stations and do not represent hydrological warning thresholds, regulatory standards, or established rainfall hazard levels.

3. Results and Discussion

The daily rainfall data used in this study cover the period from April 1, 2025, to March 1, 2026, across seven observation stations. During the preprocessing stage, data points coded as 8888 and 9999 were treated as invalid and converted to missing values prior to analysis. The resulting number of missing observations was 28 at Maritim Tanjung Priok, 45 at Kemayoran, 21 at Halim Perdana Kusuma Jakarta, 22 at Penggung, 31 at Citeko, 66 at Geofisika Bandung, and 49 at Kertajati.

To maintain temporal consistency across stations and ensure that each multivariate observation represented rainfall conditions on the same date, dates with missing observations at any station were subsequently excluded from the multivariate dataset. After this complete-case filtering and date alignment, 193 complete daily observations remained across the seven stations. The resulting 193×7 dataset was used as the basis for the subsequent marginal distribution estimation, R-vine copula modeling, simulation, and regional rainfall risk analysis.

Table 1 shows that average daily rainfall ranges from 6.789 mm at Stasiun Geofisika Bandung to 14.744 mm at Halim Perdana Kusuma Jakarta. All stations recorded a minimum value of 0 mm and exhibited relatively high variability, with CV values ranging from 1.600 to 2.353. The maximum values, which are substantially higher than the medians across all stations, also indicate a right-skewed distribution and the occurrence of extreme rainfall events.

Table 2 summarizes the selected marginal distributions for positive daily rainfall amounts based on maximum likelihood estimation, log-likelihood, and the Akaike information criterion

Table 1: Descriptive statistics of daily rainfall at each observation station

Station	N	Mean	Median	Min	Max	SD	CV
Stasiun Meteorologi Maritim Tanjung Priok	193	10.940	0.1	0.0	235.2	25.740	2.353
Stasiun Meteorologi Kemayoran	193	12.288	0.8	0.0	213.4	26.631	2.167
Halim Perdana Kusuma Jakarta	193	14.744	2.1	0.0	179.7	28.920	1.962
Pos Meteorologi Penggung	193	8.506	0.6	0.0	110.6	18.805	2.211
Stasiun Meteorologi Citeko	193	12.249	2.7	0.0	111.0	21.015	1.716
Stasiun Geofisika Bandung	193	6.789	1.5	0.0	53.3	10.863	1.600
Stasiun Meteorologi Kertajati	193	10.683	1.0	0.0	128.3	21.703	2.032

(AIC). The Lognormal distribution is selected for Kemayoran, Kertajati, and Penggung, while the Weibull distribution is selected for Halim Perdana Kusuma Jakarta, Tanjung Priok, Geofisika Bandung, and Citeko. The variation across stations indicates distinct distributional characteristics of positive rainfall and supports the use of station-specific marginal models. These continuous distributions are then combined with the probability mass at 0 mm within a mixed discrete-continuous framework defined in Eq. (1), before applying the probability integral transform (PIT) and estimating the R-vine copula.

Table 2: Best-Fitted Marginal Distributions for Each Station

Station	Distribution	Log-Likelihood	AIC
Stasiun Meteorologi Maritim Tanjung Priok	Weibull	-386.6194	777.2388
Stasiun Meteorologi Kemayoran	Lognormal	-411.5001	827.0001
Halim Perdana Kusuma Jakarta	Weibull	-487.8840	979.7680
Pos Meteorologi Penggung	Lognormal	-377.2963	758.5926
Stasiun Meteorologi Citeko	Weibull	-508.1208	1020.2416
Stasiun Geofisika Bandung	Weibull	-426.6432	857.2865
Stasiun Meteorologi Kertajati	Lognormal	-421.0840	846.1680

Table 3 presents the in-sample coverage assessment using the Kupiec test at the 95% and 99% confidence levels for the seven rainfall stations. Since the VaR thresholds and violations are evaluated using the same historical observations, the results are interpreted as an in-sample assessment rather than an independent out-of-sample backtest. At the 95% level, the expected number of violations is 9.65 per station. Six stations record 10 violations and Station 6 records 9, with violation rates of 0.0518 and 0.0466, respectively. The p-values range from 0.8282 to 0.9085, indicating no significant departure from the nominal 5% exceedance rate. The station-level VaR values range from 29.700 mm (Station 6) to 72.920 mm (Station 3).

At the 99% level, the expected number of violations is 1.93 per station, while all stations record 2 violations (0.0104). The p-value is 0.9598 for all stations, indicating no significant departure from the nominal 1% exceedance rate. The VaR values range from 50.080 mm (Station 6) to 166.832 mm (Station 3). Overall, the results indicate consistency between the observed and nominal exceedance rates within the historical sample. However, the 99% assessment should be interpreted cautiously because of the limited number of complete observations and exceedances.

Based on the classification scheme, Geofisika Bandung and Penggung were categorized as Low Risk, with station-level VaR_{95} values of 29.70 and 42.24 mm, respectively, while Tanjung Priok, Kemayoran, and Citeko were classified as Moderate Risk, with values ranging from 50.38 to 57.06 mm. Halim Perdana Kusuma Jakarta and Kertajati were classified as High Risk, with VaR_{95} values of 72.92 and 59.52 mm, respectively. Figure 1 maps these station-level univariate VaR_{95} values to illustrate the spatial variation of local rainfall risk. This GIS visualization represents station-specific risk and should not be interpreted as a spatial decomposition of the R-vine-based multivariate VaR, which is expressed as a single regional aggregate value derived from the joint rainfall distribution across all seven stations.

After selecting the best-fitting marginal distributions, the rainfall data were modeled using

Table 3: In-Sample VaR Coverage Results Using the Kupiec Test

Station	Confidence	VaR	Violations	Expected	Violation Rate	P-value	Decision
1	95%	50.380	10	9.65	0.0518	0.9085	Valid
2	95%	56.080	10	9.65	0.0518	0.9085	Valid
3	95%	72.920	10	9.65	0.0518	0.9085	Valid
4	95%	42.240	10	9.65	0.0518	0.9085	Valid
5	95%	57.060	10	9.65	0.0518	0.9085	Valid
6	95%	29.700	9	9.65	0.0466	0.8282	Valid
7	95%	59.520	10	9.65	0.0518	0.9085	Valid
1	99%	99.268	2	1.93	0.0104	0.9598	Valid
2	99%	121.000	2	1.93	0.0104	0.9598	Valid
3	99%	166.832	2	1.93	0.0104	0.9598	Valid
4	99%	97.468	2	1.93	0.0104	0.9598	Valid
5	99%	97.048	2	1.93	0.0104	0.9598	Valid
6	99%	50.080	2	1.93	0.0104	0.9598	Valid
7	99%	89.580	2	1.93	0.0104	0.9598	Valid

Note: Station codes are defined as follows: 1 = Stasiun Meteorologi Maritim Tanjung Priok; 2 = Stasiun Meteorologi Kemayoran; 3 = Halim Perdana Kusuma Jakarta; 4 = Pos Meteorologi Penggung; 5 = Stasiun Meteorologi Citeko; 6 = Stasiun Geofisika Bandung; and 7 = Stasiun Meteorologi Kertajati.

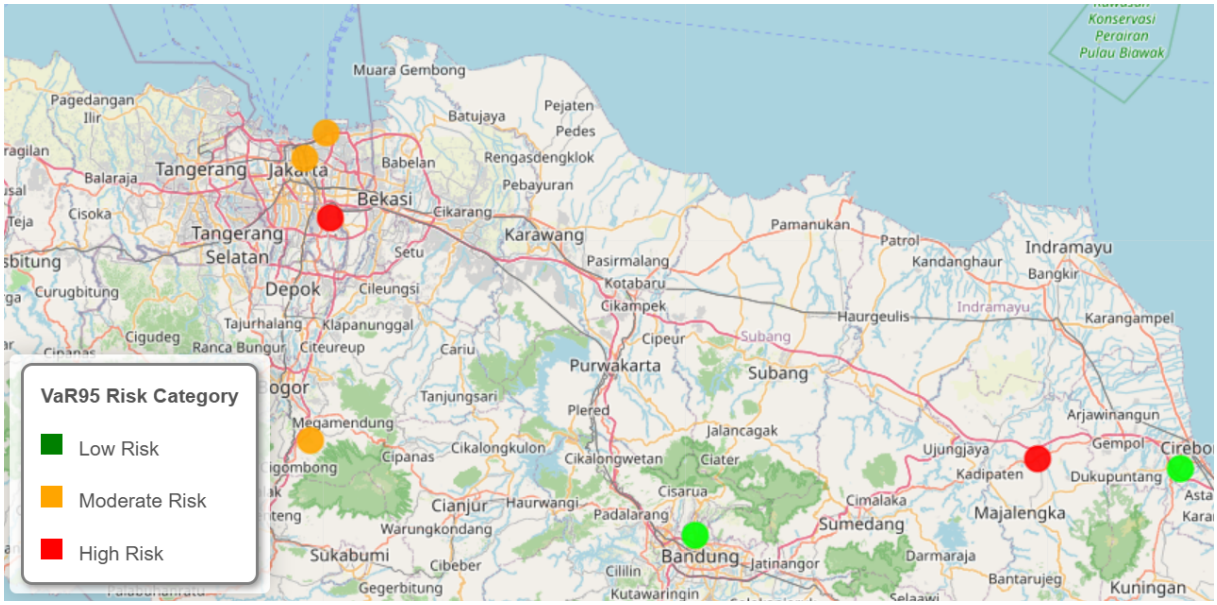


Fig. 1: Spatial distribution of station-level univariate Value-at-Risk

a mixed discrete-continuous framework to account for zero rainfall. The marginal distribution at each station consists of a discrete probability mass at 0 mm and a continuous distribution for positive rainfall amounts. The probability integral transform (PIT) was then applied to the mixed marginal distributions, with the discrete component explicitly accounting for zero-rainfall observations and the fitted continuous distribution applied to positive rainfall amounts. This treatment avoids imposing a continuous distribution on zero rainfall and preserves the occurrence probability of non-rainfall conditions.

The transformed data were subsequently used to construct the R-vine copula dependence structure among the rainfall stations. The joint density of the R-vine is represented as a product of conditional pair-copula densities across the vine tree levels, as shown in Eq. (2). During simulation, the fitted R-vine model was used to generate dependent uniform variables that preserve the estimated dependence structure. These simulated uniform variables were then transformed back to the rainfall scale using the fitted mixed discrete-continuous marginal distributions, allowing both zero and positive rainfall outcomes to be generated.

Table 4 presents the structure and selected pair-copula families of the fitted R-vine model,

while the corresponding vine structure is illustrated in the copula structure plot. The seven rainfall stations form an R-vine with six tree levels and 21 pair-copulas. In Tree 1, which represents the strongest and most direct pairwise dependencies, the selected families include BB8, Frank, Gaussian, and Tawn type 1 copulas. In the higher trees, the dependence structure is represented by conditional pair-copulas, with conditioning sets increasing according to the tree level.

Table 4: Structure and selected pair-copula families of the R-vine model

Pair	Tree	Variable 1	Variable 2	Conditioning Set	Copula	Par	Par2
1	1	2	1	–	BB8	4.30	0.95
2	1	3	2	–	Frank	6.47	–
3	1	5	6	–	Frank	3.84	–
4	1	3	5	–	Gaussian	0.59	–
5	1	7	3	–	Frank	2.85	–
6	1	7	4	–	Tawn type 1	2.05	0.66
7	2	3	1	2	Student-t	0.11	6.67
8	2	5	2	3	Tawn type 1	2.03	0.21
9	2	3	6	5	Survival BB7	1.03	0.22
10	2	7	5	3	Rotated Tawn type 2 (180°)	20.00	0.01
11	2	4	3	7	Tawn type 1	19.82	0.02
12	3	5	1	3,2	Independence	–	–
13	3	6	2	5,3	Independence	–	–
14	3	7	6	3,5	Student-t	0.21	5.97
15	3	4	5	7,3	Gumbel	1.07	–
16	4	6	1	5,3,2	Frank	-1.72	–
17	4	7	2	6,5,3	Survival BB8	1.13	1.00
18	4	4	6	7,3,5	Independence	–	–
19	5	7	1	6,5,3,2	Independence	–	–
20	5	4	2	7,6,5,3	Rotated Tawn type 1 (180°)	15.66	0.01
21	6	4	1	7,6,5,3,2	Tawn type 2	6.41	0.05

Note: Station codes are defined as follows: 1 = Stasiun Meteorologi Maritim Tanjung Priok; 2 = Stasiun Meteorologi Kemayoran; 3 = Halim Perdana Kusuma Jakarta; 4 = Pos Meteorologi Penggung; 5 = Stasiun Meteorologi Citeko; 6 = Stasiun Geofisika Bandung; and 7 = Stasiun Meteorologi Kertajati.

The R-vine structure and pair-copula families were selected automatically in R using the `RVineStructureSelect()` function from the `VineCopula` package (version 2.6.1). The setting `familyset = NA` was used, with the Akaike information criterion (AIC) as the selection criterion. With this setting, the selection procedure considers the copula families supported by the function, including the independence, Gaussian, Student-t, Clayton, Gumbel, Frank, Joe, BB1, BB6, BB7, BB8, and Tawn copulas, together with their applicable rotated or survival forms. The final model selected Gaussian, Student-t, Gumbel, Frank, BB8, BB7, and Tawn copulas, including rotated or survival variants. In addition, the independence copula was selected for several conditional pairs in Trees 3–5, indicating that these pairs did not require an additional dependence structure according to the AIC-based selection procedure. The selected R-vine dependence structure is illustrated in Figure 2.

Table 5 compares Kendall's tau between the observed and simulated rainfall data for all 21 station pairs. The simulated data generally preserve the positive dependence structure observed in the historical data, although the values of Kendall's tau are slightly lower for all station pairs. The differences range from -0.005 for Stations 2–6 to -0.184 for Stations 1–4, indicating that the R-vine simulation tends to slightly underestimate the strength of dependence, particularly for several station pairs with moderate observed association.

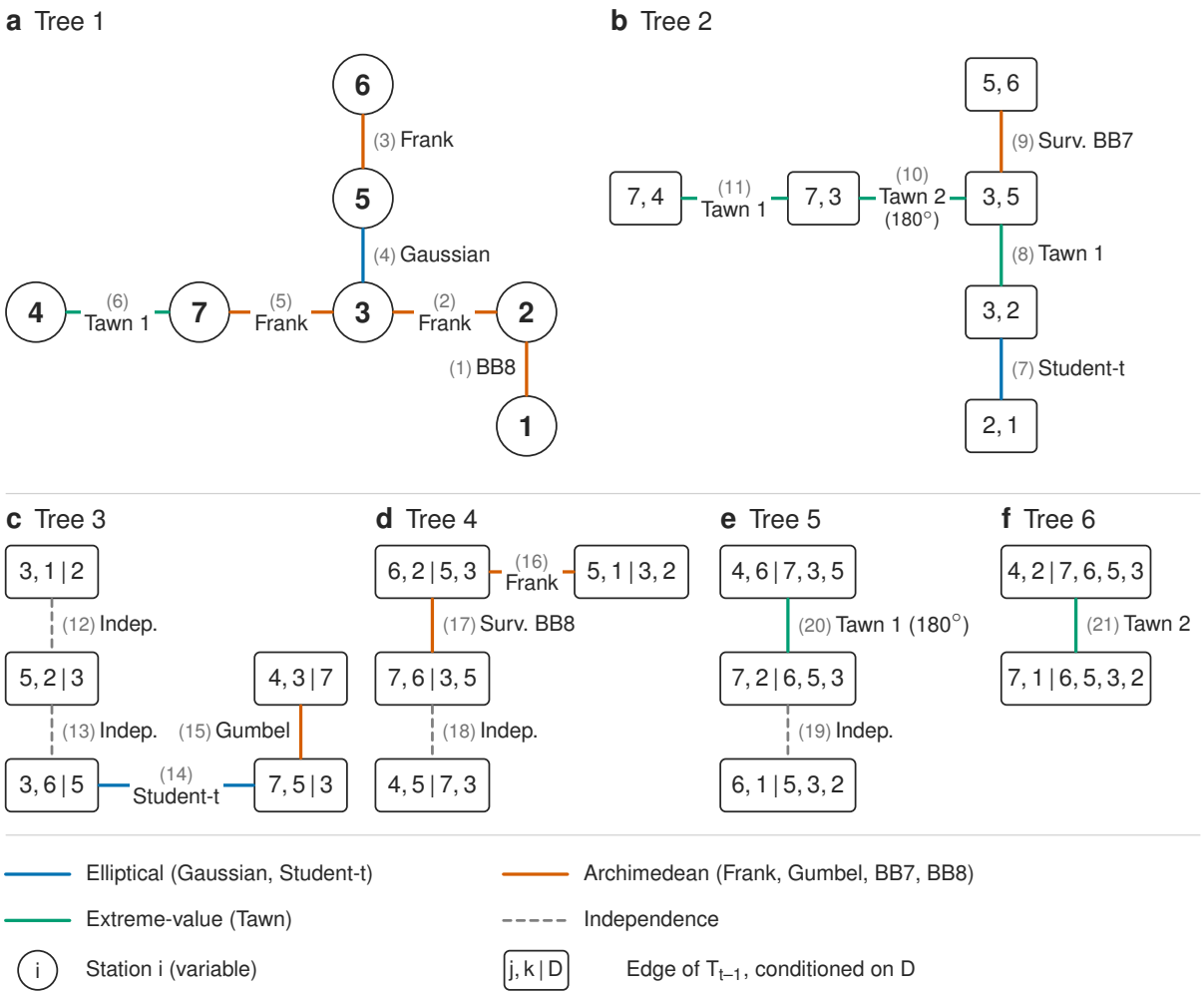


Fig. 2: Complete structure of the R-vine copula model across six tree levels, showing pairwise dependencies and conditioning sets.

Table 5: Comparison of Kendall's tau between observed and simulated data

Station Pair	τ_{obs}	τ_{sim}	$\Delta\tau$
1–2	0.737	0.679	−0.058
1–3	0.561	0.483	−0.078
1–4	0.332	0.148	−0.184
1–5	0.432	0.348	−0.084
1–6	0.203	0.152	−0.050
1–7	0.323	0.199	−0.124
2–3	0.619	0.552	−0.067
2–4	0.294	0.164	−0.131
2–5	0.454	0.400	−0.054
2–6	0.265	0.260	−0.005
2–7	0.303	0.250	−0.053
3–4	0.284	0.174	−0.110
3–5	0.462	0.405	−0.057
3–6	0.336	0.297	−0.039
3–7	0.334	0.299	−0.035
4–5	0.262	0.141	−0.121
4–6	0.211	0.155	−0.056
4–7	0.560	0.406	−0.154

Continued on next page

Table 5 – continued from previous page

Station Pair	τ_{obs}	τ_{sim}	$\Delta\tau$
5–6	0.424	0.395	−0.029
5–7	0.310	0.167	−0.143
6–7	0.314	0.231	−0.083

Note: τ_{obs} denotes Kendall's tau based on observed data, τ_{sim} denotes Kendall's tau based on simulated data, and $\Delta\tau = \tau_{\text{sim}} - \tau_{\text{obs}}$.

The strongest observed dependence occurs between Stations 1–2 ($\tau_{\text{obs}} = 0.737$) and Stations 2–3 ($\tau_{\text{obs}} = 0.619$), which are also relatively well reproduced by the simulation, with $\tau_{\text{sim}} = 0.679$ and 0.552, respectively. Overall, the similarity between the observed and simulated Kendall's tau values indicates that the fitted R-vine model adequately preserves the dependence structure among rainfall stations. The RMSE of Kendall's tau is 0.0935, indicating a relatively small overall discrepancy between the observed and simulated dependence measures and supporting the adequacy of the fitted model for generating multivariate rainfall scenarios.

Table 6 presents the estimated multivariate Value-at-Risk (VaR) at the 95% and 99% confidence levels based on 100,000 Monte Carlo simulations. The estimated VaR is 46.0744 mm at the 95% confidence level and increases to 92.3355 mm at the 99% confidence level. The higher VaR at the 99% confidence level reflects the greater magnitude of rainfall associated with more extreme joint events. These estimates represent joint rainfall risk across the seven stations rather than station-specific VaR estimates. Thus, the multivariate VaR provides a single quantitative measure of joint rainfall risk that incorporates the dependence structure captured by the R-vine model. The results indicate that the proposed framework can generate multivariate rainfall scenarios while accounting for dependence among the seven rainfall stations.

Table 6: Multivariate VaR Estimation at 95% and 99% Confidence Levels

Confidence Level	Number of simulations	VaR (mm)	CI (mm)
95%	100,000	46.0744	45.4340–46.7030
99%	100,000	92.3355	90.4392–94.6717

The uncertainty of the regional aggregate VaR estimates was assessed using bootstrap resampling of the 100,000 Monte Carlo-simulated aggregate rainfall values, with the fitted marginal and R-vine models kept fixed. Based on $B = 1,000$ bootstrap replications, the resulting confidence intervals were 45.4340–46.7030 mm for VaR_{95} and 90.4392–94.6717 mm for VaR_{99}. These intervals represent uncertainty conditional on the fitted model and do not account for uncertainty in marginal parameter estimation, copula-family selection, vine structure, or copula parameter estimation. The wider interval at the 99% level is consistent with the greater uncertainty associated with the upper-tail rainfall quantile. In addition, because the analysis is based on only 193 complete observations, the 99% estimate is supported by limited empirical information in the far upper tail and should therefore be interpreted cautiously.

In addition, the ECDF and QQ plots were used to compare the distributional behavior of the observed and simulated rainfall data. The ECDF plots in Figure 3 show generally consistent cumulative distributional patterns across the seven stations. The close alignment of the empirical cumulative distributions suggests that the R-vine-based simulation captures the overall rainfall distribution reasonably well.

The QQ plots in Figure 4 also show reasonable agreement over the central portion of the distributions, while some discrepancies are observed in the upper tail. These upper-tail discrepancies suggest limitations in reproducing extreme rainfall magnitudes and warrant cautious interpretation of the 99% regional aggregate VaR, which is more sensitive to tail behavior. Given that only 193 complete observations are available, the far upper tail is supported by limited empirical information and should therefore be interpreted with caution.

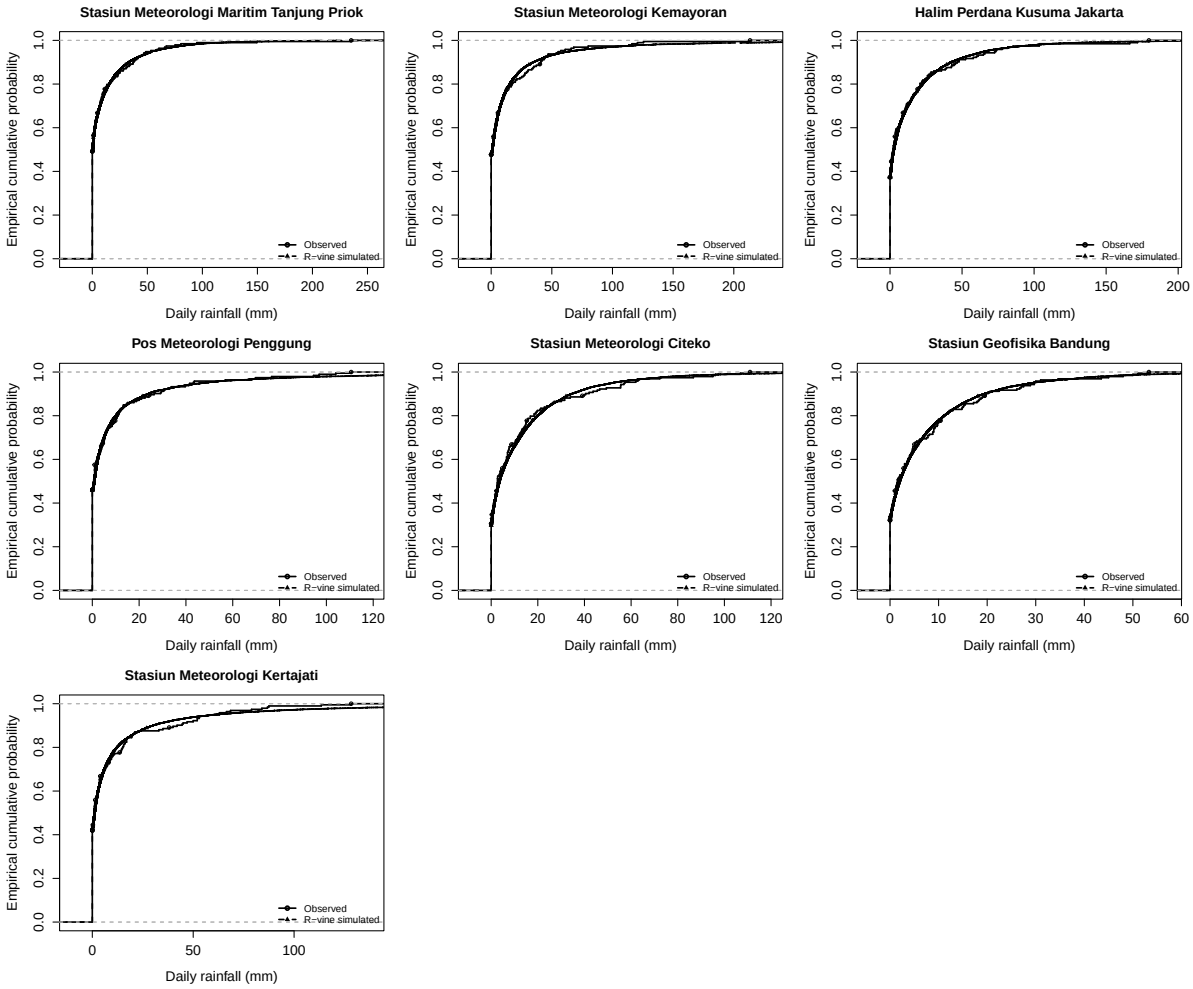


Fig. 3: ECDF comparison between observed and R-vine-simulated rainfall data.

Overall, the graphical diagnostics indicate that the R-vine-based simulation reproduces the main distributional and dependence characteristics of the observed rainfall data, with some discrepancies in the far upper tail. The Kendall's tau comparison provides an assessment of the reproduced dependence structure, while the ECDF and QQ plots assess the distributional behavior. The upper-tail discrepancies are particularly relevant for interpreting the 99% regional aggregate VaR, which may be sensitive to the modeled tail behavior and involve extrapolation beyond the observed sample range. The bootstrap confidence intervals quantify uncertainty conditional on the fitted marginal and R-vine models and do not account for uncertainty in marginal parameter estimation, copula-family selection, vine structure selection, or copula parameter estimation. The Kupiec test provides a separate in-sample assessment of unconditional coverage for the station-level univariate VaR estimates. Nevertheless, the combined diagnostic results provide complementary evidence regarding the adequacy of the fitted model for representing rainfall distributions and inter-station dependence. The resulting multivariate VaR therefore provides a model-based scalar measure of joint rainfall risk derived from the dependence structure among the study stations. The estimates at more extreme quantile levels, particularly the 99% multivariate VaR, should nevertheless be interpreted with caution because they are more sensitive to the modeled upper-tail behavior and may involve extrapolation beyond the observed sample range.

Sensitivity analysis was conducted using five random seeds (123, 456, 789, 2026, and 31415), with seed 123 used for the main analysis and as the reference for comparison. The selected R-vine structures and pair-copula families varied across some seeds. Relative to seed 123, the RMSE of Kendall's tau ranged from 0.0211 to 0.0322, with maximum absolute differences of

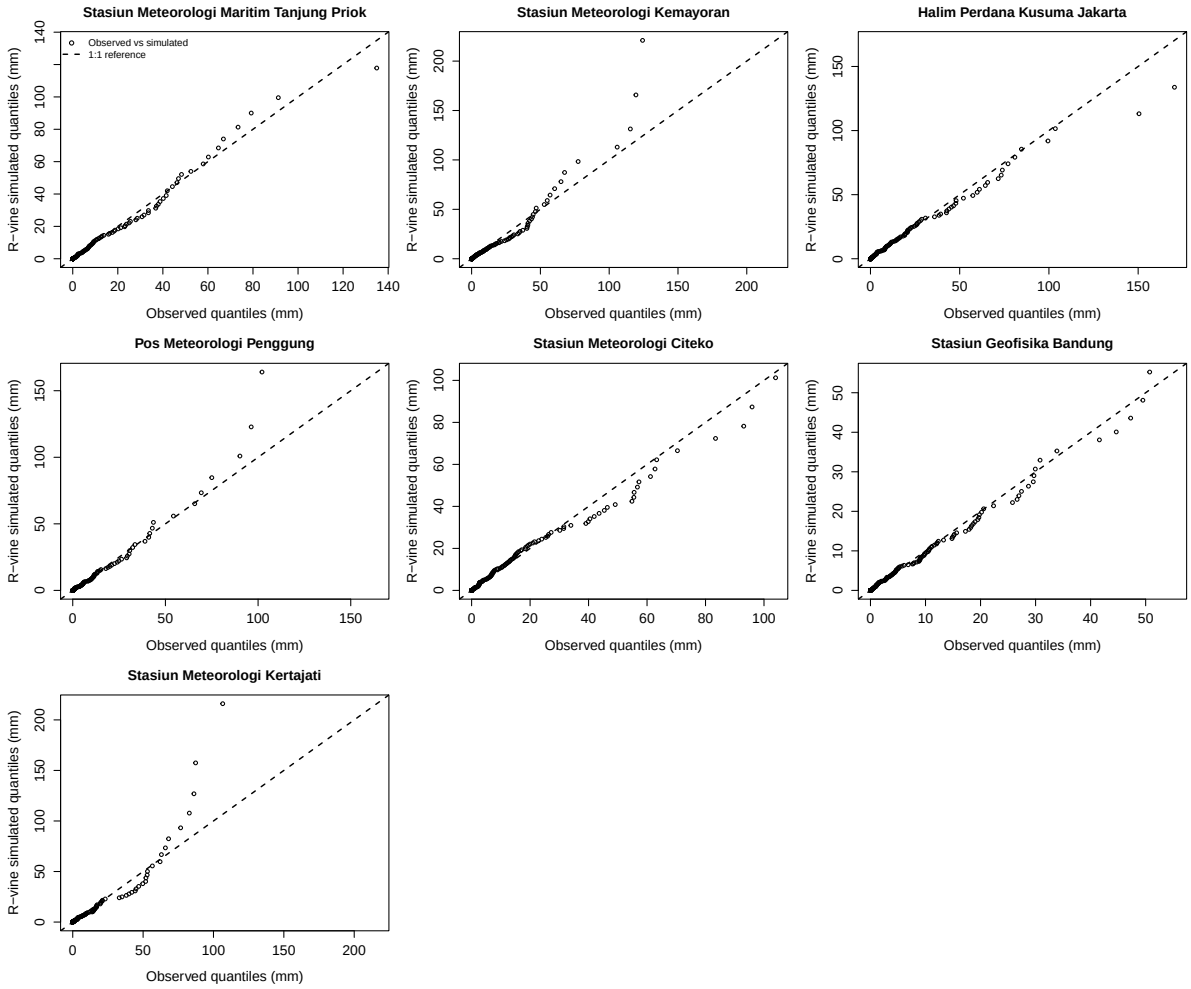


Fig. 4: QQ plot comparison between observed and R-vine-simulated rainfall data.

0.0475–0.0805. The resulting VaR estimates ranged from 45.9138 to 46.6937 mm at the 95% confidence level and from 92.4158 to 99.1207 mm at the 99% confidence level. All VaR estimates at the 95% confidence level fell within the corresponding confidence interval (45.4340–46.7030 mm), whereas some estimates at the 99% confidence level fell outside the corresponding interval (90.4392–94.6717 mm). These findings suggest that VaR_{95} estimates are relatively consistent across the tested seeds, while the VaR_{99} estimates are more sensitive to seed-dependent model selection and should therefore be interpreted with caution.

Validation of joint extreme rainfall was conducted by evaluating the frequency of events where rainfall at a minimum number of stations simultaneously exceeded the VaR value for each respective station. The thresholds employed were the 95% VaR and 99% VaR, calculated individually for each station. This approach enables an evaluation of the R-vine copula model's ability to represent inter-station dependence under extreme conditions. The validation results are presented in Table 7.

Based on Table 7, at the 95% VaR level, the observed proportion of events where at least two stations simultaneously exceeded the threshold was 8.81%, whereas the simulation yielded a proportion of 9.86%. The difference of 1.05 percentage points indicates that the model produced a joint extreme event frequency relatively close to observed conditions. For events involving at least three stations, the observed proportion was 3.11% compared to 3.79% in the simulation, a difference of 0.68 percentage points. For four to six stations, the differences between observation and simulation remained relatively small, with absolute differences of 0.23, 0.14, and 0.43 percentage points, respectively.

Table 7: Joint Extreme Rainfall Exceedance under VaR 95% and VaR 99%

Minimum Number of Stations	Observed VaR95	Simulated VaR95	Observed VaR99	Simulated VaR99
2	0.0881	0.0986	0.0104	0.0288
3	0.0311	0.0379	0.0000	0.0060
4	0.0155	0.0132	0.0000	0.0011
5	0.0052	0.0038	0.0000	0.0002
6	0.0052	0.0008	0.0000	0.0000
7	0.0000	0.0001	0.0000	0.0000

At the 99% VaR level, joint extreme events became rarer, as expected given the higher threshold used. For at least two stations, the observed proportion was 1.04%, while the simulation yielded 2.88%. Although the simulation tended to produce a higher frequency of joint exceedances at this level, the absolute difference was 1.85 percentage points. For at least three stations, the observed proportion was 0%, while the simulation showed 0.6%. For events involving four or more stations, the observed joint exceedance proportion was also very low, while simulation results indicated frequencies approaching zero. Overall, the results indicate that the R-vine copula model is capable of replicating joint extreme rainfall patterns reasonably well—particularly at the 95% VaR level—with relatively small discrepancies between observed and simulated proportions across most joint occurrence levels. At the 99% VaR level, larger discrepancies observed for two- or three-station events reflect the limitations posed by the scarcity of extreme data points; consequently, validation results concerning the extreme tails of the distribution require cautious interpretation.

4. Conclusion

This study developed an R-vine copula-based framework for assessing joint extreme rainfall risk across seven rainfall stations in DKI Jakarta and West Java. The marginal analysis showed that the rainfall characteristics differ across stations, with the Lognormal and Weibull distributions providing the best fit for positive rainfall amounts. The occurrence of zero rainfall was explicitly incorporated through a mixed discrete-continuous marginal framework, allowing the subsequent dependence modeling and simulation to account for both non-rainfall and positive rainfall conditions.

The fitted R-vine model consisted of six tree levels and 21 pair-copulas, providing a flexible representation of both unconditional and conditional dependence among the seven rainfall stations. Different copula families were selected across the tree levels, reflecting heterogeneous dependence patterns among the stations. The comparison between observed and simulated Kendall's tau yielded an RMSE of 0.0935, indicating a relatively small overall discrepancy and suggesting that the R-vine simulation reasonably reproduced the main dependence characteristics of the observed rainfall data. The ECDF and QQ plot comparisons further supported the ability of the model to reproduce the main distributional characteristics, although some discrepancies were observed in the upper tail.

Based on 100,000 Monte Carlo simulations, the estimated regional aggregate VaR was 46.0744 mm at the 95% confidence level and 92.3355 mm at the 99% confidence level. Bootstrap confidence intervals were obtained using $B = 1,000$ replications by resampling the simulated aggregate rainfall values with replacement, while keeping the fitted marginal distributions and R-vine model fixed. The resulting confidence intervals were 45.4340–46.7030 mm and 90.4392–94.6717 mm for the 95% and 99% VaR estimates, respectively. The higher VaR at the 99% confidence level reflects the greater severity associated with more extreme joint rainfall conditions. However, the 99% VaR should be interpreted with caution because the model is based on only 193 complete daily observations, which provide limited information about the far upper tail. The upper-tail

discrepancies observed in the QQ plots further indicate that the extreme-tail estimates may be sensitive to the modeled tail behavior and may involve extrapolation beyond the observed sample range.

The Kupiec test provided an in-sample coverage assessment of the station-level univariate VaR estimates. At both the 95% and 99% confidence levels, the corresponding p-values exceeded 0.05, indicating no significant departure from the expected unconditional exceedance frequencies within the historical sample. These results should be interpreted specifically as evidence regarding the marginal VaR coverage and not as an independent out-of-sample validation. In particular, the Kupiec test does not constitute a direct backtest of the seven-dimensional joint rainfall model or the regional aggregate VaR.

Overall, the proposed R-vine copula framework provides a flexible approach for representing dependence among rainfall stations and quantifying regional aggregate rainfall risk. The framework integrates station-specific mixed discrete-continuous marginal distributions, R-vine dependence modeling, Monte Carlo simulation, and multivariate VaR estimation while explicitly accounting for dependence among stations. The multivariate Value-at-Risk estimates provide a single quantitative measure of joint rainfall risk across the study region. However, the 99% estimate should be interpreted cautiously because only 193 complete observations are available and the QQ plots show some upper-tail discrepancies, indicating that extreme-tail estimates may be sensitive to the fitted tail behavior and may involve extrapolation beyond the observed sample range. Further studies using longer records and additional extreme-tail validation would be valuable for improving the assessment of very high quantile rainfall risk.

CRediT Authorship Contribution Statement

Lailatul Maziyah Wildan Mufaridho: Conceptualization, Methodology, Writing–Original Draft. **Dewi Susanawati:** Data Curation, Formal Analysis, Writing–Review & Editing. **Wahyu Joko Saputro:** Software, Visualization, Writing–Review & Editing.

Declaration of Generative AI and AI-assisted technologies

Generative AI and AI-assisted technologies were not used for data analysis, statistical computation, model development, or the generation of research results in this study. If any AI-assisted tool was used during manuscript preparation, it was limited to language editing and formatting assistance only, and the authors take full responsibility for the content of the manuscript.

Declaration of Competing Interest

The authors declare no competing interests.

Ethics Approval and Consent to Participate

Ethics approval was not required for this study.

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Data and Code Availability

The dataset analyzed during this study is publicly available at https://dataonline.bmkg.go.id/beranda.

References

[1] Fadhila Febriyanti Najamuddin, Erna Tri Herdiani, and Andi Kresna Jaya. "Value at Risk Estimation Using Extreme Value Theory Approach in Indonesia Stock Exchange". In: *BAREKENG: Jurnal Ilmu Matematika dan Terapan* 18.2 (2024), pp. 695–706. DOI: 10.30598/barekengvol18iss2pp0695-0706.

[2] Zhengying He, Mitsuyoshi Akiyama, Putri S. Firdaus, Yu Huang, Dan M. Frangopol, and Koki Aoki. "Probabilistic Connectivity Assessment of Road Networks Exposed to Spatially Correlated Rainfall-Triggered Landslides". In: *Reliability Engineering & System Safety* 257 (2025), p. 110800. DOI: 10.1016/j.ress.2025.110800.

[3] Dedi Setiawan. "Analisis Curah Hujan di Indonesia untuk Memetakan Daerah Potensi Banjir dan Tanah Longsor dengan Metode Cluster Fuzzy C-Means dan Singular Value Decompotition (SVD)". In: *Engineering, Mathematics and Computer Science (EMACS) Journal* 3.3 (2021), pp. 115–120. DOI: 10.21512/emacsjournal.v3i3.7428.

[4] Suhadi, Faizatul Mabruroh, Adis Wiyanto, and Ikra. "Analisis Fenomena Perubahan Iklim terhadap Curah Hujan Ekstrim". In: *OPTIKA: Jurnal Pendidikan Fisika* 7.1 (2023), pp. 94–100. DOI: 10.37478/optika.v7i1.2738.

[5] Anik Djuraidah, Cici Suheni, and Banan Nabila. "Peramalan Curah Hujan Ekstrim di Provinsi Banten dengan Model Ekstrim Spasial". In: *Media Statistika* 12.1 (2019), pp. 50–62. DOI: 10.14710/medstat.12.1.50-62.

[6] Yuningsih Yuningsih, Masita Dwi Mandini Manessa, and Hafid Setiadi. "Analisis Curah Hujan Dominan Desa Ciputri: Risiko Bencana Hidroklimatologi". In: *Media Komunikasi Geografi* 25.1 (2024), pp. 150–166. DOI: 10.23887/mkg.v25i1.73029.

[7] David E. Allen, Michael McAleer, and Abhay K. Singh. "Risk Measurement and Risk Modelling Using Applications of Vine Copulas". In: *Sustainability* 9.10 (2017), p. 1762. DOI: 10.3390/su9101762.

[8] Fatih Tosunoğlu, Faruk Gürbüz, and Muhammet Nuri İspirli. "Multivariate Modeling of Flood Characteristics Using Vine Copulas". In: *Environmental Earth Sciences* 79.19 (2020), p. 459. DOI: 10.1007/s12665-020-09199-6.

[9] Christian Genest and Johanna Nešlehová. "Copulas and Copula Models". In: *Wiley StatsRef: Statistics Reference Online*. John Wiley & Sons, Ltd, 2014. DOI: 10.1002/9781118445112.stat07523.

[10] George Casella and Roger L. Berger. *Statistical Inference*. 2nd ed. Boca Raton: Chapman and Hall/CRC, 2024. DOI: 10.1201/9781003456285.

[11] Simon Michael Papalexiou. "Rainfall Generation Revisited: Introducing CoSMoS-2s and Advancing Copula-Based Intermittent Time Series Modeling". In: *Water Resources Research* 58.6 (2022), pp. 1–33. DOI: 10.1029/2021WR031641.

[12] Ludger Rüschendorf. "On the distributional transform, Sklar's theorem, and the empirical copula process". In: *Journal of Statistical Planning and Inference* 139.11 (2009), pp. 3921–3927. DOI: 10.1016/j.jspi.2009.05.030.

[13] Joshua Größer and Ostap Okhrin. "Copulae: An Overview and Recent Developments". In: *WIREs Computational Statistics* 14.3 (2022), e1557. DOI: 10.1002/wics.1557.

[14] Thomas Nagler. "Simplified Vine Copula Models: State of Science and Affairs". In: *Risk Sciences* 1.2 (2025), p. 100022. DOI: 10.1016/j.risk.2025.100022.

[15] Lan Zhang and V. P. Singh. "Rainfall Frequency Analysis". In: *Copulas and their Applications in Water Resources Engineering*. Cambridge: Cambridge University Press, 2019, pp. 367–395. DOI: 10.1017/9781108565103.011.

[16] Eike Christian Brechmann and Ulf Schepsmeier. "Modeling Dependence with C- and D-Vine Copulas: The R Package CDVine". In: *Journal of Statistical Software* 52.3 (2013), pp. 1–27. DOI: 10.18637/jss.v052.i03.

[17] Jacinto Martín, M. Isabel Parra, Mario M. Pizarro, and Eva L. Sanjuán. "A New Bayesian Method for Estimation of Value at Risk and Conditional Value at Risk". In: *Empirical Economics* 68.3 (2025), pp. 1171–1189. DOI: 10.1007/s00181-024-02664-2.

[18] David C. M. Dickson, Mary R. Hardy, and Howard R. Waters. *Actuarial Mathematics for Life Contingent Risks*. Cambridge: Cambridge University Press, 2009. DOI: 10.1017/CBO9780511800146.

[19] Khreshna Syuhada, Rizka Puspitasari, I Kadek Darma Arnawa, Lailatul Mufaridho, Elonasari Elonasari, Miftahul Jannah, and Aniq Rohmawati. "Enhancing Value-at-Risk with Credible Expected Risk Models". In: *International Journal of Financial Studies* 12.3 (2024), p. 80. DOI: 10.3390/ijfs12030080.