



Elliptical Orbits Mode Application for Approximation of Fuel Volume Change

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ABSTRACT

This article discusses the Elliptical Orbits Mode (EOM) as a method of approximating the function of changing the volume of fuel in the Underground Tank (UT). This research was conducted at the 45.507.21 Candirejo Tuntang Pertamina Gas Station. The calculation of the approximation method will be applied to the measuring book data from the Semarang Metrology Regency specifically for the Peralite (Fuel Product of Pertamina) buried tank, because the calculation of the gas station is not smooth, it is necessary for a smoother data fitting by considering Residual Square Error (RSS) and Mean Square Error (MSE). The result of this research is the application of EOM(θ) measuring book with elliptical height control produces smaller RSS and MSE compared to using COM, EOM, Least Square degree two and three.

Keywords: orbits mode, data fitting, ellips, fuel, approximation

INTRODUCTION

Based on the assumptions given in [1] and [2] the previous Orbits Mode Data Fitting research which was used to calibrate the Dipstick measuring instrument that converts the height to the volume of fuel in the buried tank, it is explained that the approximation function of the volume of fuel in the tank is only based on height.

In [1] and [2] it is explained that the Orbits Mode Data Fitting-based calibration is limited by several assumptions and field conditions, including the following:

1. The resulting approximation function is the change in the volume of fuel in the UT which only depends on the variable height of the fuel in the UT.
2. Orbits Mode Data Fitting proposed by the researcher is used only for the distribution of data that forms a semicircle or ellipse in the first quadrant.
3. The data to be approximated is the fuel measurement manual in the UT from the Semarang Regency Metrology Agency.
4. It is assumed that UT is not tilted or flat during measurement, including tank trucks that deliver fuel to gas stations or are filling supplies at gas stations.
5. UT for the right and left have the same shape alias symmetrical, due to the second assumption.

The approximation function follows the pattern of the UT cross-section, where the data used is the change in the volume of fuel in the tank based on changes in the height of the fuel in the UT in units (cm).

Orbits Mode Data Fitting is a method proposed by the author in approximating the function of the data which tends to be in the form of a semi-circle or half an ellipse. In [1]

and [2] the researcher introduced the Orbits Mode Data Fitting method only in a circle shape, then compared it with Cubic Spline Interpolation and Least Square Data Fitting, but this time the researchers made the Orbits Mode Data Fitting method in the shape of an ellipse too, because in the value approach there is a volume of fuel which has not been detected in the function.

In [3] curve fitting which is applied to approximate the shape of an island on the map, then in [4] curve fitting which is used to detect enlarged and shrinking eye retina, research in [3] and [4] has its own additional algorithm to approach the desired result, which will also be applied to the Orbits Mode Data Fitting starting from the proposed method and the object applied to calibration is something new.

In [5] Hyper Least Square or HyperLS was also introduced and [6] also calibrated data in the form of curves, but using an orthogonal matrix where the more data the more complicated, so the method will be difficult for large data.

Therefore, the researcher proposes the following method because the calculation is simpler for small and large data and is smoother, although only for data that tends to be semicircular or elliptical, to approach the value of the fuel volume with minimized errors.

For example $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$, with n the amount of data, it will approximate a function that $y = f(x)$ is close to the actual value, namely, and for the calibration case using a dipstick, namely changing the oil level into the volume of fuel oil.

Definition 1 (Ellips Equation)

In [7] the ellipse equation is presented in equation (1) which (p, q) is the center point of the ellipse with the major and minor axes adjusting a and b ,

$$\frac{(x - p)^2}{a^2} + \frac{(y - q)^2}{b^2} = 1 \quad (1)$$

Definition 2 (A_i set for Ellips Mode)

Next, a set of points will be formed between the equations of 2 ellipses, denoted as follows,

$$A_i = \left\{ (x, y) \mid d_{i1} < \left(\frac{xw}{\frac{w}{2}} \right)^2 + \left(\frac{yl}{\frac{l}{2}} \right)^2 < d_{i2} \right\}, \quad (2)$$

With $i = 1, 2, \dots, k$. the set (13) it can be visualized as follows,

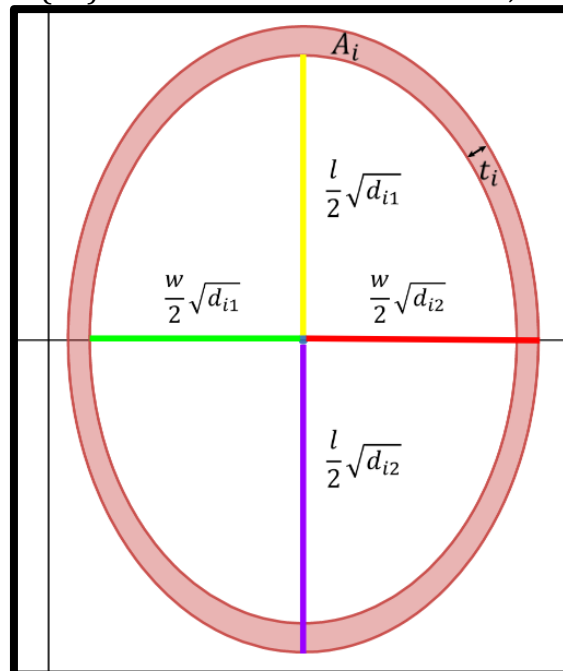


Figure 1. Set Visualization A_i (2)

with $x_{\frac{w}{2}} = \left(x - \frac{w}{2}\right)$ and $w = \text{UT height}$ and $l = \text{UT width}$.

Defined $\frac{w}{2}(\sqrt{d_{i2}} - \sqrt{d_{i1}}) = \frac{l}{2}(\sqrt{d_{i2}} - \sqrt{d_{i1}}) = t_i$ (thickness) the thickness of the ellipse from the partition interval taken from the maximum and minimum values of the volume change $[\Delta V(h)_{\min}, \Delta V(h)_{\max}]$ will be divided by several partitions where the thickness with the most points is sought, then

$$[\Delta V(h)_{\min}, \Delta V(h)_{\max}] = [d_{11}, d_{12}] \cup [d_{21}, d_{22}] \cup [d_{31}, d_{32}] \cup \dots \cup [d_{k1}, d_{k2}] \quad (3)$$

with $d_{i2} = d_{(i+1)1}$ and the intersection of the respective sub-blankets of the minimum and maximum volume intervals in equation (3) is denoted $\forall [d_{i1}, d_{i2}] \cap [d_{(i+1)1}, d_{(i+1)2}] = d_{i2}$ or $d_{(i+1)1}$, where $i = 1, 2, \dots, k$ with k is the number of blankets dividing the maximum and minimum intervals of the volume change.

Definition 3 (Partition of A_i set)

The next step is to divide the partition to become a set of points between 2 ellipse equations, defined as follows,

$$\begin{aligned} A_1 &= \left\{ (x, y) \mid d_{11} < \left(\frac{x_{\frac{w}{2}}}{\left(\frac{w}{2}\right)}\right)^2 + \left(\frac{y}{\left(\frac{l}{2}\right)}\right)^2 < d_{12} \right\} \\ A_2 &= \left\{ (x, y) \mid d_{21} < \left(\frac{x_{\frac{w}{2}}}{\left(\frac{w}{2}\right)}\right)^2 + \left(\frac{y}{\left(\frac{l}{2}\right)}\right)^2 < d_{22} \right\} \\ A_3 &= \left\{ (x, y) \mid d_{31} < \left(\frac{x_{\frac{w}{2}}}{\left(\frac{w}{2}\right)}\right)^2 + \left(\frac{y}{\left(\frac{l}{2}\right)}\right)^2 < d_{32} \right\} \\ &\vdots \\ A_k &= \left\{ (x, y) \mid d_{k1} < \left(\frac{x_{\frac{w}{2}}}{\left(\frac{w}{2}\right)}\right)^2 + \left(\frac{y}{\left(\frac{l}{2}\right)}\right)^2 < d_{k2} \right\} \end{aligned} \quad (4)$$

From the set of partitions (4) can be described as follows,

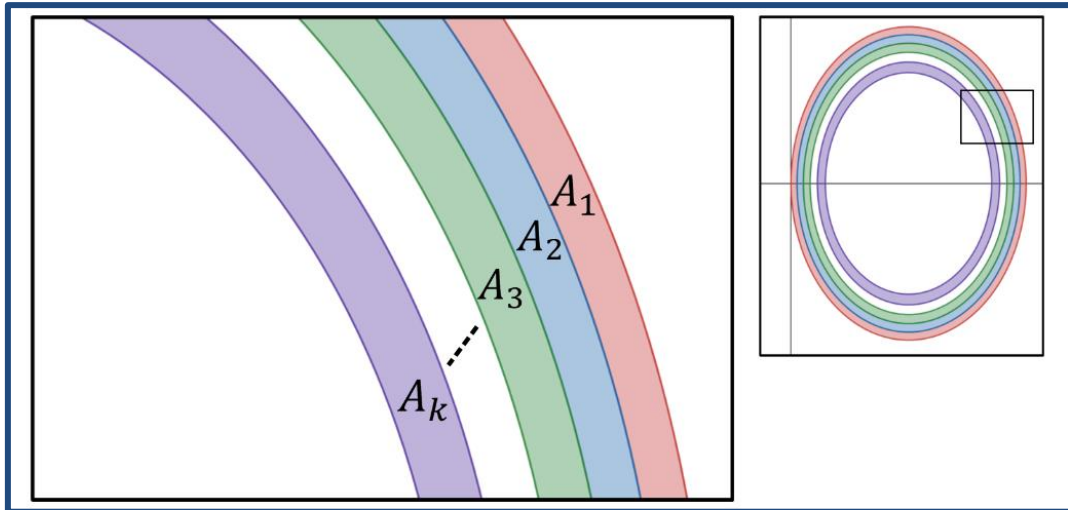


Figure 2. Visualization of Set Partitions A_1, A_2, A_3 up to A_k (4)

Definition 4 (Elliptical Orbits Mode)

After that it is searched, which one A_i has the most points, defined as follows:

$$\text{Max}(n(A_i)) = \text{Max}(n(A_1), n(A_2), n(A_3), \dots, n(A_k)) \quad (5)$$

with $i = 1, 2, \dots, k$. If there is a condition where $\text{Max}(n(A_i)) = n(A_{k_1}) = \dots = n(A_{k_m})$, then the average ellipse inequality $A_{k_1}, A_{k_2}, \dots, A_{k_m}$ is taken so that it is chosen for $\text{Max}(n(A_i)) = n(\overline{A_m})$. Therefore, the inequality whose ellipse will change is defined $\overline{A_m}$ as follows,

$$\overline{A_m} = \left\{ (x, y) \mid \left(\frac{d_{k_1 1} + \dots + d_{k_m 1}}{m} \right) < \left(\frac{xw}{\left(\frac{w}{2}\right)} \right)^2 + \left(\frac{y}{\left(\frac{l}{2}\right)} \right)^2 < \left(\frac{d_{k_1 2} + \dots + d_{k_m 2}}{m} \right) \right\}$$

The next step, because we have obtained A_i or $\overline{A_m}$, then we get the approximated ellipse equation, divided by two cases:

Case 1 [$\text{Max}(n(A_i)) = n(A_m)$]

Based on the set with the maximum number of points between 2 ellipses, choose $A_m = \left\{ (x, y) \mid d_{m1} < \left(\frac{xw}{\left(\frac{w}{2}\right)} \right)^2 + \left(\frac{y}{\left(\frac{l}{2}\right)} \right)^2 < d_{m2} \right\}$, so that we get:

$$\begin{aligned} d_{m1} &< \left(\frac{xw}{\left(\frac{w}{2}\right)} \right)^2 + \left(\frac{y}{\left(\frac{l}{2}\right)} \right)^2 < d_{m2} \\ \Rightarrow \left(\frac{xw}{\left(\frac{w}{2}\right)} \right)^2 + \left(\frac{y}{\left(\frac{l}{2}\right)} \right)^2 &= \left(\frac{d_{m1} + d_{m2}}{2} \right) \\ \Rightarrow y &= \left(\frac{l}{2} \right) \sqrt{\left(\frac{d_{m1} + d_{m2}}{2} \right) - \left(\frac{xw}{\left(\frac{w}{2}\right)} \right)^2} \end{aligned} \quad (6)$$

The steps in (6) can be described as follows,

$$d_{m1} < \left(\frac{xw}{\left(\frac{w}{2}\right)} \right)^2 + \left(\frac{y}{\left(\frac{l}{2}\right)} \right)^2 < d_{m2} \quad \left(\frac{xw}{\left(\frac{w}{2}\right)} \right)^2 + \left(\frac{y}{\left(\frac{l}{2}\right)} \right)^2 = \left(\frac{d_{m1} + d_{m2}}{2} \right) \quad y = \left(\frac{l}{2} \right) \sqrt{\left(\frac{d_{m1} + d_{m2}}{2} \right) - \left(\frac{xw}{\left(\frac{w}{2}\right)} \right)^2}$$

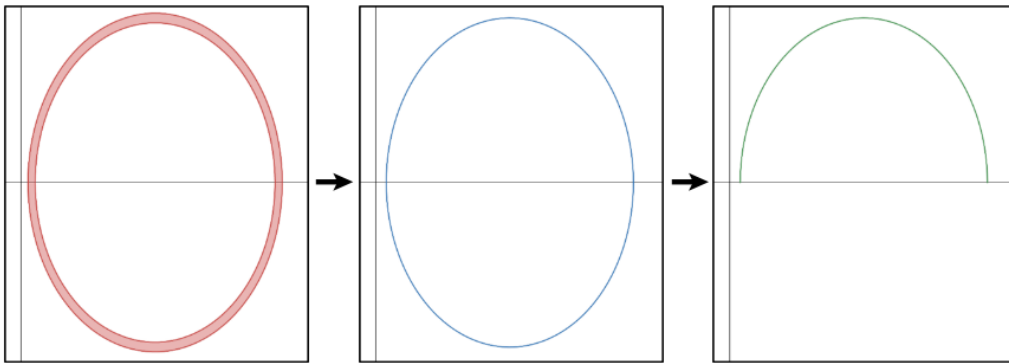


Figure 3. Visualization Steps in (6)

Therefore, from (6) the result of the elliptical orbital mode is a semicircular function by substituting $x_{\frac{w}{2}} = x_{d_m}$, as follows:

$$y = \left(\frac{l}{2}\right) \sqrt{\left(\frac{d_{m1} + d_{m2}}{2}\right) - \left(\frac{x_{d_m}}{\left(\frac{w}{2}\right)}\right)^2} \quad (7)$$

with x = the fuel level in UT and $x_{d_m} = x - \left(\frac{w}{2}\right) \sqrt{\frac{d_{m1} + d_{m2}}{2}}$.

Theorem 1 (Defined Intervals for Elliptical Orbits Mode Approximation Function)

If substitute $x_{\frac{w}{2}} = x_{d_m}$ to $y = \left(\frac{l}{2}\right) \sqrt{\left(\frac{d_{m1} + d_{m2}}{2}\right) - \left(\frac{x_{d_m}}{\left(\frac{w}{2}\right)}\right)^2}$ then y is defined in $\mathbb{R}$, for interval $0 \leq x \leq \frac{w}{2} - \left(\frac{w}{2}\right) \sqrt{\frac{d_{m1} + d_{m2}}{2}}$.

Proof:

Substitute $x_{\frac{w}{2}} = x_{d_m}$ to $y = \left(\frac{l}{2}\right) \sqrt{\left(\frac{d_{m1} + d_{m2}}{2}\right) - \left(\frac{x_{d_m}}{\left(\frac{w}{2}\right)}\right)^2}$ so that it is obtained:

$$\begin{aligned} y &= \left(\frac{l}{2}\right) \sqrt{\left(\frac{d_{m1} + d_{m2}}{2}\right) - \left(\frac{x_{d_m}}{\left(\frac{w}{2}\right)}\right)^2} \Leftrightarrow y = \left(\frac{l}{2}\right) \sqrt{\left(\frac{d_{m1} + d_{m2}}{2}\right) - \left(\frac{x - \left(\frac{w}{2}\right) \sqrt{\frac{d_{m1} + d_{m2}}{2}}}{\left(\frac{w}{2}\right)}\right)^2} \\ &\Leftrightarrow y = \left(\frac{l}{2}\right) \sqrt{\left(\frac{d_{m1} + d_{m2}}{2}\right) - \left(\frac{x^2 - wx \sqrt{\frac{d_{m1} + d_{m2}}{2}} + \left(\frac{w}{2}\right)^2 \left(\frac{d_{m1} + d_{m2}}{2}\right)}{\left(\frac{w}{2}\right)^2}\right)} \\ &\Leftrightarrow y = \left(\frac{l}{2}\right) \sqrt{\left(\frac{d_{m1} + d_{m2}}{2}\right) - \left(\frac{d_{m1} + d_{m2}}{2}\right) - \left(\frac{x^2 - wx \sqrt{\frac{d_{m1} + d_{m2}}{2}}}{\left(\frac{w}{2}\right)^2}\right)} \\ &\Leftrightarrow y = \left(\frac{l}{2}\right) \sqrt{\frac{x \left(w \sqrt{\frac{d_{m1} + d_{m2}}{2}} - x\right)}{\left(\frac{w}{2}\right)^2}} \\ &\Leftrightarrow y = \left(\frac{l}{w}\right) \sqrt{x \left(w \sqrt{\frac{d_{m1} + d_{m2}}{2}} - x\right)} \end{aligned}$$

with $w, d_{m1}, d_{m2} \in \mathbb{R}^+$ obviously y defined in $\mathbb{R}$ since $x \left(w \sqrt{\frac{d_{m1}+d_{m2}}{2}} - x \right) \geq 0$, then x must on interval $0 \leq x \leq w \sqrt{\frac{d_{m1}+d_{m2}}{2}}$ likewise for interval $0 \leq x \leq \frac{w}{2} - \left(\frac{w}{2}\right) \sqrt{\frac{d_{m1}+d_{m2}}{2}}$, because $w \sqrt{\frac{d_{m1}+d_{m2}}{2}} > \frac{w}{2} - \left(\frac{w}{2}\right) \sqrt{\frac{d_{m1}+d_{m2}}{2}}$. ■

Substitution $x_{\frac{w}{2}} = x_{d_m}$ so that the value y is defined in $0 \leq x \leq \frac{w}{2} - \left(\frac{w}{2}\right) \sqrt{\frac{d_{m1}+d_{m2}}{2}}$, so that (7) the function of volume change to fuel level in the UT can be visualized as follows,

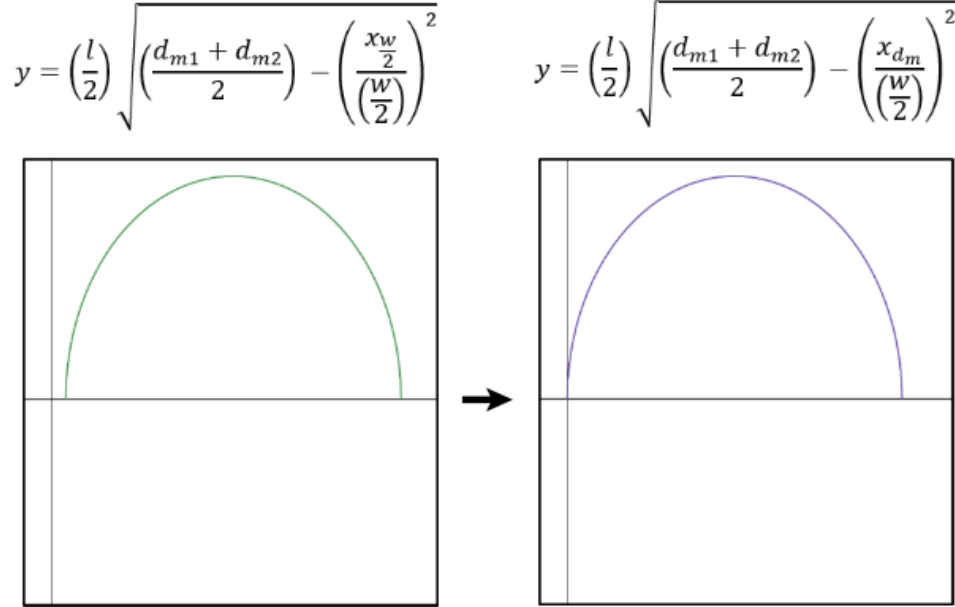


Figure 4. Visualization of Steps in (6) to (7)

The function is formed from the half ellipse selected for the calibration of the buried tank whose cross-sectional area is circular but convex, so that the volume of UT will be calculated as a function of the change in volume with respect to height, which then the coordinates are taken from the change in units (cm).

Case 2 [$\text{Max}(n(A_i)) = n(A_{k_1}) = \dots = n(A_{k_m})$]

Based on the set with the maximum number of points between 2 ellipse equations,

$\overline{A_m} = \left\{ (x, y) \mid \left(\frac{d_{k_11} + \dots + d_{k_m1}}{m}\right) < \left(\frac{x_{\frac{w}{2}}}{\left(\frac{w}{2}\right)}\right)^2 + \left(\frac{y}{\left(\frac{l}{2}\right)}\right)^2 < \left(\frac{d_{k_12} + \dots + d_{k_m2}}{m}\right) \right\}$ then obtained:

$$\begin{aligned} \left(\frac{d_{k_11} + \dots + d_{k_m1}}{m}\right) &< \left(\frac{x_{\frac{w}{2}}}{\left(\frac{w}{2}\right)}\right)^2 + \left(\frac{y}{\left(\frac{l}{2}\right)}\right)^2 < \left(\frac{d_{k_12} + \dots + d_{k_m2}}{m}\right) \\ \Rightarrow \left(\frac{x_{\frac{w}{2}}}{\left(\frac{w}{2}\right)}\right)^2 + \left(\frac{y}{\left(\frac{l}{2}\right)}\right)^2 &= \left(\frac{\left(\frac{d_{k_11} + \dots + d_{k_m1}}{m}\right) + \left(\frac{d_{k_12} + \dots + d_{k_m2}}{m}\right)}{2}\right) \\ \Rightarrow \left(\frac{x_{\frac{w}{2}}}{\left(\frac{w}{2}\right)}\right)^2 + \left(\frac{y}{\left(\frac{l}{2}\right)}\right)^2 &= \left(\frac{d_{m1} + d_{m2}}{2}\right) \end{aligned} \quad (8)$$

The steps in (8) are visualized similarly in Figure 11 with $d_{m1} = \left(\frac{d_{k11} + \dots + d_{km1}}{m}\right)$ and $d_{m2} = \left(\frac{d_{k12} + \dots + d_{km2}}{m}\right)$, then

$$\Rightarrow y = \left(\frac{l}{2}\right) \sqrt{\left(\frac{d_{m1} + d_{m2}}{2}\right) - \left(\frac{x_{\frac{w}{2}}}{\left(\frac{w}{2}\right)}\right)^2} \quad (9)$$

Therefore, the result of the orbital mode ellipse is a half-ellipse function, as follows:

$$y = \left(\frac{l}{2}\right) \sqrt{\left(\frac{d_{m1} + d_{m2}}{2}\right) - \left(\frac{x_{d_m}}{\left(\frac{w}{2}\right)}\right)^2} \quad (10)$$

With x = the fuel level in UT and $x_{d_m} = x - \left(\frac{w}{2}\right) \sqrt{\frac{d_{m1} + d_{m2}}{2}}$.

As Theorem 1 about defined interval for (10) mention for substituting $x_{\frac{w}{2}} = x_{d_m}$ for y the defined values since $0 \leq x \leq \frac{w}{2} - \left(\frac{w}{2}\right) \sqrt{\frac{d_{m1} + d_{m2}}{2}}$, we get (10) the function of the change in volume to the fuel level in the UT which is visualized similarly in Figure 4. This function is formed from the half ellipse selected for the calibration of the buried tank whose cross-sectional area is circular but convex, so that the volume of the UT will be calculated on the function of the change in volume with respect to height, which then the coordinates of the points are taken from the change in units (cm).

METHODS

Research Steps

1. Construction of mathematical model Elliptical Orbits Mode methods with the follow steps:
 - a) Construction of mathematical model Elliptical Orbits Mode.
 - Review for Ellips Equation
 - Define A_i set for Ellips Mode
 - Make Partition for A_i set (Definition)
 - Choose Partition Partition of A_i set with $n(A_i)$ is the maximum value
 - Divided on 2 cases singular and plural maximum value
 - Create function $y = f(x)$ from chosen A_i sets
 - Translations $y = f(x)$ so that y define on 0 and so on.
 - And controlling height for
2. Construction of mathematical model Elliptical Orbits Mode will be apply on Data from Candirejo Gas Station, Measuring book from Governmet Metrology Agency specific for Peralite (Fuel Product of Pertamina) only and visualize it.
3. Measuring Performance using Residual Sum of Square (RSS) [8] and Mean Square Error (MSE) [9,10] and compare with Circle Orbits Mode from [1], Least Square with $n = 2$ and $n = 3$ from [11], and Elipptical with Height Control.

RESULTS AND DISCUSSION

Pertalite Measuring Book Data

Calculation of gas station 45.507.21 Candirejo Tuntang using a measuring book to determine the volume of fuel in the buried tank obtained the following data:

Table 1. Fuel Volume Measuring Book Data for Pertalite Tanks

Height (x)	Volume	Diff (y)	Height (x)	Volume	Diff (y)	Height (x)	Volume	Diff (y)
0	0.0	0.0	75	7085.9	117.7	150	16124.4	111.1
1	237.1	237.1	76	7203.5	117.6	151	16235.6	111.2
2	294.3	57.2	77	7321.2	117.7	152	16346.7	111.1
3	351.4	57.1	78	7438.8	117.6	153	16457.8	111.1
4	409.4	58.0	79	7556.5	117.7	154	16568.9	111.1
5	468.2	58.8	80	7674.1	117.6	155	16680.0	111.1
6	527.1	58.9	81	7791.8	117.7	156	16791.1	111.1
7	586.1	59.0	82	7909.4	117.6	157	16902.2	111.1
8	646.7	60.6	83	8027.1	117.7	158	17013.3	111.1
9	707.3	60.6	84	8144.7	117.6	159	17124.4	111.1
10	767.9	60.6	85	8262.4	117.7	160	17232.6	108.2
11	831.6	63.7	86	8380.0	117.6	161	17337.9	105.3
12	896.1	64.5	87	8497.6	117.6	162	17443.2	105.3
13	960.6	64.5	88	8615.3	117.7	163	17548.4	105.2
14	1026.7	66.1	89	8732.9	117.6	164	17653.7	105.3
15	1093.3	66.6	90	8855.0	122.1	165	17758.9	105.2
16	1160.0	66.7	91	8980.0	125.0	166	17864.2	105.3
17	1228.3	68.3	92	9105.0	125.0	167	17969.5	105.3
18	1297.2	68.9	93	9230.0	125.0	168	18065.7	96.2
19	1366.2	69.0	94	9355.0	125.0	169	18161.0	95.3
20	1439.3	73.1	95	9480.0	125.0	170	18256.2	95.2
21	1513.3	74.0	96	9605.0	125.0	171	18351.4	95.2
22	1588.0	74.7	97	9730.0	125.0	172	18446.7	95.3
23	1668.0	80.0	98	9855.0	125.0	173	18541.9	95.2
24	1748.0	80.0	99	9980.0	125.0	174	18637.1	95.2
25	1830.0	82.0	100	10105.0	125.0	175	18732.4	95.3
26	1913.3	83.3	101	10230.0	125.0	176	18825.5	93.1
27	1997.4	84.1	102	10355.0	125.0	177	18916.4	90.9
28	2084.3	86.9	103	10480.0	125.0	178	19007.3	90.9
29	2171.3	87.0	104	10605.0	125.0	179	19098.2	90.9
30	2261.8	90.5	105	10730.0	125.0	180	19189.1	90.9
31	2352.7	90.9	106	10855.0	125.0	181	19280.0	90.9
32	2446.7	94.0	107	10980.0	125.0	182	19370.9	90.9
33	2541.9	95.2	108	11105.0	125.0	183	19458.3	87.4
34	2637.1	95.2	109	11230.0	125.0	184	19545.2	86.9
35	2732.4	95.3	110	11355.0	125.0	185	19632.2	87.0
36	2830.0	97.6	111	11480.0	125.0	186	19719.1	86.9

37	2930.0	100.0	112	11605.0	125.0	187	19804.0	84.9
38	3030.0	100.0	113	11730.0	125.0	188	19884.0	80.0
39	3130.0	100.0	114	11855.0	125.0	189	19964.0	80.0
40	3230.0	100.0	115	11980.0	125.0	190	20044.0	80.0
41	3330.0	100.0	116	12105.0	125.0	191	20124.0	80.0
42	3430.0	100.0	117	12230.0	125.0	192	20202.2	78.2
43	3530.0	100.0	118	12355.0	125.0	193	20276.3	74.1
44	3630.0	100.0	119	12480.0	125.0	194	20350.4	74.1
45	3730.0	100.0	120	12605.0	125.0	195	20420.0	69.6
46	3832.6	102.6	121	12730.0	125.0	196	20486.7	66.7
47	3937.9	105.3	122	12855.0	125.0	197	20553.3	66.6
48	4043.2	105.3	123	12980.0	125.0	198	20617.5	64.2
49	4148.4	105.2	124	13105.0	125.0	199	20680.0	62.5
50	4257.8	109.4	125	13227.1	122.1	200	20742.5	62.5
51	4368.9	111.1	126	13344.7	117.6	201	20802.9	60.4
52	4480.0	111.1	127	13462.4	117.7	202	20860.0	57.1
53	4591.1	111.1	128	13580.0	117.6	203	20917.1	57.1
54	4702.2	111.1	129	13697.6	117.6	204	20974.3	57.2
55	4813.3	111.1	130	13815.3	117.7	205	21028.6	54.3
56	4924.4	111.1	131	13932.9	117.6	206	21082.7	54.1
57	5035.6	111.2	132	14050.6	117.7	207	21136.8	54.1
58	5146.7	111.1	133	14168.2	117.6	208	21187.1	50.3
59	5257.8	111.1	134	14285.9	117.7	209	21222.9	35.8
60	5368.9	111.1	135	14403.5	117.6	210	21258.6	35.7
61	5480.0	111.1	136	14521.2	117.7	211	21294.3	35.7
62	5591.1	111.1	137	14638.8	117.6	212	21330.0	35.7
63	5702.2	111.1	138	14756.5	117.7	213	21365.7	35.7
64	5813.3	111.1	139	14874.1	117.6	214	21387.1	21.4
65	5924.4	111.1	140	14991.8	117.7	215	21399.1	12.0
66	6035.6	111.2	141	15109.4	117.6	216	21411.0	11.9
67	6146.7	111.1	142	15227.1	117.7	217	21422.9	11.9
68	6262.4	115.7	143	15344.7	117.6	218	21434.8	11.9
69	6380.0	117.6	144	15457.8	113.1	219	21446.7	11.9
70	6497.6	117.6	145	15568.9	111.1	220	21458.6	11.9
71	6615.3	117.7	146	15680.0	111.1	221	21470.5	11.9
72	6732.9	117.6	147	15791.1	111.1	222	21482.4	11.9
73	6850.6	117.7	148	15902.2	111.1	222.3	21486.0	3.6
74	6968.2	117.6	149	16013.3	111.1			

3.3. Approximation using Elliptical Orbits Mode (EOM)

Using Table 1 as a sample that will be approximated using Elliptical Orbits Mode. To obtain a half-ellipse function from the smallest to the largest abscissa, choose,

$$\frac{w}{2} = \frac{\text{maximum height on underground tank}}{2} = \frac{222.3}{2} = 111.15$$

$\frac{l}{2} = \text{maximum volume change on underground tank} = 125$

After that, select the difference d_{i1} and d_{i2} for the prefix of the A_i set which is $t_i = 0.2$ defined as follows,

$$A_i = \left\{ (x, y) \mid 0.8 < \frac{(x - 111.15)^2}{(111.15)^2} + \frac{y^2}{(125)^2} < 1 \right\}$$

with step-size = 0.02 then the number of partitions is obtained $\frac{t_i}{\text{step-size}} = 10$, so that the partitions are obtained from the A_i set with $i = 1, 2, \dots, 10$,

$$\begin{aligned} A_1 &= \left\{ (x, y) \mid 0.98 < \frac{(x - 111.15)^2}{(111.15)^2} + \frac{y^2}{(125)^2} < 1 \right\} \\ A_2 &= \left\{ (x, y) \mid 0.96 < \frac{(x - 111.15)^2}{(111.15)^2} + \frac{y^2}{(125)^2} < 0.98 \right\} \\ A_3 &= \left\{ (x, y) \mid 0.94 < \frac{(x - 111.15)^2}{(111.15)^2} + \frac{y^2}{(125)^2} < 0.96 \right\} \\ A_4 &= \left\{ (x, y) \mid 0.92 < \frac{(x - 111.15)^2}{(111.15)^2} + \frac{y^2}{(125)^2} < 0.94 \right\} \\ A_5 &= \left\{ (x, y) \mid 0.90 < \frac{(x - 111.15)^2}{(111.15)^2} + \frac{y^2}{(125)^2} < 0.92 \right\} \\ A_6 &= \left\{ (x, y) \mid 0.88 < \frac{(x - 111.15)^2}{(111.15)^2} + \frac{y^2}{(125)^2} < 0.90 \right\} \\ A_7 &= \left\{ (x, y) \mid 0.86 < \frac{(x - 111.15)^2}{(111.15)^2} + \frac{y^2}{(125)^2} < 0.88 \right\} \\ A_8 &= \left\{ (x, y) \mid 0.84 < \frac{(x - 111.15)^2}{(111.15)^2} + \frac{y^2}{(125)^2} < 0.86 \right\} \\ A_9 &= \left\{ (x, y) \mid 0.82 < \frac{(x - 111.15)^2}{(111.15)^2} + \frac{y^2}{(125)^2} < 0.84 \right\} \\ A_{10} &= \left\{ (x, y) \mid 0.80 < \frac{(x - 111.15)^2}{(111.15)^2} + \frac{y^2}{(125)^2} < 0.82 \right\} \end{aligned}$$

So that

Table 2. Calculation of Elliptical Orbits Moden(A_i)

A_i	$n(A_i)$
A_1	11
A_2	15
A_3	16
A_4	26
A_5	25
A_6	19
A_7	9
A_8	3
A_9	0
A_{10}	0

According to Table 2 it is obtained $Max(n(A_i)) = n(A_4)$, with $i = 1, 2, \dots, 10$ the selected A_4 set, after that from the A_4 set the following functions $y = f(x)$ will be formed,

$$\begin{aligned} 0.92 &< \frac{(x - 111.15)^2}{(111.15)^2} + \frac{y^2}{(125)^2} < 0.94 \\ \Rightarrow \frac{(x - 111.15)^2}{(111.15)^2} + \frac{y^2}{(125)^2} &= \left(\frac{0.92 + 0.94}{2} \right) \\ \Rightarrow \frac{(x - 111.15)^2}{(111.15)^2} + \frac{y^2}{(125)^2} &= 0.93 \\ \Rightarrow y &= 125 \sqrt{0.93 - \frac{(x - 111.15)^2}{(111.15)^2}} \end{aligned}$$

Then the translation $y = f(x)$ to be defined at $0 \leq x \leq 111.15 - 111.15\sqrt{0.93}$ or around $0 \leq x \leq 3,96$, substitution $x_{\frac{w}{2}} = (x - 111.5)$ with $x_{d_m} = (x - 111.15\sqrt{0.93})$, so that we get:

$$y = 125 \sqrt{0.93 - \frac{(x - 111.15\sqrt{0.93})^2}{(111.15)^2}} \quad (11)$$

with y = change in the volume of fuel in the tank, for visualization of the graph of changes in the volume of fuel obtained:

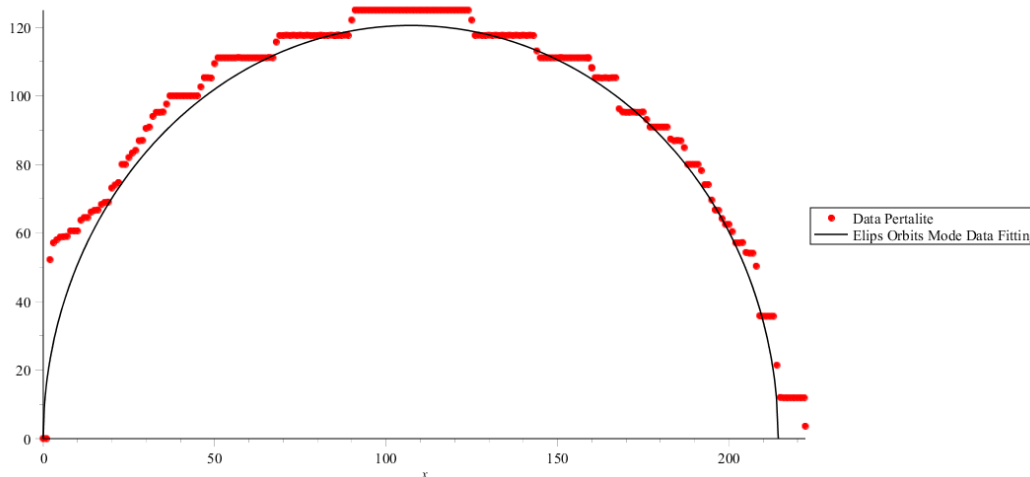


Figure 5 Graph of Change in Fuel Volume by EOM

Based on Figure 5 $y = f(x)$, the EOM results produce a function that is fit to the peralite data, then look for the volume function with the integral, we get:

$$V(h) = \int_0^h 125 \sqrt{0.93 - \frac{(x - 111.15\sqrt{0.93})^2}{(111.15)^2}} dx$$

Using the help of Maple 2015 the integral results are obtained as follows,

$$\begin{aligned}
 V(h) := & \frac{535945891}{8000000000000000000} \sqrt{18792746045928961} \\
 & + \frac{116250000}{17040701} \sqrt{896879} \arcsin\left(\frac{10182971929}{930000000000000} \sqrt{896879} \sqrt{93}\right) \\
 & + \frac{1}{1600000000000} \\
 & (-809433297500000000000000 h^2 + 17352497993348222900000000 h \\
 & + 18792746045928961)^{1/2} h \\
 & - \frac{535945891}{8000000000000000000} \\
 & (-809433297500000000000000 h^2 + 17352497993348222900000000 h \\
 & + 18792746045928961)^{1/2} \\
 & + \frac{116250000}{17040701} \sqrt{896879} \arcsin\left(\frac{19}{930000000000000} \sqrt{896879} \sqrt{93}\right) (5000000 h \\
 & - 535945891);
 \end{aligned} \tag{12}$$

where $V(h)$ is defined on the interval $0 \leq h \leq 222.3\sqrt{0.93}$. The EOM version of the fuel volume calculation uses (12) with $V_{max} = 20.296.55$ liters.

Elliptical Orbits Mode with elliptical height control on Data Peralite

Based on Figure 5, it can be seen that the Volume change function according to EOM will regress more pertalite data if the ellipse height is higher, so it is necessary to control the ellipse height.

The EOM result at (11) has an elliptical height 125 which represents the equation so that it has a volume change function with respect to the fuel level in the UT, with the general form of (11):

$$EOM(\theta, x) = \theta \cdot \sqrt{0.93 - \frac{(x - 111.15\sqrt{0.93})^2}{(111.15)^2}} \tag{13}$$

With θ is the height of the ellipse, defined on the interval $0 \leq x \leq 222.3\sqrt{0.93}$. Choose θ the one that minimizes the residual sum square

$$\begin{aligned}
 E &= \sum_{i=1}^{\lfloor 222.3\sqrt{0.93} \rfloor} (y_i - EOM(\theta, x_i))^2 \\
 E &= \sum_{i=1}^{214} (y_i^2 - 2EOM(\theta, x_i)y_i + EOM^2(\theta, x_i)) \\
 E &= \sum_{i=1}^{214} y_i^2 - 2 \sum_{i=1}^{214} EOM(\theta, x_i)y_i + \sum_{i=1}^{214} (EOM(\theta, x_i))^2 \\
 E &= \sum_{i=1}^{214} y_i^2 - 2 \sum_{i=1}^{214} y_i \cdot \theta \sqrt{0.93 - \frac{(x - 111.15\sqrt{0.93})^2}{(111.15)^2}} + \sum_{i=1}^{214} \left(\theta \sqrt{0.93 - \frac{(x - 111.15\sqrt{0.93})^2}{(111.15)^2}} \right)^2 \\
 E &= \sum_{i=1}^{214} y_i^2 - 2 \sum_{i=1}^{214} y_i \cdot \theta \sqrt{0.93 - \frac{(x - 111.15\sqrt{0.93})^2}{(111.15)^2}} + \sum_{i=1}^{214} \theta^2 \left(0.93 - \frac{(x - 111.15\sqrt{0.93})^2}{(111.15)^2} \right)
 \end{aligned}$$

Then, to find θ the minimization of E , find the solution of the equation $\frac{\partial E}{\partial \theta} = 0$, we get:

$$\begin{aligned}
 &\Leftrightarrow -2 \sum_{i=1}^{216} y_i \sqrt{0.93 - \frac{(x - 111.15\sqrt{0.93})^2}{(111.15)^2}} + 2\theta \sum_{i=1}^{216} \left(0.93 - \frac{(x - 111.15\sqrt{0.93})^2}{(111.15)^2}\right) = 0 \\
 &\Leftrightarrow \frac{-2 \sum_{i=1}^{216} y_i \sqrt{0.93 - \frac{(x - 111.15\sqrt{0.93})^2}{(111.15)^2}} + 2\theta \sum_{i=1}^{216} \left(0.93 - \frac{(x - 111.15\sqrt{0.93})^2}{(111.15)^2}\right)}{2 \sqrt{0.93 - \frac{(x - 111.15\sqrt{0.93})^2}{(111.15)^2}}} = 0 \\
 &\Leftrightarrow -\sum_{i=1}^{216} y_i + \theta \sum_{i=1}^{216} \sqrt{0.93 - \frac{(x - 111.15\sqrt{0.93})^2}{(111.15)^2}} = 0 \\
 &\Leftrightarrow \theta = \frac{\sum_{i=1}^{216} y_i}{\sum_{i=1}^{216} \sqrt{0.93 - \frac{(x - 111.15\sqrt{0.93})^2}{(111.15)^2}}}
 \end{aligned}$$

By using the data in Table 1, it is obtained $\theta \approx 130,37$ that from (27) it is obtained:

$$EOM(130,37, x) = 130,37 \cdot \sqrt{0.93 - \frac{(x - 111.15\sqrt{0.93})^2}{(111.15)^2}}$$

and visualized the graph of the function of the change in fuel to the level of fuel in the reservoir as follows:

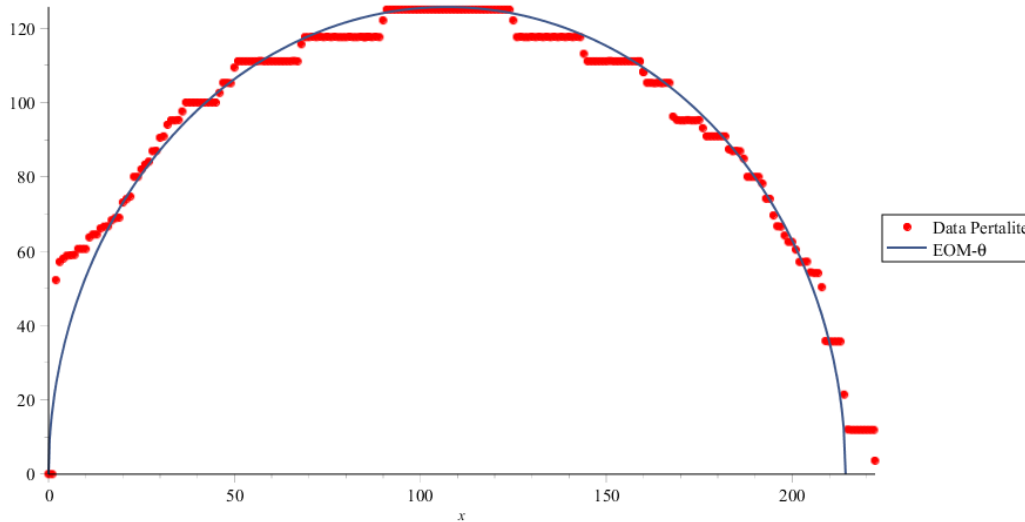


Figure 6 Graph of Changes in Fuel Volume by $EOM(\theta)$

Based on Figure 6 $y = f(x)$ the $EOM(\theta)$ results produce a function that is more fit to the peralite data than the EOM results in Figure 5, then look for the volume function $V(h)$ with integral and with (24) obtained:

$$V_{\theta}(h) = \frac{130,37}{125} \cdot V(h)$$

where $V_{\theta}(h)$ is defined on the interval $0 \leq h \leq 222.3$. Calculation of the volume of the fuel $EOM(\theta)$ version has $V_{max} = 21.166,71$ liters.

Comparison of approximate visualization results

Data visualization results from Circle Orbits Mode, Elliptical Orbits Mode, $EOM(\theta)$, Least Square Data Fitting $n = 2$, and Least Square Data Fitting $n = 3$, as follows:

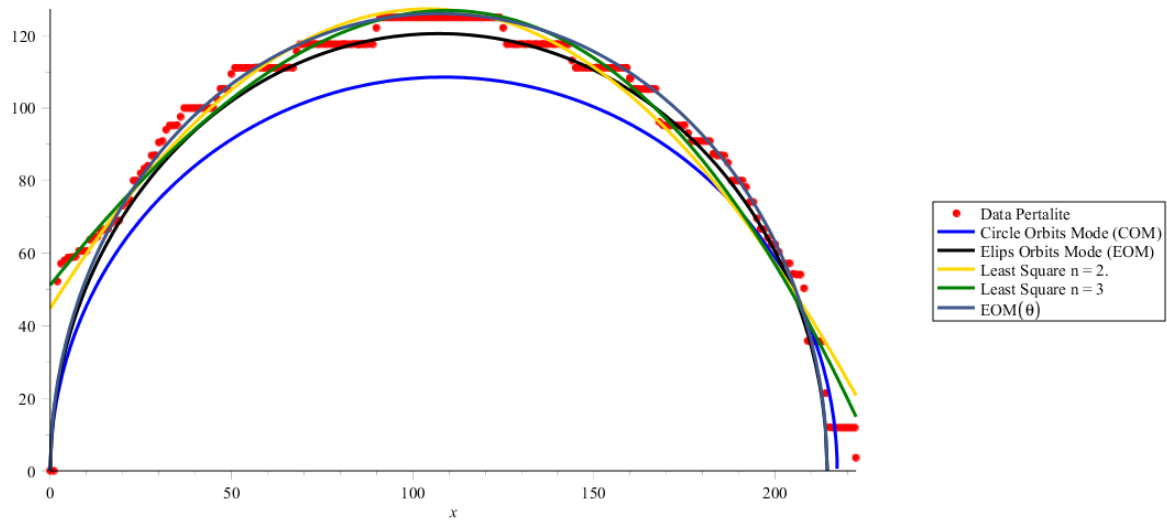


Figure 7 Comparison Graph of Approximation Method Results

The results of the calculation of changes in the volume of fuel based on the height of the fuel in the UT will be applied to the Peralite Data to search for RSS and MSE from each result.

Peralite Data Approximation RSS and MSE Calculation

Comparison of the results between the proposed method and other methods can be seen from the calculation of RSS and MSE in Table 5 below:

Table 5 Peralite Data Approximation RSS and MSE Calculation

Method	<i>RSS</i>	<i>MSE</i>
<i>COM</i>	40.390,49	185,28
<i>EOM</i>	8.529,37	39,67
<i>LS</i> ($n = 2$)	8.980,63	40,09
<i>LS</i> ($n = 3$)	7.574,51	33,81
<i>EOM</i> (θ)	6.415,32	29,84

Based on Table 5 the calculation of COM which has the largest RSS and MSE, for EOM has RSS and MSE which is slightly below $LS(n = 2)$, but still above $LS(n = 3)$, then by controlling the height of the ellipse looking for θ the one that minimizes the RSS we get the minimum i.e. 6.415,32 and its MSE 29,84. The smallest value RSS and MSE are obtained when using $EOM(\theta)$. For the calculation of RSS and MSE the Peralite data approximation is not compared to the calculation of gas stations and Cubic Spline Interpolation because the RSS and MSE are definitely 0.

Defined intervals and maximum volume

In the Orbits Mode Data Fitting construction, there is a reduction in the BBM altitude domain in the UT, so that it is only defined at a certain height. The results of the comparison of the defined interval and the maximum volume of each approximation method are as follows:

Table 6 Defined interval and maximum volume approximation method

Approximation Method	Defined interval (cm)	V_{max} (maximum volume)
Gas Station Calculation	$0 \leq h \leq 222,3$	$V_{max} = 21.486.00$ liter.
<i>Circle Orbits Mode</i>	$0 \leq h \leq 217,1$	$V_{max} = 18.508,85$ liter.
<i>Elliptical Orbits Mode</i>	$0 \leq h \leq 222,3\sqrt{0,93}$	$V_{max} = 20.296,55$ liter.
$EOM(\theta)$	$0 \leq h \leq 222,3\sqrt{0,93}$	$V_{max} = 21.166,04$ liter.
<i>Least Square $n = 2$</i>	$0 \leq h \leq 222,3$	$V_{max} = 21.248,90$ liter.
<i>Least Square $n = 3$</i>	$0 \leq h \leq 222,3$	$V_{max} = 21.256,66$ liter.

The calculation result of Circle Orbits Mode is only defined to altitude 217,1 cm there is a reduction of 5.2 cm and Elliptical Orbits Mode is only defined to a height of $222,3\sqrt{0,93}$ cm or about 214,382 cm, there is a reduction of $(222,3 - 222,3\sqrt{0,93})$ cm or about 7,92 cm.

For further research, this has no effect on the application of Daily Sales Data (According to Dispenser) because according to the data, the maximum height of fuel is 209.10 cm. So that the value is defined for all data as well as each Approximation Method as well.

CONCLUSIONS

Based on the results and discussion, it can be concluded that the method of approximating the pertalite data with the smallest RSS and MSE is $EOM(\theta)$ by $\theta \approx 130,37$, resulting in RSS and MSE respectively are 6.415,32 and 29,81. $EOM(\theta)$ also produces a more fit half-ellipse function than other approximation methods as visualized in Figure 7. The results of the comparison of the approximation of the pertalite data are compared with COM , EOM , $LS(n = 2)$, and $LS(n = 3)$ which are represented in Table 5.

Although $EOM(\theta)$ produces RSS and MSE, which are smaller than other methods, there is a reduction in the altitude domain and has a different maximum volume compared to the calculation of gas stations. According to the Gas Station Metrology Measurement Book, the height of the UT is 222.3 cm and has a maximum volume of 21,486 liters, but $EOM(\theta)$ only detects the volume of fuel up to a height of about 214.1 cm and the maximum volume is below the calculation of the gas station.

The author hopes for the development of this research, applied to different types of fuel such as Pertamina and Dexlite. As well as for a more real problem under study, use data on changes in the height and volume of BBM based on Daily Sales according to the BBM Dispenser which must first be tested for the accuracy of the BBM Dispenser used. As well as calculating errors using MAPE, MAD, and other error calculations.

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