



Local Stability and Bifurcation Analysis of a Mangrove Detritus Small Fish Model with Dynamic Harvesting Effort Considering Beddington DeAngelis Functional Response

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Abstract

This study develops a three-dimensional mangrove detritus small fish model with a dynamic harvesting effort governed by a threshold rule and a Beddington DeAngelis functional response. The main objective is to understand how adaptive harvesting and consumer interference shape long-term dynamics and stability. Equilibrium points are derived and their local stability is examined using the Jacobian matrix and eigenvalue-based criteria, supported by numerical simulations. Bifurcation analysis with respect to the threshold parameter is performed using numerical continuation to detect qualitative transitions in system behavior. The results show the existence of biologically feasible coexistence equilibria with active harvesting, including a stable branch and an unstable branch. A Hopf bifurcation is detected at approximately $a = 6.6159$, confirming the emergence of sustained oscillations limit cycles in fish density and harvesting effort, while a fold limit point occurs near $a = 22.001$, indicating changes in equilibrium structure. Moreover, simulation scenarios demonstrate that reducing environmental capacity and increasing fish mortality can drive the system toward a no-effort equilibrium where harvesting collapses. These findings highlight the threshold parameter as a key control factor that can switch the system between stable harvesting, effort extinction, and oscillatory harvesting regimes.

Keywords: Beddington–DeAngelis; hopf–bifurcation; harvesting; limit–cycle; mangrove.

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1 Introduction

The mangrove ecosystem ranks among the most productive coastal systems and is crucial for sustaining biological resources and the overall health of the coastal environment. Mangroves provide habitat, serve as nursery grounds, and bolster food webs that sustain diverse groups of flora and fauna with both ecological and economic significance [1]. One key mechanism linking mangrove productivity to consumer communities is the detritus pathway, where mangrove leaf litter decomposes into detritus and becomes an energy source for higher trophic levels [2]. Differences in mangrove species and habitat features may also correlate with variations in associated biota composition within the ecosystem [3]. In addition, mangrove ecosystems support fish populations, including small fish species that utilize mangrove regions [4].

Conversely, mangrove ecosystems are subjected to human-induced pressures that can modify their ecological conditions and functions, such as land conversion and intensive utilization

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activities . The clearing of land for aquaculture and other forms of exploitation may lead to the depletion of mangrove resources and threaten ecosystem sustainability [5]. Small-scale fisheries in mangrove areas can also alter fish community composition and the abundance of particular groups [4]. More generally, harvesting pressure and its consequences for fish stocks remain critical issues in aquatic ecosystem management [6].

Biomathematical modeling offers a quantitative approach for elucidating interactions among ecosystem components and the effects of disturbances on long-term system dynamics. Prey predator and food-chain models have been employed to investigate system responses to management strategies, such as implementing protection zones that can influence exploitation rates and population trends [7]. Predator-prey models incorporating harvesting are also frequently used to analyze equilibrium existence and local stability [8]. In the fisheries context, prey-predator-based fishery models can exhibit complex dynamics and regime shifts driven by changes in parameters and interaction structures [9]. Moreover, periodic behavior and oscillations in prey-predator systems are commonly observed and are important for stability studies and long-term predictions [10].

A crucial aspect of prey predator modeling is the choice of functional response. Although linear predation terms are often used due to their simplicity, they may be unrealistic when consumption saturates at high resource availability or when predator interference occurs at high predator densities. The Beddington DeAngelis functional response explicitly accounts for saturation/handling and predator interference, making it more representative in such situations [11]. Nonlinear functional responses are known to affect coexistence conditions and local stability properties of ecological systems [12]. In various ecological model variants, changes in interaction mechanisms may generate instabilities and qualitative shifts that can be examined using bifurcation frameworks [13]. Even in food-web models, incorporating behavioral mechanisms such as fear effects may further modify system dynamics and stability [14].

Besides consumption interactions, harvesting is a major factor shaping fish population dynamics, particularly for communities associated with mangrove habitats. Harvesting is commonly modeled through a catch effort relationship that depends on target biomass and harvesting effort. Predator prey systems with harvesting (including prey harvesting) have been widely investigated to understand their implications for stability and persistence [15]. In addition, threshold-based effort dynamics provide a mechanism for representing adaptive entry-exit behavior -: effort increases when the target biomass exceeds a viability threshold and decreases when it falls below that threshold [16]. Optimal harvesting studies also emphasize the importance of accounting for stock dynamics and effort responses in sustainable management [17]. In related settings, equilibria in shared-effort systems suggest that the organization of effort and thresholds can affect long-term outcomes [18].

Despite extensive studies on prey predator/food-chain modeling, nonlinear functional responses, and harvesting, specific research gaps remain for detritus-based mangrove environments. To enhance model relevance to mangrove ecosystems, it is essential to explicitly incorporate the detritus pathway as a foundational resource that channels energy to higher trophic levels [2]. Many food-chain models study trophic structures and conservation strategies, yet often do not integrate threshold-based dynamic effort mechanisms [7]. Dynamic effort can more realistically represent adaptive responses of fishers to fluctuations in target biomass [16]. Moreover, Beddington DeAngelis functional responses may enrich system dynamics through saturation and interference, motivating investigation within the mangrove detritus small fish framework [11]. Stability and bifurcation analyses are therefore necessary to describe qualitative changes in system behavior induced by management parameters [12].

Based on this context, this study develops and analyzes a preypredator mathematical model for a mangrove ecosystem incorporating a detritus pathway and adaptive harvesting effort. We employ local stability analysis to characterize biologically feasible equilibria and use bifurcation analysis as a primary tool to describe dynamic transitions with respect to the threshold parameter

[12]. Hopf bifurcation in food-chain systems provides a theoretical basis for understanding the emergence of periodic oscillations (limit cycles) in nonlinear ecological models [19]. Continuation-based bifurcation analysis further reveals that parameter variations may produce BP/LP/Hopf events and alter equilibrium stability [13], [20]. Numerical schemes that preserve qualitative properties support the verification of analytical results through simulations [21].

2 Methods

This section describes the methodological framework used in this study, including the literature review, model formulation, equilibrium analysis, local stability assessment, numerical simulation, and bifurcation analysis conducted to investigate the mangrove detritus–small fish system under dynamic harvesting effort.

2.1 Literature Study

The study begins with a literature review on predator–prey and resource–consumer models applied to coastal and mangrove ecosystems, with particular emphasis on detritus-based food webs and harvesting dynamics. Previous works on functional responses such as linear (Lotka–Volterra) and Holling type II forms are reviewed to understand consumer saturation effects. Additionally, studies employing the Beddington–DeAngelis functional response are examined to capture predator interference, which may arise from competition and crowding among consumers. The reviewed literature provides the theoretical basis for selecting the functional response, defining biological assumptions, and identifying meaningful ranges of model parameters relevant to mangrove detritus and small fish populations.

2.2 Model Construction

Based on the literature review, an ordinary differential equation (ODE) model is constructed to describe the interaction between mangrove detritus $x(t)$, small fish population $y(t)$, and harvesting effort $E(t)$. The detritus compartment is assumed to grow logistically with intrinsic growth rate r and carrying capacity k , reflecting limited environmental resources and detritus regeneration. The consumption of detritus by small fish is modeled using the Beddington–DeAngelis functional response to incorporate both saturation with respect to resource density and interference among consumers. Fish growth is driven by assimilation from detritus intake, while fish losses include natural mortality and harvesting mortality. Harvesting is represented through a catchability coefficient q and a dynamic effort variable $E(t)$. The effort dynamics follow a threshold mechanism: harvesting effort expands when fish density exceeds a biomass threshold a , and contracts when fish density falls below this threshold. This rule reflects an adaptive response of fishing activity to perceived stock availability.

2.3 Equilibrium points

The equilibrium points of the model are obtained by setting the time derivatives of all state variables equal to zero, i.e., $\dot{x} = \dot{y} = \dot{E} = 0$. Solving the resulting algebraic system yields boundary equilibria (including extinction and fish-free states) as well as coexistence equilibria. Because the effort equation implies $E = 0$ or $y = a$ at equilibrium, two coexistence regimes are possible: a coexistence equilibrium without harvesting ($E = 0$) and a coexistence equilibrium with active harvesting ($E > 0$, hence $y = a$). When explicit closed-form expressions are cumbersome, equilibrium values are characterized implicitly (e.g., as positive roots of algebraic equations) and subsequently approximated numerically. Only equilibria in the biologically feasible region $x \geq 0$, $y \geq 0$, and $E \geq 0$ are considered for ecological interpretation.

2.4 Local Stability Analysis

Local stability of each equilibrium point is investigated by linearizing the system around the equilibrium using the Jacobian matrix. The eigenvalues of the Jacobian determine whether an equilibrium is locally asymptotically stable or unstable (at least one eigenvalue has positive real part). For boundary equilibria, stability conditions are derived directly from the signs of the eigenvalues. For the coexistence equilibrium with active harvesting, the Jacobian generally yields a cubic characteristic polynomial. Stability is assessed either through analytical criteria (e.g., Routh Hurwitz conditions) or by computing eigenvalues numerically using symbolic numerical support. In particular, the no-harvesting coexistence equilibrium ($E = 0$) has stability in the effort direction governed by the sign of $uq(y - a)$, which provides an explicit threshold-based criterion for the persistence or collapse of harvesting effort near equilibrium.

2.5 Numerical Simulation

Numerical simulations are performed to support and illustrate the analytical results. The ODE system is solved numerically under multiple nonnegative initial conditions to ensure biological relevance and to verify convergence toward predicted equilibria. Several simulation scenarios are considered by varying the threshold parameter a , since it directly controls the adaptive harvesting response. Time-series trajectories and phase-plane projections are used to examine transient dynamics, long-term behavior, and consistency with local stability predictions.

3 Results and Discussion

This section presents the analytical and numerical results of the three-dimensional mangrove ecosystem model developed to describe the dynamics between detritus, small fish predators, and harvesting effort. First, the equilibrium points of the system are identified to describe the possible long-term population and bionomic states. Next, the local stability properties of each equilibrium are analyzed using the Jacobian matrix and the Routh-Hurwitz criterion to determine conditions for system stability. The theoretical findings are then supported by numerical simulations under various parameter scenarios, focusing particularly on the influence of the Beddington-DeAngelis predator interference and the harvesting threshold. Finally, the results are discussed to interpret their ecological implications and to evaluate sustainable harvesting management strategies in mangrove areas.

3.1 Mathematics Model

This prey predator interaction model was developed to describe the population dynamics within a mangrove ecosystem, specifically focusing on the interaction between leaf litter detritus and small fish. The model incorporates a Beddington-DeAngelis functional response to represent the predation process and accounts for human intervention through bionomic harvesting effort. In this framework, the Beddington-DeAngelis response is used to describe the consumption of detritus by small fish, taking into account both the handling time by the predator and the mutual interference among predators. Furthermore, the harvesting effort is modeled as a dynamic variable that changes based on the density of the fish population relative to an economic or biological threshold. Based on these biological assumptions, the mathematical model can be

written as the following system of ordinary differential equations:

$$\begin{aligned}\frac{dx}{dt} &= rx \left(1 - \frac{x}{k}\right) - \frac{\alpha xy}{1 + cy + bx}, \\ \frac{dy}{dt} &= \frac{\beta xy}{1 + cy + bx} - \mu y - uqEy, \\ \frac{dE}{dt} &= uqE(y - a).\end{aligned}\tag{1}$$

In the model (1), x , y , and E represent the density of mangrove leaf litter detritus, the population of small fish, and the harvesting effort, respectively. The parameters $r, k, \alpha, c, b, \beta, \mu, u, q$, and a represent the detritus growth rate, environmental carrying capacity, predation rate, predator interference coefficient, handling time, conversion efficiency, natural mortality rate of the fish, economic gain factor, catchability coefficient, and the population threshold for harvesting profitability. Model (1) is given under the initial conditions $x(0) > 0, y(0) > 0$, and $E(0) > 0$, where all parameters are assumed to be positive.

3.2 Positivity and Boundedness of Solutions

Lemma 3.1. Suppose $x(0) \geq 0, y(0) \geq 0$, and $E(0) \geq 0$. Then the solutions $(x(t), y(t), E(t))$ of system (1) remain nonnegative for all $t \geq 0$.

Proof. On the boundary planes we have

$$\dot{x}|_{x=0} = 0, \quad \dot{y}|_{y=0} = 0, \quad \dot{E}|_{E=0} = 0.$$

Hence, trajectories cannot cross from the nonnegative orthant into the negative region. Therefore, any solution starting in $\mathbb{R}_+^3$ remains in $\mathbb{R}_+^3$ for all $t \geq 0$. $\square$

Lemma 3.2. Let $x(0) \geq 0$. Then

$$0 \leq x(t) \leq \max\{x(0), k\} \quad \text{for all } t \geq 0.$$

In particular, if $x(0) \leq k$, then $x(t) \leq k$ for all $t \geq 0$.

Proof. Since $\frac{\alpha xy}{1 + bx + cy} \geq 0$ for $x, y \geq 0$, we obtain

$$\dot{x} = rx \left(1 - \frac{x}{k}\right) - \frac{\alpha xy}{1 + bx + cy} \leq rx \left(1 - \frac{x}{k}\right).$$

By the comparison theorem with the logistic equation, $x(t)$ cannot exceed $\max\{x(0), k\}$ for $t \geq 0$, which proves the claim. $\square$

Lemma 3.3. Assume all parameters in system (3.1) are positive with $\beta > 0$. Then all solutions of the system are uniformly bounded in a compact region $\Omega \subset \mathbb{R}_+^3$.

Proof. Define the function

$$W(t) = x(t) + \frac{\alpha}{\beta}y(t) + \frac{\alpha}{\beta}E(t).$$

Using system (3.1) and writing $D = 1 + bx + cy$, we obtain

$$\begin{aligned} \frac{dW}{dt} &= \dot{x} + \frac{\alpha}{\beta}\dot{y} + \frac{\alpha}{\beta}\dot{E} \\ &= rx \left(1 - \frac{x}{k}\right) - \frac{\alpha xy}{D} + \frac{\alpha}{\beta} \left(\frac{\beta xy}{D} - \mu y - uqEy \right) + \frac{\alpha}{\beta} (uqE(y - a)) \\ &= rx \left(1 - \frac{x}{k}\right) - \frac{\alpha\mu}{\beta}y - \frac{\alpha uqa}{\beta}E. \end{aligned}$$

Let $0 < \eta \leq \min\{\mu, uqa\}$. Then

$$\begin{aligned} \frac{dW}{dt} + \eta W &= rx \left(1 - \frac{x}{k}\right) + \eta x + \frac{\alpha}{\beta}(\eta - \mu)y + \frac{\alpha}{\beta}(\eta - uqa)E \\ &\leq (r + \eta)x - \frac{r}{k}x^2. \end{aligned}$$

The right-hand side attains its maximum at $x = \frac{k(r + \eta)}{2r}$, and thus

$$(r + \eta)x - \frac{r}{k}x^2 \leq \frac{k(r + \eta)^2}{4r} =: C.$$

Hence,

$$\frac{dW}{dt} + \eta W \leq C.$$

By the differential inequality theorem, $W(t)$ is uniformly bounded for $t \geq 0$, and consequently

$$x(t) \leq W(t), \quad y(t) \leq \frac{\beta}{\alpha}W(t), \quad E(t) \leq \frac{\beta}{\alpha}W(t),$$

which implies that $(x(t), y(t), E(t))$ is uniformly bounded in $\mathbb{R}_+^3$. $\square$

3.3 Equilibrium points

The equilibrium points of system (3.1) are obtained by setting

$$\dot{x} = 0, \quad \dot{y} = 0, \quad \dot{E} = 0. \quad (2)$$

Equivalently, we solve

$$rx \left(1 - \frac{x}{k}\right) - \frac{\alpha xy}{1 + bx + cy} = 0, \quad (3)$$

$$\frac{\beta xy}{1 + bx + cy} - \mu y - uqEy = 0, \quad (4)$$

$$uqE(y - a) = 0. \quad (5)$$

For convenience, define $D = 1 + bx + cy$. From (5), any equilibrium must satisfy either $E = 0$ (no-harvesting branch) or $y = a$ (active-harvesting branch with $E > 0$). The biologically feasible equilibria are listed below.

1. The trivial equilibrium

$$E_0 = (0, 0, 0),$$

which represents the extinction of detritus, small fish, and harvesting effort.

2. The detritus-only equilibrium

$$E_1 = (k, 0, 0),$$

which represents persistence of detritus at the carrying capacity in the absence of fish and harvesting.

3. The coexistence equilibrium without harvesting effort

$$E_2 = (x_2, y_2, 0),$$

which corresponds to coexistence of detritus and fish when harvesting effort is absent ($E = 0$). For $E = 0$ and $y_2 > 0$, dividing (4) by y yields

$$\frac{\beta x}{1 + bx + cy} = \mu, \quad (6)$$

so that

$$y_2 = \frac{x_2(\beta - \mu b) - \mu}{\mu c}. \quad (7)$$

Substituting (7) into (3) gives an algebraic equation (quadratic) for x_2 . The equilibrium E_2 is biologically feasible provided that $x_2 > 0$ and $y_2 > 0$.

4. The coexistence equilibrium with active harvesting

$$E_3 = (x^*, y^*, E^*), \quad x^* > 0, y^* > 0, E^* > 0.$$

Since $E^* > 0$, equation (5) implies

$$y^* = a. \quad (8)$$

Substituting (8) into (3) (and dividing by $x > 0$) yields

$$r \left(1 - \frac{x}{k} \right) = \frac{\alpha a}{1 + bx + ca}, \quad (9)$$

which can be rearranged into a quadratic equation for x^* . Therefore, depending on the parameter values, equation (9) may admit *two distinct positive roots* $x_3^{(1)}$ and $x_3^{(2)}$. For each positive root $x_3^{(i)}$ ($i = 1, 2$), substituting $(x, y) = (x_3^{(i)}, a)$ into (4) and dividing by $a > 0$ gives

$$E_3^{(i)} = \frac{1}{uq} \left(\frac{\beta x_3^{(i)}}{1 + bx_3^{(i)} + ca} - \mu \right), \quad i = 1, 2. \quad (10)$$

Hence, the active-harvesting equilibrium may consist of two biologically feasible equilibria

$$E_3^{(1)} = (x_3^{(1)}, a, E_3^{(1)}) \quad \text{and} \quad E_3^{(2)} = (x_3^{(2)}, a, E_3^{(2)}),$$

provided that $x_3^{(i)} > 0$ and $E_3^{(i)} > 0$ for $i = 1, 2$.

Remark. The algebraic system (3)–(5) may admit additional solutions with negative components. Such equilibria are mathematically admissible but biologically infeasible, and thus are omitted.

3.4 Local Stability

Local stability of each equilibrium point is investigated by linearizing system (3.1) around the equilibrium using the Jacobian matrix. For convenience, define

$$D = 1 + bx + cy.$$

Let

$$\dot{x} = f_1(x, y, E), \quad \dot{y} = f_2(x, y, E), \quad \dot{E} = f_3(x, y, E),$$

where

$$f_1 = rx \left(1 - \frac{x}{k}\right) - \frac{\alpha xy}{D}, \quad f_2 = \frac{\beta xy}{D} - \mu y - uqEy, \quad f_3 = uqE(y - a).$$

The Jacobian matrix is

$$J(x, y, E) = \begin{pmatrix} D_{11} & D_{12} & 0 \\ D_{21} & D_{22} & D_{23} \\ 0 & D_{32} & D_{33} \end{pmatrix}, \quad (11)$$

with entries given by

$$D_{11} = r \left(1 - \frac{2x}{k}\right) - \alpha y \frac{D - bx}{D^2},$$

$$D_{12} = -\alpha x \frac{D - cy}{D^2},$$

$$D_{21} = \beta y \frac{D - bx}{D^2},$$

$$D_{22} = \beta x \frac{D - cy}{D^2} - \mu - uqE,$$

$$D_{23} = -uqy, \quad D_{32} = uqE, \quad D_{33} = uq(y - a).$$

Theorem 3.3. The equilibrium point $E_0 = (0, 0, 0)$ is unstable.

Proof. Substituting $E_0 = (0, 0, 0)$ into (11) yields

$$J(E_0) = \begin{pmatrix} r & 0 & 0 \\ 0 & -\mu & 0 \\ 0 & 0 & -uqa \end{pmatrix}.$$

Hence, the eigenvalues are $\lambda_1 = r > 0$, $\lambda_2 = -\mu < 0$, and $\lambda_3 = -uqa < 0$. Since one eigenvalue is positive, E_0 is unstable. $\square$

Theorem 3.4. The boundary equilibrium point $E_1 = (k, 0, 0)$ is locally asymptotically stable if

$$\frac{\beta k}{1 + bk} < \mu,$$

and unstable if $\frac{\beta k}{1 + bk} > \mu$.

Proof. Substituting $E_1 = (k, 0, 0)$ into (11) gives

$$J(E_1) = \begin{pmatrix} -r & -\frac{\alpha k}{1 + bk} & 0 \\ 0 & \frac{\beta k}{1 + bk} - \mu & 0 \\ 0 & 0 & -uqa \end{pmatrix}.$$

Thus, the eigenvalues are $\lambda_1 = -r < 0$, $\lambda_2 = \frac{\beta k}{1 + bk} - \mu$, and $\lambda_3 = -uqa < 0$. Therefore, E_1 is locally asymptotically stable if $\lambda_2 < 0$, i.e. $\frac{\beta k}{1 + bk} < \mu$. $\square$

Theorem 3.5. Let $E_2 = (x_2, y_2, 0)$ be the coexistence equilibrium without harvesting effort. Then E_2 is locally asymptotically stable if $y_2 < a$, $\text{tr}(J_2) < 0$, and $\det(J_2) > 0$.

Proof. Evaluating (11) at $E_2 = (x_2, y_2, 0)$ yields

$$J(E_2) = \begin{pmatrix} D_{11}^{(2)} & D_{12}^{(2)} & 0 \\ D_{21}^{(2)} & D_{22}^{(2)} & -uqy_2 \\ 0 & 0 & uq(y_2 - a) \end{pmatrix},$$

where $D_{ij}^{(2)}$ denotes D_{ij} evaluated at $(x, y, E) = (x_2, y_2, 0)$. This matrix is block triangular, hence one eigenvalue is $\lambda_3 = uq(y_2 - a)$, which is negative if $y_2 < a$. The remaining two eigenvalues are the roots of the characteristic polynomial of the 2×2 block

$$J_2 = \begin{pmatrix} D_{11}^{(2)} & D_{12}^{(2)} \\ D_{21}^{(2)} & D_{22}^{(2)} \end{pmatrix}.$$

For a planar system, both eigenvalues have negative real parts if $\text{tr}(J_2) < 0$ and $\det(J_2) > 0$. Therefore, E_2 is locally asymptotically stable if $y_2 < a$, $\text{tr}(J_2) < 0$, and $\det(J_2) > 0$. $\square$

Theorem 3.6. Let $E_3 = (x^*, a, E^*)$ be the coexistence equilibrium with active harvesting ($E^* > 0$). The equilibrium E_3 is locally asymptotically stable if the Routh–Hurwitz conditions

$$A_1 > 0, \quad A_2 > 0, \quad A_3 > 0, \quad \text{and} \quad A_1 A_2 > A_3$$

hold for the characteristic polynomial of $J(E_3)$.

Proof. Since $E^* > 0$, we have $y^* = a$, and thus $D_{33} = uq(y - a) = 0$ at E_3 . Hence,

$$J(E_3) = \begin{pmatrix} D_{11}^* & D_{12}^* & 0 \\ D_{21}^* & D_{22}^* & -uqa \\ 0 & uqE^* & 0 \end{pmatrix},$$

where D_{ij}^* denotes D_{ij} evaluated at $(x, y, E) = (x^*, a, E^*)$. The characteristic polynomial can be written as

$$\lambda^3 + A_1 \lambda^2 + A_2 \lambda + A_3 = 0,$$

with

$$A_1 = -(D_{11}^* + D_{22}^*), \quad A_2 = (D_{11}^* D_{22}^* - D_{12}^* D_{21}^*) + u^2 q^2 a E^*, \quad A_3 = -D_{11}^* u^2 q^2 a E^*.$$

By the Routh–Hurwitz criterion for a cubic polynomial, all roots have negative real parts if and only if $A_1 > 0$, $A_2 > 0$, $A_3 > 0$, and $A_1 A_2 > A_3$. This completes the proof. $\square$

3.5 Numerical Simulation

To ensure reproducibility of the numerical simulations, the baseline parameter values used throughout this study are summarized in Table 1. Numerical simulations are carried out to illustrate changes in system dynamics around the equilibrium points. In particular, the evolution of mangrove detritus, small fish, and harvesting effort is visualized through time-series plots and phase portraits.

Table 1: Baseline parameter values used in the numerical simulations.

Parameter	Description	Value
r	Intrinsic detritus regeneration rate	3
k	Carrying capacity of detritus	100
α	Maximum consumption rate	1.8
β	Conversion efficiency (detritus $\rightarrow$ fish)	0.7
μ	Natural mortality rate of fish	0.2
q	Catchability coefficient	1
u	intensity factor	1
b	Saturation/handling parameter	0.3
c	Interference parameter	0.4
a	Fish-density threshold	6

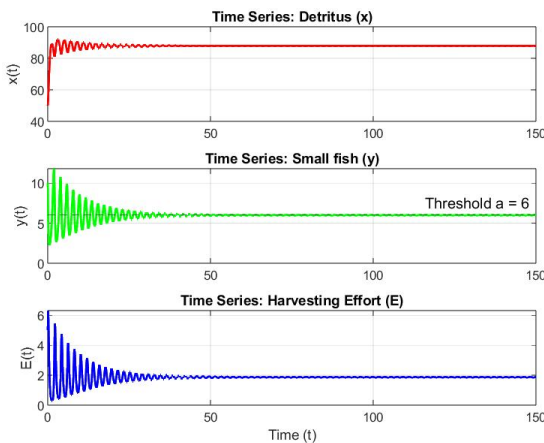
In this section, numerical simulations are performed using the baseline parameter values in Table 1. For these parameter values, the system admits the biologically feasible active-harvesting equilibrium

$$E_3 = (x^*, y^*, E^*) = (87.32249179, 6, 1.952561457),$$

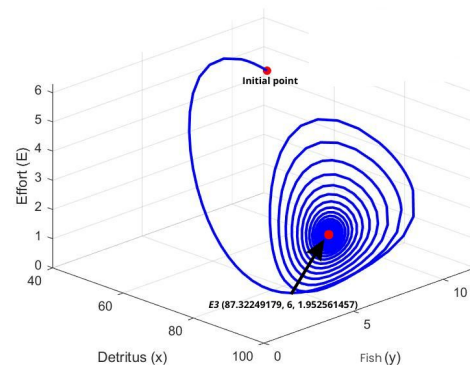
which is locally asymptotically stable. This is confirmed by the eigenvalues of the Jacobian at E_3 ,

$$\lambda_1 = -2.26055240, \quad \lambda_{2,3} = -0.04961301 \pm 3.42866255 i,$$

whose real parts are all negative.



(a) Time series of $x(t)$, $y(t)$, and $E(t)$.



(b) 3D phase portrait in the (x, y, E) -space.

Figure 1: Numerical dynamics of system (3.1) for the baseline parameter set in Table 1. The solution exhibits transient oscillations with decreasing amplitudes and converges to the stable equilibrium $E_3 = (87.32249179, 6, 1.952561457)$.

Based on Fig. 1, the trajectories of the detritus density $x(t)$, the small fish density $y(t)$, and the harvesting effort $E(t)$ exhibit transient oscillations with gradually decreasing amplitudes before settling to constant values. This pattern indicates that the solution is attracted to the active-harvesting equilibrium $E_3 = (87.32249179, 6, 1.952561457)$. In particular, the fish density approaches the threshold level $y = a = 6$, while the effort converges to a positive constant, suggesting persistence of harvesting activity in the long term under the chosen parameter set.

The three-dimensional phase portrait in Fig. 1 provides a geometric confirmation of this convergence. Starting from a positive initial condition, the trajectory forms a spiral that contracts toward the equilibrium point E_3 . This spiral-in behavior is consistent with the complex conjugate eigenvalues with negative real parts, implying that the system approaches equilibrium through damped oscillations rather than monotone convergence. Biologically, the transient

fluctuations reflect short-term adjustments in the detritus fish interaction and the adaptive response of harvesting effort; however, these fluctuations are eventually regulated, leading to a stable coexistence regime in which detritus, small fish, and a positive harvesting effort persist.

Overall, the time-series plots and the phase portrait consistently demonstrate that, for $a = 6$ and the baseline parameter values in Table 1, system (3.1) converges to the stable equilibrium E_3 , indicating stable coexistence with active harvesting.

To further examine how environmental capacity and fish mortality influence the persistence of harvesting activity, we next consider a second simulation scenario. While the baseline parameter set leads to convergence toward the active-harvesting equilibrium E_3 with positive effort, a reduction in the detritus carrying capacity and an increase in fish natural mortality may suppress harvesting viability. Therefore, we modify only two parameters, namely the carrying capacity $k = 10$ and the fish mortality rate $\mu = 0.6$, and investigate whether the system transitions to a no-effort equilibrium where harvesting effort vanishes.

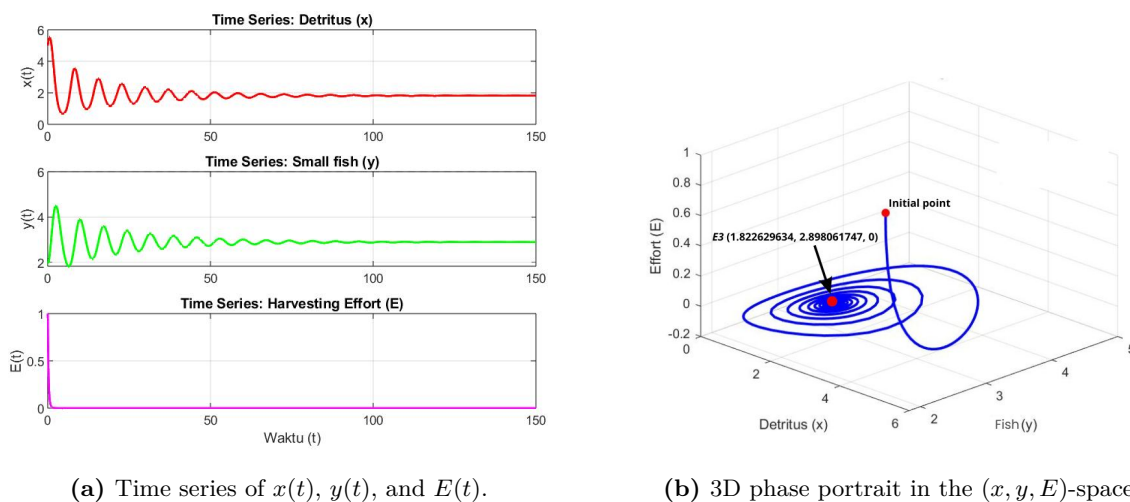


Figure 2: Numerical dynamics of system (3.1) under the modified parameters $k = 10$ and $\mu = 0.6$ (other parameters fixed). The trajectory converges to the no-effort equilibrium $E_2 = (1.822629634, 2.898061747, 0)$.

Figure 2(a) shows that the detritus density $x(t)$ and the fish density $y(t)$ undergo damped oscillations and eventually approach constant values. In contrast, the harvesting effort quickly decreases to $E(t) \approx 0$ and remains at zero thereafter. This indicates that the system is attracted to the no-effort coexistence equilibrium $E_2 = (1.822629634, 2.898061747, 0)$, where detritus and fish persist but harvesting activity collapses.

The convergence is further illustrated by the three-dimensional phase portrait in Fig. 2(b). Starting from a positive initial condition, the trajectory spirals toward the equilibrium point lying on the plane $E = 0$. Biologically, decreasing the carrying capacity k limits detritus availability, while increasing the mortality rate μ reduces fish persistence; together, these effects weaken the stock-effort feedback and drive the adaptive harvesting effort toward extinction. Hence, compared with the baseline case leading to a stable positive-effort equilibrium E_3 , the modified scenario demonstrates that changes in environmental capacity and mortality can shift the system to a stable regime without harvesting.

Overall, the two simulation scenarios indicate that the model may switch between an active-harvesting steady state (E_3) and a no-effort regime (E_2) depending on environmental capacity and fish mortality.

3.6 Bifurcation with respect to the threshold parameter a

This subsection investigates qualitative changes in the system dynamics when the threshold parameter a is treated as the bifurcation parameter. The continuation is performed using

311 MATCONT to track equilibrium branches and detect critical bifurcation points that indicate
 312 changes in stability and long-term behavior.

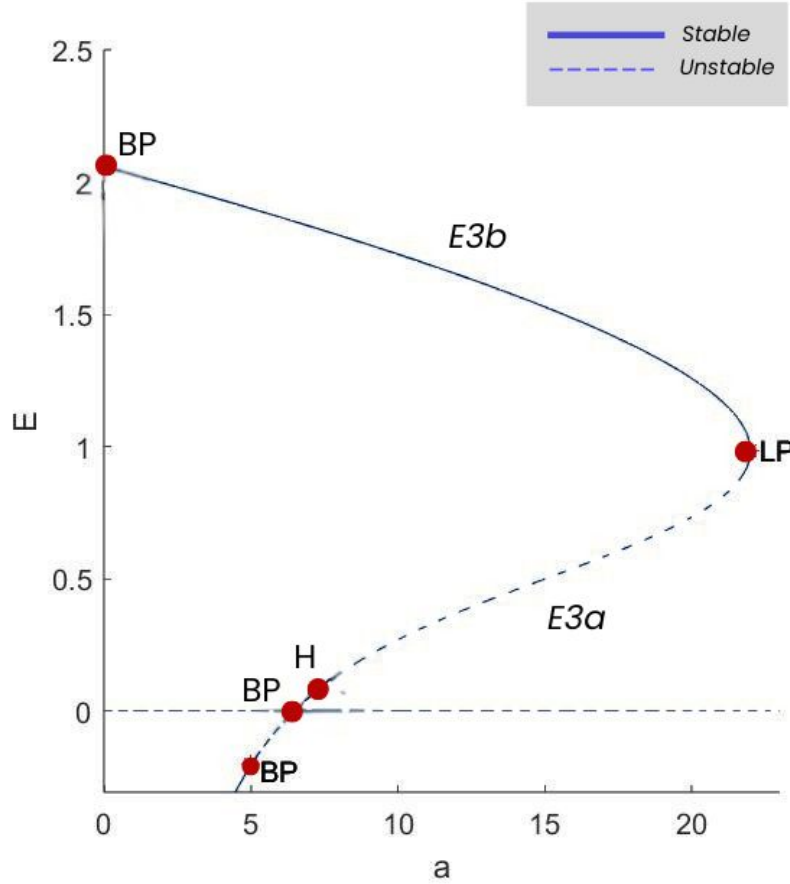


Figure 3: Bifurcation diagram in the (a, E) -plane obtained by MATCONT. Solid and dashed curves represent stable and unstable equilibrium segments, respectively. Detected singular points include branch points (BP), a Hopf bifurcation (H), and a limit point (LP).

313 Figure 3 shows the equilibrium continuation results in the (a, E) -plane. Two branches of
 314 biologically feasible active-harvesting equilibria are observed, labelled E_{3b} (stable, solid curve)
 315 and E_{3a} (unstable, dashed curve). The existence of two distinct branches indicates that, for
 316 certain ranges of a , the model may admit multiple equilibrium solutions with positive effort,
 317 implying that the asymptotic outcome can depend on the system's initial state.

318 A fold (saddle-node) bifurcation is detected at the limit point (LP) ... located at approximately
 319 $a \approx 22.000996$ with equilibrium coordinates

$$(x, y, E, a) \approx (33.666010, 22.000996, 0.984220, 22.000996).$$

321 This point marks a turning of the equilibrium branch and indicates a parameter threshold at
 322 which the number of equilibrium solutions may change. In addition, branch points (BP) are
 323 detected near the boundary branch $E \approx 0$, including a biologically relevant BP at $a \approx 6.460557$
 324 with coordinates $(x, y, E, a) \approx (1.120070, 6.460557, 0, 6.460557)$, suggesting a change in the
 325 equilibrium branch structure near the no-effort regime. A Hopf bifurcation is detected on
 326 the lower (unstable) branch at approximately $a \approx 6.615926$ with coordinates $(x, y, E, a) \approx$
 327 $(1.243956, 6.615926, 0.016633, 6.615926)$. Moreover, MATCONT computes the first Lyapunov
 328 coefficient $l_1 \approx 6.337488 \times 10^{-2} > 0$, which indicates a subcritical Hopf bifurcation and suggests
 329 a sensitive transition in the neighborhood of the Hopf point. Solutions with negative effort (e.g.,
 330 BP points yielding $E < 0$) are mathematically admissible but biologically infeasible and are
 331 therefore not considered.

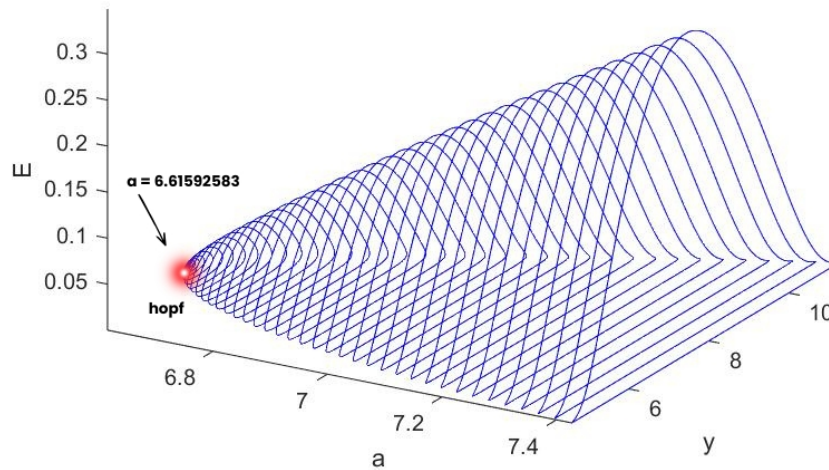


Figure 4: Limit-cycle dynamics near the Hopf bifurcation point ($a \approx 6.61592583$). The closed periodic orbit confirms the emergence of sustained oscillations in fish density y and harvesting effort E in a neighborhood of the Hopf point.

Figure 4 provides numerical evidence supporting the Hopf bifurcation detected in Fig. 3. Near $a \approx 6.61592583$, trajectories do not converge to a steady state; instead, they approach a closed orbit, indicating a periodic solution (limit cycle). Biologically, this periodic behavior represents recurrent oscillations driven by feedback between fish abundance and adaptive harvesting effort governed by the threshold rule $\dot{E} = uqE(y - a)$. When fish density increases above the threshold, effort tends to rise, which strengthens harvesting pressure and reduces fish density; as fish density decreases, effort declines, allowing the stock to recover. This feedback loop repeats, producing sustained boom bust fluctuations in both $y(t)$ and $E(t)$.

Overall, Figs. 3–4 show that the threshold parameter a plays a central role in shaping the long-term harvesting dynamics. The bifurcation diagram reveals stability changes and the presence of multiple equilibrium branches (stable E_{3b} and unstable E_{3a}), while the limit-cycle plot confirms that oscillatory harvesting regimes can arise near the Hopf point. These results emphasize that small changes in the threshold may shift the system between steady harvesting equilibria and persistent oscillations, which is important for interpreting stability and sustainability in the mangrove detritus small fish system.

4 Conclusion

This study developed and analyzed a three-dimensional mangrove detritus small fish model with dynamic harvesting effort under a Beddington DeAngelis functional response. The main objective was to understand how consumer interference and threshold-driven adaptive effort shape the long-term dynamics and stability of the system. Analytical results showed that the model admits biologically feasible equilibrium states, including a stable coexistence equilibrium with positive harvesting effort and a coexistence equilibrium without effort harvesting extinction under certain conditions. Local stability analysis using the Jacobian matrix and eigenvalue criteria confirmed the stability properties of these equilibria.

Numerical simulations supported the analytical findings by illustrating convergence to the stable active-harvesting equilibrium under the baseline parameter set, while a modified en-

environmental capacity and fish mortality scenario drove the trajectories toward the no-effort equilibrium. Furthermore, numerical continuation with respect to the threshold parameter a revealed critical bifurcation structures, including branch points and a fold limit point near $a \approx 22.001$. Importantly, a Hopf bifurcation was detected near $a \approx 6.616$, and the associated limit cycle demonstrated that the system can shift from steady-state coexistence to persistent oscillations in fish density and harvesting effort.

Overall, the results highlight the threshold parameter as a key control factor that can switch the system among stable harvesting, harvesting collapse, and oscillatory harvesting regimes. From a management perspective, this implies that threshold-based harvesting rules may stabilize the fishery under certain conditions but may also induce undesirable oscillations when parameters approach critical values. Future work may incorporate additional ecological complexity such as seasonal forcing, stochastic disturbances, or data-informed parameter estimation to further improve the applicability of the model to real mangrove ecosystems.

CRediT Authorship Contribution Statement

Donna Kurniasih: Conceptualization, Methodology, Formal Analysis, Investigation, Software (Maple/MATCONT/MATLAB), Writing Original Draft, Visualization. **Dian Savitri:** Conceptualization, Methodology, Supervision, Validation, Writing Review & Editing.

Declaration of Generative AI and AI-assisted technologies

Generative AI tools (including ChatGPT) were used in a limited manner to assist with language refinement, grammar editing, and improving clarity of presentation. All mathematical modeling, analytical derivations, numerical computations, simulation outputs, interpretation of results, and scientific conclusions were developed and verified by the authors. No generative AI tool was used to generate or validate the core scientific results.

Declaration of Competing Interest

The authors declare no competing interests.

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Data Availability

This study does not use field or experimental datasets. The numerical values of parameters are used as illustrative case-study settings to demonstrate the qualitative dynamics of the proposed model. Simulation codes and generated figures are available from the corresponding author upon reasonable request.

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