CAUCHY – Jurnal Matematika Murni dan Aplikasi
Volume 11 (1) (2026), Pages 427-439
p-ISSN: 2086-0382; e-ISSN: 2477-3344

Total Edge Irregularity Strength of Cycle Snake Graphs

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Abstract

Let G be a simple undirected graph with vertex set and edge set. A total labeling that maps from the union of the vertex set and the edge set to the set of positive integers from 1 to k is called a k-total labeling. The weight of an edge uv, denoted by $w(uv)$, is defined as the sum of the labels of the two vertices of u, v and the label of the edge uv itself. A k-total labeling is called an edge irregular total k-labeling of G if the weights of all distinct edges are different. The minimum k for which every edge of G has a distinct weight is called the total edge irregularity strength of the graph, denoted by $tes(G)$. A cycle snake graph is obtained from a path graph P_n by replacing each edge with a cycle graph C_m, denoted by $CS_{m,n}$. In this paper, we study the cycle snake graphs $CS_{3,n}$ and $CS_{m,n}$. Furthermore, we determine the total edge irregularity strength of these graphs. The results of this study contribute to the development of graph labeling theory by extending the investigation of total edge irregularity strength to the class of cycle snake graphs in general.

Keywords: cycle snake graph; edge irregular total k-labeling; total edge irregularity strength.

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1 Introduction

Graph labeling is one of the fundamental topics in graph theory. A labeling of a graph G is defined as a function from the elements of G to a set of positive integers. Depending on the domain of the mapping, graph labelings are typically classified into three categories: vertex labelings, edge labelings, and total labelings, where labels are assigned to vertices, edges, or both, respectively [1]. Graph labeling has been widely studied due to its applications in various fields, including vehicle routing [2], coding theory [3], electrical networks [4], and brain tumor [5].

Total labeling has received considerable attention from researchers because it involves the assignment of labels to both vertices and edges. Let $G = (V, E)$ be a simple undirected graph with vertex set $V(G)$ and edge set $E(G)$, and let ∂ be a labeling from $V(G) \cup E(G)$ to the integer set $\{1, 2, \ldots, k\}$, called a k-total labeling. The weight of a vertex u, denoted by $w(u)$, is defined as $w(u) = \partial(u) + \sum_{uv \in E(G)} \partial(uv)$. The weight of an edge uv, denoted by $w(uv)$, is defined as $w(uv) = \partial(u) + \partial(v) + \partial(uv)$ [1]. One of the well-studied types of total labeling is total irregular labeling, introduced by M. Bača et al. in 2007 [6]. A k-total labeling of G is said to induce vertex irregularity if $w(u) \neq w(v)$ for all distinct $u, v \in V(G)$. Similarly, it induces edge irregularity if $w(uv) \neq w(ef)$ for all distinct $uv, ef \in E(G)$. The total vertex irregularity strength of a graph G, denoted by $tvs(G)$, is defined as $tvs(G) = \min\{k \in \mathbb{N} : w(u) \neq w(v) \text{ for all } u \neq v \in V(G)\}$.

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Submitted: ... Reviewed: ... Accepted: ...
DOI: https://doi.org/10.18860/cauchy.v11i1.32446

Analogously, the total edge irregularity strength of a graph G, denoted by $tes(G)$, is defined as $tes(G) = \min\{k \in \mathbb{N} : w(uv) \neq w(ef) \text{ for all } uv \neq ef \in E(G)\}$. The total vertex and edge irregularity strengths for several simple graph families, including paths, cycles, and stars, have been established [6]. This concept provides a fundamental basis for further research on the total vertex irregularity strength of various graph classes, as seen in [7], [8], [9]. It also underpins studies on the total edge irregularity strength of several other classes, such as complete and complete bipartite graphs [10], generalized prism graphs [11], and uniform centralized theta graphs [12].

The concepts of total vertex irregularity strength and total edge irregularity strength were further developed for classes of snake graph. A snake graph is defined as a path or cycle graph in which each edge is replaced by a graph from a specific class. The total vertex irregularity strength for several classes of snake graphs, including triangular snakes and double-triangular snakes, was examined by Ahmad et al. in 2014 [13]. The total edge irregularity strength of quintet snake graphs constructed by replacing each edge of a path P_n with a cycle C_5, denoted by $CS_{5,n}$ has been determined in 2021 by Salama [14]. In 2021, Salama and Elanin examined the total edge irregularity strength of several specific types of uniform theta snake graphs [15]. The total edge irregularity strength of heptagonal snake graphs, denoted by $CS_{7,n}$ has also been analyzed in 2022 by Salama [16]. More recently, the total edge irregularity strength for a general form of snake graphs derived from star graphs was established in 2025 by Attiya et al [17]. Based on some limited previous studies [14], [16], this article determines $CS_{3,n}$ and $CS_{m,n}$.

2 Methods

To facilitate the labeling process, first we need to consider the theorems the bounds on the total edge irregularity strength of graphs.

Theorem 1 [6] Let $G = (V, E)$ be a graph with vertex set $V(G)$ and a non-empty edge set $E(G)$, then

$$\left\lceil \frac{|E(G)| + 2}{3} \right\rceil \leq tes(G) \leq |E(G)|.$$

Theorem 2 [6] Let $G = (V, E)$ be a graph with maximum degree $\Delta = \Delta(G)$, then

1. $tes(G) \geq \left\lceil \frac{\Delta + 1}{2} \right\rceil$, and
2. $tes(G) \leq |E(G)| - \Delta$ if $\Delta \leq \frac{|E(G)| - 1}{2}$.

Based on our obejectif in this article, next we review definition of the cycle snake graphs $CS_{m,n}$, be studied throughout this article.

Definition 1 A cycle snake graph, denoted by $CS_{m,n}$, is constructed from a path graph P_n by replacing every edge with a copy of the cycle graph C_m for $m \geq 3$ and $n \geq 2$.

Lastly, the steps for determining the total edge irregularity strength of a cycle graph $CS_{3,n}$ and $CS_{m,n}$ for $m \geq 4$ are as follows:

i. Determine the lower bound of the total edge irregularity strength of the graph by constructing the lower bound as stated in **Theorem 1** and **Theorem 2**.

ii. Construct an explicit total edge irregular labeling for the graph such that the weight of every edge is distinct, beginning with graphs of small order and size until the labeling pattern can be identified.

iii. Construct a formula that represents the total edge irregular labeling for the graph.

iv. The minimum value k obtained from this formula serves as the upper bound of the total edge irregularity strength of the graph.

v. The upper and lower bounds are verified. If both bounds coincide, then this value is the exact total edge irregularity strength. If the bounds do not coincide, the step (ii) is

repeated.

vi. The obtained results are stated as a theorem together with its proof.

The findings of this research advance graph labeling theory by extending the concept of total edge irregularity strength to cycle snake graphs.

3 Results and Discussion

In this section, we present the main results regarding the total edge irregularity strength of cycle snake graphs. We first consider the special case of cycle snake graphs with C_3, which is stated in the following lemma.

Lemma 1 Let $CS_{3,n}$ be a cycle snake graph with $n \geq 2$, then

$$tes(CS_{3,n}) = n + 1$$

Proof. The cycle snake graph $CS_{3,n}$ has order $2n + 1$ vertices, size $3n$ edges, and maximum degree $\Delta(CS_{3,n}) = 4$. According to Theorem 1, we have

$$tes(CS_{3,n}) \geq n + 1.$$

Let the vertex and edge set of $CS_{3,n}$ be defined as follows:

$$V(CS_{3,n}) = \{x_i : 1 \leq i \leq n+1\} \cup \{y_i : 1 \leq i \leq n\},$$

$$E(CS_{3,n}) = \{x_i x_{i+1} : 1 \leq i \leq n\} \cup \{x_i y_i : 1 \leq i \leq n\} \cup \{x_{i+1} y_i : 1 \leq i \leq n\},$$

and define $\partial : V(CS_{3,n}) \cup E(CS_3, n) \to \{1, 2, ..., k\}$ as a total k-labeling given by

$$\begin{aligned} \partial(x_i) &= i, \quad \text{for } 1 \leq i \leq n+1. \\ \partial(y_i) = \partial(x_i y_i) &= \partial(x_{i+1} y_i) = i, \quad \text{for } 1 \leq i \leq n. \\ \partial(x_i x_{i+1}) &= i + 1, \quad \text{for } 1 \leq i \leq n. \end{aligned}$$

The edge weights of $CS_{3,n}$ are given as follows:

$$\begin{aligned} w(x_i y_i) &= 3i, \quad \text{for } 1 \leq i \leq n. \\ w(x_{i+1} y_i) &= 3i + 1, \quad \text{for } 1 \leq i \leq n. \\ w(x_i x_{i+1}) &= 3i + 2, \quad \text{for } 1 \leq i \leq n. \end{aligned}$$

Clearly, all edge weights are distinct, taking the consecutive integer values $3, 4, \ldots, 3n+2$. Hence the total edge irregularity strength of $CS_{3,n}$ is $tes(CS_{3,n}) = n + 1$. □

Example 1:

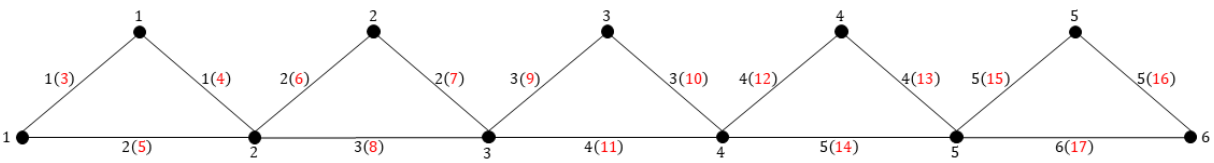

Figure 1: Cycle snake graph $CS_{3,5}$ with $tes = 6$

Figure 1 illustrates the total labeling of $CS_{3,5}$ based on Lemma 1. The maximum vertex and edge labels are 6, and the edge weights are distinct and form the set $\{3, 4, \ldots, 17\}$.

We now extend the previous results on snake cycle graphs with three vertices per cycle to a general case, in which each cycle contains $m \geq 4$ and $n \geq 2$ vertices. The following theorem presents the total edge irregularity strength for this broader class of snake cycle graphs.

Theorem 3 Let $CS_{m,n}$ be a cycle snake graph with $m \geq 4$, then the following relation holds.

$$tes(CS_{m,n}) = \left\lceil \frac{mn+2}{3} \right\rceil$$

Proof. The cycle snake graph has order $n(m-1)+1$ vertices, size mn edges, and maximum degree $\Delta(CS_{m,n}) = 4$. According to Theorem 1, we obtain the inequality

$$tes(CS_{m,n}) \geq \left\lceil \frac{mn+2}{3} \right\rceil.$$

Let the vertex and edge set of $CS_{m,n}$ be defined as follows:

$$V(CS_{m,n}) = \{x_i : 1 \leq i \leq n+1\} \cup \left\{ y_j^i : 1 \leq i \leq n, 1 \leq j \leq \left\lceil \frac{m-2}{2} \right\rceil \right\}$$
$$\cup \left\{ z_j^i : 1 \leq i \leq n, 1 \leq j \leq \left\lfloor \frac{m-2}{2} \right\rfloor \right\},$$

$$E(CS_{m,n}) = \{x_i y_j^i, x_i z_j^i : 1 \leq i \leq n, j = 1\} \cup \left\{ y_j^i y_{j+1}^i : 1 \leq i \leq n, 1 \leq j \leq \left\lceil \frac{m-2}{2} \right\rceil - 1 \right\}$$
$$\cup \left\{ z_j^i z_{j+1}^i : 1 \leq i \leq n, 1 \leq j \leq \left\lfloor \frac{m-2}{2} \right\rfloor - 1 \right\} \cup \left\{ x_{i+1} y_j^i : 1 \leq i \leq n, j = \left\lceil \frac{m-2}{2} \right\rceil \right\}$$
$$\cup \left\{ x_{i+1} z_j^i : 1 \leq i \leq n, j = \left\lfloor \frac{m-2}{2} \right\rfloor \right\},$$

and define $\partial : V(CS_{m,n}) \cup E(CS_{m,n}) \to \{1, 2, \ldots, k\}$ be a total k-labeling defined by the following cases.

Case 1: $m \equiv 0 (\mathrm{mod}\ 3)$.

A total edge irregular labeling is defined as follows:

$$\partial(x_i) = \partial(x_i y_1^i) = \partial(x_i z_1^i) = (i-1)\left(\frac{m}{3}\right) + 1, \quad \text{for } 1 \leq i \leq n.$$

$$\partial(x_{n+1}) = n\left(\frac{m}{3}\right) + 1.$$

$$\partial(y_j^i) = (i-1)\left(\frac{m}{3}\right) + j - \left\lfloor \frac{j-1}{3} \right\rfloor, \quad \text{for } 1 \leq i \leq n, 1 \leq j \leq \left\lceil \frac{m-2}{2} \right\rceil.$$

$$\partial(z_j^i) = (i-1)\left(\frac{m}{3}\right) + j - \left\lceil \frac{j-1}{3} \right\rceil + 1, \quad \text{for } 1 \leq i \leq n, 1 \leq j \leq \left\lfloor \frac{m-2}{2} \right\rfloor.$$

$$\partial(y_j^i y_{j+1}^i) = (i-1)\left(\frac{m}{3}\right) + j - \left\lceil \frac{j-1}{3} \right\rceil + 1, \quad \text{for } 1 \leq i \leq n, 1 \leq j \leq \left\lceil \frac{m-2}{2} \right\rceil - 1.$$

$$\partial(x_{i+1} y_j^i) = \begin{cases} \frac{im}{3}, & \text{for } 1 \leq i \leq n, j = \frac{m-2}{2}, m \equiv 0 (\mathrm{mod}\ 2), \\ \frac{im}{3} + 1, & \text{for } 1 \leq i \leq n, j = \left\lceil \frac{m-2}{2} \right\rceil, m \equiv 1 (\mathrm{mod}\ 2). \end{cases}$$

$$\partial(z_j^i z_{j+1}^i) = (i-1)\left(\frac{m}{3}\right) + j - \left\lfloor \frac{j}{3} \right\rfloor + 1, \quad \text{for } 1 \le i \le n, 1 \le j \le \left\lfloor \frac{m-2}{2} \right\rfloor - 1.$$

For $1 \le i \le n$ and $j = \left\lfloor \frac{m-2}{2} \right\rfloor$,

$$\partial(x_{i+1} z_j^i) = \begin{cases} \frac{im}{3} + 1, & \text{for } m \equiv 0 (\text{mod } 2), \\ \frac{im}{3}, & \text{for } m \equiv 1 (\text{mod } 2). \end{cases}$$

The edge weights of $CS_{m,n}$ are given as follows:
For $1 \le i \le n$,

$$w(y_j^i y_{j+1}^i) = \begin{cases} m(i-1) + 3\left(j - \left\lceil \frac{j-1}{3} \right\rceil + 1\right), & \text{for } j \equiv 0 (\text{mod } 3), \\ m(i-1) + 3\left(j - \left\lceil \frac{j-1}{3} \right\rceil\right) + 2, & \text{for } j \equiv 1 (\text{mod } 3), \\ m(i-1) + 3\left(j - \left\lceil \frac{j-1}{3} \right\rceil\right) + 4, & \text{for } j \equiv 2 (\text{mod } 3). \end{cases}$$

For $1 \le i \le n$ and $j = \left\lceil \frac{m-2}{2} \right\rceil$,

$$w(x_{i+1} y_j^i) = \begin{cases} im + 1, & \text{for } m \equiv 0 (\text{mod } 2), \\ im + 2, & \text{for } m \equiv 1 (\text{mod } 2). \end{cases}$$

$$w(x_i y_1^i) = m(i-1) + 3, \quad \text{for } 1 \le i \le n.$$

$$w(x_i z_1^i) = m(i-1) + 4, \quad \text{for } 1 \le i \le n.$$

For $1 \le i \le n$,

$$w(z_j^i z_{j+1}^i) = \begin{cases} m(i-1) + 3\left(j - \left\lfloor \frac{j}{3} \right\rfloor\right) + 4, & \text{for } j \equiv 0 (\text{mod } 3), \\ m(i-1) + 3\left(j - \left\lfloor \frac{j}{3} \right\rfloor + 1\right), & \text{for } j \equiv 1 (\text{mod } 3), \\ m(i-1) + 3\left(j - \left\lfloor \frac{j}{3} \right\rfloor\right) + 2, & \text{for } j \equiv 2 (\text{mod } 3). \end{cases}$$

For $1 \le i \le n$ and $j = \left\lfloor \frac{m-2}{2} \right\rfloor$

$$w(x_{i+1} z_j^i) = \begin{cases} im + 2, & \text{for } m \equiv 0 (\text{mod } 2), \\ im + 1, & \text{for } m \equiv 1 (\text{mod } 2). \end{cases}$$

Clearly, the edge weights are all distinct and form consecutive integers in the set $\{3, 4, \ldots, mn+2\}$.

Case 2: $m \equiv 1 (\text{mod } 3)$.
Define the total edge irregular labeling as follows:

$$\partial(x_i) = (i-1)\left\lfloor \frac{m}{3} \right\rfloor + \left\lfloor \frac{i}{3} \right\rfloor + 1, \quad \text{for } 1 \le i \le n+1.$$

For $1 \leq j \leq \left\lceil \frac{m-2}{2} \right\rceil$,

$$\partial(y_j^i) = \begin{cases} (i-1)\left\lfloor \frac{m}{3} \right\rfloor + \left\lfloor \frac{i}{3} \right\rfloor - \left\lfloor \frac{j}{3} \right\rfloor + j, & \text{for } i \equiv 0 (\text{mod } 3), \\ (i-1)\left\lfloor \frac{m}{3} \right\rfloor + \left\lfloor \frac{i}{3} \right\rfloor - \left\lfloor \frac{j-1}{3} \right\rfloor + j, & \text{for } i \equiv 1 (\text{mod } 3), \\ (i-1)\left\lfloor \frac{m}{3} \right\rfloor + \left\lfloor \frac{i}{3} \right\rfloor - \left\lfloor \frac{j+1}{3} \right\rfloor + j + 1, & \text{for } i \equiv 2 (\text{mod } 3). \end{cases}$$

For $1 \leq j \leq \left\lfloor \frac{m-2}{2} \right\rfloor$,

$$\partial(z_j^i) = \begin{cases} (i-1)\left\lfloor \frac{m}{3} \right\rfloor + \left\lfloor \frac{i}{3} \right\rfloor - \left\lfloor \frac{j-1}{3} \right\rfloor + j, & \text{for } i \equiv 0 (\text{mod } 3), \\ (i-1)\left\lfloor \frac{m}{3} \right\rfloor + \left\lfloor \frac{i}{3} \right\rfloor - \left\lfloor \frac{j+1}{3} \right\rfloor + j + 1, & \text{for } i \equiv 1 (\text{mod } 3), \\ (i-1)\left\lfloor \frac{m}{3} \right\rfloor + \left\lfloor \frac{i+1}{3} \right\rfloor - \left\lfloor \frac{j}{3} \right\rfloor + j, & \text{for } i \equiv 2 (\text{mod } 3). \end{cases}$$

For $1 \leq j \leq \left\lceil \frac{m-2}{2} \right\rceil - 1$,

$$\partial(y_j^i y_{j+1}^i) = \begin{cases} (i-1)\left\lfloor \frac{m}{3} \right\rfloor + \left\lfloor \frac{i}{3} \right\rfloor - \left\lfloor \frac{j-1}{3} \right\rfloor + j, & \text{for } i \equiv 0 (\text{mod } 3), \\ (i-1)\left\lfloor \frac{m}{3} \right\rfloor + \left\lfloor \frac{i}{3} \right\rfloor - \left\lfloor \frac{j+1}{3} \right\rfloor + j + 1, & \text{for } i \equiv 1 (\text{mod } 3), \\ (i-1)\left\lfloor \frac{m}{3} \right\rfloor + \left\lfloor \frac{i+1}{3} \right\rfloor - \left\lfloor \frac{j}{3} \right\rfloor + j, & \text{for } i \equiv 2 (\text{mod } 3). \end{cases}$$

For $j = \left\lceil \frac{m-2}{2} \right\rceil$,

$$\partial(x_{i+1} y_j^i) = \begin{cases} i\left\lfloor \frac{m}{3} \right\rfloor + \left\lfloor \frac{i}{3} \right\rfloor, & \text{for } i \equiv 0, 2 (\text{mod } 3), m \equiv 0 (\text{mod } 2), \\ i\left\lfloor \frac{m}{3} \right\rfloor + \left\lfloor \frac{i}{3} \right\rfloor + 1, & \text{for } \begin{cases} i \equiv 1 (\text{mod } 3), m \equiv 0 (\text{mod } 2), \\ 1 \leq i \leq n, m \equiv 1 (\text{mod } 2). \end{cases} \end{cases}$$

$$\partial(x_i y_1^i) = \begin{cases} (i-1)\left\lfloor \frac{m}{3} \right\rfloor + \left\lfloor \frac{i}{3} \right\rfloor, & \text{for } i \equiv 0 (\text{mod } 3), \\ (i-1)\left\lfloor \frac{m}{3} \right\rfloor + \left\lfloor \frac{i}{3} \right\rfloor + 1, & \text{for } i \equiv 1, 2 (\text{mod } 3). \end{cases}$$

$$\partial(x_i z_1^i) = \begin{cases} (i-1)\left\lfloor \frac{m}{3} \right\rfloor + \left\lfloor \frac{i}{3} \right\rfloor + 1, & \text{for } i \equiv 0, 1 (\text{mod } 3), \\ (i-1)\left\lfloor \frac{m}{3} \right\rfloor + \left\lfloor \frac{i}{3} \right\rfloor + 2, & \text{for } i \equiv 2 (\text{mod } 3). \end{cases}$$

For $1 \leq j \leq \left\lfloor \frac{m-2}{2} \right\rfloor - 1$,

$$\partial(z_j^i z_{j+1}^i) = \begin{cases} (i-1)\left\lfloor \frac{m}{3} \right\rfloor + \left\lfloor \frac{i}{3} \right\rfloor - \left\lfloor \frac{j+1}{3} \right\rfloor + j + 1, & \text{for } i \equiv 0 (\text{mod } 3), \\ (i-1)\left\lfloor \frac{m}{3} \right\rfloor + \left\lfloor \frac{i+2}{3} \right\rfloor - \left\lfloor \frac{j}{3} \right\rfloor + j, & \text{for } i \equiv 1 (\text{mod } 3), \\ (i-1)\left\lfloor \frac{m}{3} \right\rfloor + \left\lfloor \frac{i+1}{3} \right\rfloor - \left\lfloor \frac{j-1}{3} \right\rfloor + j, & \text{for } i \equiv 2 (\text{mod } 3). \end{cases}$$

For $j = \left\lfloor \frac{m-2}{2} \right\rfloor$

$$\partial(x_{i+1}z_j^i) = \begin{cases} i\left\lfloor \frac{m}{3} \right\rfloor + \left\lfloor \frac{i}{3} \right\rfloor + 1, & \text{for } \begin{cases} 1 \le i \le n, m \equiv 0 (\text{mod } 2), \\ i \equiv 1 (\text{mod } 3), m \equiv 1 (\text{mod } 2), \end{cases} \\ i\left\lfloor \frac{m}{3} \right\rfloor + \left\lfloor \frac{i}{3} \right\rfloor, & \text{for } i \equiv 0, 2 (\text{mod } 3), m \equiv 1 (\text{mod } 2). \end{cases}$$

The edge weights of $CS_{m,n}$ are given as follows:
For $j = \left\lceil \frac{m-2}{2} \right\rceil$,

$$w(x_{i+1}y_j^i) = \begin{cases} 3\left(i\left\lfloor \frac{m}{3} \right\rfloor + \left\lfloor \frac{i}{3} \right\rfloor\right) + 1, & \text{for } i \equiv 0 (\text{mod } 3), m \equiv 0 (\text{mod } 2), \\ 3\left(i\left\lfloor \frac{m}{3} \right\rfloor + \left\lfloor \frac{i}{3} \right\rfloor\right) + 2, & \text{for } \begin{cases} i \equiv 1 (\text{mod } 3), m \equiv 0 (\text{mod } 2), \\ i \equiv 0 (\text{mod } 3), m \equiv 1 (\text{mod } 2), \end{cases} \\ 3\left(i\left\lfloor \frac{m}{3} \right\rfloor + \left\lfloor \frac{i}{3} \right\rfloor + 1\right), & \text{for } \begin{cases} i \equiv 2 (\text{mod } 3), m \equiv 0 (\text{mod } 2), \\ i \equiv 1 (\text{mod } 3), m \equiv 1 (\text{mod } 2), \end{cases} \\ 3\left(i\left\lfloor \frac{m}{3} \right\rfloor + \left\lfloor \frac{i}{3} \right\rfloor\right) + 4, & \text{for } i \equiv 2 (\text{mod } 3), m \equiv 1 (\text{mod } 2). \end{cases}$$

$$w(x_i y_1^i) = \begin{cases} 3\left((i-1)\left\lfloor \frac{m}{3} \right\rfloor + \left\lfloor \frac{i}{3} \right\rfloor\right) + 2, & \text{for } i \equiv 0 (\text{mod } 3), \\ 3\left((i-1)\left\lfloor \frac{m}{3} \right\rfloor + \left\lfloor \frac{i}{3} \right\rfloor + 1\right), & \text{for } i \equiv 1 (\text{mod } 3), \\ 3\left((i-1)\left\lfloor \frac{m}{3} \right\rfloor + \left\lfloor \frac{i}{3} \right\rfloor\right) + 4, & \text{for } i \equiv 2 (\text{mod } 3). \end{cases}$$

$$w(y_j^i y_{j+1}^i) = \begin{cases} 3\left((i-1)\left\lfloor \frac{m}{3} \right\rfloor + \left\lfloor \frac{i}{3} \right\rfloor - \left\lfloor \frac{j}{3} \right\rfloor + j\right) + 2, & \text{for } \begin{cases} i \equiv 0 (\text{mod } 3), j \equiv 0 (\text{mod } 3), \\ i \equiv 1 (\text{mod } 3), j \equiv 1 (\text{mod } 3), \\ i \equiv 2 (\text{mod } 3), j \equiv 2 (\text{mod } 3), \end{cases} \\ 3\left((i-1)\left\lfloor \frac{m}{3} \right\rfloor + \left\lfloor \frac{i}{3} \right\rfloor - \left\lfloor \frac{j-1}{3} \right\rfloor + j\right) + 1, & \text{for } \begin{cases} i \equiv 0 (\text{mod } 3), j \equiv 1 (\text{mod } 3), \\ i \equiv 1 (\text{mod } 3), j \equiv 2 (\text{mod } 3), \\ i \equiv 2 (\text{mod } 3), j \equiv 0 (\text{mod } 3), \end{cases} \\ 3\left((i-1)\left\lfloor \frac{m}{3} \right\rfloor + \left\lfloor \frac{i-1}{3} \right\rfloor - \left\lfloor \frac{j}{3} \right\rfloor + j\right) + 3, & \text{for } \begin{cases} i \equiv 0 (\text{mod } 3), j \equiv 2 (\text{mod } 3), \\ i \equiv 1 (\text{mod } 3), j \equiv 0 (\text{mod } 3), \\ i \equiv 2 (\text{mod } 3), j \equiv 1 (\text{mod } 3). \end{cases} \end{cases}$$

$$w(x_i z_1^i) = \begin{cases} 3\left((i-1)\left\lfloor \frac{m}{3} \right\rfloor + \left\lfloor \frac{i}{3} \right\rfloor + 1\right), & \text{for } i \equiv 0 (\text{mod } 3), \\ 3\left((i-1)\left\lfloor \frac{m}{3} \right\rfloor + \left\lfloor \frac{i}{3} \right\rfloor\right) + 4, & \text{for } i \equiv 1 (\text{mod } 3), \\ 3\left((i-1)\left\lfloor \frac{m}{3} \right\rfloor + \left\lfloor \frac{i}{3} \right\rfloor\right) + 5, & \text{for } i \equiv 2 (\text{mod } 3). \end{cases}$$

For $j=\left\lfloor \frac{m-2}{2} \right\rfloor$,

$$w(x_{i+1}z_j^i)=\begin{cases} 3\left(i\left\lfloor \frac{m}{3}\right\rfloor+\left\lfloor \frac{i}{3}\right\rfloor\right)+2, & \text{for } \begin{cases} i\equiv 0(\text{mod } 3), m\equiv 0(\text{mod } 2),\\ i\equiv 1(\text{mod } 3), m\equiv 1(\text{mod } 2),\end{cases}\\ 3\left(i\left\lfloor \frac{m}{3}\right\rfloor+\left\lfloor \frac{i}{3}\right\rfloor+1\right), & \text{for } \begin{cases} i\equiv 1(\text{mod } 3), m\equiv 0(\text{mod } 2),\\ i\equiv 2(\text{mod } 3), m\equiv 1(\text{mod } 2),\end{cases}\\ 3\left(i\left\lfloor \frac{m}{3}\right\rfloor+\left\lfloor \frac{i}{3}\right\rfloor\right)+4, & \text{for } i\equiv 2(\text{mod } 3), m\equiv 0(\text{mod } 2),\\ 3\left(i\left\lfloor \frac{m}{3}\right\rfloor+\left\lfloor \frac{i}{3}\right\rfloor\right)+1, & \text{for } i\equiv 0(\text{mod } 3), m\equiv 1(\text{mod } 2).\end{cases}$$

$$w(z_j^iz_{j+1}^i)=\begin{cases} 3\left((i-1)\left\lfloor \frac{m}{3}\right\rfloor+\left\lfloor \frac{i+3}{3}\right\rfloor-\left\lfloor \frac{j}{3}\right\rfloor+j\right), & \text{for } \begin{cases} i\equiv 0(\text{mod } 3), j\equiv 0(\text{mod } 3),\\ i\equiv 1(\text{mod } 3), j\equiv 1(\text{mod } 3),\\ i\equiv 2(\text{mod } 3), j\equiv 2(\text{mod } 3),\end{cases}\\ 3\left((i-1)\left\lfloor \frac{m}{3}\right\rfloor+\left\lfloor \frac{i}{3}\right\rfloor-\left\lfloor \frac{j-1}{3}\right\rfloor+j\right)+2, & \text{for } \begin{cases} i\equiv 0(\text{mod } 3), j\equiv 1(\text{mod } 3),\\ i\equiv 1(\text{mod } 3), j\equiv 2(\text{mod } 3),\\ i\equiv 2(\text{mod } 3), j\equiv 0(\text{mod } 3),\end{cases}\\ 3\left((i-1)\left\lfloor \frac{m}{3}\right\rfloor+\left\lfloor \frac{i+2}{3}\right\rfloor-\left\lfloor \frac{j}{3}\right\rfloor+j\right)+1, & \text{for } \begin{cases} i\equiv 0(\text{mod } 3), j\equiv 2(\text{mod } 3),\\ i\equiv 1(\text{mod } 3), j\equiv 0(\text{mod } 3),\\ i\equiv 2(\text{mod } 3), j\equiv 1(\text{mod } 3).\end{cases}\end{cases}$$

It's clear that all edge weights are distinct, forming the consecutive set $\{3,4,\ldots,mn+2\}$.

Case 3: $m\equiv 2(\text{mod } 3)$

The total edge irregular labeling is defined as follows.

$$\partial(x_i)=(i-1)\left\lfloor \frac{m+3}{3}\right\rfloor-\left\lfloor \frac{i}{3}\right\rfloor+1, \quad \text{for } 1\le i\le n+1.$$

For $1\le j\le \left\lceil \frac{m-2}{2}\right\rceil$,

$$\partial(y_j^i)=\begin{cases} (i-1)\left\lfloor \frac{m+3}{3}\right\rfloor-\left\lfloor \frac{i}{3}\right\rfloor-\left\lfloor \frac{j+1}{3}\right\rfloor+j+1, & \text{for } i\equiv 0(\text{mod } 3),\\ (i-1)\left\lfloor \frac{m+3}{3}\right\rfloor-\left\lfloor \frac{i}{3}\right\rfloor-\left\lfloor \frac{j-1}{3}\right\rfloor+j, & \text{for } i\equiv 1(\text{mod } 3),\\ (i-1)\left\lfloor \frac{m+3}{3}\right\rfloor-\left\lfloor \frac{i}{3}\right\rfloor-\left\lfloor \frac{j}{3}\right\rfloor+j, & \text{for } i\equiv 2(\text{mod } 3).\end{cases}$$

For $1\le j\le \left\lfloor \frac{m-2}{2}\right\rfloor$,

$$\partial(z_j^i)=\begin{cases} (i-1)\left\lfloor \frac{m+3}{3}\right\rfloor-\left\lfloor \frac{i}{3}\right\rfloor-\left\lfloor \frac{j}{3}\right\rfloor+j+1, & \text{for } i\equiv 0(\text{mod } 3),\\ (i-1)\left\lfloor \frac{m+3}{3}\right\rfloor-\left\lfloor \frac{i}{3}\right\rfloor-\left\lfloor \frac{j+1}{3}\right\rfloor+j+1, & \text{for } i\equiv 1(\text{mod } 3),\\ (i-1)\left\lfloor \frac{m+3}{3}\right\rfloor-\left\lfloor \frac{i}{3}\right\rfloor-\left\lfloor \frac{j-1}{3}\right\rfloor+j, & \text{for } i\equiv 2(\text{mod } 3).\end{cases}$$

For $j=\left\lceil \frac{m-2}{2}\right\rceil$

$$\partial(x_{i+1}y_j^i)=\begin{cases} i\left\lfloor\frac{m+3}{3}\right\rfloor-\left\lfloor\frac{i-1}{3}\right\rfloor-1, & \text{for } i\equiv 0,1(\text{mod }3), m\equiv 0(\text{mod }2),\\ i\left\lfloor\frac{m+3}{3}\right\rfloor-\left\lfloor\frac{i-1}{3}\right\rfloor, & \text{for } \begin{cases} i\equiv 2(\text{mod }3), m\equiv 0(\text{mod }2),\\ 1\le i\le n, m\equiv 1(\text{mod }2).\end{cases}\end{cases}$$

$$\partial(x_iy_1^i)=\begin{cases}(i-1)\left\lfloor\frac{m+3}{3}\right\rfloor-\left\lfloor\frac{i}{3}\right\rfloor+1, & \text{for } i\equiv 0,1(\text{mod }3),\\ (i-1)\left\lfloor\frac{m+3}{3}\right\rfloor-\left\lfloor\frac{i}{3}\right\rfloor, & \text{for } i\equiv 2(\text{mod }3).\end{cases}$$

For $1\le j\le \left\lceil\frac{m-2}{2}\right\rceil-1$,

$$\partial(y_j^iy_{j+1}^i)=\begin{cases}(i-1)\left\lfloor\frac{m+3}{3}\right\rfloor-\left\lfloor\frac{i}{3}\right\rfloor-\left\lfloor\frac{j}{3}\right\rfloor+j+1, & \text{for } i\equiv 0(\text{mod }3),\\ (i-1)\left\lfloor\frac{m+3}{3}\right\rfloor-\left\lfloor\frac{i}{3}\right\rfloor-\left\lfloor\frac{j+1}{3}\right\rfloor+j+1, & \text{for } i\equiv 1(\text{mod }3),\\ (i-1)\left\lfloor\frac{m+3}{3}\right\rfloor-\left\lfloor\frac{i}{3}\right\rfloor-\left\lfloor\frac{j-1}{3}\right\rfloor+j, & \text{for } i\equiv 2(\text{mod }3).\end{cases}$$

$$\partial(x_iz_1^i)=\begin{cases}(i-1)\left\lfloor\frac{m+3}{3}\right\rfloor-\left\lfloor\frac{i}{3}\right\rfloor+2, & \text{for } i\equiv 0(\text{mod }3),\\ (i-1)\left\lfloor\frac{m+3}{3}\right\rfloor-\left\lfloor\frac{i}{3}\right\rfloor+1, & \text{for } i\equiv 1,2(\text{mod }3).\end{cases}$$

For $1\le j\le \left\lfloor\frac{m-2}{2}\right\rfloor-1$,

$$\partial(z_j^iz_{j+1}^i)=\begin{cases}(i-1)\left\lfloor\frac{m+3}{3}\right\rfloor-\left\lfloor\frac{i}{3}\right\rfloor-\left\lfloor\frac{j-1}{3}\right\rfloor+j+1, & \text{for } i\equiv 0(\text{mod }3),\\ (i-1)\left\lfloor\frac{m+3}{3}\right\rfloor-\left\lfloor\frac{i}{3}\right\rfloor-\left\lfloor\frac{j}{3}\right\rfloor+j+1, & \text{for } i\equiv 1(\text{mod }3),\\ (i-1)\left\lfloor\frac{m+3}{3}\right\rfloor-\left\lfloor\frac{i}{3}\right\rfloor-\left\lfloor\frac{j+1}{3}\right\rfloor+j+1, & \text{for } i\equiv 2(\text{mod }3).\end{cases}$$

For $j=\left\lfloor\frac{m-2}{2}\right\rfloor$,

$$\partial(x_{i+1}z_j^i)=\begin{cases} i\left\lfloor\frac{m+3}{3}\right\rfloor-\left\lfloor\frac{i-1}{3}\right\rfloor, & \text{for } \begin{cases} 1\le i\le n, m\equiv 0(\text{mod }2),\\ i\equiv 2(\text{mod }3), m\equiv 1(\text{mod }2).\end{cases}\\ i\left\lfloor\frac{m+3}{3}\right\rfloor-\left\lfloor\frac{i-1}{3}\right\rfloor-1, & \text{for } i\equiv 0,1(\text{mod }3), m\equiv 1(\text{mod }2).\end{cases}$$

The edge weights of $CS_{m,n}$ are given as follows.

$$w(x_iy_1^i)=\begin{cases}3\left((i-1)\left\lfloor\frac{m+3}{3}\right\rfloor-\left\lfloor\frac{i}{3}\right\rfloor\right)+4, & \text{for } i\equiv 0(\text{mod }3),\\ 3\left((i-1)\left\lfloor\frac{m+3}{3}\right\rfloor-\left\lfloor\frac{i}{3}\right\rfloor+1\right), & \text{for } i\equiv 1(\text{mod }3),\\ 3\left((i-1)\left\lfloor\frac{m+3}{3}\right\rfloor-\left\lfloor\frac{i}{3}\right\rfloor\right)+2, & \text{for } i\equiv 2(\text{mod }3).\end{cases}$$

$$w(x_i z_1^i) = \begin{cases} 3\left((i-1)\left\lfloor\frac{m+3}{3}\right\rfloor - \left\lfloor\frac{i}{3}\right\rfloor\right) + 5, & \text{for } i \equiv 0 (\text{mod } 3), \\ 3\left((i-1)\left\lfloor\frac{m+3}{3}\right\rfloor - \left\lfloor\frac{i}{3}\right\rfloor\right) + 4, & \text{for } i \equiv 1 (\text{mod } 3), \\ 3\left((i-1)\left\lfloor\frac{m+3}{3}\right\rfloor - \left\lfloor\frac{i}{3}\right\rfloor + 1\right), & \text{for } i \equiv 2 (\text{mod } 3). \end{cases}$$

$$w(y_j^i y_{j+1}^i) = \begin{cases} 3\left((i-1)\left\lfloor\frac{m+3}{3}\right\rfloor - \left\lfloor\frac{i}{3}\right\rfloor - \left\lfloor\frac{j}{3}\right\rfloor + j\right) + 2, & \text{for } \begin{cases} i \equiv 0 (\text{mod } 3), j \equiv 2 (\text{mod } 3), \\ i \equiv 1 (\text{mod } 3), j \equiv 1 (\text{mod } 3), \\ i \equiv 2 (\text{mod } 3), j \equiv 0 (\text{mod } 3), \end{cases} \\ 3\left((i-1)\left\lfloor\frac{m+3}{3}\right\rfloor - \left\lfloor\frac{i}{3}\right\rfloor - \left\lfloor\frac{j+2}{3}\right\rfloor + j\right) + 4, & \text{for } \begin{cases} i \equiv 0 (\text{mod } 3), j \equiv 0 (\text{mod } 3), \\ i \equiv 1 (\text{mod } 3), j \equiv 2 (\text{mod } 3), \\ i \equiv 2 (\text{mod } 3), j \equiv 1 (\text{mod } 3), \end{cases} \\ 3\left((i-1)\left\lfloor\frac{m+3}{3}\right\rfloor - \left\lfloor\frac{i}{3}\right\rfloor - \left\lfloor\frac{j+1}{3}\right\rfloor + j + 1\right), & \text{for } \begin{cases} i \equiv 0 (\text{mod } 3), j \equiv 1 (\text{mod } 3), \\ i \equiv 1 (\text{mod } 3), j \equiv 0 (\text{mod } 3), \\ i \equiv 2 (\text{mod } 3), j \equiv 2 (\text{mod } 3). \end{cases} \end{cases}$$

For $j = \left\lceil\frac{m-2}{2}\right\rceil$,

$$w(x_{i+1} y_j^i) = \begin{cases} 3\left(i\left\lfloor\frac{m+3}{3}\right\rfloor - \left\lfloor\frac{i}{3}\right\rfloor\right) + 1, & \text{for } \begin{cases} i \equiv 0 (\text{mod } 3), m \equiv 0 (\text{mod } 2), \\ i \equiv 1 (\text{mod } 3), m \equiv 1 (\text{mod } 2), \end{cases} \\ 3\left(i\left\lfloor\frac{m+3}{3}\right\rfloor - \left\lfloor\frac{i}{3}\right\rfloor\right), & \text{for } \begin{cases} i \equiv 1 (\text{mod } 3), m \equiv 0 (\text{mod } 2), \\ i \equiv 2 (\text{mod } 3), m \equiv 1 (\text{mod } 2), \end{cases} \\ 3\left(i\left\lfloor\frac{m+3}{3}\right\rfloor - \left\lfloor\frac{i}{3}\right\rfloor\right) - 1, & \text{for } i \equiv 2 (\text{mod } 3), m \equiv 0 (\text{mod } 2), \\ 3\left(i\left\lfloor\frac{m+3}{3}\right\rfloor - \left\lfloor\frac{i}{3}\right\rfloor\right) + 2, & \text{for } i \equiv 0 (\text{mod } 3), m \equiv 1 (\text{mod } 2). \end{cases}$$

For $j = \left\lfloor\frac{m-2}{2}\right\rfloor$,

$$w(x_{i+1} z_j^i) = \begin{cases} 3\left(i\left\lfloor\frac{m+3}{3}\right\rfloor - \left\lfloor\frac{i-1}{3}\right\rfloor\right) - 1, & \text{for } \begin{cases} i \equiv 0 (\text{mod } 3), m \equiv 0 (\text{mod } 2), \\ i \equiv 2 (\text{mod } 3), m \equiv 1 (\text{mod } 2), \end{cases} \\ 3\left(i\left\lfloor\frac{m+3}{3}\right\rfloor - \left\lfloor\frac{i-1}{3}\right\rfloor\right) + 1, & \text{for } i \equiv 1 (\text{mod } 3), m \equiv 0 (\text{mod } 2), \\ 3\left(i\left\lfloor\frac{m+3}{3}\right\rfloor - \left\lfloor\frac{i-1}{3}\right\rfloor\right), & \text{for } \begin{cases} i \equiv 2 (\text{mod } 3), m \equiv 0 (\text{mod } 2), \\ i \equiv 1 (\text{mod } 3), m \equiv 1 (\text{mod } 2), \end{cases} \\ 3\left(i\left\lfloor\frac{m+3}{3}\right\rfloor - \left\lfloor\frac{i-1}{3}\right\rfloor\right) - 2, & \text{for } i \equiv 0 (\text{mod } 3), m \equiv 1 (\text{mod } 2). \end{cases}$$

$$w(z_j^i z_{j+1}^i) = \begin{cases} 3\left((i-1)\left\lfloor\frac{m+3}{3}\right\rfloor - \left\lfloor\frac{i}{3}\right\rfloor - \left\lfloor\frac{j}{3}\right\rfloor + j + 1\right), & \text{for } \begin{cases} i \equiv 0(\text{mod } 3), j \equiv 2(\text{mod } 3), \\ i \equiv 1(\text{mod } 3), j \equiv 1(\text{mod } 3), \\ i \equiv 2(\text{mod } 3), j \equiv 0(\text{mod } 3), \end{cases} \\ 3\left((i-1)\left\lfloor\frac{m+3}{3}\right\rfloor - \left\lfloor\frac{i}{3}\right\rfloor - \left\lfloor\frac{j-1}{3}\right\rfloor + j\right) + 2, & \text{for } \begin{cases} i \equiv 0(\text{mod } 3), j \equiv 0(\text{mod } 3), \\ i \equiv 1(\text{mod } 3), j \equiv 2(\text{mod } 3), \\ i \equiv 2(\text{mod } 3), j \equiv 1(\text{mod } 3), \end{cases} \\ 3\left((i-1)\left\lfloor\frac{m+3}{3}\right\rfloor - \left\lfloor\frac{i}{3}\right\rfloor - \left\lfloor\frac{j+1}{3}\right\rfloor + j\right) + 4, & \text{for } \begin{cases} i \equiv 0(\text{mod } 3), j \equiv 1(\text{mod } 3), \\ i \equiv 1(\text{mod } 3), j \equiv 0(\text{mod } 3), \\ i \equiv 2(\text{mod } 3), j \equiv 2(\text{mod } 3). \end{cases} \end{cases}$$

It is clear that the edge weights are distinct, namely $\{3, 4, \ldots, mn+2\}$, hence it is proven that $tes(CS_{m,n}) = \left\lceil\frac{mn+2}{3}\right\rceil$. □

Example 2:

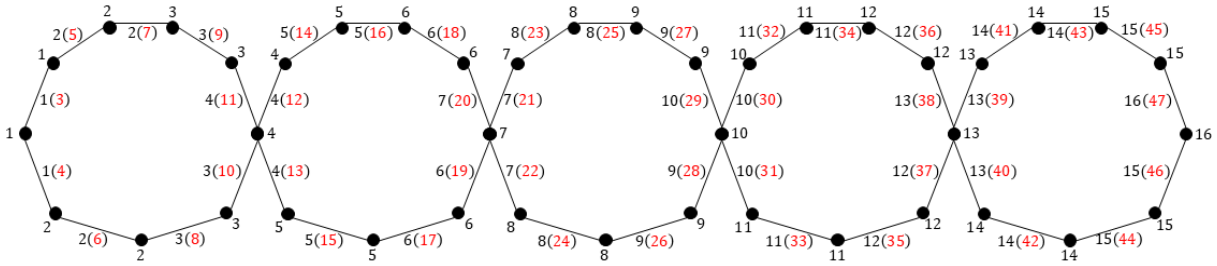

Figure 2: Cycle snake graph $CS_{9,5}$ with $tes = 16$

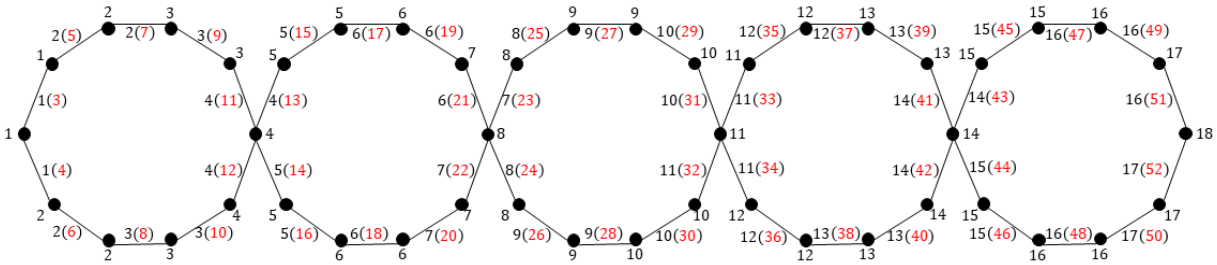

Figure 3: Cycle snake graph $CS_{10,5}$ with $tes = 18$

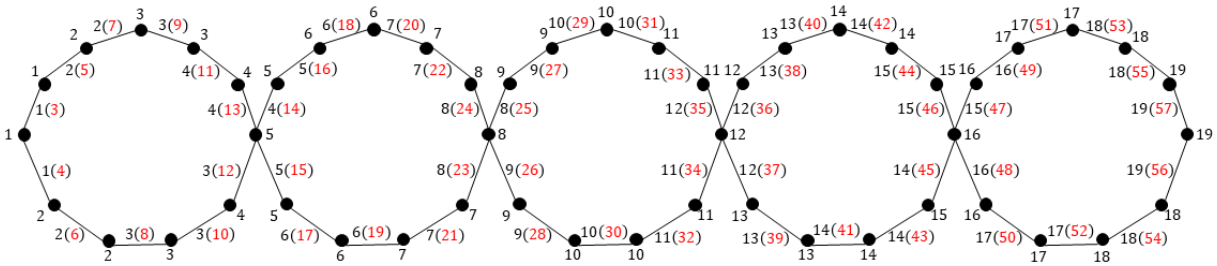

Figure 4: Cycle snake graph $CS_{11,5}$ with $tes = 19$

Figures 2, 3, and 4 show the total labeling for the cases of Theorem 3. Figure 2 depicts $CS_{9,5}$ as an example of Case 1 ($m \equiv 0(\text{mod } 3)$), whose distinct edge weights form the set $\{3, 4, \ldots, 47\}$. An example of Case 2 ($m \equiv 1(\text{mod } 3)$) is presented in Figure 3 using $CS_{10,5}$, whose distinct edge weights form the set $\{3, 4, \ldots, 52\}$. Case 3 ($m \equiv 2(\text{mod } 3)$) is showing in Figure 4 using $CS_{11,5}$, with distinct edge weights forming the set $\{3, 4, \ldots, 57\}$.

4 Conclusion

In this article, the exact values of the total edge irregularity strength of the cycle snake graphs $CS_{3,n}$ and $CS_{m,n}$ are determined as follows

$$tes(CS_{3,n}) = n + 1 \quad \text{and} \quad tes(CS_{m,n}) = \left\lceil \frac{mn+2}{3} \right\rceil .$$

The results obtained in this study contribute to the advancement of graph labeling theory by extending the investigation of the total edge irregularity strength to cycle snake graphs. Furthermore, this concept can be further extended to other classes of graphs.

CRediT Authorship Contribution Statement

Stenly Pranata: Conceptualization, Methodology, Writing–Original Draft. **Vira Hari Krisnawati:** Supervision, validation, Writing–Review & Editing. **Darmajid:** Supervision, validation, Writing–Review & Editing.

Declaration of Generative AI and AI-assisted technologies

The authors declare that no generative AI or AI-assisted technologies were used to generate or modify the results of this research. AI tools were used solely for grammatical and language editing purposes.

Declaration of Competing Interest

The authors declare no competing interests.

Funding and Acknowledgments

This research received no external funding.

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