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Modeling Maternal Mortality in East Java: A GWBZIGPR Approach with Spatial Weighting

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Abstract

Maternal mortality remains a critical indicator of health development, reflecting the quality of maternal health services and the socio-geographic conditions of a region. Data on maternal deaths during pregnancy and childbirth in East Java Province in 2025 exhibit characteristics of correlated bivariate count data, excess zeros relative to the Poisson specification, overdispersion, and spatial heterogeneity. This study applies the Geographically Weighted Bivariate Zero-Inflated Generalized Poisson Regression (GWBZIGPR) model to 38 districts and cities in East Java Province, with parameter estimation employing Maximum Likelihood Estimation and the Berndt Hall Hall Hausman (BHHH) numerical iteration. Significance testing uses the Maximum Likelihood Ratio Test (MLRT) for simultaneous testing and the Z-test for partial testing, and MLRT confirms that the model is significant overall, while the partial tests show that no individual parameter is statistically significant at the 5% level. The optimal weighting function is selected by comparing four kernels, namely Fixed Gaussian, Fixed Bisquare, Adaptive Gaussian, and Adaptive Bisquare, based on the lowest Akaike Information Criterion Corrected value (AICc). The results show that, among the four kernels, the GWBZIGPR model with the Adaptive Bisquare kernel yields the smallest AICc value of 575.2514. Because the effective degrees of freedom of the local GWBZIGPR model (23 parameters estimated separately at each of the 38 locations) differ fundamentally from those of the global BZIGPR model, the AICc values of the two models are not on a strictly equivalent penalty basis and are therefore not used to rank one model as superior to the other. Model preference is instead based on estimation precision and inferential reliability. The standard errors of the local estimates are substantially larger than the parameter estimates across locations, indicating that the GWBZIGPR model suffers from low estimation precision for this dataset and that the BZIGPR model, which estimates parameters globally, remains the preferred model for this dataset.

Keywords: BZIGPR, GWBZIGPR, Maternal Mortality, Spatial Heterogeneity, Zero-Inflated.

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1. Introduction

Maternal mortality is considered an important indicator of health development because it reflects the quality of maternal health services, access to health facilities, and the sociogeographic conditions of a region [1]. Based on data from the East Java Provincial Health Office [2], the Maternal Mortality Ratio in East Java Province was reported to reach 68.7 per 100,000 live

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births, with 399 maternal deaths recorded throughout the province; these figures were observed in 2025, sourced from the East Java Provincial Health Profile 2025, and released in 2026. These deaths are classified as deaths during pregnancy, during childbirth, and during the postpartum period.

In this study, only the first two categories are examined, namely Y1 (deaths during pregnancy) and Y2 (deaths during childbirth), which together account for 127 cases in 38 districts and cities as the bivariate response structure; the remaining postpartum and unassigned deaths are excluded, so the sum of Y1 and Y2 does not equal the provincial total of 399. Although the provincial MMR is already positioned below the SDG target of 70 per 100,000 live births, this aggregate figure conceals substantial disparities across the 38 districts and cities, with some districts still reported to experience ratios well above the target; targeted interventions are therefore still required to reduce inter-district disparities and to sustain progress toward the 2030 goal.

Since maternal mortality is measured as the number of events, data are classified as count data, for which Poisson regression is commonly used, but the assumption of equidispersion of this model is often violated in real data [3]. Such violations are observed as overdispersion, where the variance exceeds the mean, or as underdispersion, where the variance is less than the mean [4]. To overcome this problem, Consul and Jain introduced the generalized Poisson distribution model [5], and both conditions are handled simultaneously by this model [6]. Count data are also often characterized by excess zeros, and the Zero-Inflated Generalized Poisson regression model was developed by Famoye and Singh [7] by combining the Zero-Inflated Poisson model for excess zeros with the Generalized Poisson model for dispersion violations.

Because maternal deaths during pregnancy and childbirth are correlated and exhibit spatial heterogeneity, the Zero-Inflated Generalized Poisson model is extended to the bivariate case, and the Bivariate Zero-Inflated Generalized Poisson Regression (BZIGPR) model was developed by Aini et al. [8] to produce global parameter estimates for all locations. Because spatial heterogeneity between regions is not addressed by this model, the Geographically Weighted Bivariate Zero-Inflated Generalized Poisson Regression model (GWBZIGPR) was subsequently developed by Purhadi et al. [9] to produce local parameter estimates for each location and was shown to outperform BZIGPR for maternal mortality data in the Pekalongan Residence.

Based on the characteristics of maternal mortality data in East Java Province in 2025, which exhibit overdispersion, excess zeros, bivariate correlation, and spatial heterogeneity, the GWBZIGPR model is applied in this study. The research gap is identified in the underexplored behavior of the GWBZIGPR model under limited sample conditions, since previous applications were conducted on substantially larger datasets. The contribution of this study goes beyond the application of an existing model to a new geographical data set. The novelty lies in (i) the systematic comparison of four spatial weighting functions, (ii) empirical evidence that the GWBZIGPR model can exhibit low estimation precision when the sample size is small relative to the number of local parameters, and (iii) the recommendation of the global BZIGPR model for small samples, based on estimation precision rather than on AICc. The scientific contribution is to provide applied researchers with evidence-based guidance on kernel selection and data requirements for reliable local estimation. Section 2 details the methodology, Section 3 presents the results and a discussion, and Section 4 concludes the paper.

2. Methods

This section describes the data sources, the formulation of the BZIGPR and GWBZIGPR models, the parameter estimation procedure, and the diagnostic tests used to evaluate the model assumptions.

2.1. Data Description and Study Framework

This study used secondary data from the East Java Provincial Health Profile for 2025 [2], covering 38 districts and cities (29 districts and 9 cities) as observation units. The coordinate points were the centroid latitude and longitude of each region, obtained from the administrative shapefile published by the Geospatial Information Agency (*Badan Informasi Geospasial*) and projected from WGS84 (EPSG:4326) to UTM Zone 49S (EPSG:32749) because spatial weights based on Euclidean distance require linear units [10]. The provincial total of 399 maternal deaths includes cases that could not be assigned to any specific district or city due to rejected domicile records, so the sum of Y1 and Y2 across the 38 districts and cities is 127. Y1 represents the number of maternal deaths during pregnancy and Y2 represents the number of maternal deaths during childbirth, while postpartum deaths and unassigned records were excluded from the bivariate analysis.

Three of the four predictor variables (X_1, X_2, and X_4) have maximum values exceeding 100%, reflecting the administrative nature of the data, since the denominator is an estimated target population rather than an exact count. The variables used in this study are presented in Table 1. All computational procedures were implemented in R (version 2025.05.1+513) and are detailed in Subsection 2.7.

Table 1: Research Variables

Symbol	Variable	Scale
Y_1	Number of Maternal Deaths During Pregnancy	Ratio
Y_2	Number of Maternal Deaths During Childbirth	Ratio
X_1	Percentage of K1 Antenatal Care Visits	Ratio
X_2	Percentage of Td2+ Immunization Coverage	Ratio
X_3	Number of Active Integrated Health Posts	Ratio
X_4	Percentage of Facility-Based Deliveries	Ratio

2.2. Bivariate Zero-Inflated Generalized Poisson Regression Model

The BZIGPR model is a development of the Bivariate Zero-Inflated Poisson Regression (BZIPR) model by replacing the Poisson component with Generalized Poisson to handle both overdispersion and underdispersion [9]. The joint probability function of the BZIGPR model is defined as follows [9]:

$$P(Y_{1i} = y_{1i}, Y_{2i} = y_{2i}) = \begin{cases} A_i, & \text{if } y_{1i} = 0, y_{2i} = 0 \\ B_i, & \text{if } y_{1i} = 0, y_{2i} > 0 \\ C_i, & \text{if } y_{1i} > 0, y_{2i} = 0 \\ D_i, & \text{if } y_{1i} > 0, y_{2i} > 0 \end{cases} \tag{1}$$

with the mean parameters as defined in Eq. (2):

$$\mu_{ji} = \exp(\mathbf{x}_i^T \boldsymbol{\beta}_j), \quad j = 1, 2 \tag{2}$$

and the zero-inflation probabilities as defined in Eq. (3):

$$p_{ji} = \frac{\exp(\mathbf{x}_i^T \boldsymbol{\gamma}_j)}{1 + \exp(\mathbf{x}_i^T \boldsymbol{\gamma}_j)}, \quad j = 1, 2 \tag{3}$$

as well as the dispersion parameters ϕ_1, ϕ_2 and the association parameter η.

The parameter η is a multiplicative factor that controls the dependency between Y_1 and Y_2 in the joint distribution. It enters the joint probability function through the term $[1 + \eta \cdot f(y_1, y_2)]$, where $f(y_1, y_2)$ is a function of the marginal probability generating functions. When $\eta = 0$, the two response variables are independent. When $\eta \neq 0$, the two variables are dependent, with the

sign of η determining the direction of the association, where $\eta > 0$ corresponds to a positive association, while $\eta < 0$ corresponds to a negative association [8, 9].

It is important to note that the association parameter η is not equal to the Pearson correlation coefficient ρ between Y_1 and Y_2. Rather, η enters the model as a multiplicative factor in the joint probability function and determines the direction and strength of the dependency between the two variables through the marginal distributions. The sign of η is consistent with the sign of the correlation: positive η implies a positive correlation, and negative η implies a negative correlation. However, the magnitude of η does not directly correspond to the magnitude of the correlation coefficient, as the latter also depends on the marginal means and the dispersion parameters [9].

2.3. Geographically Weighted Bivariate Zero-Inflated Generalized Poisson Regression Model

The GWBZIGPR model is a development of the BZIGPR model by integrating the Geographically Weighted Regression approach, so that all regression parameters are expressed as functions of geographic coordinates (u_i, v_i) [9]. The joint probability function of the GWBZIGPR model is divided into four conditions based on the combination of the values of the two response variables [9]. The local mean and zero-inflation parameters are defined in Eq. (4):

$$\mu_j(u_i, v_i) = \exp(\mathbf{x}_i^T \boldsymbol{\beta}_j(u_i, v_i)), \quad p_j(u_i, v_i) = \frac{\exp(\mathbf{x}_i^T \boldsymbol{\gamma}_j(u_i, v_i))}{1 + \exp(\mathbf{x}_i^T \boldsymbol{\gamma}_j(u_i, v_i))}, \quad j = 1, 2 \tag{4}$$

The four conditions of the joint probability function are denoted as A_i, B_i, C_i, and D_i, corresponding to the four possible combinations of zero and positive values of Y_{1i} and Y_{2i}, following the same structure as Eq. (1). For brevity, the explicit forms are omitted here and can be found in Purhadi et al. [9]. The function g_j is the marginal probability generating function of the Generalized Poisson distribution, as defined in Eq. (5):

$$g_j = \exp\left[\frac{e^{\mathbf{x}_i^T \boldsymbol{\beta}_j(u_i,v_i)}(s_j - 1)}{1 + \phi_j e^{\mathbf{x}_i^T \boldsymbol{\beta}_j(u_i,v_i)}}\right] \tag{5}$$

2.4. Likelihood Function and Parameter Estimation

Parameter estimation for the GWBZIGPR model was carried out using the Maximum Likelihood Estimation (MLE) method with a weighted likelihood function [9], as defined in Eq. (6):

$$L(\Theta(u_i, v_i)) = \prod_{i=1}^{n} \left[A_i^{I_0(y_{1i},y_{2i})} B_i^{I_1(y_{1i},y_{2i})} C_i^{I_2(y_{1i},y_{2i})} D_i^{I_3(y_{1i},y_{2i})}\right]^{w_i(u_i,v_i)} \tag{6}$$

where $\Theta(u_i, v_i) = [\boldsymbol{\gamma}_1^T, \boldsymbol{\gamma}_2^T, \boldsymbol{\beta}_1^T, \boldsymbol{\beta}_2^T, \phi_1, \phi_2, \eta]^T$. The log-likelihood function is defined in Eq. (7):

$$\ell(\Theta(u_i, v_i)) = \sum_{i=1}^{n} w_i[I_0 \ln A_i + I_1 \ln B_i + I_2 \ln C_i + I_3 \ln D_i] \tag{7}$$

where I_0, I_1, I_2, I_3 are indicator functions for the four response combinations.

Because the first derivative has no closed-form solution, the Berndt Hall Hall Hausman (BHHH) algorithm is used [9]. The iteration equation for this algorithm is given in Eq. (8):

$$\hat{\Theta}^{(m+1)} = \hat{\Theta}^{(m)} - \mathbf{H}^{-1}(\hat{\Theta}^{(m)})\mathbf{g}(\hat{\Theta}^{(m)}) \tag{8}$$

where $\mathbf{H}(\hat{\Theta}^{(m)}) = -\sum_{i=1}^{n} \mathbf{g}_i \mathbf{g}_i^T$ approximates the Hessian matrix, and the negative sign ensures that the update follows the standard Newton-Raphson form. The iteration was terminated when $\|\Theta^{(m+1)} - \Theta^{(m)}\| < 10^{-6}$.

2.5. Spatial Weighting Functions

Four kernel weighting functions were used: Fixed Gaussian, Fixed Bisquare, Adaptive Gaussian, and Adaptive Bisquare [9]. The Fixed Gaussian and Fixed Bisquare kernels are defined in Eq. (9):

$$w_{ik} = \exp\left(-\frac{1}{2}\left(\frac{d_{ik}}{h}\right)^2\right), \quad w_{ik} = \begin{cases} \left(1-(d_{ik}/h)^2\right)^2, & d_{ik} \le h \\ 0, & d_{ik} > h \end{cases} \tag{9}$$

while the Adaptive Gaussian and Adaptive Bisquare kernels replace h with a location-specific bandwidth h_i, as defined in Eq. (10):

$$w_{ik} = \exp\left(-\frac{1}{2}\left(\frac{d_{ik}}{h_i}\right)^2\right), \quad w_{ik} = \begin{cases} \left(1-(d_{ik}/h_i)^2\right)^2, & d_{ik} \le h_i \\ 0, & d_{ik} > h_i \end{cases} \tag{10}$$

where $d_{ik} = \sqrt{(u_i-u_k)^2+(v_i-v_k)^2}$ is the Euclidean distance between locations i and k, and h and h_i are the bandwidths. The Euclidean distance was chosen because the coordinates were projected to UTM Zone 49S (EPSG:32749), which uses linear units in meters and preserves local distances at the scale of East Java Province [10].

For the fixed kernels, h is constant across locations, while for the adaptive kernels, h_i is the distance from location i to its k-th nearest neighbor. The optimal bandwidth was selected by minimizing the Generalized Cross-Validation (GCV) criterion as defined in Eq. (11):

$$\mathrm{GCV}(h) = \frac{n\sum_{i=1}^{n}(y_i-\hat{y}_i(h))^2}{(n-\mathrm{tr}(\mathbf{S}))^2} \tag{11}$$

where $\hat{y}_i(h)$ is the fitted value at location i, and $\mathrm{tr}(\mathbf{S})$ is the trace of the hat matrix. The best weighting function was selected based on the smallest AICc value [9].

2.6. Diagnostic Tests and Model Selection

Four diagnostic tests were conducted prior to model fitting. First, overdispersion or underdispersion was detected using the Variance Test [9] as defined in Eq. (12):

$$VT = \frac{s^2}{\bar{y}}, \quad s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i-\bar{y})^2 \tag{12}$$

where data are overdispersed when $VT > 1$ and underdispersed when $VT < 1$. Second, the Pearson correlation test was used to assess the relationship between Y_1 and Y_2 [9], with the test statistic defined in Eq. (13):

$$t = \frac{r_{Y_1,Y_2}\sqrt{n-2}}{\sqrt{1-r_{Y_1,Y_2}^2}} \tag{13}$$

with H_0 rejected if $|t| > t_{\alpha/2;n-2}$.

Third, multicollinearity was identified using the Variance Inflation Factor [9] as defined in Eq. (14):

$$VIF_k = \frac{1}{1-R_k^2} \tag{14}$$

with values above 10 indicating serious multicollinearity. Fourth, model selection was based on the Corrected Akaike Information Criterion [9] as defined in Eq. (15):

$$AICc = -2\ln(\hat{L}) + 2k + \frac{2k(k+1)}{n-k-1} \tag{15}$$

where $\hat{L}$ is the maximized likelihood, n is the number of observations, and k is the number of parameters. For the global BZIGPR model, $k = 23$. For the GWBZIGPR model, $k = 23$ refers

to the number of parameters estimated at each location, and the effective degrees of freedom of the local model are not equal to this nominal value. Therefore, AICc is used to compare kernels within the GWBZIGPR family, not to rank GWBZIGPR against BZIGPR.

Two spatial assumption tests were also conducted. Spatial autocorrelation was tested using Moran's I index [11] as defined in Eq. (16):

$$I = \frac{n}{\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}} \cdot \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}(e_i - \bar{e})(e_j - \bar{e})}{\sum_{i=1}^{n}(e_i - \bar{e})^2} \tag{16}$$

where $w_{ij} = 1/d_{ij}$ for $i \neq j$ and $w_{ij} = 0$ for $i = j$, row-standardized so that $\sum_j w_{ij}^* = 1$, and e_i is the residual from the global BZIGPR model. Under spatial randomness, $E[I] = -1/(n-1)$. Spatial heterogeneity was tested using the Glejser method [9] as defined in Eq. (17):

$$G = -\left[n - q - 1 - \frac{1}{2}(j - q + 1)\right] \ln\left(\frac{|\hat{\Sigma}_\Omega|}{|\hat{\Sigma}_\omega|}\right) \tag{17}$$

with H_0 rejected when $G > \chi^2_{(\alpha,jq)}$, indicating spatial heterogeneity.

2.7. Computational Details

All statistical analyses were performed using R software (version 2025.05.1+513) on a standard desktop computer, with the following packages: `readxl` for reading the Excel data file; `gamlss`, `MASS`, `hypergeo`, `goft`, `maxLik`, `AER`, and `car` for generalized Poisson and related regression modeling; `spgwr` for bandwidth selection using the `ggwr.sel` function with the Poisson family; `sf` for coordinate transformation from WGS84 (EPSG:4326) to UTM Zone 49S (EPSG:32749); and `xtable` and `openxlsx` for exporting tables of estimation results. The BZIGPR and GWBZIGPR models were estimated using a custom implementation of the BHHH algorithm in R. Initial values for the regression coefficients were obtained from univariate Poisson regression via `glm`, while the zero-inflation parameters were initialized at 0.01, the dispersion parameters at 1.065 and 1.210, and the association parameter at 0.269.

The gradient was computed analytically, and the Hessian was approximated by the negative outer product of the gradients. For the GWBZIGPR model, the BHHH algorithm was applied independently at each of the 38 locations, with the global BZIGPR estimates used as starting values. A ridge regularization term $\lambda\mathbf{I}$ with $\lambda = 10^{-3}$ was added to the Hessian approximation, and for locations with fewer than 15 effective neighbors, λ was increased to 10^{-2}. Standard errors were obtained from the square root of the diagonal elements of the inverse Hessian matrix. The analysis code in R, including the custom BHHH implementation for both BZIGPR and GWBZIGPR, is available from the corresponding author upon reasonable request.

3. Results and Discussion

This section presents the empirical findings of the study, starting from the descriptive statistics and diagnostic tests, followed by the selection of the spatial weighting function, the GWBZIGPR modeling results, and a comprehensive discussion of the model comparison.

3.1. Descriptive Statistics and Diagnostic Tests

The descriptive statistics for the research variables are summarized in Table 2. The mean number of maternal deaths during pregnancy (Y_1) is 1.868 with a variance of 3.631, while the mean number of maternal deaths during childbirth (Y_2) is 1.474 with a variance of 2.797. The variance being greater than the mean for both response variables indicates overdispersion. Among the predictor variables, X_3 has the largest variability with a standard deviation of 686.825, indicating very large differences between districts and cities in terms of the number of active integrated health posts. It is worth noting that three predictor variables (X_1, X_2, and X_4) have maximum

Table 2: Descriptive Statistics of Research Variables

Variable	Min	Max	Mean	Var	Std. Dev.
Y_1	0.000	8.000	1.868	3.631	1.905
Y_2	0.000	7.000	1.474	2.797	1.672
X_1	58.20	102.53	80.68	185.92	13.635
X_2	50.20	104.30	80.15	198.12	14.075
X_3	108.00	2888.00	1162.13	471727.86	686.825
X_4	55.50	100.30	79.58	208.32	14.433

values slightly exceeding 100%, which reflects the administrative nature of the data as discussed in Subsection 2.1.

Based on the Variance Test [9], $VT_1 = 3.631/1.868 = 1.943$ and $VT_2 = 2.797/1.474 = 1.898$. Since both values exceed 1, both response variables experience overdispersion, indicating that the Poisson regression model is not suitable and a model capable of handling overdispersion, such as Generalized Poisson, is needed [12, 13]. Maternal mortality counts are also characterized by a preponderance of zero observations, with 26.32 percent of the districts/cities recording no maternal deaths during pregnancy (Y_1) and 34.21 percent recording no maternal deaths during childbirth (Y_2). To assess whether these proportions exceed the expected level, the zero probability under a standard Poisson model was computed as $P(Y = 0) = \exp(-\hat{\lambda})$, where $\hat{\lambda}$ is the Poisson mean. Based on the sample means in Table 2, the obtained zero probabilities are $\exp(-1.868) = 15.44\%$ for Y_1 and $\exp(-1.474) = 22.90\%$ for Y_2. The observed zero proportions (26.32% and 34.21%) therefore exceed the Poisson-expected proportions by 10.88 and 11.31 percentage points, respectively.

This comparison confirms the presence of excess zeros beyond what a standard Poisson model can accommodate and is visualized in Figure 1. While the observed zero counts cannot, on their own, distinguish between structural zeros and sampling zeros, the combination of excess zeros with overdispersion motivates the use of a zero-inflated approach, which explicitly accounts for both types of zeros. This aligns with the rationale for zero-inflated count data, as explained by Lambert [14] and Famoye and Singh [7], and is consistent with the findings of Purhadi et al. [9] and Aini et al. [8].

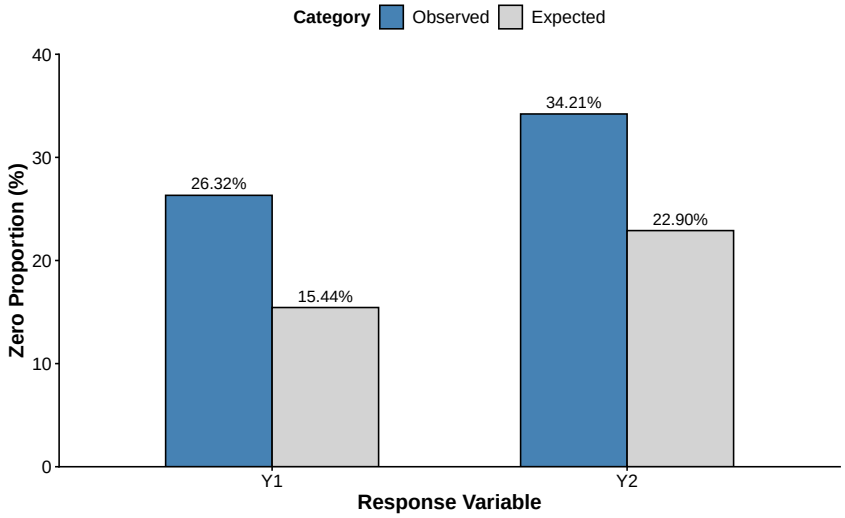


Fig. 1: Comparison of observed and Poisson-expected zero proportions for Y_1 and Y_2.

The Pearson correlation test yields $r_{Y_1,Y_2} = 0.342$ and $t = 2.186$, which exceeds $t_{(0.025;36)} = 2.028$, so the null hypothesis is rejected. This means there is a significant correlation between the two response variables at the 5 percent significance level with a p-value of 0.035, indicating that the bivariate modeling approach is more appropriate [15, 16]. The VIF values for X_1, X_2,

X_3, and X_4 are 8.317, 8.463, 1.076, and 2.334, respectively. Although the VIF values for X_1 and X_2 are quite high, since all VIF values are less than 10, it can be concluded that there is no serious multicollinearity [17]. However, the relatively high correlation between X_1 and X_2 with $r = 0.884$ needs attention in model interpretation.

Before applying the GWBZIGPR model, two spatial assumption tests were conducted. The Moran's I test yielded a statistic of 0.421 with a p-value of 0.008, indicating significant spatial autocorrelation in the residuals of the BZIGPR model. The Glejser test produced a statistic of 27.842 with a p-value of 0.003, confirming the presence of significant spatial heterogeneity across observation locations. These findings justify the use of the GWR approach [9, 17, 18].

3.2. Distribution of Response Variables

Figure 2 shows the distribution of maternal deaths during pregnancy and childbirth in East Java Province in 2025. The distribution is not uniform. Areas with the highest cases are concentrated in the southern and eastern parts, namely Malang Regency with $Y_1 = 5$ and $Y_2 = 7$, Jember Regency with $Y_1 = 8$ and $Y_2 = 3$, and Banyuwangi Regency with $Y_1 = 2$ and $Y_2 = 3$. Areas with low or zero cases are concentrated in the northern part, namely Bojonegoro Regency, Tuban Regency, and Kediri City. This non-uniform distribution pattern indicates spatial heterogeneity between regions, so modeling with the GWBZIGPR approach is appropriate [9, 19].

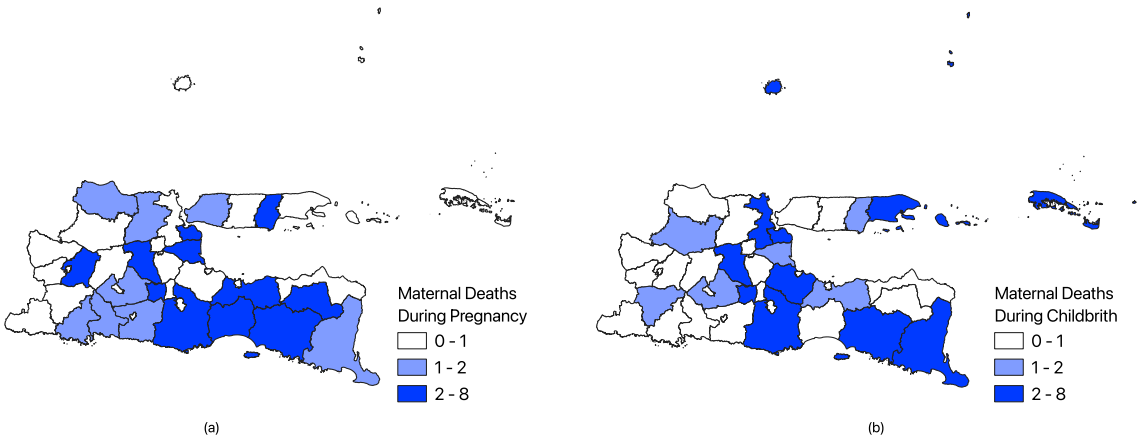


Fig. 2: Distribution of Maternal Deaths During Pregnancy (a) and During Childbirth (b) in East Java Province in 2025

3.3. Selection of Spatial Weighting Function

The selection of the best weighting function was carried out by comparing the AICc values of the GWBZIGPR model for each kernel function [9]. The comparison results are presented in Table 3. Based on Table 3, the Adaptive Bisquare weighting function produced the smallest AICc value of 575.2514, so it was used as the weighting function in the GWBZIGPR modeling. This finding differs from Purhadi et al. [9], who found that the Fixed Bisquare kernel was the best for the Pekalongan Residency data, likely due to the different spatial distribution of observation locations. The observation units in East Java Province are more unevenly distributed compared to Pekalongan Residency, with some areas having high concentrations of observation points and others having sparse distribution. The Adaptive Bisquare kernel, which allows the bandwidth to vary across locations, is better able to capture local variations in areas with sparse observations.

It is also important to note that the study by Purhadi et al. [9] utilized a substantially larger dataset comprising 91 sub-districts in the Pekalongan Residency, whereas the present study analyzes 38 districts and cities in East Java Province. This difference in sample size has direct implications for the applicability of the GWBZIGPR model. Geographically weighted

Table 3: AICc Values and Optimal Bandwidth for Each Spatial Kernel

Kernel	AICc	Bandwidth (m)	Neighbors
Fixed Gaussian	578.9654	156,343.5	30.59*
Fixed Bisquare	691.1917	231,176.7	18.09*
Adaptive Gaussian	639.4413	121,546.0†	33.43‡
Adaptive Bisquare	**575.2514**	230,560.6†	36

*Effective neighborhood size. †Median across 38 locations. ‡Median effective neighborhood size (range: 30.53–35.41).

models estimate a separate set of parameters at each observation location, and therefore require a sufficient number of observations within each local neighborhood to produce stable estimates. When the sample size is small relative to the number of parameters, the local estimation procedure lacks adequate degrees of freedom, leading to inflated standard errors and unreliable inference. Consequently, while the GWBZIGPR framework remains a valid and valuable methodological contribution, its application must be carefully matched to the data at hand, and alternative models such as the global BZIGPR may be more appropriate when the sample size is limited.

3.4. GWBZIGPR Modeling Results

The GWBZIGPR model with Adaptive Bisquare weighting achieved convergence for all 38 observation locations (100% convergence rate), with a log-likelihood of -225.1971, AIC of 496.3942, and AICc of 575.2514. The MSE values are 2.113924 for Y_1 and 1.506720 for Y_2, indicating an adequate in-sample fit. It should be noted that these MSE values are computed from the same data used for estimation and therefore reflect in-sample goodness of fit rather than out-of-sample predictive performance.

3.4.1. Simultaneous Significance Test

The overall significance of the GWBZIGPR model parameters was evaluated simultaneously using the Maximum Likelihood Ratio Test (MLRT). The test yielded a likelihood ratio statistic of $G^2 = 111.0431$ with 23 degrees of freedom. The computed p-value is < 0.0001, which is far below the 5% significance level, so the null hypothesis is rejected. This result confirms that the GWBZIGPR model is statistically significant overall, indicating that at least one predictor variable significantly affects maternal mortality. However, as discussed in the subsequent partial tests, no individual local parameter was statistically significant at the 5% level.

The dispersion parameters ψ_1 and ψ_2 range from 0.001 to 0.2513 and from 0.001 to 0.0919, respectively. In the Generalized Poisson parameterization adopted in this study, $\psi = 0$ corresponds to equidispersion, $\psi > 0$ indicates overdispersion, and $\psi < 0$ indicates underdispersion. The minimum values approaching 0 indicate near-Poisson behavior in some locations, while the positive maximum values indicate mild overdispersion in locations with the highest dispersion. Notably, none of the estimated dispersion parameters are negative, indicating that underdispersion is not present in this dataset.

The association parameter η ranges from -0.990 to 0.3085. The predominantly negative values of η across locations indicate a negative association direction between the two response variables at the local level, while the positive values observed in some locations indicate a positive association direction. This sign variation suggests that the local dependency structure between Y_1 and Y_2 is not uniform across locations.

However, η is a multiplicative factor in the joint probability function, not a Pearson correlation coefficient; its magnitude does not measure correlation strength, and its sign is not required to agree with the global Pearson correlation $r_{Y_1,Y_2} = 0.342$. The apparent contrast between predominantly negative local η and a positive global r therefore reflects the difference between a local dependency parameter and a global linear correlation, and should not be attributed to

spatial heterogeneity without further evidence [8, 9]. The GWBZIGPR model produces different parameter estimates for each observation location. As an illustrative example, the parameter estimates for Kabupaten Blitar are presented in Table 4, where the dispersion parameters $\psi_1 = 0.0010$ and $\psi_2 = 0.0010$ indicate near-Poisson behavior for both response variables, and the association parameter $\eta = -0.9900$ indicates a negative association direction between the two response variables in this location, though its magnitude does not directly represent the strength of the Pearson correlation.

Table 4: GWBZIGPR Parameter Estimation Results in Kabupaten Blitar (Adaptive Bisquare Kernel). None of the parameters are statistically significant at $\alpha = 5\%$.

Parameter	Estimate	SE	Z	p-Value
γ_{10} (Intercept)	-0.0465	28.3013	-0.0016	0.9987
γ_{11} (X_1)	-0.2656	4.6962	-0.0566	0.9549
γ_{12} (X_2)	0.5388	6.0754	0.0887	0.9293
γ_{13} (X_3)	-0.0478	0.1839	-0.2599	0.7949
γ_{14} (X_4)	0.4752	1.4386	0.3303	0.7412
γ_{20} (Intercept)	0.5746	27.5431	0.0209	0.9834
γ_{21} (X_1)	0.5772	3.5908	0.1608	0.8723
γ_{22} (X_2)	0.6060	8.6211	0.0703	0.9440
γ_{23} (X_3)	-0.0228	0.0592	-0.3848	0.7004
γ_{24} (X_4)	-0.8595	9.9435	-0.0864	0.9311
β_{10} (Intercept)	-0.5214	11.2664	-0.0463	0.9631
β_{11} (X_1)	0.0224	0.5591	0.0400	0.9681
β_{12} (X_2)	0.0537	0.6406	0.0839	0.9332
β_{13} (X_3)	0.0005	0.0028	0.1664	0.8679
β_{14} (X_4)	-0.0740	0.3973	-0.1864	0.8522
β_{20} (Intercept)	-1.7677	10.0596	-0.1757	0.8605
β_{21} (X_1)	-0.0055	0.4818	-0.0114	0.9909
β_{22} (X_2)	0.0211	0.3477	0.0606	0.9516
β_{23} (X_3)	0.0005	0.0064	0.0733	0.9415
β_{24} (X_4)	0.0053	0.3609	0.0147	0.9882
ψ_1 (Dispersion for Y_1)	0.0010	2.6640	0.0004	0.9997
ψ_2 (Dispersion for Y_2)	0.0010	4.9798	0.0002	0.9998
η (Association)	-0.9900	29.8095	-0.0332	0.9735

The local significance of the GWBZIGPR parameters was assessed using the Z-test with a significance level of $\alpha = 5\%$. Table 5 summarizes the number of districts and cities in which each parameter is statistically significant. Based on Table 5, none of the 23 parameters are statistically significant at the 5% level across the 38 districts and cities. The standard errors of the local estimates are substantially larger than the corresponding parameter estimates across locations. This indicates low estimation precision in the local GWBZIGPR model, likely due to the small sample size ($n = 38$) and the high dimensionality of the model (23 parameters per location).

The low precision of the local estimates can be attributed to two interrelated factors. First, with only $n = 38$ observations and 23 parameters estimated at each location, the effective number of observations available for local estimation is limited. In geographically weighted models, the local estimates at each location are computed using a weighted subset of observations, which further reduces the effective sample size. When the number of parameters is large relative to the effective sample size, the local design matrix becomes ill-conditioned, leading to inflated standard errors and less precise parameter estimates. Second, the presence of high collinearity between X_1 and X_2 ($r = 0.884$) further reduces the effective information available for local estimation, as the local design matrix becomes nearly singular, which inflates the variance of the parameter estimates and makes them highly sensitive to small changes in the data or the weighting scheme.

Table 5: Summary of Local Parameter Estimates and Significance Across 38 Districts and Cities

Parameter	Min Est	Median Est	Max Est	N Sig.	Percent
$\gamma_{1,\text{Intercept}}$	−2.634	0.222	8.239	0	0%
γ_{1,X_1}	−1.997	0.083	13.934	0	0%
γ_{1,X_2}	−10.321	0.184	6.093	0	0%
γ_{1,X_3}	−1.715	−0.049	−0.001	0	0%
γ_{1,X_4}	−9.913	0.046	4.834	0	0%
$\gamma_{2,\text{Intercept}}$	−7.122	−0.125	7.462	0	0%
γ_{2,X_1}	−11.288	−0.984	10.628	0	0%
γ_{2,X_2}	−2.524	−0.351	13.122	0	0%
γ_{2,X_3}	−0.240	0.041	0.736	0	0%
γ_{2,X_4}	−12.235	−1.390	2.490	0	0%
$\beta_{1,\text{Intercept}}$	−8.006	−0.935	0.591	0	0%
β_{1,X_1}	−0.108	−0.029	0.034	0	0%
β_{1,X_2}	−0.022	0.046	0.095	0	0%
β_{1,X_3}	−0.00007	0.0004	0.003	0	0%
β_{1,X_4}	−0.074	−0.008	0.161	0	0%
$\beta_{2,\text{Intercept}}$	−2.569	−1.667	−0.634	0	0%
β_{2,X_1}	−0.018	0.035	0.127	0	0%
β_{2,X_2}	−0.116	−0.022	0.034	0	0%
β_{2,X_3}	0.0005	0.0012	0.0031	0	0%
β_{2,X_4}	−0.064	−0.005	0.014	0	0%
ψ_1	0.001	0.001	0.251	0	0%
ψ_2	0.001	0.001	0.092	0	0%
η	−0.990	−0.990	0.308	0	0%

3.5. Model Comparison

The AICc was used to select the optimal kernel among the GWBZIGPR models, while the global BZIGPR and local GWBZIGPR models were evaluated with careful attention to estimation precision and inferential reliability. The results are presented in Table 6. Table 6 reports the log-likelihood and AICc values. The GWBZIGPR model with the Adaptive Bisquare kernel produced the smallest AICc (575.2514) among the local models, while the global BZIGPR model produced AICc = 1914.814. This numerical difference must not be interpreted as evidence that GWBZIGPR substantially outperforms BZIGPR. The AICc penalty term assumes a single set of k parameters estimated from n observations; for GWBZIGPR, a separate set of 23 parameters is estimated at each of the 38 locations, so the nominal $k = 23$ does not represent the effective degrees of freedom, and the product $23 \times 38 = 874$ cannot be treated directly as the effective number of parameters either.

Because a defensible effective degrees-of-freedom correction is not available here, the AICc values are not directly comparable across the two model families, and no superiority claim is made. The lack of statistical significance for all local estimates (as shown in Table 5) further indicates that the apparent improvement in in-sample fit is achieved at the cost of unreliable inference. Therefore, the BZIGPR model remains the preferred model for this dataset because of its numerical stability and interpretability, despite its higher AICc.

Table 6: Comparison of AICc Values of Models

Model	Log-Likelihood	AICc
BZIGPR	-894.9784	1914.814
GWBZIGPR (Adaptive Bisquare)	-225.1971	**575.2514**
GWBZIGPR (Adaptive Gaussian)	-257.2921	639.4413
GWBZIGPR (Fixed Bisquare)	-283.1673	691.1917
GWBZIGPR (Fixed Gaussian)	-227.0541	578.9654

3.6. Discussion

Modeling with BZIGPR generates the same parameters for each observation location [8]. This differs from the GWBZIGPR model, which generates distinct parameter estimates for each location. These differences in characteristics among observation locations cause the factors influencing maternal and postpartum mortality to vary by location. Consequently, while the GWBZIGPR model is methodologically designed to capture this local variation, the low estimation precision of the local estimates for this dataset means that the global BZIGPR model remains the more appropriate and preferred model in practice. However, based on the comparison of four weighting functions, the Adaptive Bisquare kernel produced the smallest AICc value of 575.2514, which contrasts with Purhadi et al. [9], who found the Fixed Bisquare kernel optimal for Pekalongan Residency. This discrepancy likely stems from the uneven spatial distribution of observation units in East Java Province, which features both areas with high concentrations of observation points and areas with sparse observation points [20, 21].

As shown in Table 5, no parameter reaches statistical significance in any of the 38 districts and cities, including Kabupaten Blitar. The standard errors of the local estimates are substantially larger than the parameter estimates across locations, indicating low estimation precision due to the small sample size ($n = 38$) and the high dimensionality of the model (23 parameters per location). Consequently, no region-specific significant factors can be identified from the GWBZIGPR model for this dataset.

This finding contrasts with the study by Purhadi et al. [9], where the GWBZIGPR model successfully produced significant local estimates because it utilized a substantially larger dataset comprising 91 sub-districts. In the present study, with only 38 districts and cities, the estimation of 23 local parameters per location lacks sufficient degrees of freedom. This condition leads to ill-conditioned local design matrices, which are further exacerbated by the high collinearity between X_1 and X_2 ($r = 0.884$). Consequently, the standard errors of the local estimates become substantially inflated, rendering all local parameters statistically insignificant. This finding illustrates, for the dataset and model conditions examined here, a practical limitation of the GWBZIGPR model: its local estimates become imprecise when the sample size is small relative to the number of parameters. It should not be generalized into a universal sample-size threshold.

This finding suggests that the lack of significance is not primarily due to multiple testing, even though the total number of local tests is substantial (23 parameters tested across 38 locations, yielding 874 tests in total), but rather due to the large standard errors arising from the small sample size and high model dimensionality. A Bonferroni-type adjustment would only increase the effective significance threshold and make rejection even less likely, which confirms that the absence of significance is driven by low estimation precision rather than by multiplicity alone. This is consistent with the known limitations of geographically weighted models in small samples, where the effective degrees of freedom for local estimation are insufficient to produce stable estimates [10].

The problem is compounded by the interaction between sample size and model complexity, since with $n = 38$ observations and 23 parameters per location, the ratio of observations to parameters is approximately 1.65, which is low relative to what is typically required for stable local estimation [10]. The high correlation between X_1 and X_2 ($r = 0.884$) further reduces the effective information available for local estimation, and future studies should therefore consider either reducing the number of parameters, increasing the sample size, or using regularization techniques such as ridge regression or the LASSO to address these issues.

It should also be noted that the coefficient for X_3 (the number of active integrated health posts, or Posyandu) is near zero and not statistically significant in any of the 38 locations. This is an empirical finding and should not be interpreted causally, since the number of Posyandu does not necessarily reflect the quality of maternal health services and no conclusion can be drawn about the effect of Posyandu availability on maternal mortality from this analysis. Any proposed explanation for this pattern, such as the possibility that Posyandu density is not a direct

determinant of maternal mortality, remains speculative and would require further investigation with better measures of service quality.

3.7. Substantive Contributions Beyond Model Application

The contribution of this study extends beyond the application of the GWBZIGPR model to a new dataset. Three substantive contributions are identified. First, the systematic comparison of four spatial weighting functions reveals that the optimal kernel is data-dependent, since Adaptive Bisquare is optimal for East Java Province whereas Purhadi et al. [9] found Fixed Bisquare optimal for Pekalongan Residency. Second, the study provides empirical evidence from the present dataset that the GWBZIGPR model may exhibit low estimation precision when the sample size ($n = 38$) is small relative to the number of parameters. This finding serves as a cautionary note for similar applications, rather than a general minimum sample-size requirement. Third, the study recommends that the global BZIGPR model may be preferable to GWBZIGPR for datasets with limited sample size, balancing in-sample fit against the reliability of estimates.

This study acknowledges several limitations. First, the use of cross-sectional data restricts the ability to establish causal relationships. Second, reliance on secondary administrative data may introduce reporting errors or inconsistencies. In particular, three predictor variables (X_1, X_2, and X_4) have maximum values exceeding 100%, which reflects the fact that coverage percentages in administrative health data are calculated relative to estimated target populations rather than exact counts. While these values are not data errors and do not compromise the regression analysis, they should be interpreted as indicators of relative coverage rather than exact proportions. Third, the relatively small sample size ($n = 38$ districts and cities) may limit the generalizability of the findings. Future research should consider utilizing longitudinal data, incorporating additional relevant predictors, or integrating advanced optimization methods such as the Flower Pollination Algorithm to determine a more optimal bandwidth [22, 23].

4. Conclusion

This study applied the Geographically Weighted Bivariate Zero-Inflated Generalized Poisson Regression (GWBZIGPR) model to maternal mortality data in 38 districts and cities of East Java Province in 2025. The data exhibit overdispersion, excess zeros relative to the Poisson specification, significant bivariate correlation, and significant spatial heterogeneity, which justify the use of a bivariate zero-inflated model. Among the four spatial weighting functions compared, the Adaptive Bisquare kernel yielded the smallest AICc value of 575.2514, indicating that the optimal weighting function is data-dependent and varies across study regions.

The AICc values of the GWBZIGPR and BZIGPR models are not directly comparable because their effective degrees of freedom differ fundamentally; model preference is therefore based on estimation precision rather than on AICc. Furthermore, none of the 23 local parameters were statistically significant at the 5% level, and the standard errors of the local estimates were substantially larger than the parameter estimates across locations. This indicates that the GWBZIGPR model suffers from low estimation precision for this dataset, and that the BZIGPR model, which estimates parameters globally, provides stable estimates and remains the preferred model for this dataset.

This study has several limitations. First, the cross-sectional design does not allow for causal inference. Second, the reliance on secondary administrative data may introduce reporting errors. Third, the small sample size ($n = 38$) limits the generalizability of the findings. Future research should explore model simplification strategies, such as estimating the zero-inflation component globally rather than locally, or using regularization techniques such as ridge regression or the LASSO.

CRediT Authorship Contribution Statement

Citra Tiara Ningati: Conceptualization, Methodology, Software, Formal Analysis, Data Curation, Writing-Original Draft, Visualization.
Henny Pramoedyo: Supervision, Validation, Writing – Review & Editing.

Declaration of Generative AI and AI-assisted Technologies

The authors confirm that no generative AI tools were used to produce or alter the data, results, or analyses presented in this study. AI-assisted tools were applied solely for language polishing and formatting purposes.

Declaration of Competing Interest

The authors declare that they have no financial or personal interests that could have influenced the research or its interpretation.

Ethics Approval and Consent to Participate

Ethical approval was not required for this study, since only publicly accessible secondary data from the East Java Provincial Health Office were used. No primary data involving human participants or animals were collected.

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Data and Code Availability

Data are openly accessible through the East Java Provincial Health Profile 2025 at https://dinkes.jatimprov.go.id, while the administrative shapefile and the full R script are available from the Geospatial Information Agency and the corresponding author, respectively.

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