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The Rainbow Vertex-Connection Number of Star Fan Graphs

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ABSTRACT

A vertex-colored graph $G = (V(G), E(G))$ is said to be rainbow vertex-connected, if for every two vertices u and v in $V(G)$, there exists a $u - v$ path with all internal vertices have distinct colors. The rainbow vertex-connection number of G, denoted by $rvc(G)$, is the smallest number of colors needed to make G rainbow vertex-connected. In this paper, we determine the rainbow vertex-connection number of star fan graphs.

Keywords: Rainbow Vertex-Coloring; Rainbow Vertex-Connection Number; Star Fan Graph; Fan

INTRODUCTION

All graph considered in this paper are finite, simple, and undirected. We follow the notation and terminology of Diestel [1]. A vertex-colored graph $G = (V(G), E(G))$ is said to be rainbow vertex-connected, if for every two vertices u and v in $V(G)$, there exists a $u - v$ path with all internal vertices have distinct colors. The rainbow vertex-connection number of G, denoted by $rvc(G)$, is the smallest number of colors needed to make G rainbow vertex-connected. It was introduced by Krivelevich and Yuster [2].

Let G be a connected graph, n be the size of G, and diameter of G denoted by $diam(G)$, then they stated that

$$diam(G) - 1 \leq rvc(G) \leq n - 2 \tag{1}$$

Besides that, if G has c cut vertices, then

$$rvc(G) \geq c \tag{2}$$

In fact, by coloring the cut vertices with distinct colors, we obtain $rvc(G) \geq c$. It is defined that $rvc(G) = 0$ if G is a complete graph

There are many interesting results about rainbow vertex-connection numbers. Some of them were stated by Li and Liu[3] and Simamora and Salman[4] and Bustan [5]. Li and Liu determined the rainbow vertex-connection number of a cycle C_n of order $n \geq 3$. Based on it, they proved that for a connected graph G with a block decomposition $B_1, B_2, \dots, B_k$ and c cut vertices, $rvc(B_1) + rvc(B_2) + \cdots + rvc(B_k) + t$. In 2015 Simamora and Salman determined the rainbow vertex-connection number of pencil graph. In 2016 Bustan determined the rainbow vertex-connection number of star cycle graph.

In this paper, we introduce a new class of graph that we called star fan graphs and we determine the rainbow vertex-connection number of them. Star fan graphs are divided into two classes based on the selection of a vertex of the fan graph ie a vertex with n degree and vertex with 3 degree.

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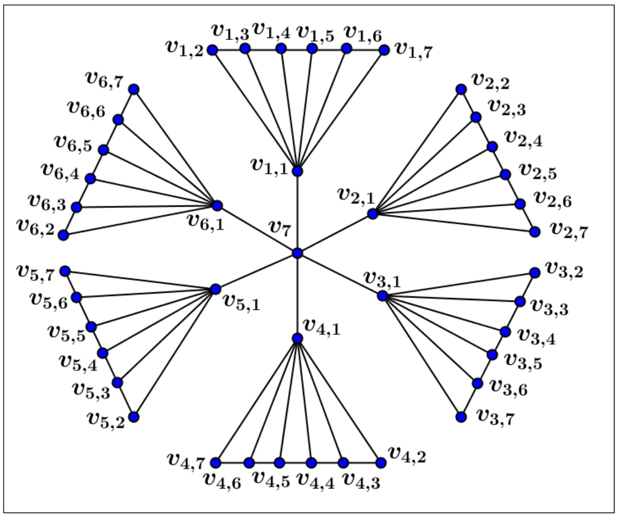

Figure 1. $S(6, F_6, v_{i,1})$

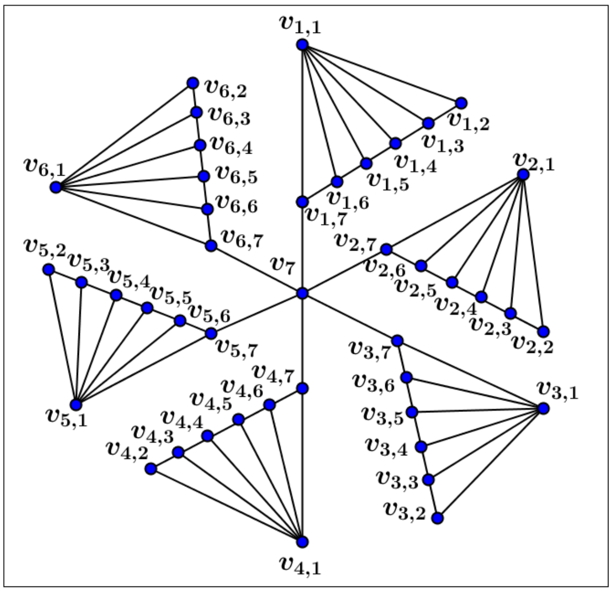

Figure 2. $S(6, F_6, v_{i,7})$

RESULTS AND DISCUSSION

Definition 1. Let m and n be two integers at least 3, S_m be a star with $m+1$ vertices. F_n be a fan with $n+1$ vertices, $v \in V(F_n)$ and v is a vertex with n degree. A star fan graph is a graph obtained by embedding a copy of F_n to each pendant of S_m, denoted by $S(m, F_n, v_{i,1})$ $i \in [1, m]$, such that the vertex set and the edge set, respectively, as follows.

$$V\left(S(m, F_n, v_{i,1})\right) = \{v_{i,j} | i \in [1, m], j \in [1, m+1]\} \cup \{v_{m+1}\},$$

$$E\left(S(m, F_n, v_{i,1})\right) = \{v_{m+1}v_{i,1} | i \in [1, m]\} \cup \{v_{i,1}v_{i,j} | \, i \in [1, m], j \in [2, m+1]\} \cup \{v_{i,j}v_{i,j+1} | \, i \in [1, m], j \in [2, m]\}$$

Theorem 1. Let m and n be two integers at least 3 and $S(m, F_n, v_{i,1})$ be a star fan graph, then

$$rvc\left(S(m, F_n, v_{i,1})\right)=m+1$$

Proof.
Based on equation (2), we have

$$rvc\left(S(m, F_n, v_{i,1})\right) \geq c = m+1 \quad (3)$$

In order to proof $rvc\left(S(m, F_n, v_{i,1})\right) \leq m+1$, define a vertex-coloring $\alpha: V\left(S(m, F_n, v_{i1})\right) \to [1, m+1]$ as follows.

$$\alpha(v_{i,j}) = \begin{cases} \boldsymbol{i}, & for\ j=1 \\ \boldsymbol{m+1}, & others \end{cases}$$

We are able to find a rainbow path for every pair vertices u and v in $V\left(S(m, F_n, v_{i,1})\right)$ as shown in table 1

Tabel 1. The rainbow vertex $u - v$ path for graph $S(m, F_n, v_{i,1})$

$\boldsymbol{u}$	$\boldsymbol{v}$	Condition	Rainbow-vertex path
u is adjacent to v			Trivial
$\boldsymbol{v_{i,j}}$	$\boldsymbol{v_{k,l}}$	$\boldsymbol{i, k \in [1, m]}$ $\boldsymbol{j, l \in [1, m+1]}$	$\boldsymbol{v_{i,j}, v_{i,1}, v_{m+1}, v_{k,1}, v_{k,l}}$

So we conclude that

$$rvc\left(S(m, F_n, v_{i,1})\right) \leq m+1 \quad (4)$$

From equation (3) and (4), we have $rvc\left(S(m, F_n, v_{i,1})\right) = m+1$

Definition 2. Let m and n be two integers at least 3, S_m be a star with $m+1$ vertices. F_n be a fan with $n+1$ vertices, $v \in V(F_n)$ and v is a vertex with 3 degree. A star fan graph is a graph obtained by embedding a copy of F_n to each pendant of S_m, denoted by $S(m, F_n, v_{i,j})$ $i \in [1, m], j \in [2, m]$ such that the vertex set and the edge set, respectively, as follows.

$V\left(S(m, F_n, v_{i,j})\right) = \{v_{i,j} | i \in [1, m], j \in [, m+1]\} \cup \{v_{m+1}\}$,

$E\left(S(m, F_n, v_{i,j})\right) = \{v_{m+1}v_{i,j} | i \in [1, m], j \in [2, m+1]\} \cup \{v_{i,1}v_{i,j} |\ i \in [1, m], j \in [2, m+1]\} \cup \{v_{i,j}v_{i,j+1} |\ i \in [1, m], j \in [2, m]\}$

Theorem 2. Let m and n be two integers at least 3 and $S(m, F_n, v_{i,j})$ be a star fan graph, then

$$rvc\left(S(m, F_n, v_{i,j})\right)=m+2$$

Proof.

- ***Case 1.*** $\boldsymbol{m=3}$

Based on equation (1), we have $rvc\left(S(3, F_3, v_{i,4})\right) \geq diam - 1 = 6 - 1 = 5$. We may define a rainbow vertex 5-coloring on $S(m, F, v_{i,7})$ as shown in Figure 2.

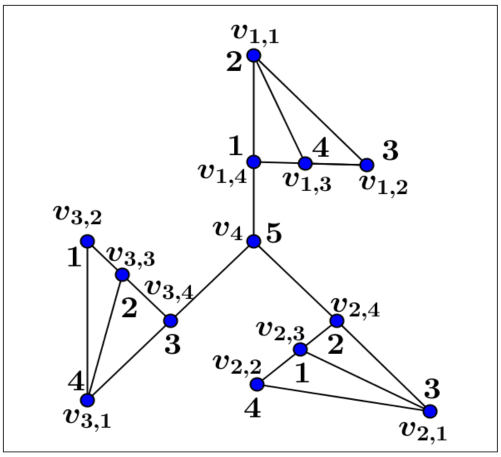

Figure 3. $S(3, F_3, v_{i,4})$

- ***Case 2.*** $\boldsymbol{m \geq 4}$

Based on equation (2), we have $rvc\left(S(m, F_n, v_{i,j})\right) \geq m+1$. Suppose that There is a rainbow vertex $m+1$-coloring on $S(m, F_n, v_{i,j})$. Without loss of generality, color the vertices as follows:

$\beta'(v_{m+1}) = m+1$
$\beta'(v_{i,m+1}) = i, i \in [1, m]$

Look at the vertex $v_{1,2}$ and $v_{2,2}$ who can not use the same color. To obtain rainbow vertex path between them, should be passed the path of $v_{1,2}, v_{1,1}, v_{1,m+1}, v_{m+1}, v_{2,m+1}, v_{2,1}, v_{2,2}$. Certainly $v_{1,1}$ should be colored by the color which used at the cut vertices, beside color $1, m+1$ and 2. Suppose that $v_{1,1}$ being color with k. It's impacted there is no rainbow vertex path between vertex $v_{k,2}$ and $v_{i,2}$ for $i \neq k$. So that, graph $S(m, F_n, v_{i,j})$ cannot be colored with $m+1$ colors, so we obtain

$$rvc\left(S(m, F_n, v_{i,j})\right) \geq m+2 \qquad (5)$$

In order to proof $rvc\left(S(m, F_n, v_{i,j})\right) \leq m+2$, define a vertex-coloring $\beta: V\left(S(m, F_n, v_{i,j})\right) \to [1, m+2]$ as follows.

$\beta(v_{m+1}) = m+2$
$\beta(v_{i,j}) = (i+j) mod (m+1), i \in [1, m], j \in [1, m+1]$

We are able to find a rainbow path for every pair vertices u and v in $V\left(S(m, F_n, v_{i,j})\right)$ as shown in table 2.

Tabel 2. The rainbow vertex $u - v$ path for graph $S(m, F_n, v_{i,j})$

$\boldsymbol{u}$	$\boldsymbol{v}$	Condition	Rainbow-vertex path
u is adjacent to v			Trivial
$\boldsymbol{v_{i,j}}$	$\boldsymbol{v_{k,l}}$	$\boldsymbol{i, k \in [1, m]}$ $\boldsymbol{j, l \in [1, m+1]}$ $\boldsymbol{k \neq i+l, k \neq i-l, k \in [2, m-1]}$	$\boldsymbol{u, v_{i,j+1}, v_{i,j+2}, \dots, v_{i,m+1}, v_{m+1}, v_{k,m+1}, v_{k,}}$
		If $\boldsymbol{i = 1, k \neq m}$ others	$\boldsymbol{u, v_{i,1}, v_{i,m+1}, v_{m+1}, v_{k,m+1}, v_{k,1}, v}$

So we conclude that

$$rvc\left(S(m, F_n, v_{i,j})\right) \leq m + 2 \qquad (6)$$

From equation (5) and (6), we have $rvc\left(S(m, F_n, v_{i,j})\right) = m + 2$

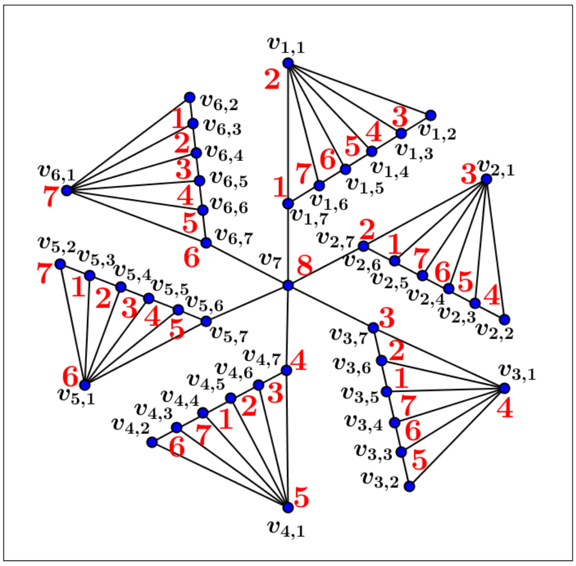

Figure 4. $S(6, F_6, v_{i,7})$

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