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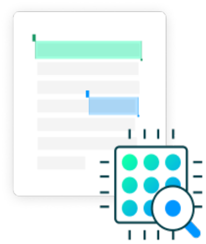

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Estimation of Gompertz-Mortality Parameter Models on Indonesian Population Mortality Table 2023

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ABSTRACT

The research article discuss Gompertz Mortality Law parameter estimation using several methods to get the best models. The data based from Indonesian population mortality table or called Tabel Mortalitas Penduduk Indonesia (TMPI) 2023. Parameter estimation using several methods, includes Nonlinear Least Square (NLLS) with the Gauss-Newton algorithm, Weighted Least Squares (WLS), and Poisson Regression. Model validation is done by calculating root mean square error (RMSE) to determine the most accurate method. The analysis includes calculation of values in the mortality table, transformation of the gompertz model, estimated parameters with each method, and RMSE calculation. In the WLS method, the estimation is carried out by transformation of natural logarithms from the force of mortality function, then minimizes the number of squares of error, with d_x as weight and forming the d_x function and maximizing the log-ordered function on Poisson regression. Model accuracy is assessed from the suitability between the q_x function value of the model results with the q_x value in TMPI, both visually and mathematically through RMSE. The analysis results show that the NLLS method with the Gauss-Newton algorithm produces the most accurate Gompertz model.

Keywords: Gompertz mortality law; Parameter estimation; RMSE; TMPI 2023

INTRODUCTION

Life and death are two inseparable things, where death is often unpredictable with certainty both time and cause [1]. Life insurance is present as a solution to provide financial protection to heirs from the insured in the face of unexpected risk of death [2]. Actuarial calculations in life insurance are very dependent on the mortality table that reflects the actual events, but for more flexible applications a mortality law approach such as Gompertz and Makeham law is needed.

Mortality law such as Gompertz is used to model the death rate that increases exponentially as we get older and are very useful in the development of life insurance products and long-term financial planning [3]. In Indonesia, the application of this law began to be used by insurance companies to perfect mortality assumptions in actuarial calculations, such as in the development of life insurance products and evaluating technical reserves [4]. The Gompertz model helps project life expectations more realistically in avoiding the risk of lack of funds, both in the insurance sector and public policy such as social security and elderly health financing (Hong et al., 2021). Nevertheless, the main challenge in Indonesia is the limited data of quality mortality, so that collaboration between industries, academics, and the government is needed to

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support the development of accurate and applicative mortality models [5].

Gompertz Mortality Law Model, which assumes the force of mortality increase exponentially to age has two main parameters, such as α and β [6]. The parameter can be estimated by several methods, such as Nonlinear Least Squares (NLLS), Weighted Least Squares (WLS), and Poisson Regression. This study using the Indonesian Population Mortality Table or called as Tabel Mortalitas Penduduk Indonesia (TMPI) 2023 as a reference data and applies the three methods comprehensively to suspect the Gompertz parameter with the aim of obtaining the most accurate mortality model and in accordance with the characteristics of the Indonesian population.

Model validation is done by comparing the estimated results of actual data using the measurement of root mean square error (RMSE) to assess the predictive performance of the model [7]. Previous studies by Tai and Noymer in 2018 and Putra et al. in 2019 provided an important foundation related to the superiority of the WLS and Poisson regression methods in producing valid Gompertz mortality models, so that this study tried to develop and integrate the approach with a more applicative analysis based on conditions in Indonesia.

This study made a scientific contribution by presenting comparative methods in the Gompertz parameter estimation that had never been previously reviewed in mortality in Indonesia. The result based on national database actuarial formulation that is not only statistically valid, but also applicative in supporting the development of life insurance products in Indonesia and strengthening the synergy between mortality theory and actuarial practice in the field. Until now, there has been no research that uses the Gompertz parameter value that is referred directly from Indonesian mortality data (data-driven concepts) both for academic purposes and application in industry related to mortality in Indonesia, thus indicating a significant gap that began to be answered through this study. Furthermore, this study will also prove that the Gompertz model tends to be valid and relevant for use in adult age groups, especially the age range of 35 to 60 years, where the pattern of increased risk of death follows the tendency of exponential in accordance with the characteristics of the model [9].

METHODS

The data used is secondary data and generation data. The secondary data used is the Mortality Data of the Indonesian population summarized in the TMPI with 111 data samples [5]. The table contains the value of the q_x function which will be the basis of the calculation of the functions in the mortality table.

Force of Mortality Calculations (m_x)

In this step, function value m_x calculated based on the value of the function q_x. Calculation begins by initializing the value of l_0 is 100,000 which indicates that it is assumed to cover the mortality rate per 100,000 people [8]. Next, the value can be calculated ℓ_x for x = 1,2,3, and etc, with n = 1, then obtained

$$q_x = \frac{\ell_x - \ell_{x+1}}{\ell_x}$$

$$q_x \ell_x = \ell_x - \ell_{x+1}$$

$$\ell_{x+1} = (1 - q_x)\ell_x \quad (1)$$

where ℓ_x states that the population that remains alive right at the age of x originating from the initial population (ℓ_0) [10]. Furthermore, the value of the d_x function which states the number of people who died exactly between the ages of x and x + n [11] from the initial population l_0 [12], that is

$$d_0 = \ell_0 - \ell_1 \tag{2}$$

and ℓ_x which states the number of years of life that someone lives x between x to $x + n$.

$$_n\mathcal{L}_x = n(\ell_{x+n} + {}_na_x \cdot {}_nd_x$$

$$\mathcal{L}_0 = \ell_1 + 0{,}5\, d_0$$

$$\mathcal{L}_0 = \ell_1 + 0{,}5\, d_0 \tag{3}$$

The value of 0.5 is the assumption that death occurs in the middle of the year and the value is not calculated by the value in the mortality table [13]. Furthermore, the value of the m_x function will be calculated which states the force of mortality measured per unit time [14].

$$_nm_x = \frac{_nd_x}{_n\mathcal{L}_x} \tag{4}$$

Gompertz Mortality Law

Gompertz Mortality Law was first introduced by Benjamin Gompertz in 1825 [15]. This model has two parameters, namely the parameter α as the initial value of the function of the force of mortality and parameter β as a speed of increasing the force of mortality to age [16]. This law modeling that the rate of death or force of mortality (μ_x) increases exponentially with age [6].

The law of Gompertz mortality is defined by the rate of death as

$$\mu_x = \alpha\beta^x, \tag{5}$$

for $x > 0$, $\alpha > 0$, and $\beta > 1$. The Gompertz Mortality Law SDF is

$$S_X(x) = \exp\left[\int_0^x \mu_y \, dy\right] = \exp\left[\frac{\alpha}{\ln \beta}(1 - \beta^x)\right]. \tag{6}$$

Furthermore, it can be calculated Probability Density Function (PDF) Gompertz mortality by multiplying the rate of death and the survival distribution function [17]. Obtained PDF for Gompertz mortality

$$f_X(x) = \mu_x \cdot S_X(x) = \alpha\beta^x \cdot \exp\left[\frac{\alpha}{\ln \beta}(1 - \beta^x)\right].$$

Estimation of parameters α and β on the Gompertz model

Parameters α and β on the law of mortality Gompertz were conducted using three methods, namely NLLS through Gauss-Newton algorithm, WLS, and Poisson regression algorithms.

- **Non-Linear Least Square (NLLS)**

Estimated parameters of α and β carried out with an NLLS approach, namely by minimizing the number of quadratic differences between empirical values and the prediction results of the Gompertz mortality model

$$S(\alpha,\beta) = \sum_{i=1}^{n} (m_i - \alpha \cdot \beta^{x_i})^2 \tag{7}$$

Then, the partial derivative of function of equation (7) for the α and β parameters become

$$\frac{\partial S}{\partial \alpha} = -2 \sum_{i=1}^{n} (m_i - \alpha \cdot \beta^{x_i})\, \beta^{x_i}$$
$$\frac{\partial S}{\partial \beta} = -2\alpha \sum_{i=1}^{n} (m_i - \alpha \cdot \beta^{x_i})\, x_i \beta^{x_i - 1}$$

so that the NLLS equation system is formed, namely

$$\sum_{i=1}^{n} (m_i - \alpha \cdot \beta^{x_i})\, \beta^{x_i} = 0$$
$$\sum_{i=1}^{n} (m_i - \alpha \cdot \beta^{x_i})\, x_i \beta^{x_i - 1} = 0$$

The equation has no exact solution, so a numerical approach is needed to solve the problem. The method used in this study is the Gauss-Newton method. The Gauss-Newton method is an iterative algorithm to estimate the parameters α dan β in the Gompertz mortality model

$$m_i = \alpha\beta^{x_i}$$

by minimizing the square error between data and models. The steps for Gauss-Newton iterations are [18]:

1. Determine the initial guess of the parameter α_0 and β_0.
2. Calculate the residual

$$r_i = m_i - \alpha\beta^{x_i}$$

which is the difference between data and models.

3. Calculate the Jacobian matrix containing a partial residual derivative to α and β:

$$\frac{\partial r_i}{\partial \alpha} = -\beta^{x_i}, \quad \frac{\partial r_i}{\partial \beta} = -\alpha x_i \beta^{x_i - 1}$$

4. Update the parameter with the formula

$$\theta^{(k+1)} = \theta^{(k)} - (J^T J)^{-1} J^T \mathbf{r}$$

   with Jacobian matrix $\theta = (\alpha, \beta)^T$, J, and residual vector $\boldsymbol{r}$.

5. Repeat steps 2-4 until the relative changes of parameters between iterations are very small (convergent), namely

$$\left|\frac{\hat{\theta}_{k+1} - \hat{\theta}_k}{\hat{\theta}_k}\right| < \delta$$

   Through this process, the estimated parameters are improved in stages until they reach the optimum value and the minimum error.

- **Weighted Least Square (WLS)**

According to [19], linear regression model parameters with non -constant variants of errors can be estimated using the WLS method. In this method, the quadratic function of the error is given weight w_i so that the function is minimized to be

$$S(\beta_0, \beta_1) = \sum_{i=1}^{n} w_i (y_i - \beta_0 - \beta_1 z_i)^2 . \tag{8}$$

Based on the equation (8) WLS normal equation can be obtained, namely

$$\hat{\beta}_0 \sum_{i=1}^{n} w_i + \hat{\beta}_1 \sum_{i=1}^{n} w_i\, z_i \& = \sum_{i=1}^{n} w_i\, y_i, \tag{9}$$

$$\hat{\beta}_0 \sum_{i=1}^{n} w_i\, z_i + \hat{\beta}_1 \sum_{i=1}^{n} w_i\, z_i^2 \& = \sum_{i=1}^{n} w_i\, y_i z_i. \tag{10}$$

By completing the equation (9) and (10), obtained parameter estimator

$$\hat{\beta}_1 = \frac{(\sum_{i=1}^{n} w_i\, y_i z_i) \sum_{i=1}^{n} w_i - (\sum_{i=1}^{n} w_i\, y_i)(\sum_{i=1}^{n} w_i\, z_i)}{(\sum_{i=1}^{n} w_i\, z_i^2) \sum_{i=1}^{n} w_i - (\sum_{i=1}^{n} w_i\, z_i)^2},$$

$$\hat{\beta}_0 = \frac{\sum_{i=1}^{n} w_i\, y_i - \hat{\beta}_1 \sum_{i=1}^{n} w_i\, z_i}{\sum_{i=1}^{n} w_i}.$$

In the Gompertz model, the quadratic function of the error weighted for linear regression in the natural logarithm transformation data is defined as

$$S(\alpha^*, \beta^*) = \sum_{i=1}^{n} w_i \left(\ln \mu_{x_i} - \alpha^* - \beta^* x_i\right)^2 \tag{11}$$

where the weight w_i is usually taken from the number of deaths d_{x_i} at the age x_i. By equating these variables to become

$$y_i = \ln\mu_{x_i}, \quad z_i = x_i, \quad \beta_0 = \alpha^*, \quad \beta_1 = \beta^*,$$

then the Gompertz parameter estimator is obtained by the WLS method, namely

$$\hat{\beta}^* = \frac{\left(\sum_{i=1}^{n} d_{x_i} y_i z_i\right)\sum_{i=1}^{n} d_{x_i} - \left(\sum_{i=1}^{n} d_{x_i} y_i\right)\left(\sum_{i=1}^{n} d_{x_i} z_i\right)}{\left(\sum_{i=1}^{n} d_{x_i} z_i^2\right)\sum_{i=1}^{n} d_{x_i} - \left(\sum_{i=1}^{n} d_{x_i} z_i\right)^2}, \tag{12}$$

$$\hat{\alpha}^* = \frac{\sum_{i=1}^{n} d_{x_i} y_i - \hat{\beta}^* \sum_{i=1}^{n} d_{x_i} z_i}{\sum_{i=1}^{n} d_{x_i}}. \tag{13}$$

The data in the mortality table is μ_{x_i} and d_{x_i} can be directly used to calculate the parameter estimation of the Gompertz model with the equation (12) and (13).

- **Poisson Regression**

According to [19], Poisson regression is one of the models used to explain the relationship between observational data in the form of a count (number of events) with predictor variables. In this model, it is assumed that the variable response y_i in the form of chopped numbers, namely $y_i = 0, 1, 2, \ldots$. Count is a statistical data type that describes the number of events that can be calculated and have a non-negative integer value [20]. The Gompertz model in this case can be written as

$$\mu_x = \alpha\beta^x = e^{\ln\alpha + x\ln\beta}. \tag{14}$$

Based on the equation (14), obtained

$$\frac{d_x}{l_x} = e^{\ln\alpha + x\ln\beta}$$

$$d_x = e^{\ln\alpha + x\ln\beta + \ln l_x} \tag{15}$$

Equations (15) can be considered a deterministic model. To enter stochastic aspects in accordance with observation data [21], the model was developed into

$$d_x = e^{\ln\alpha + x\ln\beta + \ln l_x} + \varepsilon,$$

with ε as an error. For example, $\alpha^* = \ln$ and $\beta^* = \ln$ and variable d_{x_i} is assumed to distribute Poisson with parameters λ_i, so

$$d_{x_i} = e^{\alpha^* + \beta^* x_i + \ln l_{x_i}} + \varepsilon_i$$
$$\lambda_i = e^{\alpha^* + \beta^* x_i + \ln l_{x_i}}.$$

Then, the logarithmic function for the data d_{x_i} is

$$\ln L(\alpha^*, \beta^*) = \sum_{i=1}^{n} \left(-\lambda_i + d_{x_i}\ln\lambda_i - \ln\left(d_{x_i}!\right)\right)$$

$$\ln L(\alpha^*, \beta^*) = \sum_{i=1}^{n} \left(-e^{\alpha^*+\beta^* x_i+\ln l_{x_i}} + d_{x_i}\left(\alpha^* + \beta^* x_i + \ln l_{x_i}\right) - \ln\left(d_{x_i}!\right)\right)$$

To find the values α^* and β^* which maximizes lnL, done by completing the equation

$$\frac{\partial \ln L}{\partial \alpha^*} = \sum_{i=1}^{n} \left(-e^{\alpha^*+\beta^* x_i+\ln l_{x_i}} + d_{x_i}\right) = 0,$$

$$\frac{\partial \ln L}{\partial \beta^*} = \sum_{i=1}^{n} x_i \left(-e^{\alpha^*+\beta^* x_i+\ln l_{x_i}} + d_{x_i}\right) = 0.$$

Because the equation is difficult to resolve analytically, the completion of the parameter estimation α^* and β^* are generally carried out using the help of programming software.

Calculating the Probability of Death (q_x)

After obtaining the value of each parameter estimator, q_x will be calculated for each of the resulting Gompertz models [14].

$$F_{T_x}(t) = Pr(T_x \leq t) = Pr(X \leq x + t | X > x)$$

$$F_{T_x}(t) = 1 - \frac{Pr(X > x + t)}{Pr(X > x)}$$

$$F_{T_x}(t) = 1 - \frac{S_X(x + t)}{S_X(x)}. \quad (16)$$

Calculation of q_x requires the value of the function $S_x(x)$ so that it will be calculated first using the equation (6).

Evaluating the Estimating Results

The process of evaluating predictive accuracy measurements is carried out for Gompertz parameters (α and β) are obtained from each of the estimation methods. For the performance of the estimated model, it will be measured using RMSE. According to [22], using RMSE as a model validation tool can provide an overall error distribution picture.

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} {e_i}^2}, \quad (17)$$

with e_i is error for i data, which is the difference between actual data and prediction results.

RESULTS AND DISCUSSION

Construction of Indonesia Population Mortality Table 2023

The following are the results of the construction of the calculation of the functions in the TMPI 2023

Table 1. Construction results of TMPI 2023

x	q_x	p_x	l_x	d_x	$\mathcal{L}_x$	m_x
0	0,007880	0,992120	100000,00	788,00	99606,00	0,00791
1	0,002096	0,997904	99212,00	207,95	99108,03	0,00210
2	0,000900	0,999100	99004,05	89,10	98959,50	0,00090
⋮	⋮	⋮	⋮	⋮	⋮	⋮
109	0,518532	0,481468	187,24	97,09	138,70	0,70002
110	0,559684	0,440316	90,15	50,46	64,92	0,77717
111	1,000000	0,000000	39,70	39,70	19,85	2,00000

Table 1 as the construction results will be a reference in determining the value of the Gompertz mortality parameter, especially the value of m_x which will review the force of mortality based on age.

Estimation of Gompertz Parameters

Estimation of the parameters is done using four case group model that has been formed for each estimation method used and using TMPI data as a calculated observation data.

i) Gompertz Model for male with age limitation from 35 to 100 years,
ii) Gompertz Model for female with age limitation from 35 to 100 years,
iii) Gompertz Model for male with age limitation from 0 to 111 years, and
iv) Gompertz Model for female with age limitation from 0 to 111 years.

For each case group of Gompertz model will be an estimation of parameters by the NLLS method through the Gauss-Newton algorithm where the convergence limit is 10^{-6}, WLS, and Poisson regression so that produces twelve models of the estimated process. The following is a summary of values for the α and β from each parameter.

Table 2. Gompertz model parameter estimation results

Group	$\hat{\alpha}$			$\hat{\beta}$		
	NLLS	WLS	Poisson Regression	NLLS	WLS	Poisson Regression
Male (35–100)	0,000025	0,000126	0,000200	1,099597	1,080947	1,073663
Female (35–100)	0,000007	0,000130	0,000151	1,110529	1,075340	1,072537
Male (0–111)	0,000037	0,000244	0,000247	1,095348	1,075315	1,070882
Female (0–111)	0,000003	0,000225	0,000170	1,119224	1,070744	1,071454

or if we visualized as follows.

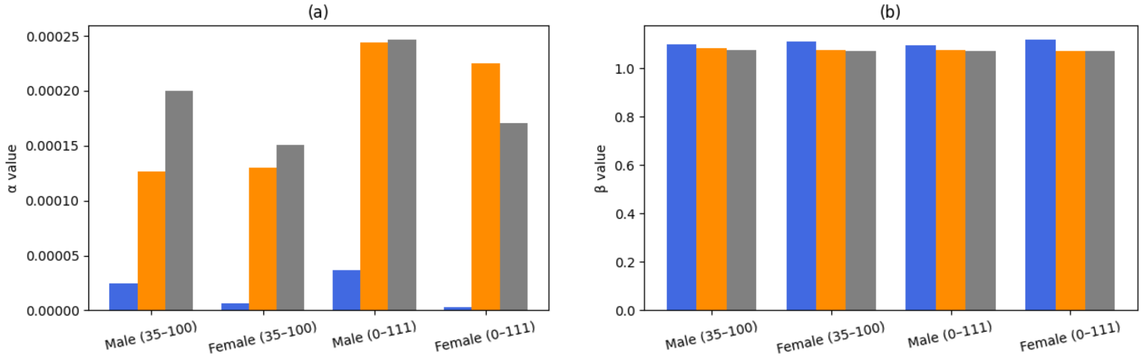

Figure 1. Estimator $\hat{\alpha}$ and $\hat{\beta}$ based on the method of Gompertz parameters

Based on Figure 1 and Table 2, the α parameter illustrates the initial mortality rate has a very small value in all groups and methods with a value that tends to be smaller in the elderly group (35–100). This is in line with theoretical expectations that in old age, the probability of death increases sharply, so that α as baseline hazard is lower. On the other hand, the β parameter, which represents the level of mortality acceleration, shows a more significant variation between methods and groups. The estimated results show that NLLS approach tends to provide an estimated acceleration of death that is higher than the WLS and Poisson regression methods, while Poisson regression gives relatively stable and moderate results. In addition, differences in the results of greater estimates in women's groups than men show the possibility of greater mortality distribution variability in women's populations, which deserve a concern in the selection of actuarial or demographic models.

When reviewed specifically in the age group 35-100, the NLLS method produces the highest $\hat{\beta}$ estimation, namely 1,099597 for male, while Poisson regression provides the lowest estimation of 1,073663. This indicates that the NLLS method estimates that the acceleration of death is faster in this group than other methods. Similar patterns are also seen in women's groups, with a value of $\hat{\beta}$ from NLLS of 1,110529 which is higher than Poisson (1,075237), and the smallest value of α on NLLS. This finding indicates that the NLLS approach tends to produce an estimated parameter that is more extreme than other methods, especially for the elderly age group.

For the age group 0–111, the difference in estimation between methods looks more moderate. Estimated parameters in male groups show consistency between methods with small variations, where the value of β ranges from 1,078082 to 1,095348. Meanwhile, in groups of women aged 0–111, variations between methods are more striking, especially in the β estimated than 1,119224 which is significantly higher than WLS and Poisson. This indicates that in women's populations with broad age coverage, the estimated Gompertz model is more sensitive to the estimated method used.

Probability of Death (q_x) based on Gompertz Estimator

Function value $S_x(x)$ required in the calculation of the function value q_x. Therefore, it is necessary to calculate the value of the function $S_x(x)$ using the equation (6) with the parameter values of α and β estimated results for each Gompertz model. For the male Gompertz model with age limits from 0 to 111 years in the NLLS method, obtained

$$S_X(0) = exp\left[\frac{0{,}0001002}{ln(1{,}0826571)}(1-(1{,}0826571)^0)\right] = 1.$$

$S_x(1)$, $S_x(2)$, and so on are calculated in the same way. Next, the function value q_x is calculated based on the equation (16). For the male Gompertz model with age limits from 0 to 111 years in the NLLS method, obtained

$$q_0 = 1 - \frac{S_X(1)}{S_X(0)} = 1 - \frac{0{,}9998957}{1} \approx 0{,}0001043$$

Value of q_1, q_2, and etc. are calculated in the same way and calculated for each other Gompertz model. Overview of the value of the function q_x each model is displayed with a graph in the Figure 2.

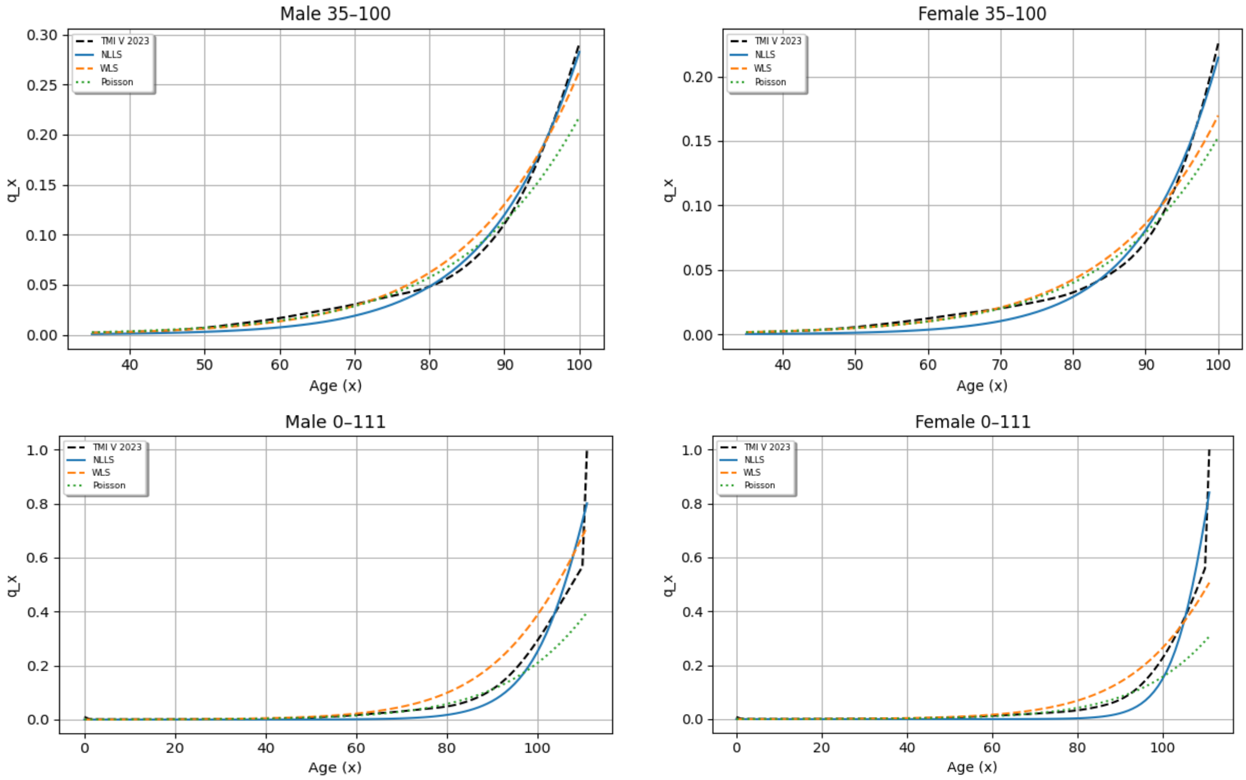

Figure 2. Function graphs of q_x for each Gompertz Model estimated

Evaluation of Estimation Results Performance

The RMSE value for each model is calculated by the equation (17). Defined the error for i data denoted with e_i, as a difference between the value of the function q_x on TMPI and the function value of q_x to i in the estimated model, namely

$$e_i = q_{x_i}\,TMPI - q_{x_i}\,model.$$

For the male Gompertz model with age limits from 0 to 111 years in the NLLS method, obtained

$$e_1 = q_0\,TMPI - q_0\,model = 0{,}00524 - 0{,}0001043 = 0{,}0051357.$$

For $e_2, e_3, \dots, e_{112}$ is calculated in the same way. Furthermore, the RMSE value is calculated using the equation (17), namely

$$\text{RMSE} = \sqrt{\frac{1}{112}\sum_{i=1}^{112} {e_i}^2} = \sqrt{0{,}00034029} \approx 0{,}007461.$$

The ratio of RMSE values can be seen in Figure 3.

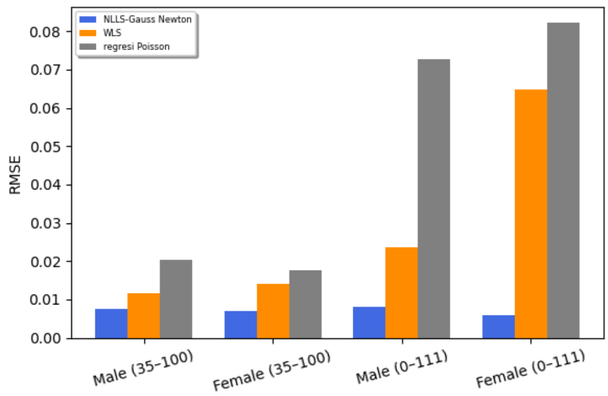

Figure 3. Comparison of RMSE values for each estimated model.

CONCLUSIONS

Based on the results and discussion of this study, it can be concluded that the estimated parameter of the Gompertz model is carried out with three methods, namely NLLS, WLS, and Poisson Regression, each of which using the function of the force of mortality (m_x) or function d_x as the main variable after the transformation of natural logarithms. The estimated parameters in NLLS and WLS are done by minimizing the number of quadratic errors, where in WLS weighted d_x, While Poisson Regression maximizes data based on count data function of d_x. The accuracy of each method is evaluated by comparing the value of death probability functions (q_x) results of the estimation of TMPI reference data through graphic analysis and RMSE calculation, which shows that the NLLS method provides the most accurate estimated Gompertz model. As an input in further research, it is recommended that the estimated parameters are carried out directly using the function formulation q_x so that it can avoid assumptions related to values ${}_na_x$ in the calculation of the mortality table. In addition, further exploration needs to be carried out on the selection of weighted values in the WLS method, for example by considering alternative weight such as Huber or Tukey, to determine its effect on the estimated results of the Gompertz model parameter.

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