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LOCAL HÖLDER REGULARITY OF WEAK SOLUTIONS FOR SINGULAR PARABOLIC SYSTEMS OF p -LAPLACIAN TYPE

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Abstract. In this article, the Hölder regularity weak solutions a singular parabolic system of p -Laplacian type is investigated. By the Poincaré inequality, we show that its weak solution within Hölder space.

Keywords: weak solutions, singular case, Hölder regularity

1 Introduction

Hölder estimates of solutions for singular parabolic equations with measurable coefficients was introduced by Chen and DiBenedetto in [3]. After that, researchers were interested to study the other case of parabolic equations. For example, the interior and boundary Hölder continuity for weak solutions and gradient solutions of degenerate parabolic equations was proved in [9] and [2]. Then, [15] investigated the same problem for bounded weak solutions of nonlinear parabolic equations for the degenerate case. DiBenedetto, et al discussed the question of the regularity of the weak solutions of singular and degenerate quasilinear parabolic equations to proving their Hölder character in [11]. In 2013, Misawa extended the Hölder regularity results by [11] for weak solutions of the p -Laplacian systems originally obtained by [3] to a larger class of right-hand side terms, the singular case was excluded here see [13]. Meanwhile, for the singular case we can see the Hölder regularity in [5]. In addition, the Hölder regularity of gradients and the existence of global weak solutions for evolutionary p -Laplacian systems with Hölder continuous coefficients and exterior force was proved in [12], [6]. In 2008, Corina and Marjono investigated the local boundedness of weak solutions of p -Laplacian type [7]. To prove the local boundedness in [7], we can adapt the method to prove energy estimates of singular parabolic equations in [10]. On the other hand, the existence of global weak solutions of p -Laplacian type which proved in [1] by using the Galerkin method. Furthermore, the intrinsic scaling method was used to treat the local Hölder regularity of weak solutions for degenerate case [8]. The intrinsic scaling method is based on [14], which used the intrinsic scaling method to approach the regularity for degenerate and singular PDEs.

Motivated by the last result of [8], in this paper, we would investigated the validity of theorem of local Hölder regularity of weak solutions for singular case.

2 Preliminaries

Let Ω be a bounded domain in $\mathbb{R}^n$, $n \geq 2$ with smooth boundary $\partial\Omega$. The unknown function $u : (0, T) \times \Omega \rightarrow \mathbb{R}^m$ is vector valued function, $u = u(z) = (u^1(z), u^2(z), \dots, u^m(z))$, where $z = (t, x_1, x_2, \dots, x_n)$. Let $\frac{2m}{m+2} < p < 2$, $m \geq 2$, we consider the following p -Laplacian type as below

$$\begin{cases} \partial_t u - \operatorname{div}(|Du|^{p-2} Du) = 0 & \text{di } (0, T) \times \Omega, \\ u(0, x) = u_0(x) & \text{pada } \partial_p(0, T) \times \Omega, \end{cases} \quad (1)$$

where $u_0(x) \in W^{1,p}(\Omega, \mathbb{R}^m)$.

The result in [1] shows that for any initial data, there exists a weak solution of (1) from Ω into $\mathbb{R}^m$. On the other hand, the local boundedness of the weak solution of (1) was proved by Corina, K., and Marjono in [7]. They modified the intrinsic scaling from the original work by DiBenedetto [10]. They used the intrinsic scaling for singular case. Their main theorem established that intrinsic scaling

well-worked to prove the local boundedness of weak solution of (1).

We now turn to the notion of Hölder continuous functions. For any $x, y \in \bar{\Omega}$, if $u \in C^0(\bar{\Omega})$ satisfy

$$\sup_{x, y \in \bar{\Omega}, x \neq y} \frac{|u(x) - u(y)|}{|x - y|^\alpha} < \infty, \quad (13)$$

then $u \in C^{0, \alpha}(\bar{\Omega})$. Corina, K. in [8] shows that the weak solution of (1) in degenerate case is locally Hölder continuous by the following theorem.

Theorem 1. Let $p \geq 2$ and let u be a weak solution of (1) in $Q(\lambda^{1-p}\rho^2, \rho)(z_0)$. Then u is locally Hölder continuous with some positive $\alpha < 1$. Moreover, for any $Q_{3\rho_0}(z'_0) \subset Q$ with $z'_0 \in Q$ and $0 < \rho_0 \leq 1$, there exist a positive constant C such that

$$|u(z) - u(z')| \leq C\{|t - t'|^\frac{\alpha}{p} + |x - x'|^\alpha\}$$

holds for any $z, z' \in Q_{\rho_0}$ where $Q_r(z_0) \subset Q(\lambda^{2-p}\rho^2, \rho)(z_0) \subset Q_{\rho_0}(z'_0) \subset Q_{3\rho_0}(z'_0)$.

3 Main Result

For singular equation (1) with $1 < p < 2$, we have the cylinder of the type

$$Q(\rho)(z_0) = (t_0 - \rho^2, t_0 + \rho^2) \times B(\lambda^{\frac{p-2}{2}}\rho, x_0),$$

By switching the cylinder $Q(\rho)(z_0)$ to $Q(1)$, we have

$$v(s, y) = \frac{u(t_0 + \lambda^{2-p}\rho^2 s, x_0 + \rho y)}{\lambda \rho}; \quad (s, y) \in B(1) \times (-1, 1) \equiv Q(1),$$

Suppose that on such a certain cylinder the relations

$$\int_{Q(\rho)(z_0)} |Du|^p dz \approx \lambda^p. \quad (2)$$

Hence, we have

$$\partial_t u - \lambda^{p-2} \operatorname{div}(Du) = 0,$$

in $Q(\rho)(z_0)$.

Let $u \in L^\infty(0, T; W^{1,p}(\Omega, \mathbb{R}^m))$ be a weak solutions of (1) in cylinder $Q = (0, T) \times \Omega$ where $z_0 = (t_0, x_0) \in Q$, $\lambda \geq 1$ and $0 < \rho < 1$ such that $Q(\rho^2, \lambda^{\frac{p-2}{2}}\rho) \subset Q$. While, our main theorem is the following.

Theorem 1. Let $\frac{2m}{m+2} < p < 2$, $m \geq 2$ and u is a weak solution of (1) in $Q(\rho^2, \lambda^{\frac{p-2}{2}}\rho)$, then u is locally α -Hölder continuous with some $0 < \alpha < 1$. Moreover, for any $Q_{3\rho_0}(z'_0) \subset Q$ with $z'_0 \in Q$ and $0 < \rho_0 \leq 1$, there is $C > 0$ such that

$$|u(z) - u(z')| \leq C\{|t - t'|^\frac{\alpha}{p} + |x - x'|^\alpha\},$$

holds for any $z, z' \in Q_{\rho_0}(z'_0)$ where $Q_r(z_0) \subset Q(\rho^2, \lambda^{\frac{p-2}{2}}\rho)(z_0) \subset Q_{\rho_0}(z'_0) \subset Q_{3\rho_0}(z'_0)$.

Proof.

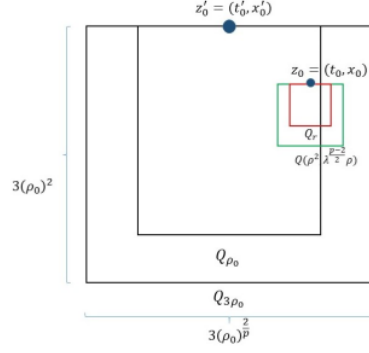


Figure 1: Any cylinder in Q

Let $Q_{3\rho_0}(z'_0) := Q(3(\rho_0)^2, 3(\rho_0)^{\frac{2}{p}}) \subset Q$ is any cylinder with center $z'_0 = (t'_0, x'_0) \in Q$, and $0 < \rho_0 \leq 1$. Let $z_0 = (t_0, x_0) \in Q_{\rho_0}(z'_0) := Q((\rho_0)^2, (\rho_0)^{\frac{2}{p}})(z'_0)$ be arbitrary and $0 < r < \rho \leq 2(\rho_0)^{\frac{2}{p}}$. We work on $Q_r(z_0) := Q(r^p, r)(z_0) \subset Q(\rho^2, \lambda^{\frac{p-2}{2}} \rho)(z_0) \subset Q_{3\rho_0}(z'_0)$, where $\lambda \geq 1$. It is easy to see that

$$\int_{Q_r(z_0)} |Du|^p dz \leq |Q_r(z_0)| \int_{Q(\rho^2, \lambda^{\frac{p-2}{2}} \rho)(z_0)} |Du|^p dz$$

By using relation (2) we have.

$$\int_{Q_r(z_0)} |Du|^p dz \leq C r^{m+p\alpha}, \quad (3)$$

where $0 < r < 1$, $0 < \alpha < 1$, and $C = \pi r^{2-m}$.

Next, we substitute (3) to Poincaré inequality to have

$$\begin{aligned} \int_{Q_r(z_0)} |u - u_r|^p dz &\leq C r^p \int_{Q_r(z_0)} |Du|^p dz \\ &\leq C r^{m+p+p\alpha} \end{aligned}$$

holds for any $z_0 \in Q_{\rho_0}(z'_0)$ and $0 < r < 2(\rho_0)^{\frac{2}{p}}$.

We invoke the characterization of Hölder continuous functions due to Campanato, which implies that for $u \in L^p(Q_{\rho_0}(z'_0))$ and

$$\sup_{Q_{\rho_0}(z'_0)} \frac{1}{r^{m+p+p\alpha}} \int_{Q_r(z_0)} |u - u_r|^p dz \leq C, \quad (4)$$

then

$$u \in C^{0,\alpha}(Q_{\rho_0}(z'_0)).$$

Thus, we have for any two points $(t, x), (t', x') \in Q_{\rho_0}(z'_0)$ with $|t - t'| = r^p$,

$$|u(t, x) - u(t', x)| \leq C |t - t'|^{\frac{\alpha}{p}}, \quad (5)$$

and let $|x - x'| = r$, then

$$|u(t, x') - u(t, x)| \leq C |x - x'|^\alpha. \quad (6)$$

Moreover, we conclude from (5), (6), and triangle inequality that

$$|u(z) - u(z')| \leq C \{|t - t'|^{\frac{\alpha}{p}} + |x - x'|^\alpha\}$$

in $Q_{\rho_0}(z'_0)$ or we have

$$u \in C^{\frac{\alpha}{p}, \alpha}(Q_{\rho_0}(z'_0)).$$

Since, $Q_{3\rho_0}(z'_0) \subset Q$ is arbitrary, then $u \in C^{\frac{\alpha}{p}, \alpha}_{loc}(Q, \mathbb{R}^n)$.

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