

SUPPLEMENTARY INFORMATION

CIF-SWARA-TOPSIS

Five experts were asked to evaluate five alternatives with respect to seven criteria. Each expert provided assessments based on the predefined linguistic scale (Table 1.), expressing their opinions regarding the performance of each alternative under every criterion.

Criteria	Experts	A ₁	A ₂	A ₃	A ₄	A ₅
C ₁	$\mathcal{E}_1$	EHL	VHL	VHL	VHL	VHL
	$\mathcal{E}_2$	VHL	VHL	VHL	HL	VHL
	$\mathcal{E}_3$	EHL	VHL	VHL	VHL	EHL
	$\mathcal{E}_4$	EHL	VHL	EHL	AAL	HL
	$\mathcal{E}_5$	EHL	EHL	HL	HL	HL
C ₂	$\mathcal{E}_1$	EHL	VHL	HL	AL	AAL
	$\mathcal{E}_2$	EHL	EHL	HL	AAL	VHL
	$\mathcal{E}_3$	EHL	EHL	EHL	VHL	EHL
	$\mathcal{E}_4$	EHL	EHL	EHL	AAL	HL
	$\mathcal{E}_5$	VHL	VHL	HL	HL	HL
C ₃	$\mathcal{E}_1$	EHL	EHL	HL	HL	VHL
	$\mathcal{E}_2$	EHL	EHL	EHL	HL	VHL
	$\mathcal{E}_3$	EHL	VHL	VHL	VHL	VHL
	$\mathcal{E}_4$	EHL	EHL	EHL	AAL	HL
	$\mathcal{E}_5$	VHL	VHL	HL	HL	HL
C ₄	$\mathcal{E}_1$	EHL	VHL	EHL	VHL	EHL
	$\mathcal{E}_2$	EHL	HL	EHL	EHL	EHL
	$\mathcal{E}_3$	EHL	HL	VHL	VHL	EHL
	$\mathcal{E}_4$	EHL	EHL	EHL	HL	VHL

	$\mathcal{E}_5$	VHL	VHL	VHL	VHL	VHL
$\mathbf{C}_5$	$\mathcal{E}_1$	EHL	VHL	VHL	HL	VHL
	$\mathcal{E}_2$	EHL	EHL	EHL	VHL	EHL
	$\mathcal{E}_3$	EHL	EHL	EHL	EHL	EHL
	$\mathcal{E}_4$	EHL	EHL	EHL	AAL	VHL
	$\mathcal{E}_5$	VHL	VHL	VHL	HL	VHL
$\mathbf{C}_6$	$\mathcal{E}_1$	ELL	VLL	VLL	LL	VLL
	$\mathcal{E}_2$	ELL	LL	BAL	AL	BAL
	$\mathcal{E}_3$	VLL	LL	LL	LL	LL
	$\mathcal{E}_4$	VLL	LL	LL	BAL	BAL
	$\mathcal{E}_5$	VLL	LL	LL	LL	LL
$\mathbf{C}_7$	$\mathcal{E}_1$	ELL	ELL	VLL	LL	LL
	$\mathcal{E}_2$	VLL	VLL	BAL	LL	LL
	$\mathcal{E}_3$	ELL	ELL	VLL	VLL	ELL
	$\mathcal{E}_4$	VLL	VLL	AAL	AAL	VLL
	$\mathcal{E}_5$	VLL	VLL	BAL	BAL	BAL

Subsequently, the table is converted into a CIF number based on the linguistic scale presented in Table 1.

Criteria	Exp	Alternatives									
		A1		A2		A3		A4		A5	
C1	E1	0,900	0,100	0,800	0,150	0,800	0,150	0,800	0,150	0,800	0,150
	E2	0,800	0,150	0,800	0,150	0,800	0,150	0,700	0,250	0,800	0,150
	E3	0,900	0,100	0,800	0,150	0,800	0,150	0,800	0,150	0,900	0,100
	E4	0,900	0,100	0,800	0,150	0,900	0,100	0,600	0,350	0,700	0,250
	E5	0,900	0,100	0,900	0,100	0,700	0,250	0,700	0,250	0,700	0,250
C2	E1	0,900	0,100	0,800	0,150	0,700	0,250	0,500	0,450	0,600	0,350
	E2	0,900	0,100	0,900	0,100	0,700	0,250	0,600	0,350	0,800	0,150
	E3	0,900	0,100	0,900	0,100	0,900	0,100	0,800	0,150	0,900	0,100

	E4	0,900	0,100	0,900	0,100	0,900	0,100	0,600	0,350	0,700	0,250
	E5	0,800	0,150	0,800	0,150	0,700	0,250	0,700	0,250	0,700	0,250
C3	E1	0,900	0,100	0,900	0,100	0,700	0,250	0,700	0,250	0,800	0,150
	E2	0,900	0,100	0,900	0,100	0,900	0,100	0,700	0,250	0,800	0,150
	E3	0,900	0,100	0,800	0,150	0,800	0,150	0,800	0,150	0,800	0,150
	E4	0,900	0,100	0,900	0,100	0,900	0,100	0,600	0,350	0,700	0,250
	E5	0,800	0,150	0,800	0,150	0,700	0,250	0,700	0,250	0,700	0,250
C4	E1	0,900	0,100	0,800	0,150	0,900	0,100	0,800	0,150	0,900	0,100
	E2	0,900	0,100	0,700	0,250	0,900	0,100	0,900	0,150	0,900	0,100
	E3	0,900	0,100	0,700	0,250	0,800	0,150	0,800	0,150	0,900	0,100
	E4	0,900	0,100	0,900	0,100	0,900	0,100	0,700	0,250	0,800	0,150
	E5	0,800	0,150	0,800	0,150	0,800	0,150	0,800	0,150	0,800	0,150
C5	E1	0,900	0,100	0,800	0,150	0,800	0,150	0,700	0,250	0,800	0,150
	E2	0,900	0,100	0,900	0,100	0,900	0,100	0,800	0,150	0,900	0,100
	E3	0,900	0,100	0,900	0,100	0,900	0,100	0,900	0,100	0,900	0,100
	E4	0,900	0,100	0,900	0,100	0,900	0,100	0,600	0,350	0,800	0,150
	E5	0,800	0,150	0,800	0,150	0,800	0,150	0,700	0,250	0,800	0,150
C6	E1	0,100	0,900	0,200	0,750	0,200	0,750	0,300	0,650	0,200	0,750
	E2	0,100	0,900	0,300	0,650	0,400	0,550	0,500	0,450	0,400	0,550
	E3	0,200	0,750	0,300	0,650	0,300	0,650	0,300	0,650	0,300	0,650
	E4	0,200	0,750	0,300	0,650	0,300	0,650	0,400	0,550	0,400	0,550
	E5	0,200	0,750	0,300	0,650	0,300	0,650	0,300	0,650	0,300	0,650
C7	E1	0,100	0,900	0,100	0,900	0,200	0,750	0,300	0,650	0,300	0,650
	E2	0,200	0,750	0,200	0,750	0,400	0,550	0,300	0,650	0,300	0,650
	E3	0,100	0,900	0,100	0,900	0,200	0,750	0,200	0,750	0,100	0,900
	E4	0,200	0,750	0,200	0,750	0,600	0,350	0,600	0,350	0,200	0,750
	E5	0,200	0,750	0,200	0,750	0,400	0,550	0,400	0,550	0,400	0,550

Subsequently, the IF values obtained from the experts' evaluations are aggregated into CIF using Equation (9), and radius is determined based on Equation (3), as presented below.

$$\begin{aligned}
C_{1,1} &= \left\langle \frac{0,900+0,800+0,900+0,900+0,900}{5}, \frac{0,100+0,150+0,100+0,100+0,100}{5} \right\rangle = \langle 0.880, 0.110 \rangle \\
C_{1,2} &= \left\langle \frac{0,800+0,800+0,800+0,800+0,900}{5}, \frac{0,150+0,150+0,150+0,150+0,100}{5} \right\rangle = \langle 0.820, 0.140 \rangle \\
&\vdots \\
C_{7,4} &= \left\langle \frac{0,300+0,300+0,200+0,600+0,400}{5}, \frac{0,650+0,650+0,750+0,350+0,550}{5} \right\rangle = \langle 0.360, 0.590 \rangle \\
C_{7,5} &= \left\langle \frac{0,300+0,300+0,100+0,200+0,400}{5}, \frac{0,650+0,650+0,900+0,750+0,550}{5} \right\rangle = \langle 0.260, 0.700 \rangle
\end{aligned}$$

$$r_{1,1} = \sqrt{\frac{(0.880-0.900)^2 + (0.110-0.100)^2 + (0.880-0.900)^2 + (0.110-0.150)^2 + \dots + (0.880-0.900)^2 + (0.110-0.100)^2}{5}} = 0.045$$

$$r_{1,2} = \sqrt{\frac{(0.820-0.800)^2 + (0.140-0.150)^2 + (0.820-0.800)^2 + (0.140-0.150)^2 + \dots + (0.880-0.900)^2 + (0.880-0.100)^2}{5}} = 0.045$$

⋮

$$r_{7,4} = \sqrt{\frac{(0.360-0.300)^2 + (0.590-0.650)^2 + (0.360-0.300)^2 + (0.590-0.650)^2 + \dots + (0.360-0.400)^2 + (0.590-0.550)^2}{5}} = 0.192$$

$$r_{7,5} = \sqrt{\frac{(0.260-0.300)^2 + (0.700-0.650)^2 + (0.260-0.300)^2 + (0.700-0.650)^2 + \dots + (0.260-0.200)^2 + (0.700-0.550)^2}{5}} = 0.156$$

The results of the aggregation and the determination of the radius are presented in the following table.

Criteria	Alternatives														
	A1			A2			A3			A4			A5		
C1	0,880	0,110	0,045	0,820	0,140	0,045	0,800	0,160	0,080	0,720	0,230	0,106	0,780	0,180	0,096
C2	0,880	0,110	0,045	0,860	0,120	0,055	0,780	0,190	0,122	0,640	0,310	0,144	0,740	0,220	0,050
C3	0,880	0,110	0,045	0,860	0,120	0,055	0,800	0,170	0,112	0,700	0,250	0,089	0,760	0,190	0,103
C4	0,880	0,110	0,045	0,780	0,180	0,096	0,860	0,120	0,055	0,800	0,170	0,075	0,860	0,120	0,055
C5	0,880	0,110	0,045	0,860	0,120	0,055	0,860	0,120	0,055	0,740	0,220	0,134	0,840	0,130	0,055
C6	0,160	0,810	0,088	0,280	0,670	0,057	0,300	0,650	0,089	0,360	0,590	0,113	0,320	0,630	0,106
C7	0,160	0,810	0,088	0,160	0,810	0,088	0,360	0,590	0,212	0,360	0,590	0,192	0,260	0,700	0,156

Subsequently, the optimistic and pessimistic matrices are determined using Equations (10) and (11), respectively.

Optimistic:

$$\mu_{1,1} = 0.880 + \frac{0.045}{2}\sqrt{2} = 0.912$$

$$v_{1,1} = 0.110 - \frac{0.045}{2}\sqrt{2} = 0.078$$

$$\mu_{1,2} = 0.820 + \frac{0.045}{2}\sqrt{2} = 0.852$$

$$v_{1,2} = 0.140 - \frac{0.045}{2}\sqrt{2} = 0.108$$

$$\mu_{1,3} = 0.800 + \frac{0.080}{2}\sqrt{2} = 0.857$$

$$v_{1,3} = 0.160 - \frac{0.080}{2}\sqrt{2} = 0.103$$

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⋮

$$\mu_{7,4} = 0.360 + \frac{0.192}{2}\sqrt{2} = 0.496$$

$$v_{7,4} = 0.590 - \frac{0.192}{2}\sqrt{2} = 0.454$$

$$\mu_{7,5} = 0.260 + \frac{0.156}{2}\sqrt{2} = 0.370$$

$$v_{7,5} = 0.700 - \frac{0.156}{2}\sqrt{2} = 0.590$$

Pessimistic:

$$\mu_{1,1} = 0.880 - \frac{0.045}{2}\sqrt{2} = 0.848$$

$$v_{1,1} = 0.110 + \frac{0.045}{2}\sqrt{2} = 0.142$$

$$\mu_{1,2} = 0.820 - \frac{0.045}{2}\sqrt{2} = 0.788$$

$$v_{1,2} = 0.140 + \frac{0.045}{2}\sqrt{2} = 0.172$$

$$\mu_{1,3} = 0.800 - \frac{0.080}{2}\sqrt{2} = 0.743$$

⋮

$$\mu_{7,4} = 0.360 - \frac{0.192}{2}\sqrt{2} = 0.224$$

$$\mu_{7,5} = 0.260 - \frac{0.156}{2}\sqrt{2} = 0.150$$

$$v_{1,3} = 0.160 + \frac{0.080}{2}\sqrt{2} = 0.217$$

⋮

$$v_{7,4} = 0.590 + \frac{0.192}{2}\sqrt{2} = 0.726$$

$$v_{7,5} = 0.700 + \frac{0.156}{2}\sqrt{2} = 0.810$$

The optimistic and pessimistic matrices are presented in the table below.

Optimistic Matrix

Criteria	Alternatives									
	A1		A2		A3		A4		A5	
C1	0,912	0,078	0,852	0,108	0,857	0,103	0,795	0,155	0,848	0,112
C2	0,912	0,078	0,899	0,081	0,867	0,103	0,742	0,208	0,775	0,185
C3	0,912	0,078	0,899	0,081	0,879	0,091	0,763	0,187	0,833	0,117
C4	0,912	0,078	0,848	0,112	0,899	0,081	0,853	0,117	0,899	0,081
C5	0,912	0,078	0,899	0,081	0,899	0,081	0,835	0,125	0,879	0,091
C6	0,222	0,748	0,320	0,630	0,363	0,587	0,440	0,510	0,395	0,555
C7	0,222	0,748	0,222	0,748	0,510	0,440	0,496	0,454	0,370	0,590

Pessimistic Matrix

Criteria	Alternatives									
	A1		A2		A3		A4		A5	
C1	0,848	0,142	0,788	0,172	0,743	0,217	0,645	0,305	0,712	0,248
C2	0,848	0,142	0,821	0,159	0,693	0,277	0,538	0,412	0,705	0,255
C3	0,848	0,142	0,821	0,159	0,721	0,249	0,637	0,313	0,687	0,263
C4	0,848	0,142	0,712	0,248	0,821	0,159	0,747	0,223	0,821	0,159
C5	0,848	0,142	0,821	0,159	0,821	0,159	0,645	0,315	0,801	0,169
C6	0,098	0,872	0,240	0,710	0,237	0,713	0,280	0,670	0,245	0,705
C7	0,098	0,872	0,098	0,872	0,210	0,740	0,224	0,726	0,150	0,810

In the next stage, the determination of the criteria weights is carried out using the SWARA method. The experts first evaluated the criteria using the linguistic terms in Table 2, and these evaluations are converted into IF values based on the same table.

Criteria	$\mathcal{E}1$	$\mathcal{E}2$	$\mathcal{E}3$	$\mathcal{E}4$	$\mathcal{E}5$
C1	VHL	VHL	AHL	AHL	AHL
C2	AHL	AHL	VHL	AHL	AHL

C3	AHL	VHL	VHL	AHL	VHL
C4	VHL	VHL	VHL	AHL	AHL
C5	VHL	AHL	AHL	AHL	AHL
C6	VHL	AHL	AHL	AHL	VHL
C7	AHL	AHL	AHL	AHL	VHL

Criteria	Experts									
	$\mathcal{E}_1$		$\mathcal{E}_2$		$\mathcal{E}_3$		$\mathcal{E}_4$		$\mathcal{E}_5$	
C1	0,750	0,150	0,750	0,150	0,850	0,050	0,850	0,050	0,850	0,050
C2	0,850	0,050	0,850	0,050	0,750	0,150	0,850	0,050	0,850	0,050
C3	0,850	0,050	0,750	0,150	0,750	0,150	0,850	0,050	0,750	0,150
C4	0,750	0,150	0,750	0,150	0,750	0,150	0,850	0,050	0,850	0,050
C5	0,750	0,150	0,850	0,050	0,850	0,050	0,850	0,050	0,850	0,050
C6	0,750	0,150	0,850	0,050	0,850	0,050	0,850	0,050	0,750	0,150
C7	0,850	0,050	0,850	0,050	0,850	0,050	0,850	0,050	0,750	0,150

The determination of the experts' weights is then carried out using Equation (12). The importance level of each expert is assessed based on Table 2, considering their position and years of experience in the respective field.

Experts	$\mathcal{E}_1$	$\mathcal{E}_2$	$\mathcal{E}_3$	$\mathcal{E}_4$	$\mathcal{E}_5$
Importance Level	AHL	AHL	VHL	VHL	HL
IF Value	(0.850,0.050)	(0.850,0.050)	(0.750,0.150)	(0.750,0.150)	(0.650,0.250)

$$\omega_1 = \frac{[0.850+0.100(\frac{0.850}{1-0.100})]}{[0.850+0.100(\frac{0.850}{1-0.100})]+[0.850+0.100(\frac{0.850}{1-0.100})]+[0.750+0.100(\frac{0.750}{1-0.100})]+[0.750+0.100(\frac{0.750}{1-0.100})]+[0.650+0.100(\frac{0.650}{1-0.100})]} = 0.221$$

$$\omega_2 = \frac{[0.850+0.100(\frac{0.850}{1-0.100})]}{[0.850+0.100(\frac{0.850}{1-0.100})]+[0.850+0.100(\frac{0.850}{1-0.100})]+[0.750+0.100(\frac{0.750}{1-0.100})]+[0.750+0.100(\frac{0.750}{1-0.100})]+[0.650+0.100(\frac{0.650}{1-0.100})]} = 0.221$$

$$\omega_3 = \frac{[0.750+0.100(\frac{0.750}{1-0.100})]}{[0.850+0.100(\frac{0.850}{1-0.100})]+[0.850+0.100(\frac{0.850}{1-0.100})]+[0.750+0.100(\frac{0.750}{1-0.100})]+[0.750+0.100(\frac{0.750}{1-0.100})]+[0.650+0.100(\frac{0.650}{1-0.100})]}$$

$$= 0.195$$

$$\omega_4 = \frac{[0.750+0.100(\frac{0.750}{1-0.100})]}{[0.850+0.100(\frac{0.850}{1-0.100})]+[0.850+0.100(\frac{0.850}{1-0.100})]+[0.750+0.100(\frac{0.750}{1-0.100})]+[0.750+0.100(\frac{0.750}{1-0.100})]+[0.650+0.100(\frac{0.650}{1-0.100})]}$$

$$= 0.195$$

$$\omega_5 = \frac{[0.650+0.100(\frac{0.650}{1-0.100})]}{[0.850+0.100(\frac{0.850}{1-0.100})]+[0.850+0.100(\frac{0.850}{1-0.100})]+[0.750+0.100(\frac{0.750}{1-0.100})]+[0.750+0.100(\frac{0.750}{1-0.100})]+[0.650+0.100(\frac{0.650}{1-0.100})]}$$

$$= 0.169$$

Experts	ε_1	ε_2	ε_3	ε_4	ε_5
Weight	0,221	0,221	0,195	0,195	0,169

The aggregation of the experts' evaluations is performed using Equation (2) and the radius calculated using Equation (3).

$$C_1 = \langle 0.750^{0.221} \cdot 0.750^{0.221} \cdot \dots \cdot 0.850^{0.169}, 1 - ((1 - 0.150)^{0.221} \cdot (1 - 0.150)^{0.221} \cdot \dots \cdot (1 - 0.050)^{0.169}) \rangle$$

$$= \langle 0.804, 0.005 \rangle$$

$$C_2 = \langle 0.850^{0.221} \cdot 0.850^{0.221} \cdot \dots \cdot 0.850^{0.169}, 1 - ((1 - 0.050)^{0.221} \cdot (1 - 0.050)^{0.221} \cdot \dots \cdot (1 - 0.050)^{0.169}) \rangle$$

$$= \langle 0.830, 0.070 \rangle$$

$$C_3 = \langle 0.850^{0.221} \cdot 0.750^{0.221} \cdot \dots \cdot 0.750^{0.169}, 1 - ((1 - 0.050)^{0.221} \cdot (1 - 0.150)^{0.221} \cdot \dots \cdot (1 - 0.150)^{0.169}) \rangle$$

$$= \langle 0.790, 0.110 \rangle$$

⋮

$$C_7 = \langle 0.850^{0.221} \cdot 0.850^{0.221} \cdot \dots \cdot 0.750^{0.169}, 1 - ((1 - 0.050)^{0.221} \cdot (1 - 0.050)^{0.221} \cdot \dots \cdot (1 - 0.150)^{0.169}) \rangle$$

$$= \langle 0.832, 0.068 \rangle$$

$$r_1 = \sqrt{\frac{(0.804-0.750)^2 + (0.096-0.150)^2 + (0.804-0.750)^2 + (0.096-0.150)^2 + \dots + (0.804-0.850)^2 + (0.110-0.050)^2}{5}} = 0.005$$

$$r_2 = \sqrt{\frac{(0.830-0.850)^2 + (0.070-0.050)^2 + (0.830-0.850)^2 + (0.070-0.050)^2 + \dots + (0.830-0.850)^2 + (0.070-0.050)^2}{5}} = 0.003$$

$$r_3 = \sqrt{\frac{(0.790-0.850)^2 + (0.110-0.050)^2 + (0.790-0.750)^2 + (0.110-0.150)^2 + \dots + (0.790-0.750)^2 + (0.110-0.150)^2}{5}} = 0.005$$

⋮

$$r_7 = \sqrt{\frac{(0.832-0.850)^2 + (0.068-0.050)^2 + (0.832-0.850)^2 + (0.068-0.050)^2 + \dots + (0.832-0.750)^2 + (0.068-0.150)^2}{5}} = 0.003$$

The aggregated evaluations of the experts for each criterion are presented in the following table.

Criteria	CIF		
C1	0,804	0,096	0,005
C2	0,830	0,070	0,003
C3	0,790	0,110	0,005
C4	0,785	0,115	0,005
C5	0,827	0,073	0,003
C6	0,810	0,090	0,005
C7	0,832	0,068	0,003

The score function of each criterion was calculated using Equation (13).

$$RSF_1 = \frac{(1-0.096)(1+0.804)+0.804}{3} \times \left(\frac{\frac{1}{0.005}}{\sqrt{\frac{1}{0.005^2} + \frac{1}{0.003^2} + \dots + \frac{1}{0.005^2} + \frac{1}{0.003^2}}} \right)^{0.01} = 0.32321$$

$$RSF_2 = \frac{(1-0.070)(1+0.830)+0.830}{3} \times \left(\frac{\frac{1}{0.003}}{\sqrt{\frac{1}{0.005^2} + \frac{1}{0.003^2} + \dots + \frac{1}{0.005^2} + \frac{1}{0.003^2}}} \right)^{0.01} = 0.32678$$

⋮

$$RSF_7 = \frac{(1-0.068)(1+0.832)+0.832}{3} \times \left(\frac{\frac{1}{0.003}}{\sqrt{\frac{1}{0.005^2} + \frac{1}{0.003^2} + \dots + \frac{1}{0.005^2} + \frac{1}{0.003^2}}} \right)^{0.01} = 0.32124$$

Criteria	RSF
C1	0,32321
C2	0,32678
C3	0,32182
C4	0,32124
C5	0,32655
C6	0,32375
C7	0,32698

Next, the RSF values are ranked from largest to smallest, and their relative importance is calculated using Equation (14), coefficient Kk using Equation (15), the weight (Qk) using Equation (16) and normalization of the weight using Equation (17).

Criteria	RSF	Sk	Kk	Qk	Qj
C7	0,32698	-	1,00000	1,00000	0,14324
C2	0,32678	0,00020	1,00020	0,99980	0,14321
C5	0,32655	0,00024	1,00024	0,99956	0,14317
C6	0,32375	0,00280	1,00280	0,99677	0,14277
C1	0,32321	0,00054	1,00054	0,99624	0,14270
C3	0,32182	0,00139	1,00139	0,99485	0,14250
C4	0,32124	0,00058	1,00058	0,99427	0,14242

After normalization, the criteria are reordered according to their original sequence, and the weights for each criterion are obtained as follows:

Criteria	Weight
C1	0,14270
C2	0,14321
C3	0,14250
C4	0,14242
C5	0,14317
C6	0,14277
C7	0,14324

The next step is to determine the PIS and NIS using Equations (18)–(20), noting that Criteria 1–5 are of the benefit type (J1), while Criteria 6–7 are of the cost type (J2).

From the optimistic matrix:

$$PIS = \langle (0.912, 0.078), (0.912, 0.078), (0.912, 0.078), (0.912, 0.078), (0.912, 0.078), (0.222, 0.748), (0.222, 0.748) \rangle$$

$$NIS = \langle (0.795, 0.155), (0.742, 0.208), (0.763, 0.187), (0.848, 0.117), (0.835, 0.125), (0.440, 0.510), (0.510, 0.440) \rangle$$

From the pessimistic matrix:

$$PIS = \langle (0.848, 0.142), (0.848, 0.142), (0.848, 0.142), (0.848, 0.142), (0.848, 0.142), (0.098, 0.872), (0.098, 0.872) \rangle$$

$$NIS = \langle (0.645, 0.305), (0.538, 0.412), (0.637, 0.313), (0.712, 0.238), (0.645, 0.315), (0.280, 0.670), (0.224, 0.726) \rangle$$

Next, the distances between each alternative and the PIS and NIS are calculated using Equations 5 and 6.

Distance from A_1 with S_o^*

$$Cr(A_1(x_1), S_o^*(x_1)) = |0.078 - 0.078| + |0.912 - 0.912| = 0$$

$$Cr(A_1(x_2), S_o^*(x_2)) = |0.078 - 0.078| + |0.912 - 0.912| = 0$$

⋮

$$Cr(A_1(x_7), S_o^*(x_7)) = |0.222 - 0.222| + |0.748 - 0.748| = 0$$

$$\begin{aligned} d_{DG}^w(A_1, S_o^*) &= \frac{3}{8} [0.14270(|0.912 - 0.912| + |0.078 - 0.078|) + 0.14270(\frac{0}{1+0}) + \dots + 0.14324(|0.222 - 0.222| \\ &\quad + |0.748 - 0.748|) + 0.14324(\frac{0}{1+0})] \\ &= 0 \end{aligned}$$

The results of the distance calculations between each alternative and the PIS and NIS are presented in the table below.

	A₁	A₂	A₃	A₄	A₅
D _i O*	0.00000	0.04523	0.10405	0.18555	0.11475
D _i O ⁻	0.18816	0.14760	0.08979	0.00351	0.08343
D _i P*	0.00000	0.07536	0.12953	0.23478	0.12873
D _i P ⁻	0.24014	0.17130	0.12339	0.00621	0.12461

Subsequently, the closeness coefficients are calculated based on the optimistic and pessimistic matrices, according to Equations (22) and (23).

Optimistic

$$CC_1^O = \frac{0.18816}{0+0.18816} = 1.0000$$

$$CC_2^O = \frac{0.14760}{0.04523+0.14760} = 0.76545$$

⋮

$$CC_5^O = \frac{0.08343}{0.11475+0.08343} = 0.42100$$

Pessimistic

$$CC_1^O = \frac{0.24014}{0+0.24014} = 1.0000$$

$$CC_2^O = \frac{0.17130}{0.07635+0.17130} = 0.69450$$

⋮

$$CC_5^O = \frac{0.12461}{0.11472+0.12461} = 0.49186$$

In the next stage, the ratio scores of each closeness coefficient are calculated using equation (24) to determine the order of the alternatives.

$$CC_1^{CR} = 0.5(1.0000) + (1 - 0.5)(1.0000) = 1.0000$$

$$CC_2^{CR} = 0.5(0.76545) + (1 - 0.5)(0.69450) = 1.0000$$

⋮

$$CC_5^{CR} = 0.5(0.42100) + (1 - 0.5)(0.49186) = 1.0000$$

The ratio scores along with the corresponding rankings are shown in the table below.

	A₁	A₂	A₃	A₄	A₅
CC _i ^{DR}	1.00000	0.72997	0.47555	0.02217	0.45643
Rank	1	2	3	5	4

SENSITIVITY ANALYSIS

1. Vary α value

Sensitivity analysis was performed by altering the α values in the ratio score computation, while all other steps of the proposed method remained unchanged.

α	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
A₁	0.93895	0.94499	0.95103	0.95708	0.96312	0.96916	0.97521	0.98125	0.98729	0.99334	0.99938
A₂	0.71707	0.71299	0.70891	0.70482	0.70074	0.69666	0.69258	0.68850	0.68442	0.68033	0.67625
A₃	0.60888	0.57838	0.54787	0.51737	0.48686	0.45635	0.42585	0.39534	0.36483	0.33433	0.30382
A₄	0.04841	0.04844	0.04846	0.04849	0.04851	0.04854	0.04857	0.04859	0.04862	0.04864	0.04867
A₅	0.55587	0.53427	0.51268	0.49109	0.46949	0.44790	0.42630	0.40471	0.38311	0.36152	0.33993

α	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Rank 1	A1	A1	A1	A1	A1	A1	A1	A1	A1	A1	A1
Rank 2	A2	A2	A2	A2	A2	A2	A2	A2	A2	A2	A2
Rank 3	A3	A3	A3	A3	A3	A3	A5	A5	A5	A5	A5
Rank 4	A5	A5	A5	A5	A5	A5	A3	A3	A3	A3	A3
Rank 5	A4	A4	A4	A4	A4	A4	A4	A4	A4	A4	A4

Kendall's τ (Vary α value)

α	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	1.00	1.00	1.00	1.00	1.00	1.00	0.80	0.80	0.80	0.80	0.80
0.1		1.00	1.00	1.00	1.00	1.00	0.80	0.80	0.80	0.80	0.80

0.2			1.00	1.00	1.00	1.00	0.80	0.80	0.80	0.80	0.80
0.3				1.00	1.00	1.00	0.80	0.80	0.80	0.80	0.80
0.4					1.00	1.00	0.80	0.80	0.80	0.80	0.80
0.5						1.00	0.80	0.80	0.80	0.80	0.80
0.6							1.00	1.00	1.00	1.00	1.00
0.7								1.00	1.00	1.00	1.00
0.8									1.00	1.00	1.00
0.9										1.00	1.00
1.0											1.00

2. Criteria Weight Perturbations

A perturbation is applied to the criterion with the highest weight (C7), and the weights are subsequently normalized to ensure that their total sum remains equal to one.

Scenario	Ranking Order
C7 – 20%	A1, A2, A3, A5, A4
C7 – 15%	A1, A2, A3, A5, A4
C7 – 10%	A1, A2, A3, A5, A4
C7 + 10%	A1, A2, A3, A5, A4
C7 + 15%	A1, A2, A3, A5, A4
C7 + 20%	A1, A2, A3, A5, A4

Kendall's τ (Criteria Weight Perturbations)

Scenario	Original	C7–20%	C7–15%	C7–10%	C7+10%	C7+15%	C7+20%
Original	1.00	1.00	1.00	1.00	1.00	1.00	1.00
C7–20%		1.00	1.00	1.00	1.00	1.00	1.00
C7–15%			1.00	1.00	1.00	1.00	1.00

C7-10%				1.00	1.00	1.00	1.00
C7+10%					1.00	1.00	1.00
C7+15%						1.00	1.00
C7+20%							1.00

3. Equal Experts Weight

Scenario	Ranking Order
$w_j = 0, 20$	A1, A2, A3, A5, A4

Kendall's τ

$$\tau = \frac{C-D}{\frac{1}{2}n(n-1)} = \frac{10-0}{10} = 1$$

4. Alternative distance measure

A sensitivity analysis is performed by employing a different distance measure, specifically the weighted Euclidean distance.

$$d(A, B) = \sqrt{\frac{1}{2} \sum_{j=1}^n w_j ((\mu_A(x_j) - \mu_B(x_j))^2 + (v_A(x_j) - v_B(x_j))^2 + (\pi_A(x_j) - \pi_B(x_j))^2)}$$

Distance Measure	Ranking Order
Garg Distance (original)	A1, A2, A3, A5, A4
Weighted Euclidean Distance	A1, A2, A5, A3, A4

Kendall's τ (Alternative distance measure)

Distance Measure	Garg Distance	Weighted Euclidean Distance
Garg Distance (original)	1.00	1.00
Weighted Euclidean Distance		1.00

Algorithm 1 CIFS-SWARA-TOPSIS Part 1

Require: m, n, p ▷ Where m - alternatives, n - criteria, p - experts
for $k = 1$ to p **do**
 $D^k = [d_{ij}^k]_{m \times n}$
end for

for $i=1$ to m **do**
 for $j=1$ to n **do**
 $\mathcal{C}i = \langle \mu_C(\mathcal{C}i), \nu_C(\mathcal{C}i) \rangle = \left\langle \frac{\sum_{j=1}^{k_i} \mu_{i,j}}{k_i}, \frac{\sum_{j=1}^{k_i} \nu_{i,j}}{k_i} \right\rangle$ ▷ Aggregation of experts' opinions
 end for
end for
for $i=1$ to m **do**
 for $j=1$ to n **do**
 $r_i = \sqrt{\frac{\sum_{j=1}^n I_{ij}^2}{n}}$ ▷ radius calculation where
 $l_{ij} = \sqrt{(\mu_C(x_i) - m_{i,j})^2 + (\nu_C(x_i) - n_{i,j})^2}$
 end for
end for
for $i= 1$ to m **do**
 for $j=1$ to n **do**
 $Q^{O_d} = \langle \mu_{ij}^O, \nu_{ij}^O \rangle = \langle \mu_{ij} + \frac{r_{ij}}{2} \sqrt{2}, \nu_{ij} - \frac{r_{ij}}{2} \sqrt{2} \rangle$ ▷ Optimistic Matrix (IF Value)
 $Q^{P_d} = \langle \mu_{ij}^P, \nu_{ij}^P \rangle = \langle \mu_{ij} - \frac{r_{ij}}{2} \sqrt{2}, \nu_{ij} + \frac{r_{ij}}{2} \sqrt{2} \rangle$ ▷ Pessimistic matrix (IF Value)
 end for
end for
Criteria rank by experts ▷ SWARA Method
for $l=1$ to p **do**
 $\omega_l = \frac{\left[\mu_l + \pi_l \left(\frac{\mu_l}{1 - \pi_l} \right) \right]}{\sum_{j=1}^l \left[\mu_j + \pi_l \left(\frac{\mu_l}{1 - \pi_l} \right) \right]}, \sum_{j=1}^p \omega_l = 1$ ▷ Experts' weight calculation where $\pi_l = 1 - \mu_l - \nu_l$.
end for
for $j= 1$ to m **do**
 for $l = 1$ to p **do**
 $C_j = \langle \prod_{j=1}^n \mu_j^{\omega_l}, 1 - \prod_{j=1}^n (1 - \nu_j)^{\omega_l} \rangle$ ▷ Aggregation of experts' opinions
 end for
end for
for $i=1$ to m **do**
 for $j=1$ to n **do**
 $r_j = \sqrt{\frac{\sum_{j=1}^n \uparrow_{ij}^2}{n}}$ ▷ radius calculation where
 $\uparrow_{ij} = \sqrt{(\mu_C(x_j) - m_{l,j})^2 + (\nu_C(x_j) - n_{l,j})^2}$
 end for
end for
for $j= 1$ to n **do**
 $RSF_j = \frac{(1 - \nu_j)(1 + \mu_j) + \mu_j}{3} \times \left(\frac{\frac{1}{r_j}}{\sqrt{\sum_{j=1}^n \frac{1}{r_j^2}}} \right)^\theta, \theta = 0.01$ ▷ Relative Significance Factors
end for

Algorithm 2 continue CIFS-SWARA-TOPSIS

for k=1 to n do
 $S_k = RSF_k - RSF_{k-1}$
end for
for k=1 to n do

$$K_k = \begin{cases} 1, k = 1 \\ S_k + 1, k > 1 \end{cases}$$

end for
for k=1 to n do

$$q_k = \begin{cases} 1, k = 1 \\ \frac{q_{k-1}}{K_k}, k > 1 \end{cases}$$

end for
for j=1 to n do
 $W_j = \frac{q_j}{\sum_{j=1}^n q_j}, j = 1, 2, \dots, n$

▷ Criteria Weights

end for
for j= 1 to n do

 if j=5 then
 $S_O^* = \langle \max_i \mu_{ij}^O, \min_i \nu_{ij}^O \rangle$
 $S_O^- = \langle \min_i \mu_{ij}^O, \max_i \nu_{ij}^O \rangle$

▷ PIS and NIS of Optimistic Matrix

 else
 $S_O^* = \langle \min_i \mu_{ij}^O, \max_i \nu_{ij}^O \rangle$
 $S_O^- = \langle \max_i \mu_{ij}^O, \min_i \nu_{ij}^O \rangle$

 end if

end for

for j= 1 to n do

 if j=5 then
 $S_P^* = \langle \max_i \mu_{ij}^P, \min_i \nu_{ij}^P \rangle$
 $S_P^- = \langle \min_i \mu_{ij}^P, \max_i \nu_{ij}^P \rangle$

▷ PIS and NIS of Pessimistic Matrix

 else
 $S_P^* = \langle \min_i \mu_{ij}^O, \max_i \nu_{ij}^P \rangle$
 $S_P^- = \langle \max_i \mu_{ij}^O, \min_i \nu_{ij}^P \rangle$

 end if

end for

for j= 1 to n do

▷ Distance calculation

$$d_{DG}^w(A, S_O^*) = \frac{3}{8} \sum_{j=1}^m \left[w_j |\mu_A(x_j) - \mu_{S_O^*}(x_j)| + |\nu_A(x_j) - \nu_{S_O^*}(x_j)| + w_j \frac{Cr(A, S_O^*)}{1 + Cr(A, S_O^*)} \right]$$

where

$$Cr(A, S_O^*) = \begin{aligned} & |\min(\mu_A(x_j), \nu_{S_O^*}(x_j)) - \min(\mu_{S_O^*}(x_j), \nu_A(x_j))| \\ & + |\max(\mu_A(x_j), \nu_{S_O^*}(x_j)) - \max(\mu_{S_O^*}(x_j), \nu_A(x_j))| \end{aligned}$$

end for

Algorithm 3 continue CIFS-SWARA-TOPSIS

for j= 1 to n **do**

$$d_{DG}^w(A, S_P^*) = \frac{3}{8} \sum_{j=1}^m \left[w_j |\mu_A(x_j) - \mu_{S_P^*}(x_j)| + |\nu_A(x_j) - \nu_{S_P^*}(x_j)| + w_j \frac{Cr(A, S_P^*)}{1 + Cr(A, S_P^*)} \right]$$

where

$$Cr(A, S_P^*) = |\min(\mu_A(x_j), \nu_{S_P^*}(x_j)) - \min(\mu_{S_P^*}(x_j), \nu_A(x_j))| \\ + |\max(\mu_A(x), \nu_{S_P^*}(x_j)) - \max(\mu_{S_P^*}(x_j), \nu_A(x_j))|$$

end for

for j= 1 to n **do**

$$d_{DG}^w(A, S_O^-) = \frac{3}{8} \sum_{j=1}^m \left[w_j |\mu_A(x_j) - \mu_{S_O^-}(x_j)| + |\nu_A(x_j) - \nu_{S_O^-}(x_j)| + w_j \frac{Cr(A, S_O^-)}{1 + Cr(A, S_O^-)} \right]$$

where

$$Cr(A, S_O^-) = |\min(\mu_A(x_j), \nu_{S_O^-}(x_j)) - \min(\mu_{S_O^-}(x_j), \nu_A(x_j))| \\ + |\max(\mu_A(x), \nu_{S_O^-}(x_j)) - \max(\mu_{S_O^-}(x_j), \nu_A(x_j))|$$

end for

for j= 1 to n **do**

$$d_{DG}^w(A, S_P^-) = \frac{3}{8} \sum_{j=1}^m \left[w_j |\mu_A(x_j) - \mu_{S_P^-}(x_j)| + |\nu_A(x_j) - \nu_{S_P^-}(x_j)| + w_j \frac{Cr(A, S_P^-)}{1 + Cr(A, S_P^-)} \right]$$

where

$$Cr(A, S_P^-) = |\min(\mu_A(x_j), \nu_{S_P^-}(x_j)) - \min(\mu_{S_P^-}(x_j), \nu_A(x_j))| \\ + |\max(\mu_A(x), \nu_{S_P^-}(x_j)) - \max(\mu_{S_P^-}(x_j), \nu_A(x_j))|$$

end for

for i=1 to m **do**

$$CC_i^O = \frac{D_i^{O-}}{D_i^{O-} + D_i^{O*}}$$

▷Closeness Coefficient based on Optimistic Matrix

end for

for i=1 to m **do**

$$CC_i^P = \frac{D_i^{P-}}{D_i^{P-} + D_i^{P*}}$$

▷Closeness Coefficient based on Pessimistic Matrix

end for

for i=1 to m **do**

$$CC_i^{CR} = \alpha \times CC_i^O + (1 - \alpha) \times CC_i^P, \alpha = 0.5$$

▷Score Ration of Alternatives

end for

$[\sim, P] = \text{sort}(CC_i^{CR}, \text{'desend'})$

$[\sim, R] = \text{sort}(P)$

for i=1 to m **do**

$$\text{Rank}(CC_i^{CR}) = R_i$$

end for
